Effective Length

- Specific Values of *K* shall be known

End conditions	*K*
Pin-Pin	*1.0*
Pin-Fixed	*0.8*
Fixed-Fixed	*0.65*
Fixed-Free	*2.1*

Recommended design values (not theoretical values)

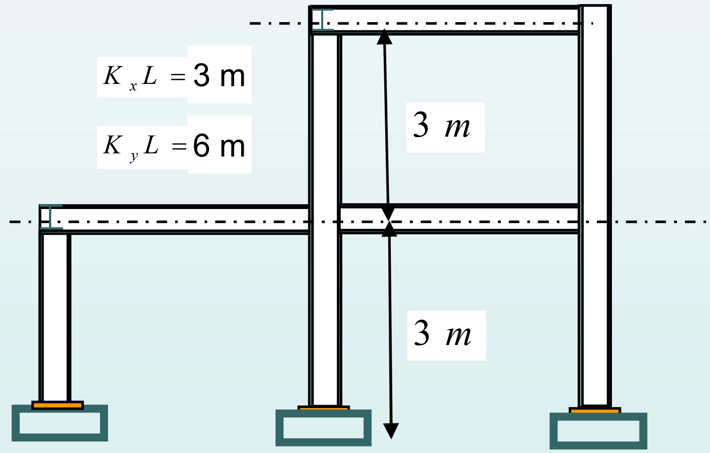
- Values for *K* for different end conditions range from 0.5 for theoretically fixed ends to 1.0 for pinned ends and are given by:

Table C-C2.2 AISC Manual

- For compression elements connected as rigid frames the effective length is a function of the relative stiffness of the element compared to the overall stiffness of the joint. This will be discussed later in this chapter

K Factor for Rigid Frames

- If we assume all connections are pinned then: $K_x L$ = 3 m and $K_y L$ = 6 m
- However the rigidity of the beams affect the rotation of the columns. Thus in rigid frames the K factor can be determined from the relative rigidity of the columns
- Determine a G factor



$$G = \frac{\sum E_c I_c / L_c}{\sum E_g I_g / L_g} \Rightarrow G = \frac{\sum I_c / L_c}{\sum I_g / L_g}$$

- Where "c" represents column and "g" represents girder
- The G value is computed at each end of the member and K is computed factor from the monograms in

AISC Manual – Figure C-C2.2

Effective Length of Columns in Frames

- So far, we have looked at the buckling strength of individual columns. These columns had various boundary conditions at the ends, but they were not connected to other members with moment (fix) connections.
- The effective length factor K for the buckling of an individual column can be obtained for the appropriate end conditions from Table C-C2.2 of the AISC Manual .
- However, when these individual columns are part of a frame, their ends are connected to other members (beams etc.).
  - Their effective length factor K will depend on the restraint offered by the other members connected at the ends.
  - Therefore, the effective length factor K will depend on the relative rigidity (stiffness) of the members connected at the ends.

Effective Length of Columns in Frames

- The effective length factor for columns in frames must be calculated as follows:
  - First, you have to determine whether the column is part of a braced frame or an unbraced (moment resisting) frame.
    - If the column is part of a braced frame then its effective length factor $0 < K \leq 1$
    - If the column is part of an unbraced frame then $1 < K \leq \infty$
  - Then, you have to determine the relative rigidity factor G for both ends of the column
    - G is defined as the ratio of the summation of the rigidity (EI/L) of all columns coming together at an end to the summation of the rigidity (EI/L) of all beams coming together at the same end.

$$G = \frac{\sum \frac{E\,I_c}{L_c}}{\sum \frac{E\,I_b}{L_b}}$$

c: for columns

It must be calculated for both ends of the column

b: for beams

Effective Length of Columns in Frames

- Then, you can determine the effective length factor K for the column using the calculated value of G at both ends, i.e., G_A and G_B and the appropriate alignment chart

- There are two alignment charts provided by the AISC manual,
  - One is for columns in braced (sidesway inhibited) frames. $0 < K \leq 1$
  - The second is for columns in unbraced (sidesway uninhibited) frames. $1 < K \leq \infty$
  - The procedure for calculating G is the same for both cases.

