Lateral-Torsional Buckling (LTB) – Uniform BM

- As soon as any portion of the cross-section reaches the yield stress F_y, the elastic LTB equation cannot be used.
  - L_r is the unbraced length that corresponds to a LTB moment

    $M_r = S_x (0.7F_y)$.
  - M_r will cause yielding of the cross-section due to residual stresses.
- When the unbraced length is less than L_r, then the elastic LTB Eq. cannot be used.
- When the unbraced length (L_b) is less than L_r but more than the plastic length L_p, then the LTB M_n is given by the Eq. below:

Lateral-Torsional Buckling – Uniform BM

- If $L_p \le L_b \le L_r$, then $M_n = \left[M_p - (M_p - M_r)\left(\dfrac{L_b - L_p}{L_r - L_p} \right) \right]$
- This is linear interpolation between (L_p, M_p) and (L_r, M_r)
- See Fig. 10 again.
- $$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_0}} \sqrt{1 + \sqrt{1 + 6.76\left(\frac{0.7F_y}{E} \frac{S_x h_0}{Jc} \right)^2}}$$
- $$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x}$$
- For a doubly symmetric I-shape: $c = 1$
- h_0 = distance between the flange centroids (mm)

Moment Capacity of Beams Subjected to Non-uniform BM

- As mentioned previously, the case with uniform bending moment is worst for lateral torsional buckling.
- For cases with non-uniform bending moment, the LTB moment is greater than that for the case with uniform moment.
- The AISC specification says that:
  - The LTB moment for non-uniform bending moment case
  - $\mathbf{C_b}$ **x** lateral torsional buckling moment for uniform moment case.

Moment Capacity of Beams Subjected to Non-uniform BM

- **C_b** is always greater than 1.0 for non-uniform bending moment.
  - **C_b** is equal to 1.0 for uniform bending moment.
  - Sometimes, if you cannot calculate or figure out **C_b**, then it can be conservatively assumed as 1.0. for doubly and singly symmetric sections

$$C_b = \frac{12.5\,M_{\max}}{2.5\,M_{\max} + 3\,M_A + 4\,M_B + 3\,M_c} < 3.0$$

M_{max} - *magnitude of maximum bending moment in* L_b

M_A - *magnitude of bending moment at quarter point of* L_b

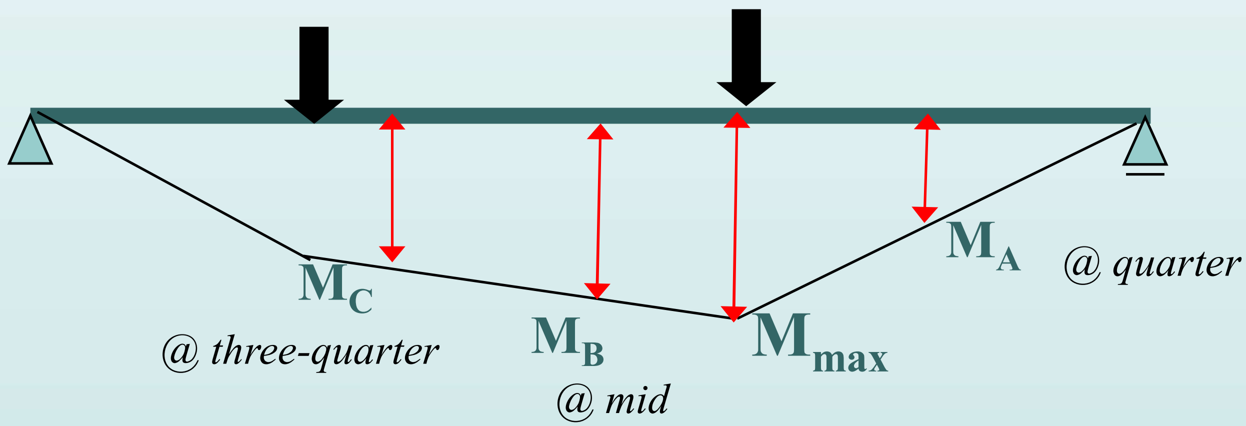
M_B - *magnitude of bending moment at half point of* L_b

