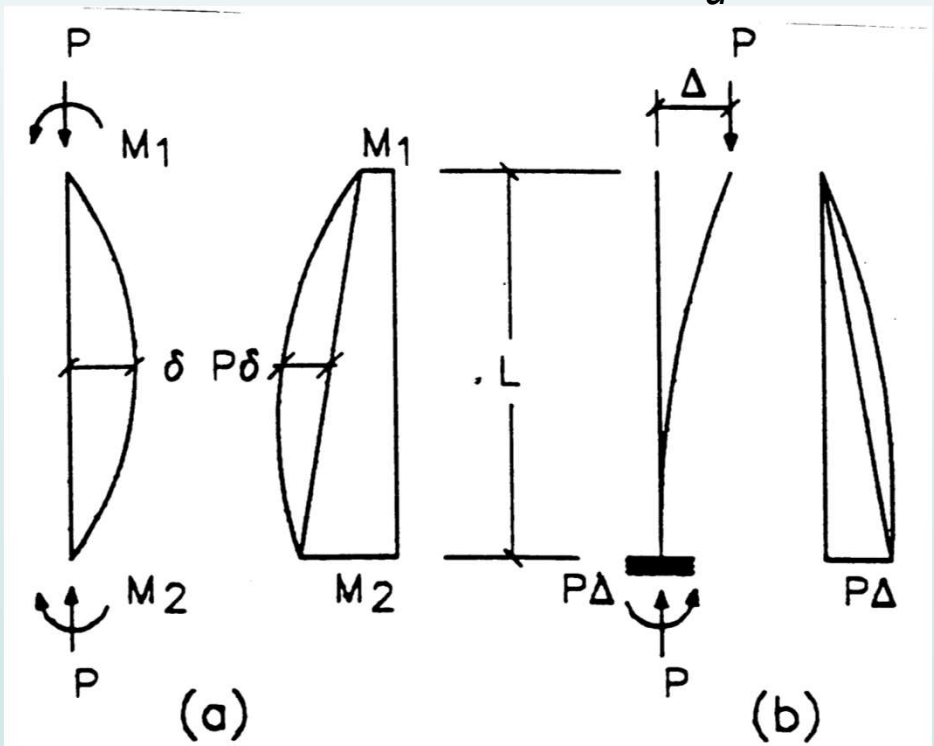
Moment Amplification

Second-order Moments, $P_u\delta$ and $P_u\Delta$



Moment amplification in column braced against sidesway

$$M_u = M_{nt} + P_u\delta$$

Moment amplification in unbraced column

$$M_u = M_{lt} + P_u\Delta$$

Moment Amplification

- Using first principles we can prove that the final moment M_{max} is amplified from M_0 as

$$M_{max} = M_0 B = M_0 \left[\frac{1}{1 - \left(\frac{P_u}{P_e} \right)} \right]$$

- The amplification factor B can be

$$B = \left[\frac{1}{1 - \left(\frac{P_u}{P_e} \right)} \right]$$

Where

$M_{max} = bending\ moment$

Second Order Analysis

OVERVIEW

Objective: Determine the maximum moment (Mu) and maximum deflection of a beam-column, based on 2nd order elastic analysis.

Definitions:

- "1st order analysis" - equilibrium equations are based on the undeformed geometry of the structure

- "2nd order analysis" - equilibrium equations are based on the deformed geometry of the structure

Second Order Analysis

Methods for 2nd Order Elastic Analysis

I. EXACT METHODS

A. Closed form mathematical solutions

- based on solution to differential equation of equilibrium, with equilibrium taken for the deformed structure
- applicable to simple cases only
- generally not useful for design; however, it is useful to study these solutions for developing an understanding of other analysis methods and to check approximate methods of 2nd order analysis

Second Order Analysis

B. Structural Analysis computer programs with capability for exact 2nd order analysis,

(Note: most commercial programs currently available which claim 2nd order analysis capability provide incomplete, approximate 2nd order analysis)

Second Order Analysis

II. Approximate Methods

A. Amplification Factors

Procedure:

- conduct 1st order analysis
- 2nd order moment ≈
  first order moment × moment amplification factor (MAF)
- 2nd order deflection ≈
  first order deflection × deflection amplification factor (DAF)

Second Order Analysis

B. Approximate Computer Methods

(a) Use computer program with approximate 2nd order effects built in to the program

(b) Use a conventional 1st order analysis computer program, combined with various "tricks" to approximate 2nd order effects.