Effective Length

Monograph or
Jackson and Moreland
Alignment Chart
for Unbraced Frame

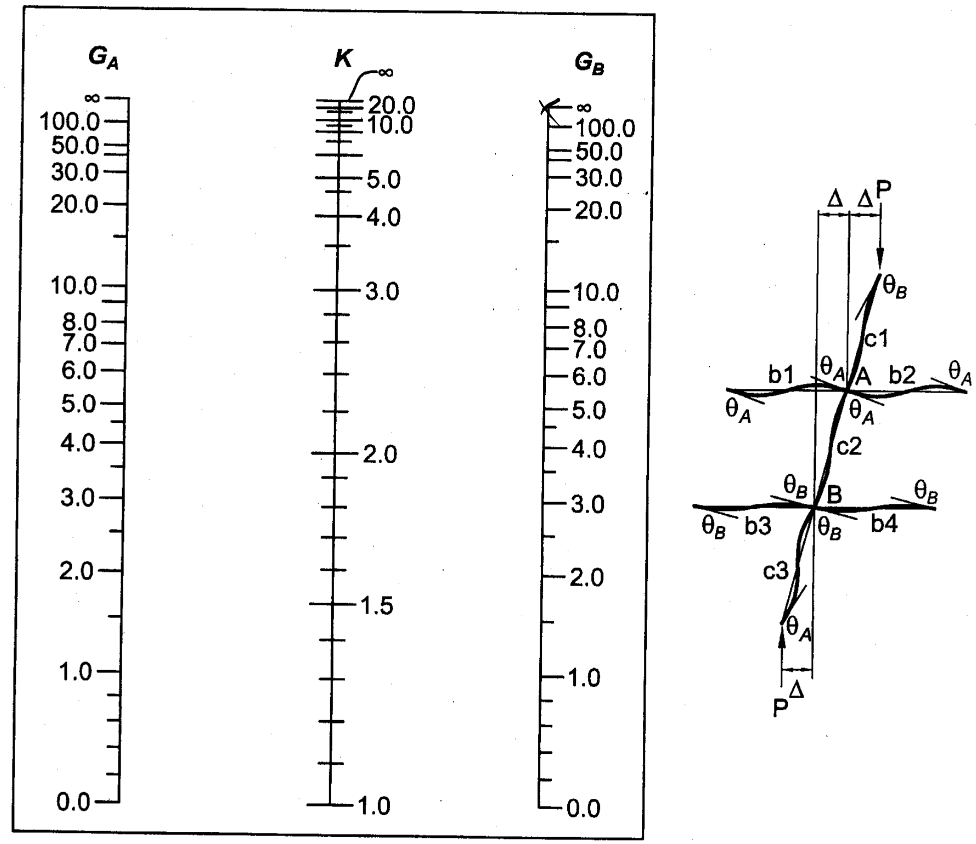


Fig. C-C2.4. Alignment chart—sidesway uninhibited (moment frame).