M_C - *magnitude of bending moment at three-quarter point of* L_b

  - Use absolute values of M

Flexural Strength of Compact Sections

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C} < 3.0$$



Moments determined between bracing points

Other quation for Cb

$$C_b = 1.75 + 1.05\left(\frac{M_1}{M_2}\right) + 0.3\left(\frac{M_1}{M_2}\right)^2 \qquad \text{(C-F1-1)}$$

where

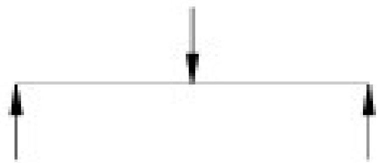
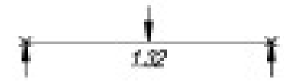
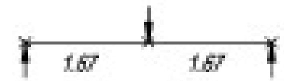
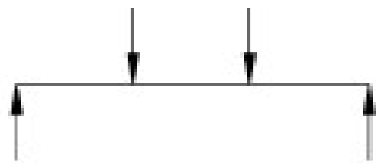
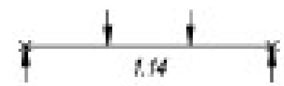
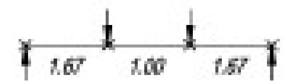
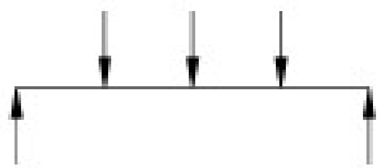
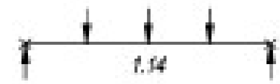
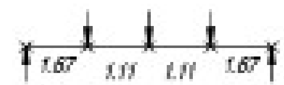
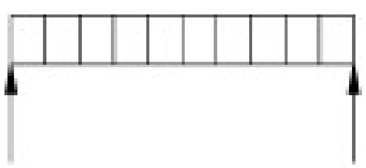
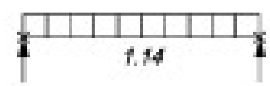
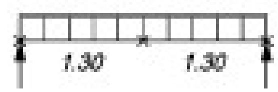
M_1 = smaller moment at end of unbraced length, kip-in. (N-mm)

M_2 = larger moment at end of unbraced length, kip-in. (N-mm)

(M_1/M_2) is positive when moments cause reverse curvature and negative for single curvature

Table 5-1.
Values of C_b for Simply Supported Beams

3-1

Load	Lateral Bracing Along Span	C_b
	None	 1.32
	At load points	 1.67 1.67
	None	 1.14
	At load points	 1.67 1.00 1.67
	None	 1.14
	At load points	 1.67 1.11 1.11 1.67
	None	 1.14
	At centerline	 1.30 1.30

Moment Capacity of Beams Subjected to Non-uniform Bending Moments

- The moment capacity M_n for the case of non-uniform bending moment
  - M_n = $\mathbf{C}_b$ x {M_n for the case of uniform bending moment} ≤ $\mathbf{M}_p$
  - Important to note that the increased moment capacity for the non-uniform moment case cannot possibly be more than $\mathbf{M}_p$.
  - Therefore, if the calculated values is greater than $\mathbf{M}_p$, then you have to reduce it to $\mathbf{M}_p$

