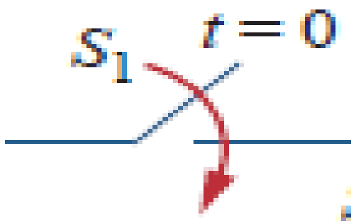
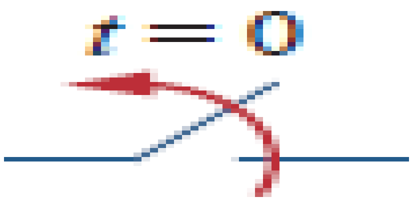
Chapter 8
Basic RL and RC circuits

Switches

2 –port switch

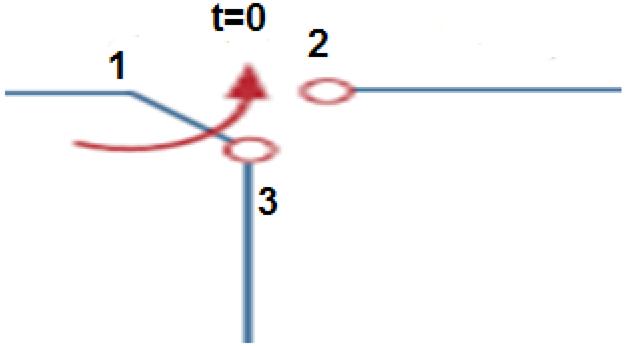




t >0 switch is closed
t<0 switch is open

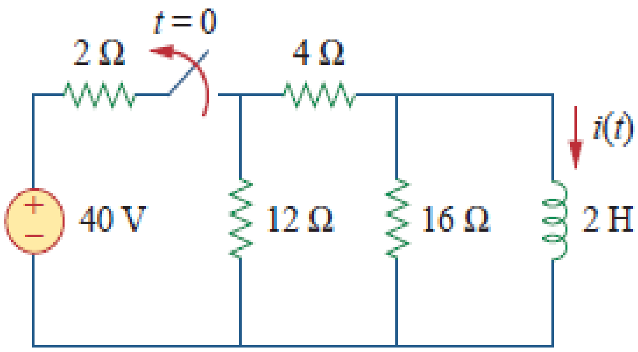
t >0 switch is open
t<0 switch is closed

3-port switch



t<0 select 3
t>0 select 2

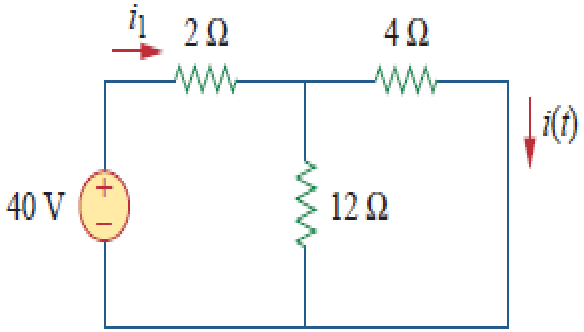
The switch in the circuit of Fig. 7.16 has been closed for a long time. At $t = 0$, the switch is opened. Calculate $i(t)$ for $t > 0$.



Solution

When $t < 0$, the switch is closed, and the inductor acts as a short

$$i_1 = \frac{40}{2+3} = 8 \text{ A}$$



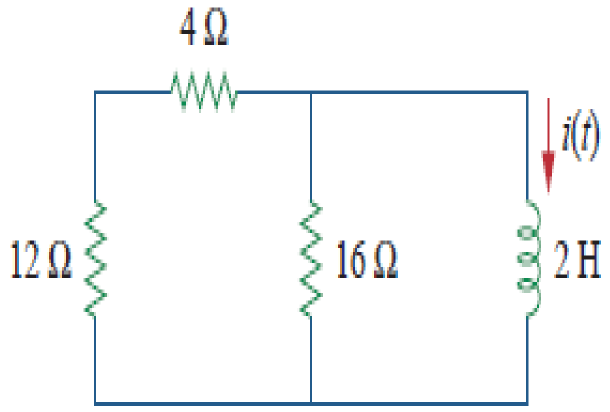
$$\frac{4 \times 12}{4 + 12} = 3\ \Omega$$

$$i(t) = \frac{12}{12+4} i_1 = 6 \text{ A}, \qquad t < 0$$

$$i(0) = i(0^-) = 6 \text{ A}$$

When $t > 0$, the switch is open and the voltage source is disconnected. We now have the source-free *RL* circuit in Fig. 7.17(b).