Specification for Structural Steel Buildings, March 9, 2005

Effective Length

Monograph or
Jackson and Moreland
Alignment Chart
for braced Frame

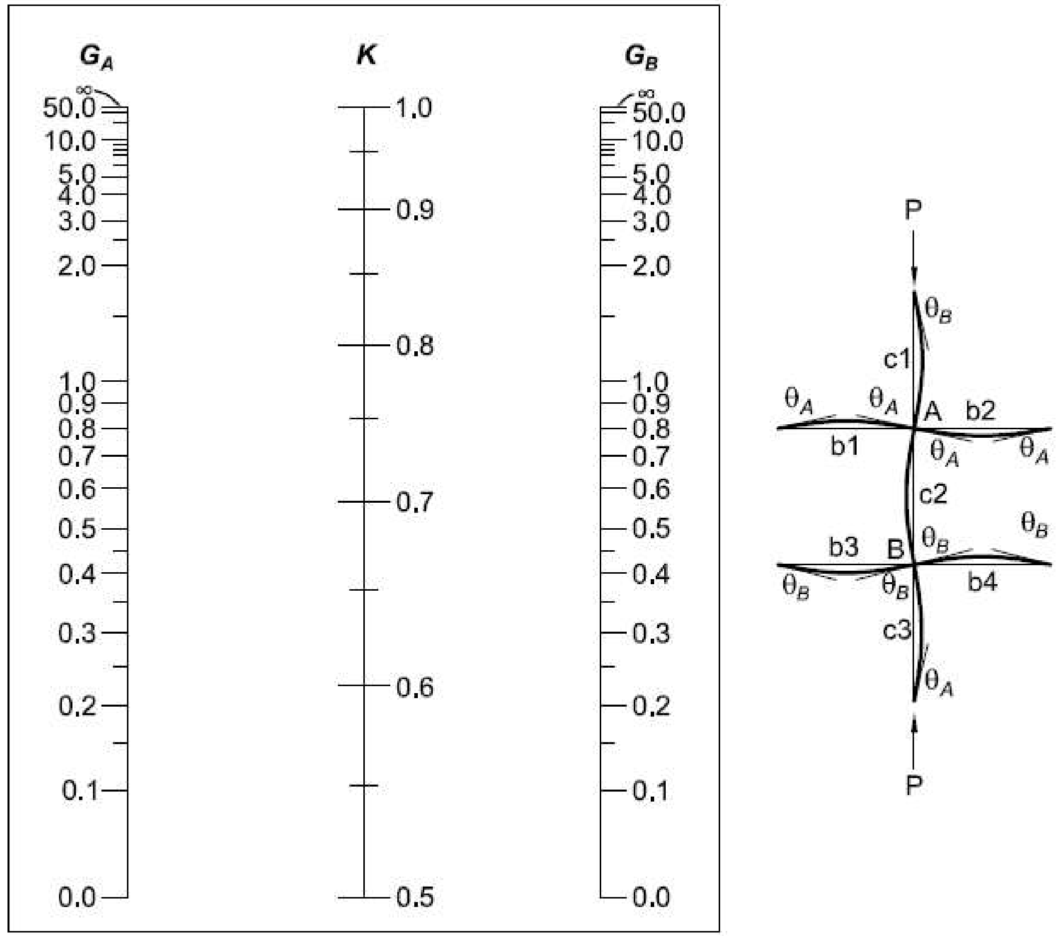
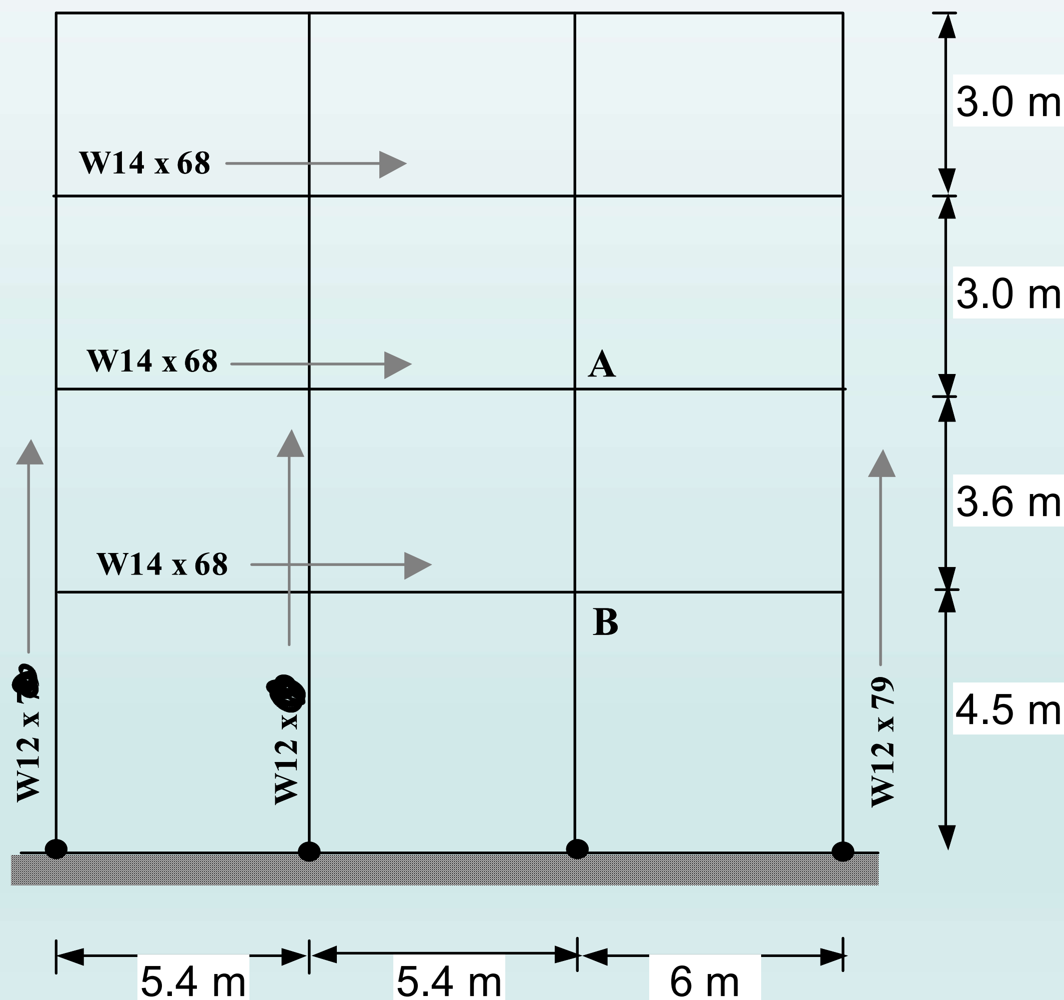