Moment Capacity of Beams Subjected to Non-uniform BM

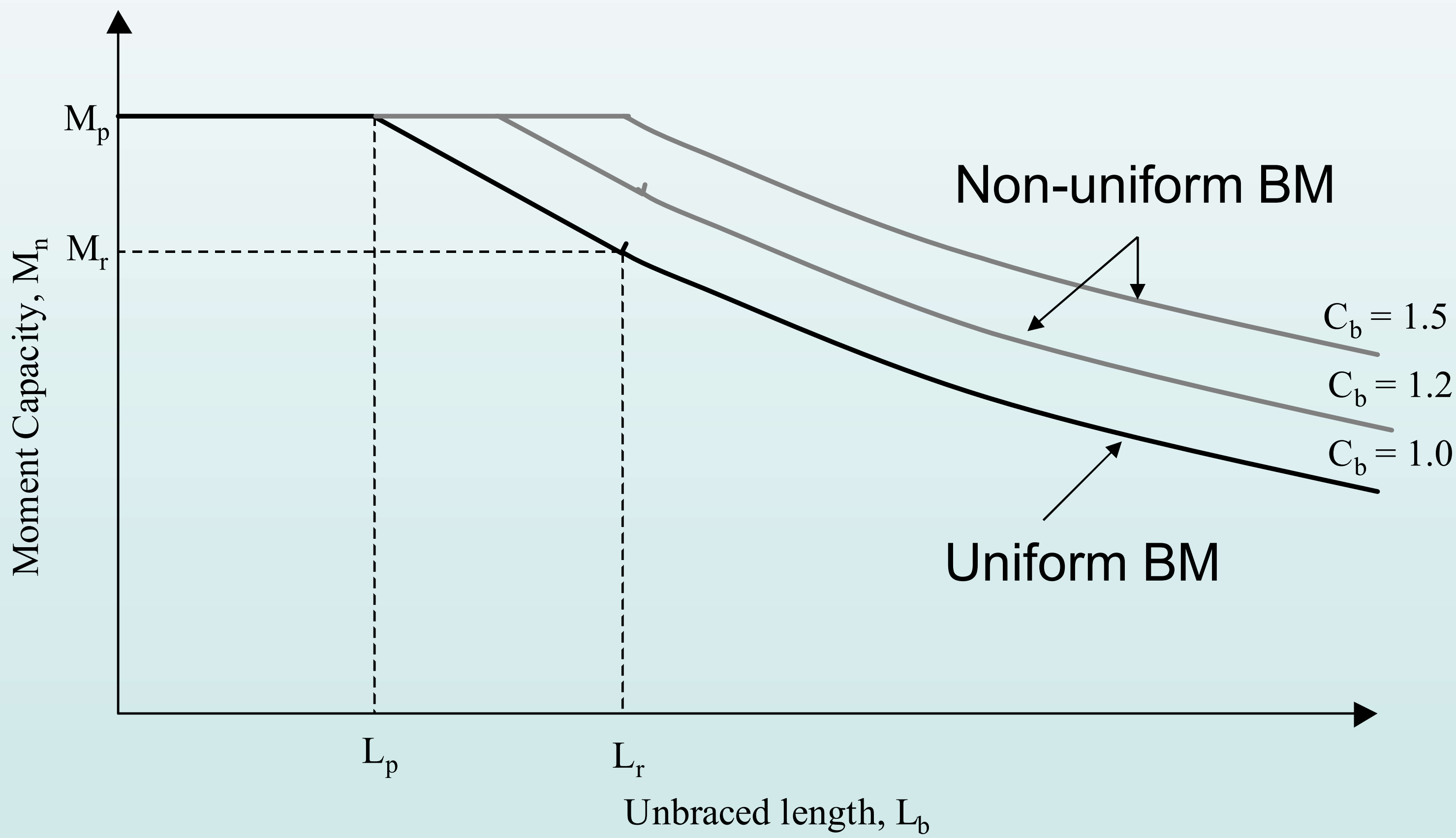


Figure 11. Moment capacity versus Lb for non-uniform moment case

C_b = 1.0 means uniform BM

Structural Design of Beams

- Steps for adequate design of beams:
  1) Compute the factored loads, factored moment and shear
  2) Determine unsupported length L_b and C_b
  3) Select a WF shape and choose Z_x assuming it is a compact section with full lateral support

$$Z_x = \frac{M_u}{\phi_b F_y}$$

$$M_n = M_p = ZF_y$$

$$M_u \leq \phi_b M_n = 0.9ZF_y$$

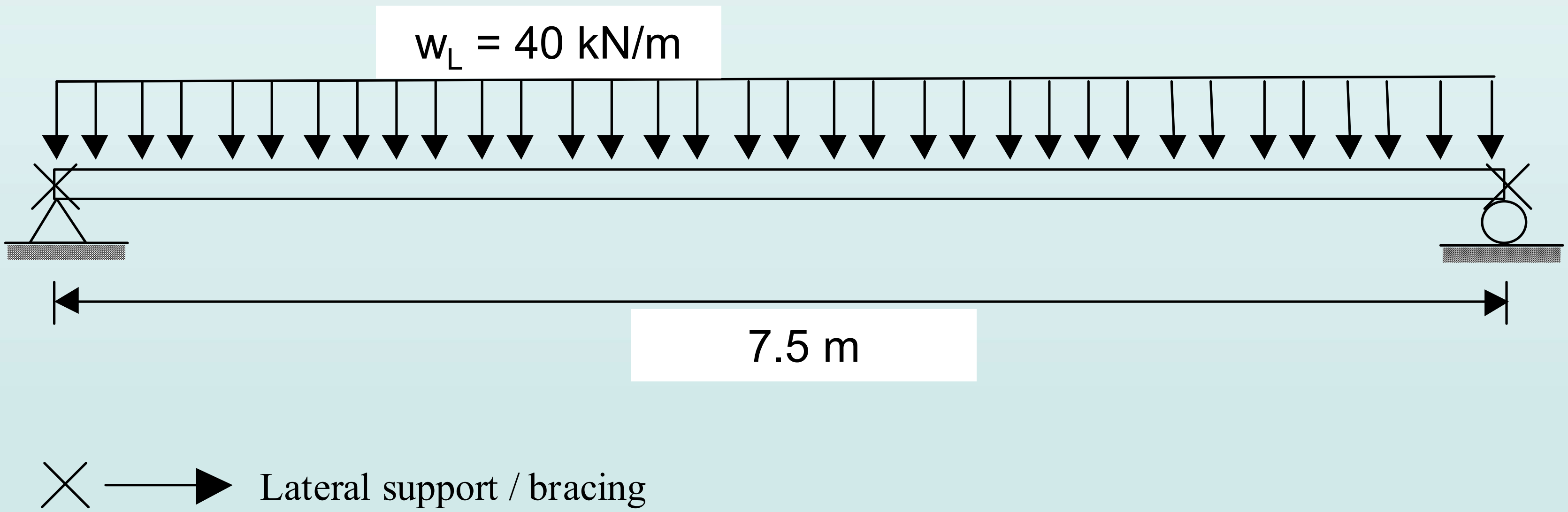
  4) Check the section dimension for compactness and determine $\phi_b M_n$

$$M_u \leq \phi_b M_n$$

  5) Use service loads to check deflection requirements

Ex. 4.4 – Beam Design

- Use Grade 50 steel to design the beam shown below. The unfactored uniformly distributed live load is equal to 40 kN/m. There is no dead load. Lateral support is provided at the end reactions. Select W16 section.