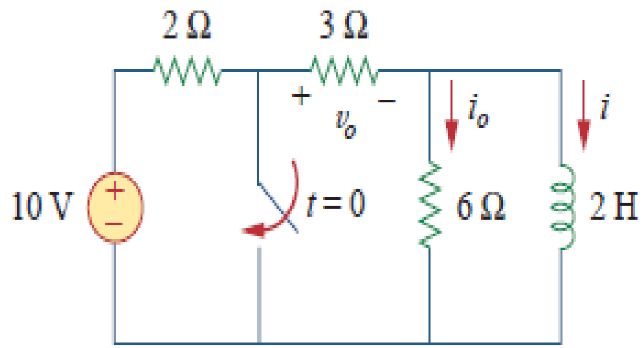
$$R_{eq} = (12 + 4) \parallel 16 = 8\ \Omega$$



$$\tau = \frac{L}{R_{eq}} = \frac{2}{8} = \frac{1}{4}\ \text{s}$$

$$i(t) = i(0)e^{-t/\tau} = 6e^{-4t}\ \text{A}$$

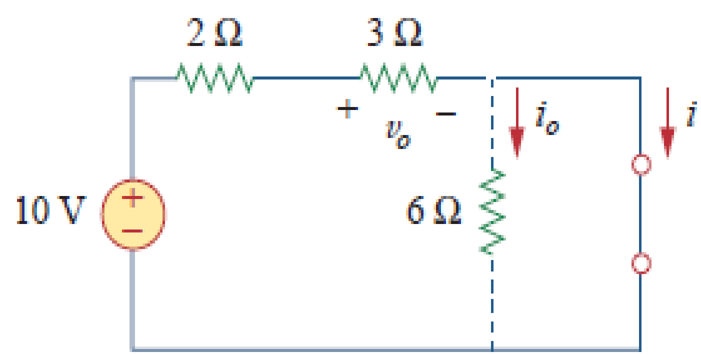
In the circuit shown in Fig. 7.19, find i_o, v_o, and i for all time, assuming that the switch was open for a long time.



For $t < 0$, the switch is open. Since the inductor acts like a short circuit to dc, the 6-Ω resistor is short-circuited, so that we have the circuit shown in Fig. 7.20(a). Hence, $i_o = 0$, and

$$i(t) = \frac{10}{2 + 3} = 2\text{ A}, \qquad t < 0$$

$$v_o(t) = 3i(t) = 6\text{ V}, \qquad t < 0$$

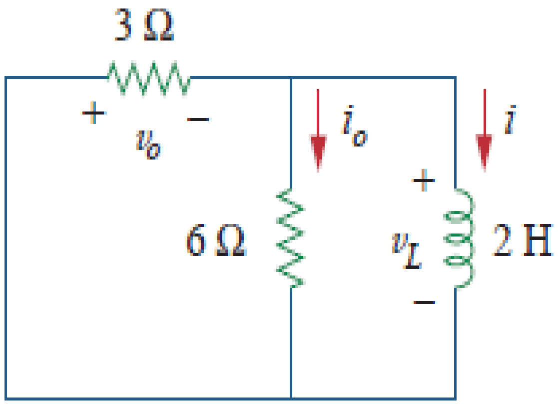


Thus, $i(0) = 2$.

