Engineering Economy

[1-2]
Introduction
Examples and Additional Concepts

Example [1]

Time Value of Money

- You have $1,000 and you want to buy a $1,000 machine

- Suppose that you can invest money at 6% interest, but the price of the machine increases only at an annual rate of 4% due to inflation. After a year, you can still buy the machine and you will have $20 left over (*earning power exceeds inflation*)

- If the price of the machine increases at an annual rate of 8% instead, you will not have enough money to buy the machine a year from today. In this case, it is better to buy it today (*inflation exceeds earning power*)

Example [2]

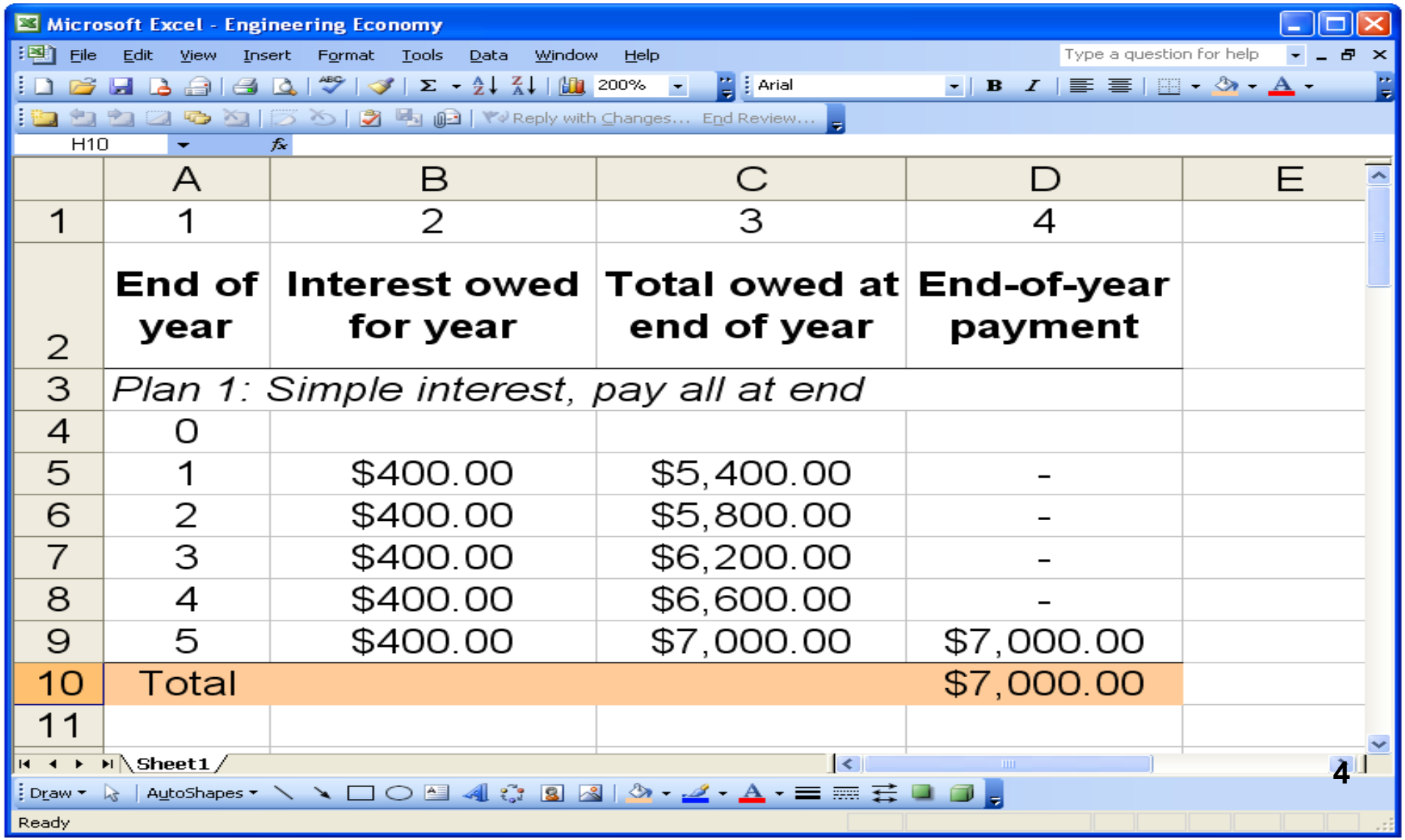
The Concept of Equivalence

- Demonstrate the concept of equivalence using the different loan repayment plans described below. Each plan repays a $5,000 loan in 5 years at 8% interest per year