Ex. 4.4 – Beam Design

- **Step I.** Calculate the factored loads assuming a reasonable self-weight.

  Assume self-weight = w_{sw} = 1.46 kN/m.

  Dead load = w_D = 0 + 1.46 = 1.46 kN/m.

  Live load = w_L = 40 kN/m.

  Ultimate load = $w_u = 1.2\ w_D + 1.6\ w_L$ = 65.8 kN/m.

  Factored ultimate moment = $M_u = w_u\ L^2/8$ = 462.3 kN-m.

Is BM uniform??	Yes	C_b =1.0
	No	Go to Step II

- **Step II.** Determine unsupported length L_b and C_b

  There is only one unsupported span with L_b = 7.5 m

  C_b = 1.14 for the parabolic bending moment diagram, See values of C_b shown in Table 3-1.

Ex. 4.4 – Beam Design

- **Step III.** Select a wide-flange shape
  - Compute Z_x = 462.3*10^6/(0.9*344) = 1493x10^6 mm^3.
  - Select W16 x 50 steel section
  - Z_x = 1508x10^3 mm^3 S_x = 1327x10^3 mm^3 r_y = 40.4 mm
  - C_w = 610x10^9 mm^6 I_y = 15.5x10^6 mm^4 J = 0.63x10^6 mm^4
  - $L_p = 1.76 r_y \sqrt{E/F_y} = \dfrac{1.76 \times 40.4 \sqrt{200000/344}}{1000} = 1.71\, m$
  - $L_r = 1.95 r_{ts} \dfrac{E}{0.7F_y} \sqrt{\dfrac{Jc}{S_x h_0}} \sqrt{1 + \sqrt{1 + 6.76 \left(\dfrac{0.7F_y}{E} \dfrac{S_x h_0}{Jc} \right)^2}}$
  - $r_{ts} = \sqrt{\dfrac{\sqrt{I_y C_w}}{S_x}} = \sqrt{\dfrac{\sqrt{15.5 \times 10^6 \times 610 \times 10^9}}{1327 \times 10^3}} = 48.1\, mm$

Ex. 4.4 – Beam Design

- $h_0 = D - T_F = 414 – 16 = 398$ mm
- $$\sqrt{1+6.76\left(\frac{0.7F_y}{E}\frac{S_x h_0}{Jc}\right)^2} = \sqrt{1+6.76\left(\frac{0.7\times 344}{200000}\frac{1327\times 10^3\times 398}{0.63\times 10^6\times 1}\right)^2} = 2.81$$
- $$L_r = 1.95\times\frac{48.1}{1000}\times\frac{200000}{0.7\times 344}\sqrt{\frac{0.63\times 10^6\times 1}{1327\times 10^3\times 398}}\sqrt{1+2.81} = 5.26\ m$$
- $L_b > L_r$

$$M_n = C_b\sqrt{\frac{\pi^2 EI_y}{L_b^2}\left(GJ+\frac{\pi^2 EC_w}{L_b^2}\right)}$$

$$= 1.14\sqrt{\frac{\pi^2\times 200\times 15.5\times 10^6}{7500^2}\left(77\times 0.63\times 10^6+\frac{\pi^2\times 200\times 610\times 10^9}{7500^2}\right)}$$

$$= 222\times 10^3\ kN.mm = 222\ kN.m \quad < M_p = \frac{1508\times 10^3\times 344}{10^6} = 518.8\ kN.m$$

Ex. 4.4 – Beam Design

- **Step IV.** Check if section is adequate
  - $M_u > \phi M_n$ Not OK
- **Step V.** Try a larger section.
  - After few trials select W16 x 67 $\phi M_n = 497.7 > M_u$ **OK**

- **Step VI.** Check for local buckling.

  $\lambda = B_f / 2T_f = 7.7$; Corresponding $\lambda_p = 0.38\,(E/F_y)^{0.5} = 9.19$

  Therefore, $\lambda < \lambda_p$ - compact flange

  $\lambda = H/T_w = 35.9$; Corresponding $\lambda_p = 3.76\,(E/F_y)^{0.5} = 90.5$

  Therefore, $\lambda < \lambda_p$ - compact web

  Compact section. - OK!

- This example demonstrates the method for designing beams and accounting for $C_b > 1.0$)
- Values for L_r and L_p can be obtained from Tables too