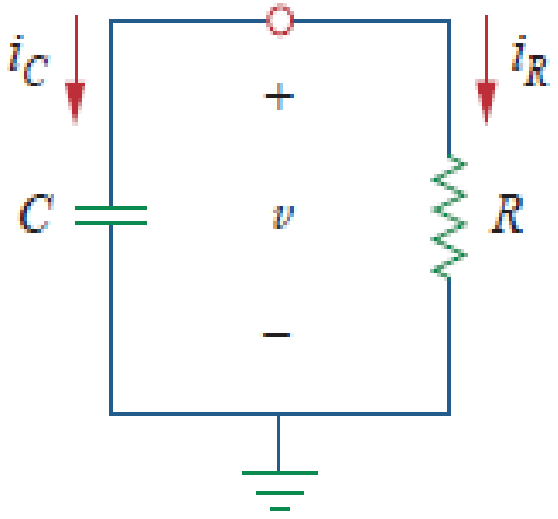
Chapter 8
Basic RL and RC circuits

The Source-Free RC Circuit

This is a circuit without source

We assume that the capacitor is fully charged to V_0



$$v(t) = v_o . e^{\frac{-t}{\tau}}$$

$v_o = v(0)$ Initial voltage

$\tau = R.C$ Time constant

$$i_c(t) = c\frac{dv(t)}{dt} = c\frac{d(v_o.e^{\frac{-t}{\tau}})}{dt}$$

$$i_c(t) = C.v_o.\frac{-1}{\tau}.e^{\frac{-t}{\tau}} = -C.v_o\,\frac{1}{R.C}.e^{\frac{-t}{\tau}} = -\frac{v_o}{R}.e^{\frac{-t}{\tau}}$$

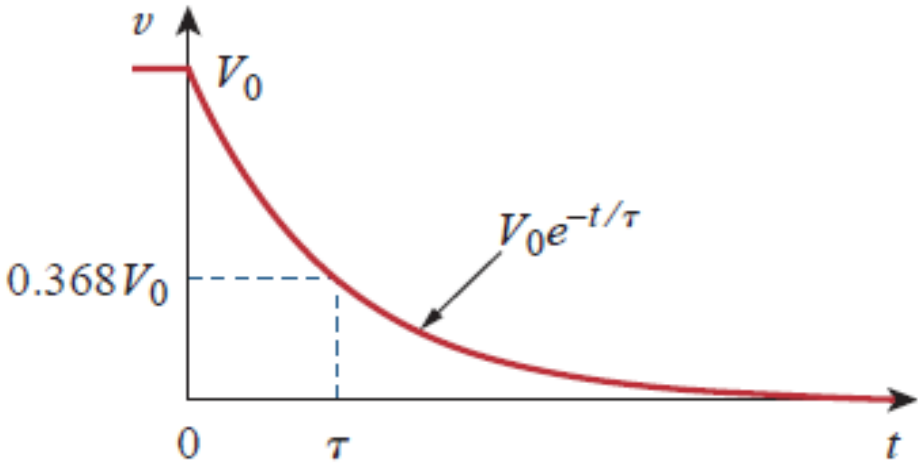
$$i_c(t) = I_o.e^{\frac{-t}{\tau}}$$

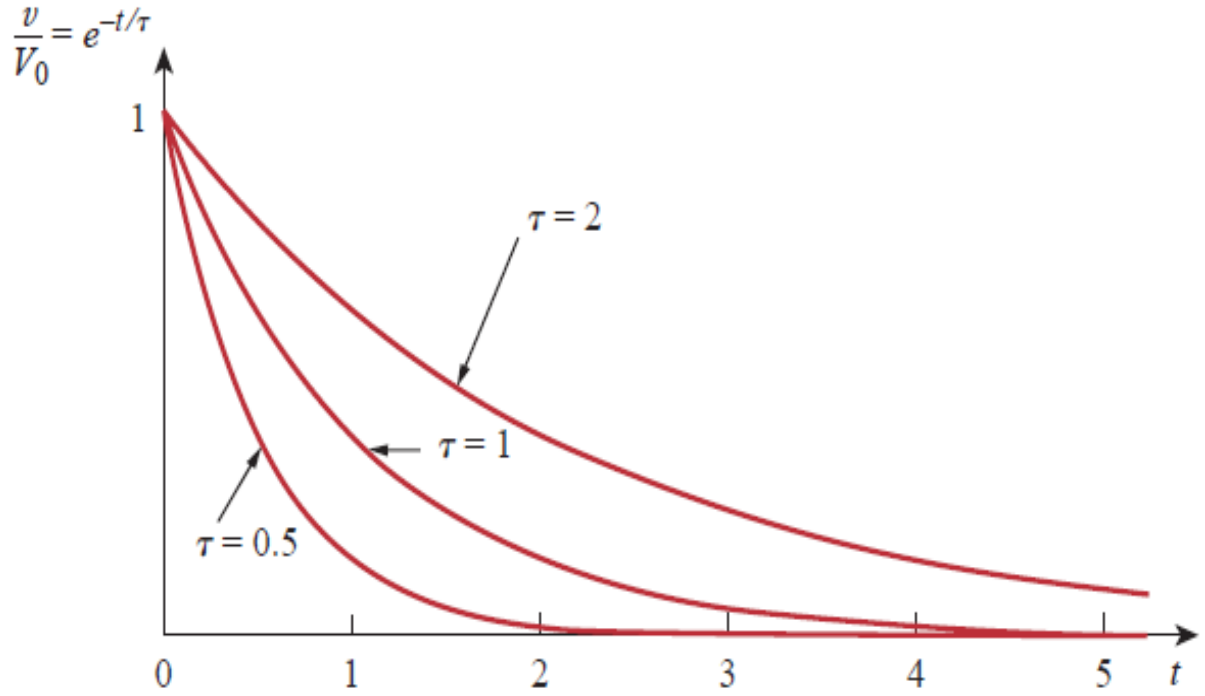
$$i_R(t) = -i_c(t) = \frac{v(t)}{R} = \frac{v_o}{R}.e^{\frac{-t}{\tau}}$$