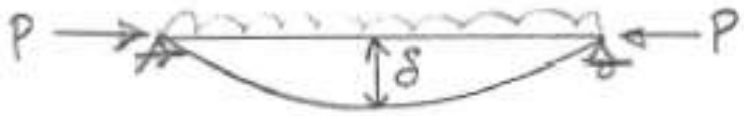
LRFD: Permits any of the above methods. LRFD (Chap C) provides approx. amplification factors.

Second Order Analysis

STUDY OF SECOND ORDER ANALYSIS

Our study of 2nd order effects will be divided into two categories:

1) 2nd order effects due to displacement between member ends:



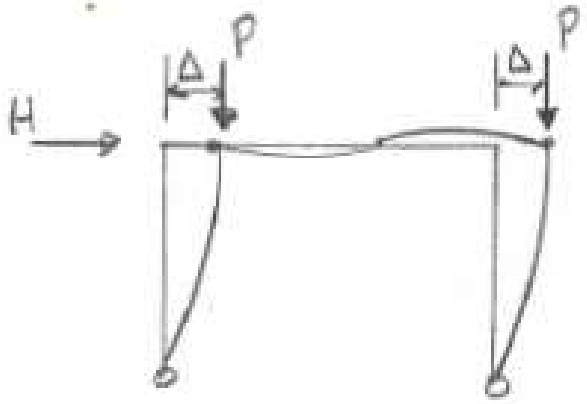
δ = displ. between member ends

2nd order effects ≡ "P-δ" effects

LRFD: B_1 = moment amplification factor for P-δ effects

Second Order Analysis

2) 2nd order effects due to lateral translation of member ends:



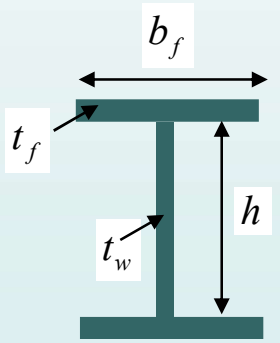
Δ = relative displacement between member ends (i.e. lateral translation of one end of the member with respect to the other end).

2nd order effects = "P-Δ" effects

LRFD: B_2 = moment amplification factor for P-Δ effects

Compact Sections for Beam-Columns

- The axial load affects the ratio for compactness. When the check for compactness for the web is performed while the web is subjected to axial load the following ratios shall be



$$\text{for } \frac{P_u}{\phi_b P_y} \le 0.125 \quad \Rightarrow \quad \lambda_p = 3.76\sqrt{\frac{E}{F_y}}\left[1 - \frac{2.75\,P_u}{\phi_b P_y}\right]$$

$$\text{for } \frac{P_u}{\phi_b P_y} > 0.125 \quad \Rightarrow \quad \lambda_p = 1.12\sqrt{\frac{E}{F_y}}\left[2.33 - \frac{P_u}{\phi_b P_y}\right] \ge 1.49\sqrt{\frac{E}{F_y}}$$

$$\text{for all } \frac{P_u}{\phi_b P_y} \quad \Rightarrow \quad \lambda_r = 5.70\sqrt{\frac{E}{F_y}}\left[1 - \frac{0.74\,P_u}{\phi_b P_y}\right]$$