- *Plan 1*: *Simple interest, pay all at end*. No interest or principal is paid until the end of year 5. Interest accumulates each year on the principal only
- *Plan 2*: *Compound interest, pay all at end*. No interest or principal is paid until the end of year 5. Interest accumulates each year on the total of principle and all accrued interest
- *Plan 3*: *Simple interest paid annually, principal repaid at end*. The accrued interest is paid each year, and the entire principal is repaid at the end of year 5
- *Plan 4*: *Compound interest and portion of principal repaid annually*. The accrued interest and one-fifth of the principal is repaid each year

Example [2]

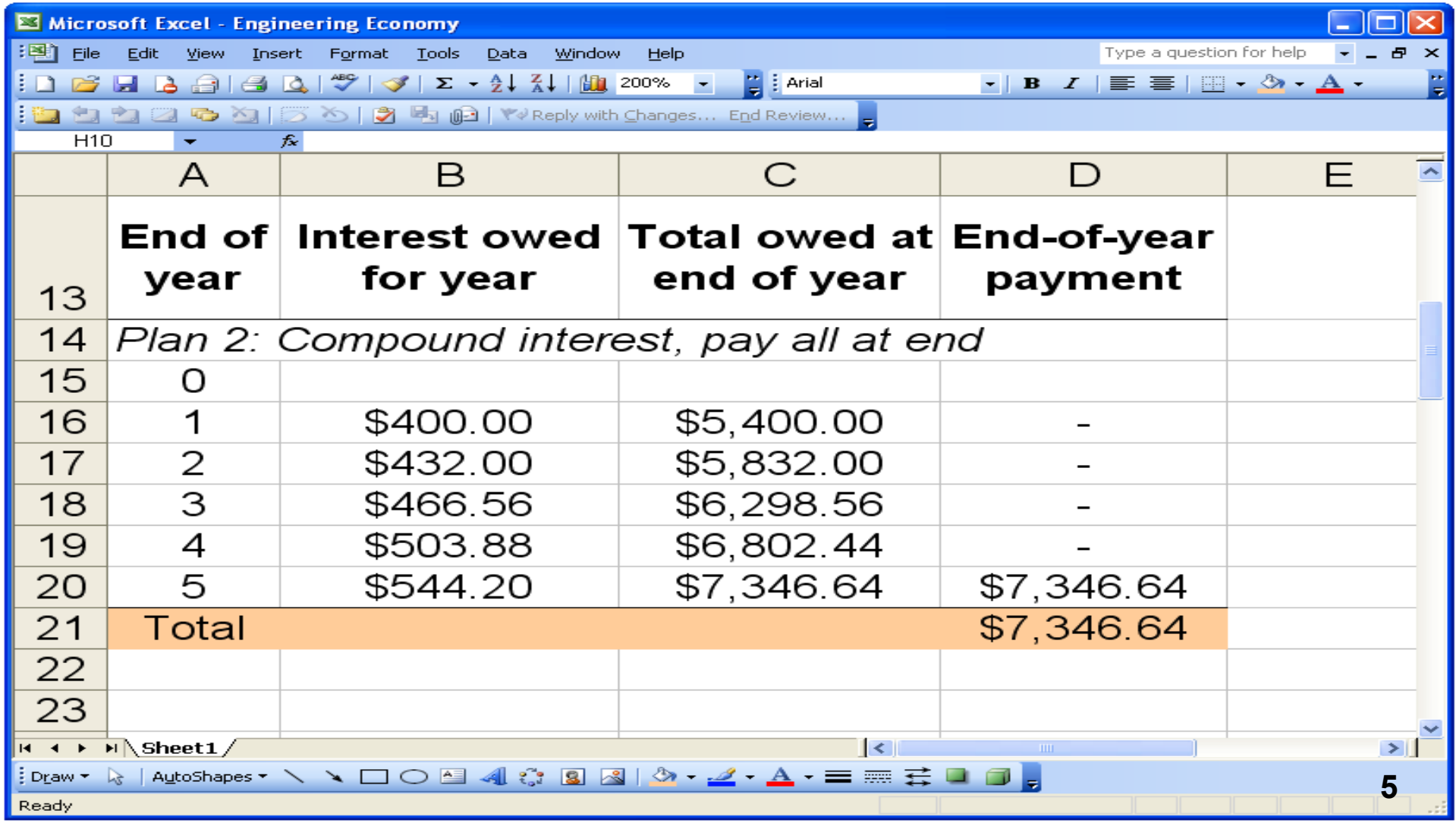
The Concept of Equivalence – Plan 1



	A	B	C	D	E
1	1	2	3	4	
2	**End of year**	**Interest owed for year**	**Total owed at end of year**	**End-of-year payment**	
3	*Plan 1: Simple interest, pay all at end*				
4	0				
5	1	$400.00	$5,400.00	-	
6	2	$400.00	$5,800.00	-	
7	3	$400.00	$6,200.00	-	
8	4	$400.00	$6,600.00	-	
9	5	$400.00	$7,000.00	$7,000.00	
10	Total			$7,000.00	
11					

Example [2]