The natural response of a circuit refers to the behavior (in terms of voltages and currents) of the circuit itself, with no external sources of excitation.

The time constant of a circuit is the time required for the response to decay to a factor of $1/e$ or 36.8 percent of its initial value.[1]

Response after τ $V_0 e^{-\tau/RC} = V_0 e^{-1} = 0.368 V_0$



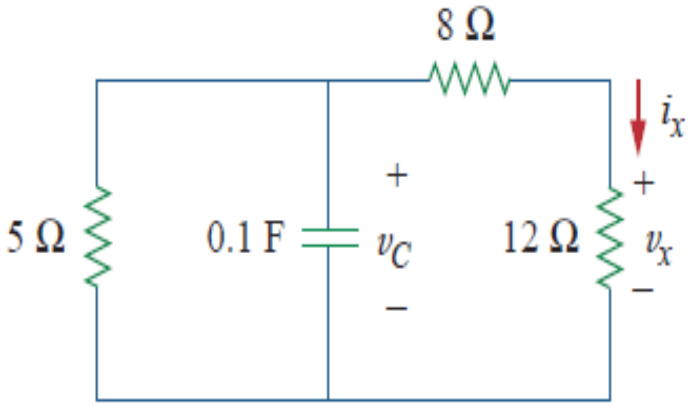


The Key to Working with a Source-free *RC* Circuit Is Finding:

1. The initial voltage $v(0) = V_0$ across the capacitor.
2. The time constant τ.

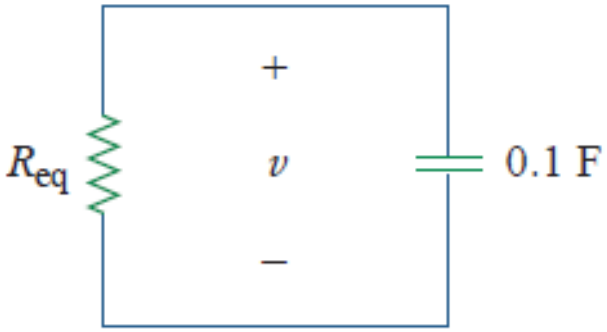
R: is the thevenin resistance computed at the terminals of the capacitor

let $v_C(0) = 15$ V. Find v_C, v_x, and i_x for $t > 0$.



$$R_{eq} = \frac{20 \times 5}{20 + 5} = 4\ \Omega$$

$$\tau = R_{eq}C = 4(0.1) = 0.4\ \text{s}$$



$$v = v(0)e^{-t/\tau} = 15e^{-t/0.4}\ \text{V}, \qquad v_C = v = 15e^{-2.5t}\ \text{V}$$