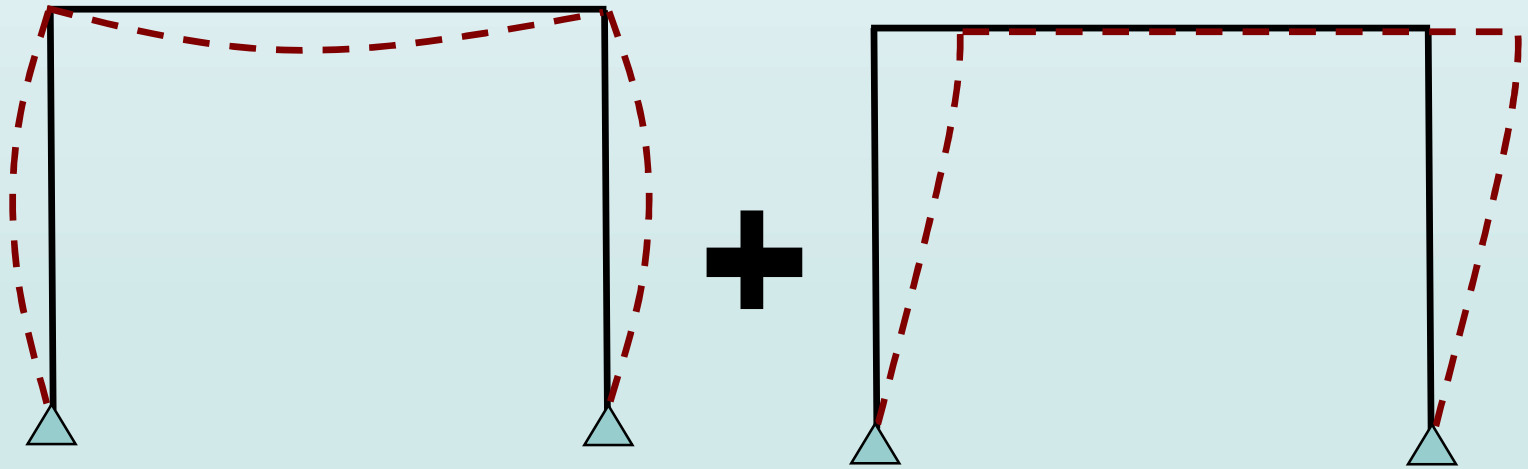
Flange limit is similar to beams

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} \qquad \lambda_r = 0.83\sqrt{\frac{E}{F_y - 68.9}}$$

Braced and Unbraced Frames

- Two components of amplification moments can be observed in unbraced frames:
  - Moment due to member deflection (similar to braced frames)
  - Moment due to sidesway of the structure

Unbraced Frames

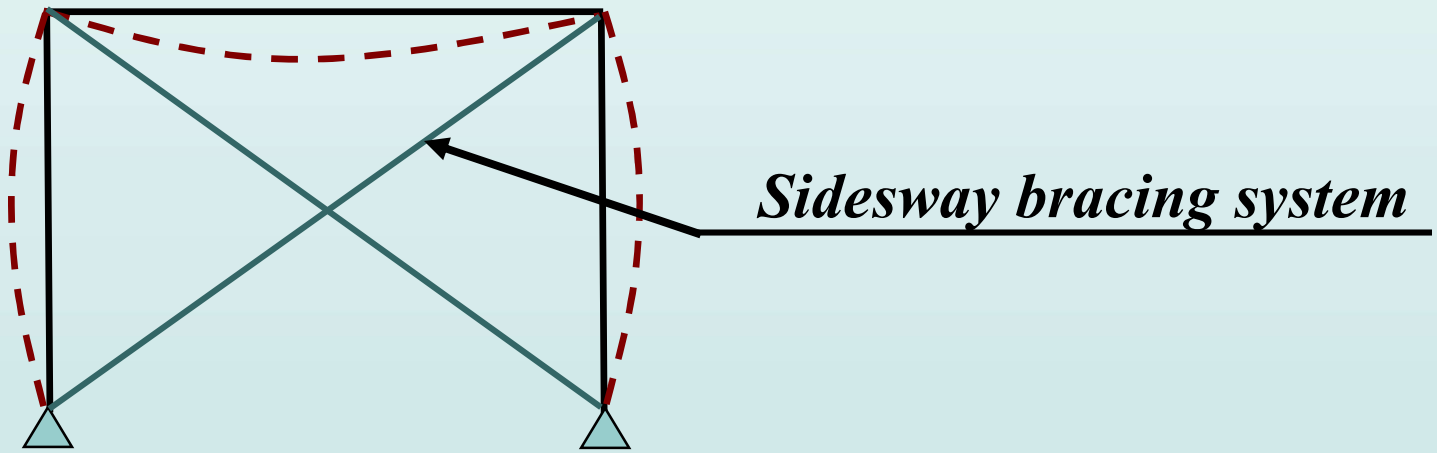


Member deflection **Member sidesway**

Unbraced and Braced Frames

- In braced frames amplification moments can only happens due to member deflection

Braced Frames



Member deflection