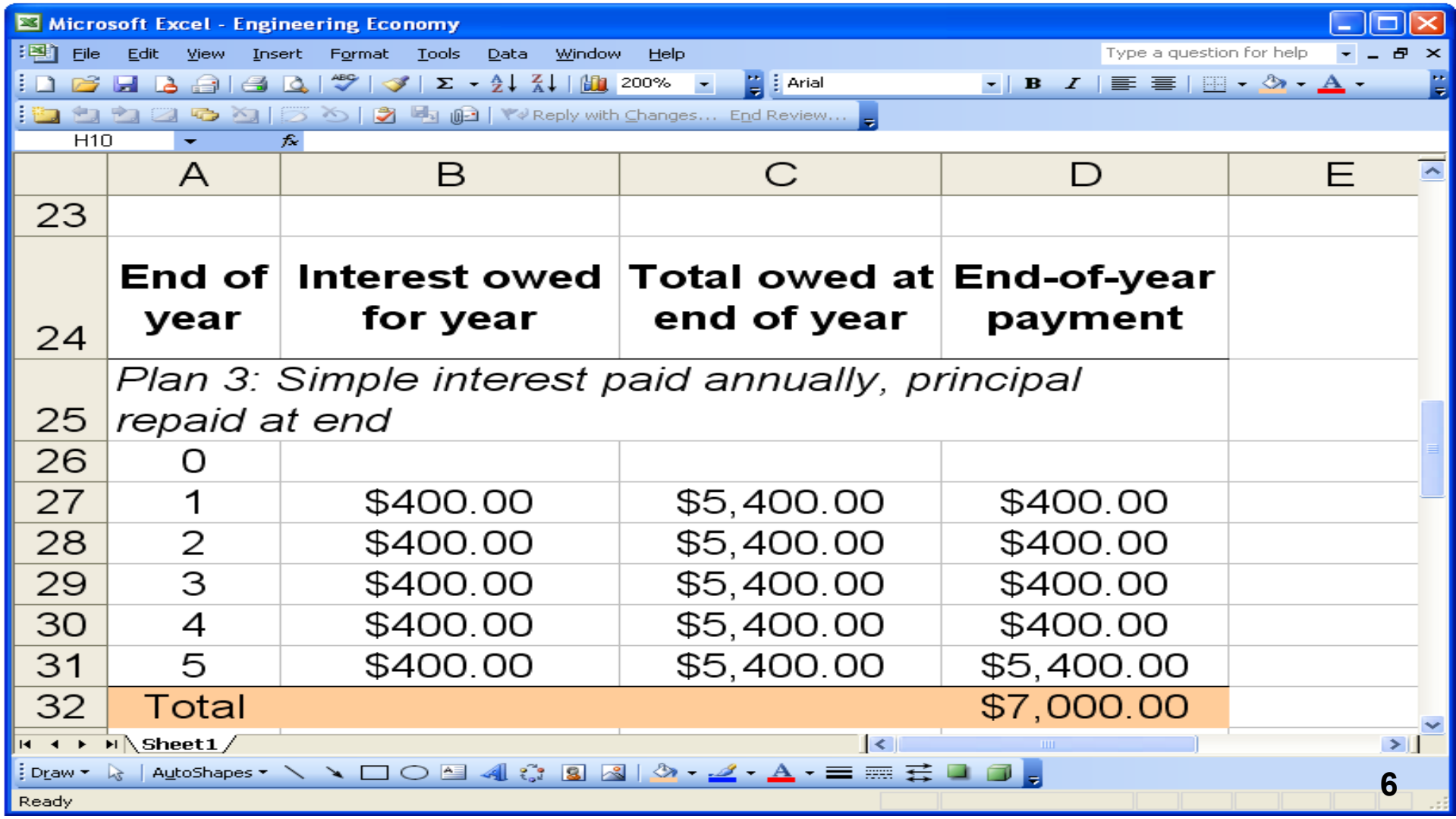
The Concept of Equivalence – Plan 2



	A	B	C	D	E
13	**End of year**	**Interest owed for year**	**Total owed at end of year**	**End-of-year payment**	
14	*Plan 2: Compound interest, pay all at end*				
15	0				
16	1	$400.00	$5,400.00	-	
17	2	$432.00	$5,832.00	-	
18	3	$466.56	$6,298.56	-	
19	4	$503.88	$6,802.44	-	
20	5	$544.20	$7,346.64	$7,346.64	
21	Total			$7,346.64	
22					
23					

Example [2]