$$v_x = \frac{12}{12 + 8}v = 0.6(15e^{-2.5t}) = 9e^{-2.5t}\ \text{V}$$

$$i_x = \frac{v_x}{12} = 0.75e^{-2.5t}\ \text{A}$$

The Source-Free *RL* Circuit

$$i(t) = I_0 e^{-t/\tau}$$

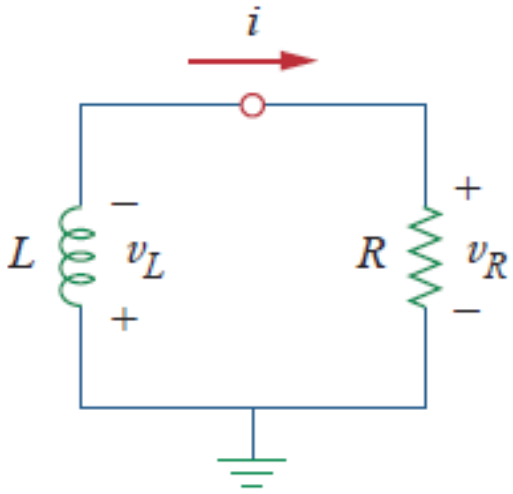
$$\tau = \frac{L}{R}$$

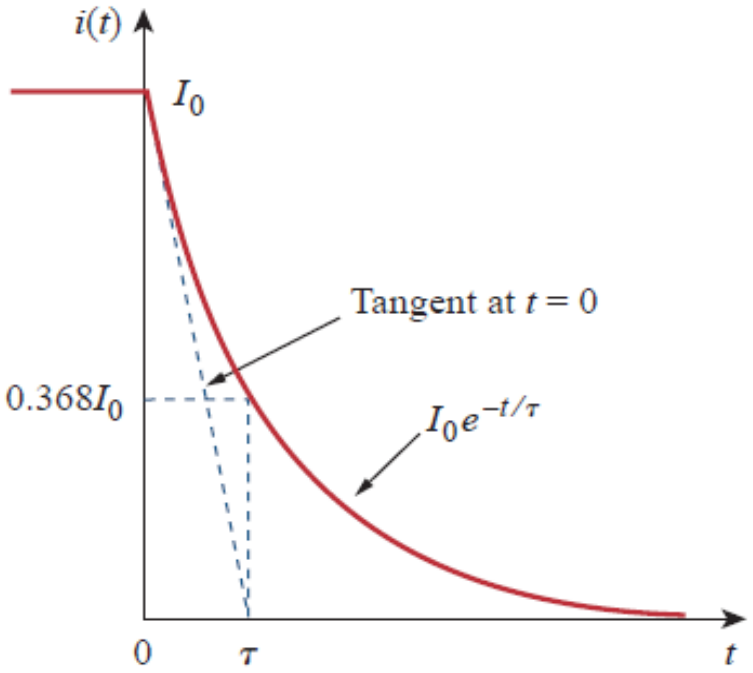
$$v_R(t) = iR = I_0 R e^{-t/\tau}$$

$$V_L(t) = -V_R(t) = -I_o.\mathrm{Re}^{\frac{-t}{\tau}}$$

$$V_L(t) = V_o.e^{\frac{-t}{\tau}}$$

$$V_o = -I_o.R$$

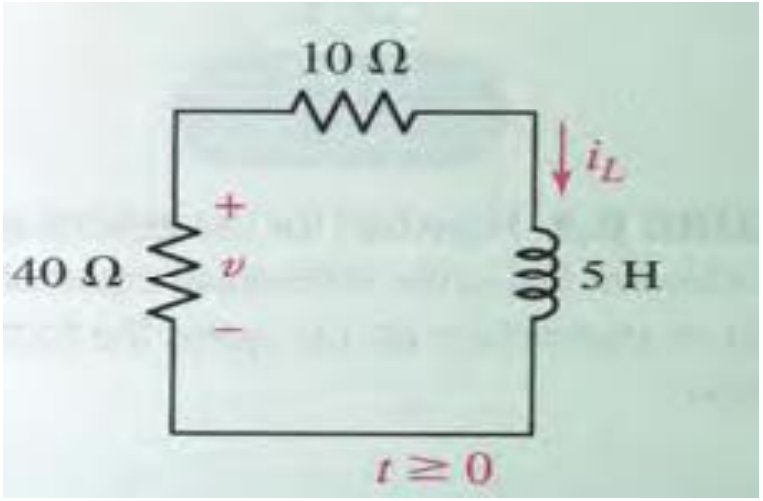




The Key to Working with a Source-free *RL* Circuit Is to Find:

1. The initial current $i(0) = I_0$ through the inductor.
2. The time constant τ of the circuit.

Find V, i_L for t>=0



$$R_T = 10 + 40 = 50\Omega \rightarrow \tau = \frac{L}{R_T} = \frac{5}{50} = \frac{1}{10}\,s$$

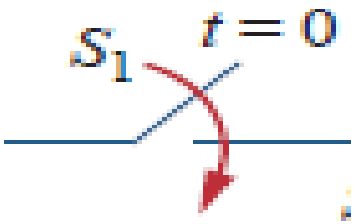
$$I_L(t) = 2.4e^{\frac{-t}{\tau}} = 2.4e^{-10t}$$

$$V_{40\Omega} = -2.4(40).e^{-10t} = -96.e^{-10t}$$

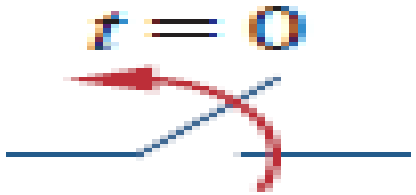
I_o=2.4 A

Switches

2 –port switch

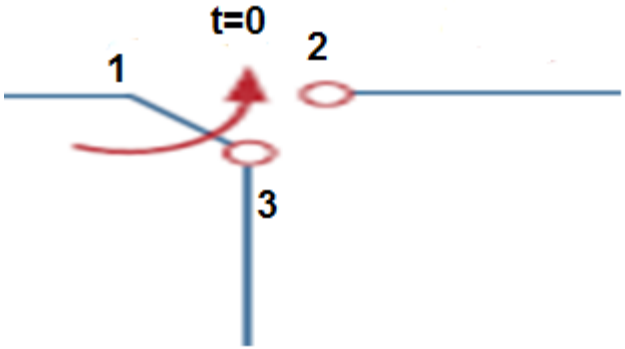


t >0 switch is closed
t<0 switch is open



t >0 switch is open
t<0 switch is closed

3-port switch



t<0 select 3

t>0 select 2