Fig. C-C2.3. Alignment chart—sidesway inhibited (braced frame).

Ex. 3.6 – Effective Length Factor

- Calculate the effective length factor for the **W12 x 53** column AB of the frame shown. Assume that the column is oriented in such a way that major axis bending occurs in the plane of the frame. Assume that the columns are braced at each story level for out-of-plane buckling. Assume that the same column section is used for the stories above and below.



Ex. 3.6 – Effective Length Factor

- **Step I.** Identify the frame type and calculate L_x, L_y, K_x, and K_y if possible.

  It is an unbraced (sidesway uninhibited) frame.

  $L_x = L_y = 12$ ft

  $K_y = 1.0$

  K_x depends on boundary conditions, which involve restraints due to beams and columns connected to the ends of column AB.

  Need to calculate K_x using alignment charts.

- **Step II.** Calculate K_x

  I_{xx} of W 12 x 53 = 425 in^4 I_{xx} of W14x68 = 753 in^4

$$G_A = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{\frac{425}{10\times 12} + \frac{425}{12\times 12}}{\frac{723}{18\times 12} + \frac{723}{20\times 12}} = \frac{6.493}{6.360} = 1.021$$

Ex. 3.6 – Effective Length Factor

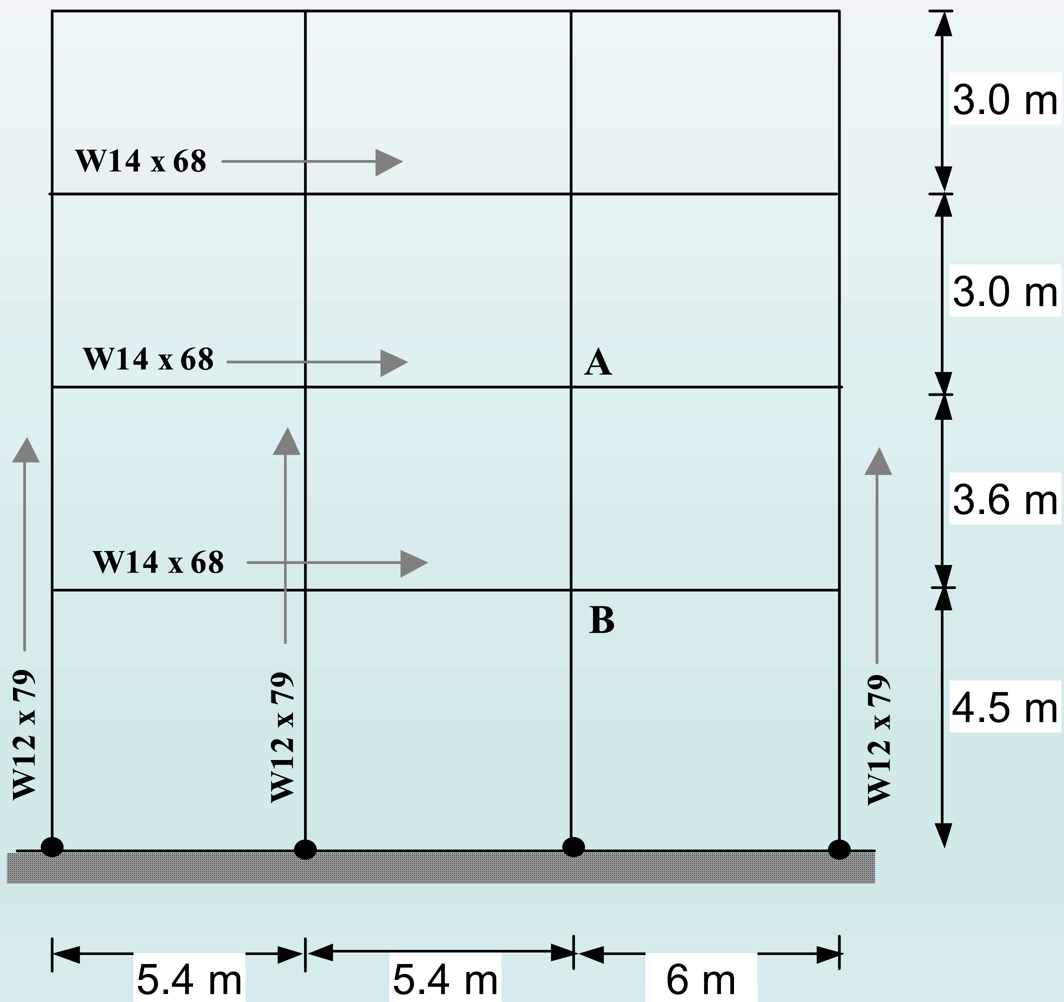
$$G_B = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{\frac{425}{12\times 12} + \frac{425}{15\times 12}}{\frac{723}{18\times 12} + \frac{723}{20\times 12}} = \frac{5.3125}{6.360} = 0.835$$