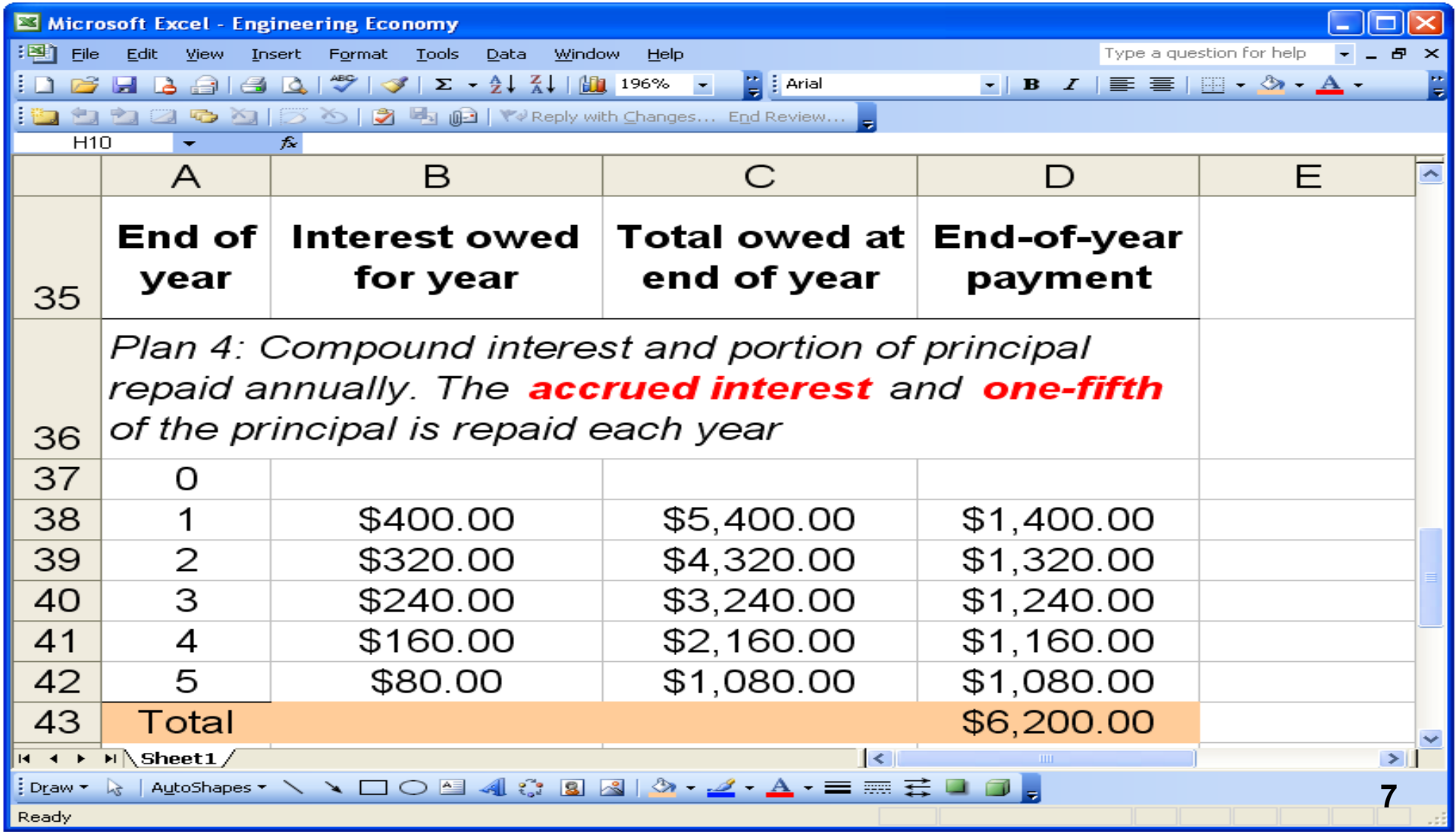
The Concept of Equivalence – Plan 3



	A	B	C	D	E
23					
24	**End of year**	**Interest owed for year**	**Total owed at end of year**	**End-of-year payment**	
25	*Plan 3: Simple interest paid annually, principal repaid at end*				
26	0				
27	1	$400.00	$5,400.00	$400.00	
28	2	$400.00	$5,400.00	$400.00	
29	3	$400.00	$5,400.00	$400.00	
30	4	$400.00	$5,400.00	$400.00	
31	5	$400.00	$5,400.00	$5,400.00	
32	Total			$7,000.00	

Example [2]

The Concept of Equivalence – Plan 4



	A	B	C	D	E
35	**End of year**	**Interest owed for year**	**Total owed at end of year**	**End-of-year payment**	
36	*Plan 4: Compound interest and portion of principal repaid annually. The **accrued interest** and **one-fifth** of the principal is repaid each year*				
37	0				
38	1	$400.00	$5,400.00	$1,400.00	
39	2	$320.00	$4,320.00	$1,320.00	
40	3	$240.00	$3,240.00	$1,240.00	
41	4	$160.00	$2,160.00	$1,160.00	
42	5	$80.00	$1,080.00	$1,080.00	
43	Total			$6,200.00	

Example [2]

The Concept of Equivalence – *Comments*

$5,000 at time 0 is equivalent to each of the following:

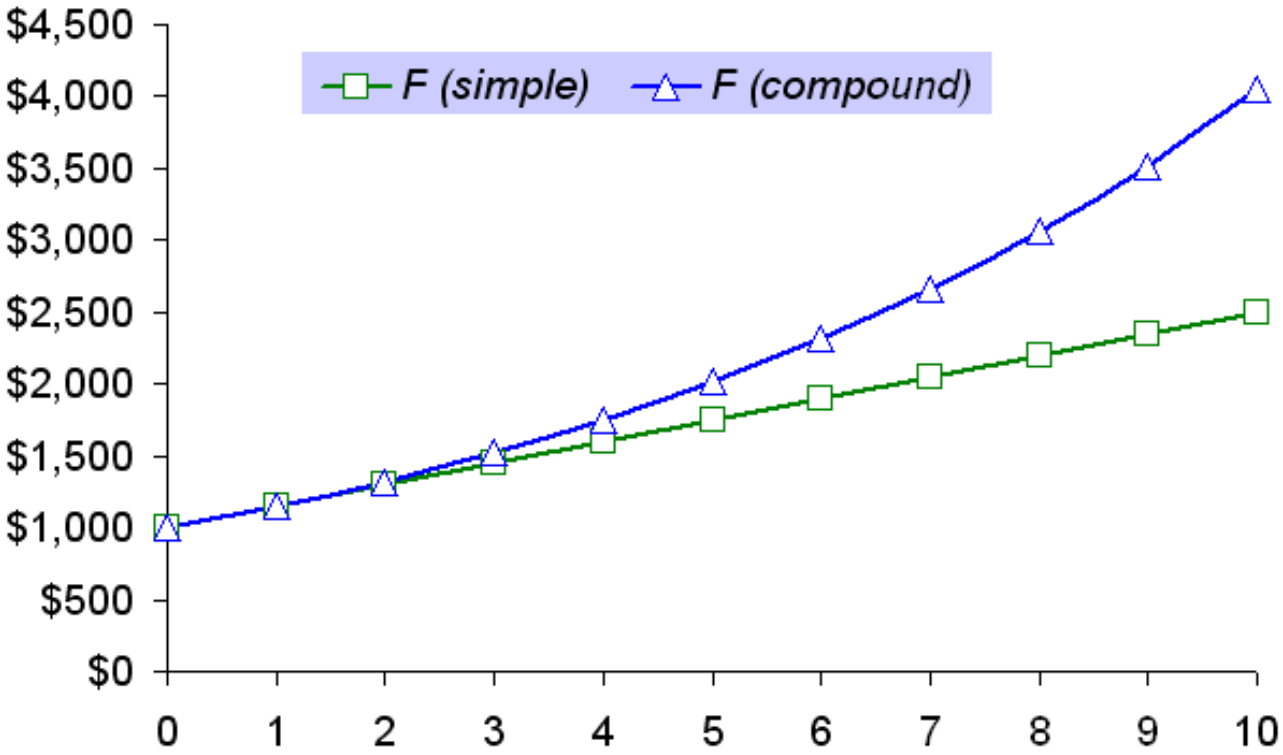
- Plan 1: $7,000 at the end of year 5 at 8% simple interest
- Plan 2: $7,346.64 at the end of year 5 at 8% compound interest
- Plan 3: $400 per year for 4 years and $5,400 at the end of year 5 at 8% simple interest
- Plan 4: Decreasing payments of interest and partial principal in years 1 ($1,400) through 5 ($1,080) at 8% compound interest

Just Keep in Mind

- Simple interest: $F = P(1+ni)$

- Compound interest: $F = P(1+i)^n$

where
F: future worth
P: present worth
i: interest rate
n: number of interest periods



Simple and Compound Interest Comparison

- The total amount with *simple* interest grows *linearly*
- The total amount with *compound* interest grows *exponentially*
- This exponential growth is referred to as the power of compounding

Terminology and Symbols

- The equations and procedures of *engineering economy* utilize the following terms and symbols:

- P: [dollars] value or amount of money at the present time (time o). P is referred to as present worth (PW), present value (PV), net present value (NPV), discounted cash flow (DCF), and capitalized cost (CC)

- F: [dollars] value or amount of money at some future time. Also F is called future worth (FW) and future value (FV)

Terminology and Symbols

- A: [dollars per year, dollars per months] series of consecutive, equal, end-of-period amounts of money. Also A is called the annual worth (AW) and equivalent uniform annual worth (EUAW)

- n: [years, months, days] number of interest periods

- i: [percent per year, percent per month, percent per day] interest rate or rate of return per time period. *Assume compound interest if not specified*

- All engineering economy problems involve the elements of interest period n and interest rate i

Example [3]

The Concept of Equivalence

- You are given the alternative of receiving either $3,000 at the end of five years or P dollars today

- What value of P would make you *indifferent* to $3,000 at the end of five years if the interest rate is 8%?