For $t > 0$, the switch is closed, so that the voltage source is short-circuited. We now have a source-free RL circuit as shown in Fig. 7.20(b). At the inductor terminals,

$$R_{Th} = 3 \parallel 6 = 2\,\Omega$$

$$\tau = \frac{L}{R_{Th}} = 1\text{ s}$$



$$i(t) = i(0)e^{-t/\tau} = 2e^{-t}\text{ A}, \qquad t > 0$$

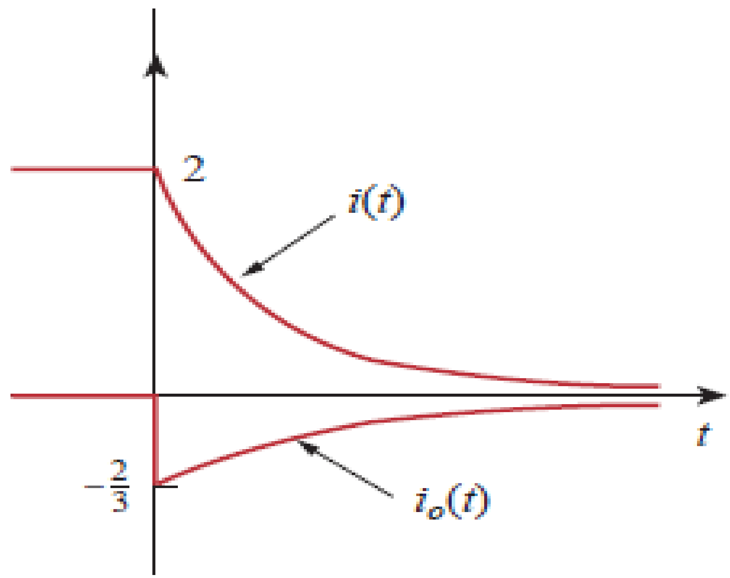
$$v_o(t) = -v_L = -L\frac{di}{dt} = -2(-2e^{-t}) = 4e^{-t}\text{ V}, \qquad t > 0$$

$$i_o(t) = \frac{v_L}{6} = -\frac{2}{3}e^{-t}\text{ A}, \qquad t > 0$$

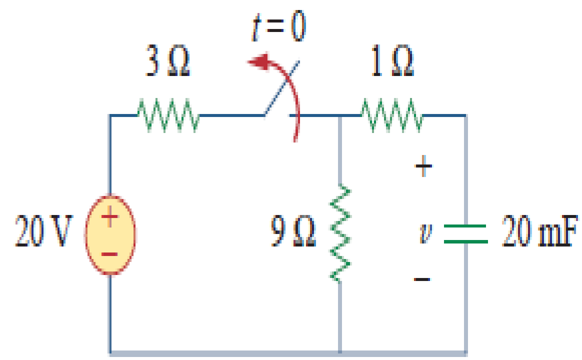
Thus, for all time,

$$i_o(t) = \begin{cases} 0\text{ A}, & t < 0 \\ -\dfrac{2}{3}e^{-t}\text{ A}, & t > 0 \end{cases}, \qquad v_o(t) = \begin{cases} 6\text{ V}, & t < 0 \\ 4e^{-t}\text{ V}, & t > 0 \end{cases}$$

$$i(t) = \begin{cases} 2\text{ A}, & t < 0 \\ 2e^{-t}\text{ A}, & t \geq 0 \end{cases}$$



The switch in the circuit in Fig. 7.8 has been closed for a long time, and it is opened at $t = 0$. Find $v(t)$ for $t \geq 0$. Calculate the initial energy stored in the capacitor.



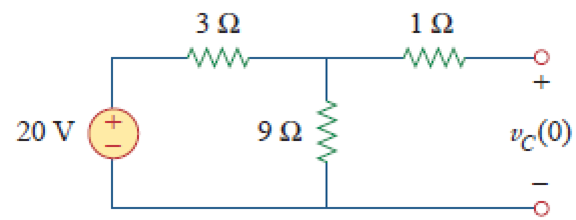
Solution:

For $t < 0$, the switch is closed; the capacitor is an open circuit to dc, as represented in Fig. 7.9(a). Using voltage division