Using G_A and G_B: $K_x = 1.3$ - from Alignment Chart on Page 16.1-242

Ex. 3.8 – Column Design

- Design Column AB of the frame shown below for a design load of 2300 kN.
- Assume that the column is oriented in such a way that major axis bending occurs in the plane of the frame.
- Assume that the columns are braced at each story level for out-of-plane buckling.
- Assume that the same column section is used for the stories above and below.
- Use A992 steel.

Ex. 3.8 – Column Design



Ex. 3.8 – Column Design

- **Step I** - Determine the design load and assume the steel material.

  Design Load = P_u = 2300 kN.

  Steel yield stress = 344 MPa (A992 material).

- **Step II.** Identify the frame type and calculate L_x, L_y, K_x, and K_y if possible.

  It is an unbraced (sidesway uninhibited) frame.

  $L_x = L_y = 3.6$ m

  $K_y = 1.0$

  K_x depends on boundary conditions, which involve restraints due to beams and columns connected to the ends of column AB.

  Need to calculate K_x using alignment charts.

  Need to select a section to calculate K_x

Ex. 3.8 – Column Design

- **Step III** - Select a column section

  Assume minor axis buckling governs.

  $K_y\ L_y = 3.6$ m

  Select section W12x53

  $K_yL_y/r_y = 57.2$ $F_e = 604.4$ $F_{cr} = 271.1$

  $\phi_c P_n$ for y-axis buckling = 2455.4 kN

- **Step IV** - Calculate K_x

  I_{xx} of W 12 x 53 = 177×10^6 mm^4

  I_{xx} of W14x68 = 301×10^6 mm^4

Ex. 3.8 – Column Design

$$G_A = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{\left(\frac{177}{3} + \frac{177}{3.6}\right)}{\frac{301}{5.4} + \frac{301}{6}} = 1.02$$

$$G_B = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{I_b}{L_b}} = \frac{\left(\frac{177}{3.6} + \frac{177}{4.5}\right)}{\frac{301}{5.4} + \frac{301}{6}} = 0.836$$

Using GA and GB: $K_x = 1.3$ - from Alignment Chart

Ex. 3.8 – Column Design

- **Step V** - Check the selected section for X-axis buckling

  $K_x L_x = 1.3 \times 3.6 = 4.68$ m

  $K_x L_x/r_x = 35.2$ $F_e = 1590.4$ $F_{cr} = 314.2$

  For this column, $\phi_c P_n$ for X-axis buckling = 2846.3

- **Step VI** - Check the local buckling limits

  For the flanges, $b_f/2t_f = 8.69 <$ $\lambda_r = 0.56 \times \sqrt{\frac{E}{F_y}} = 13.5$

  For the web, $h/t_w = 28.1$ $<$ $\lambda_r = 1.49 \times \sqrt{\frac{E}{F_y}} = 35.9$

  Therefore, the section is non-compact. OK, local buckling is not a problem