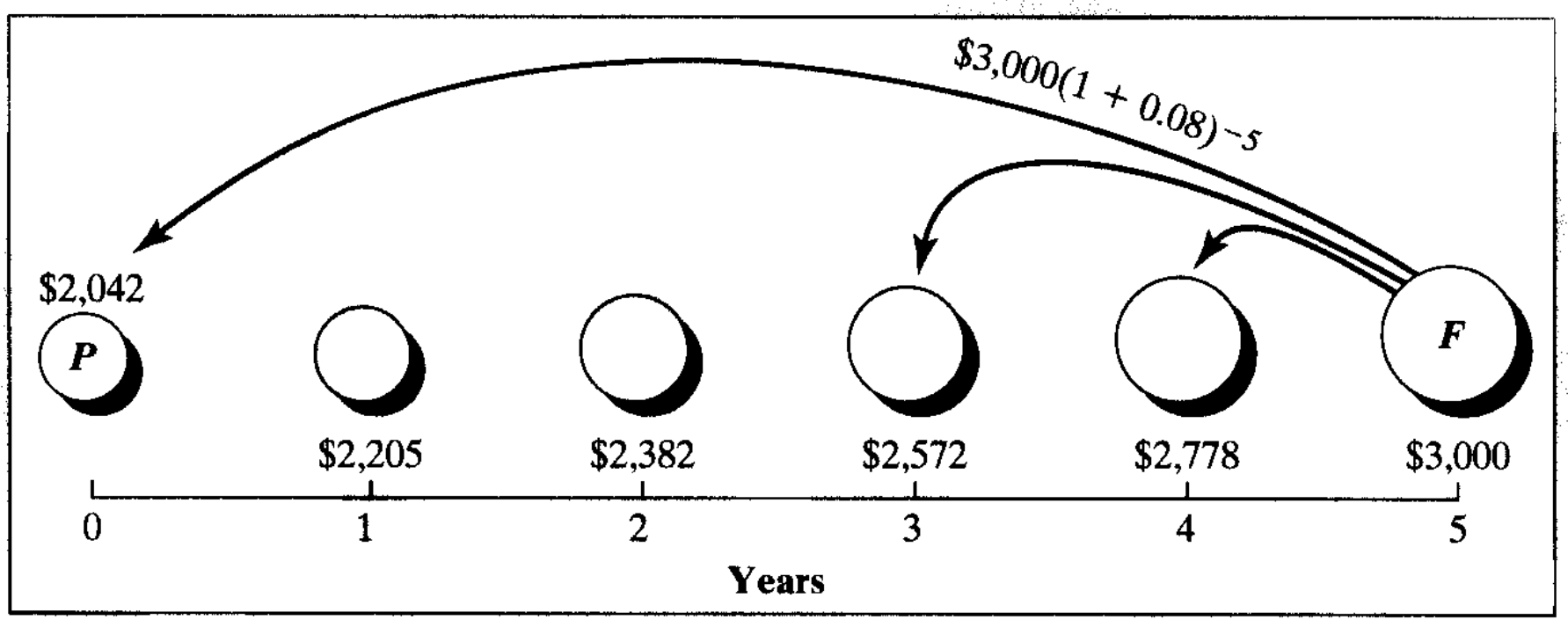
  Symbols are as follows: F = $3,000, n = 5 years, and i = 8%. Find P

  We know that $F = P(1+i)^n$

  Substituting yields P = $2,042

Example [3]

The Concept of Equivalence



Various dollar amounts that will be economically equivalent to $3,000 in five years given an interest rate of 8%