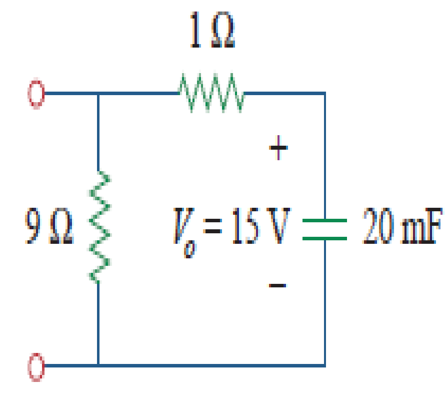
$$v_C(t) = \frac{9}{9+3}(20) = 15 \text{ V}, \qquad t < 0$$

Since the voltage across a capacitor cannot change instantaneously, the voltage across the capacitor at $t = 0^-$ is the same at $t = 0$, or

$$v_C(0) = V_0 = 15 \text{ V}$$



For $t > 0$, the switch is opened, and we have the RC circuit shown in Fig. 7.9(b). [Notice that the RC circuit in Fig. 7.9(b) is source free; the independent source in Fig. 7.8 is needed to provide V_0 or the initial energy in the capacitor.] The 1-Ω and 9-Ω resistors in series give



$$R_{eq} = 1 + 9 = 10\ \Omega$$

The time constant is

$$\tau = R_{eq}C = 10 \times 20 \times 10^{-3} = 0.2\ \text{s}$$

Thus, the voltage across the capacitor for $t \geq 0$ is

$$v(t) = v_C(0)e^{-t/\tau} = 15e^{-t/0.2}\ \text{V}$$

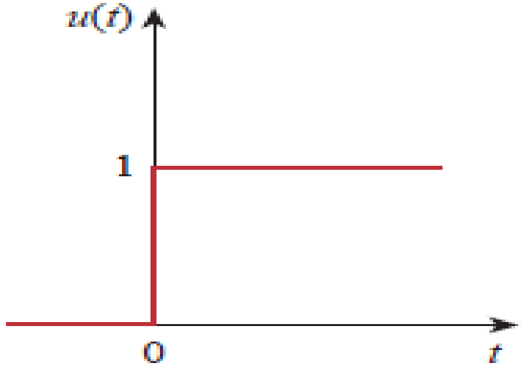
or

$$v(t) = 15e^{-5t}\ \text{V}$$

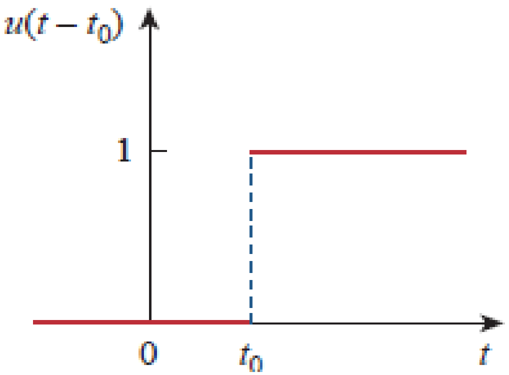
The initial energy stored in the capacitor is

$$w_C(0) = \frac{1}{2}Cv_C^2(0) = \frac{1}{2} \times 20 \times 10^{-3} \times 15^2 = 2.25\ \text{J}$$

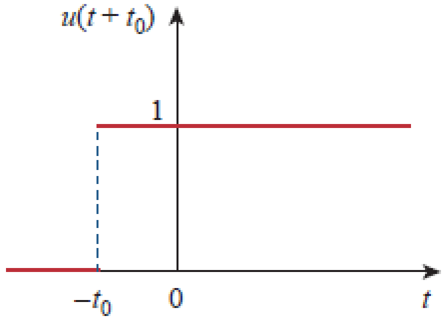
Unit step function



$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$



$$u(t - t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$$



$$u(t + t_0) = \begin{cases} 0, & t < -t_0 \\ 1, & t > -t_0 \end{cases}$$

$$v(t) = \begin{cases} 0, & t < t_0 \\ V_0, & t > t_0 \end{cases}$$

$$v(t) = V_0 u(t - t_0)$$