Example [4]

Terminology and Symbols

- Someone plans to borrow $10,000 now to help in buying a car

- He arranged to repay the entire principal plus 8% per year interest after 5 years

- *Identify the engineering economy symbols* involved and their values for the total owed after 5 years

Example [4]

Terminology and Symbols

- We have P = $10,000, i = 8%, and n = 5 years

- You must find the value of F which represents the amount to be repaid after 5 years

Example [5]

Terminology and Symbols

- Assume you borrow $2,000 now at 7% per year for 10 years and you must repay the loan in equal yearly payments

- Determine the symbols involved and their values

Example [5]

Terminology and Symbols

- We have P = $2,000, i = 7% per year, and n = 10 years

- We need to find the value of A which represents the yearly equal (uniform) payments

Example [6]

Terminology and Symbols

- Last year Jane's grandmother offered to put enough money into a savings account to generate $1,000 to help pay Jane's expenses at college

- Identify the symbols

- Calculate the amount that had to be deposited exactly 1 year ago to earn $1,000 ***in interest*** now if the rate is 6% per year

Example [6]

Terminology and Symbols

- We have i = 6% and n = 1
- We know that the amount of interest is $1,000
- Since $F = P(1+i)^n$ and F = P + interest amount
- Then P = $16,666.67 which represents the amount to be deposited at the beginning

Example [7]

- What is the present worth of a lump sum of one million dollars to be received 50 years from today if the interest rate is 10%?

- $F = P(1+i)^n$ and or $P = F(1+i)^{-n}$ →

  $P = \$1{,}000{,}000 \times (1+10\%)^{-50} \approx \$8{,}519$

Example [8]

- Show the yearly payments a borrower has to pay to the bank for the following three repayment plans. The loan is $10,000 and must be repaid to the bank in 5 years at 10% interest per year

- *Plan 1*: *Compound interest*. However, there is no interest in the third year. Pay all at end. No interest or principal is paid until the end of year 5

- *Plan 2*: *Compound interest*. However, you pay the *interest* plus *one-quarter* of the principal by the end of each year

- *Plan 3*: Same as Plan 2 except in the third year you pay only the interest

Example [8]

End of year	Interest owed for year	Total owed at end of year	End-of-year payment
Plan 1: Compound interest, pay all at end			
0			
1	$1,000.00	$11,000.00	-
2	$1,100.00	$12,100.00	-
3	$0.00	$12,100.00	-
4	$1,210.00	$13,310.00	-
5	$1,331.00	$14,641.00	$14,641.00
Total			*$14,641.00*

Example [8]

End of year	Interest owed for year	Total owed at end of year	End-of-year payment
*Plan 2: Compound interest and portion of principal repaid annually. The **accrued interest** and **one-fourth** of the principal is repaid each year*			
0			
1	$1,000.00	$11,000.00	$3,500.00
2	$750.00	$8,250.00	$3,250.00
3	$500.00	$5,500.00	$3,000.00
4	$250.00	$2,750.00	$2,750.00
5	$0.00	$0.00	$0.00
Total			*$12,500.00*

Example [8]

End of year	Interest owed for year	Total owed at end of year	End-of-year payment
*Plan 3: Compound interest and portion of principal repaid annually. The **accrued interest** and **one-fourth** of the principal is repaid each year except in the third year where you pay only the interest*			
0			
1	$1,000.00	$11,000.00	$3,500.00
2	$750.00	$8,250.00	$3,250.00
3	$500.00	$5,500.00	$500.00
4	$500.00	$5,500.00	$3,000.00
5	$250.00	$2,750.00	$2,750.00
Total			*$13,000.00*

Solution by Excel

- Using the symbols P, F, A, i, and n exactly as defined earlier, the Excel functions that most used in engineering economy are the following:

  - P: PV(i%,n,A,F)
  - F: FV(i%,n,A,P)
  - A: PMT(i%,n,P,F) Just equal values
  - n: NPER(i%,A,P,F)
  - i: RATE(n,A,P,F)