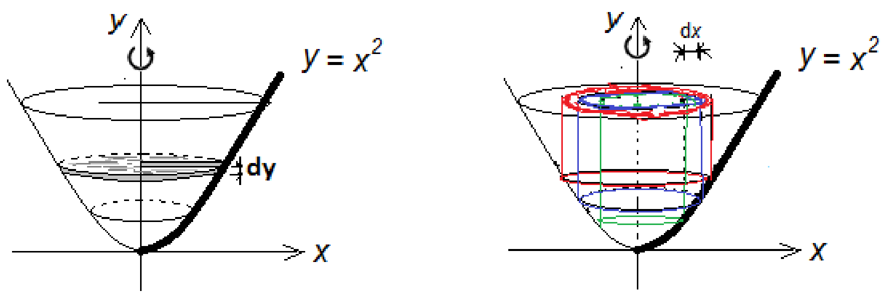
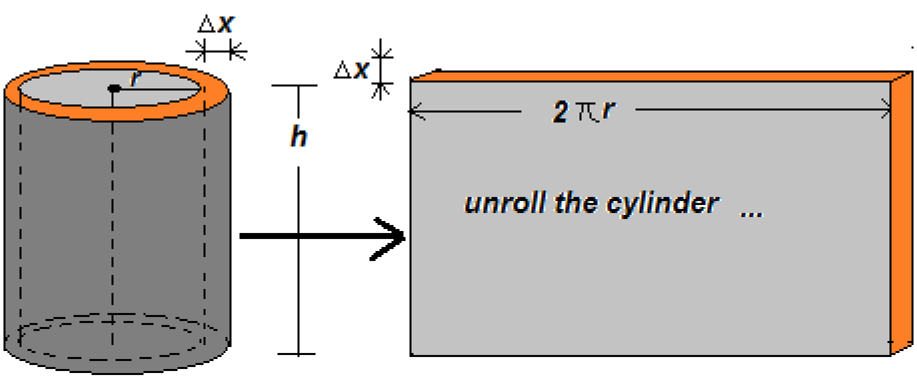
L38 Volume of Solid of Revolution II–Shell Method

Shell Method is another way to calculate the volume of a solid of revolution when the slice is parallel to the axis of revolution.

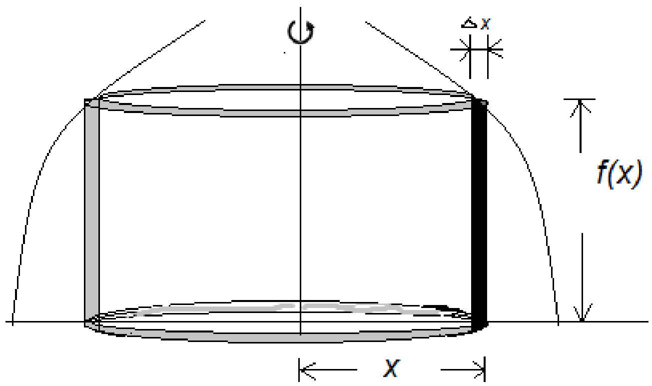
$$V = \int dV$$



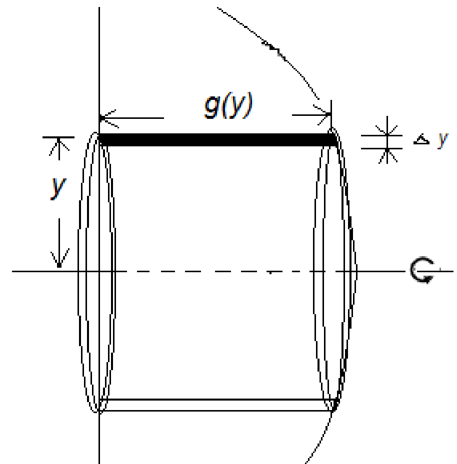


$$V_{shell} = 2\pi r h \Delta x = 2\pi(\text{shell.radius})(\text{shell.height})\,\Delta x$$

The necessary equation for calculating such a volume depends on which axis is serving as the axis of revolution.



Vertical Axis of Revolution



Horizontal Axis of revolution

$$V_{ver} = \int A(x)dx$$

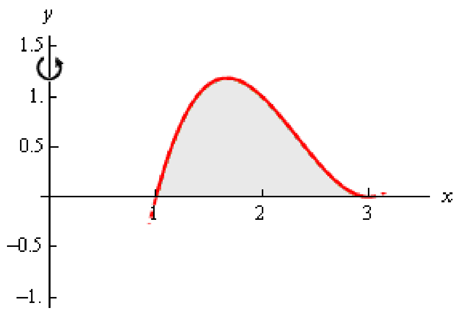
if the rotation is around a vertical axis of revolution.

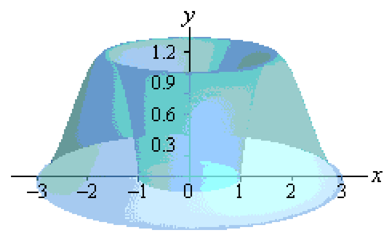
$$V_{hor} = \int A(y)dy$$

if the rotation is around a horizontal axis of revolution;

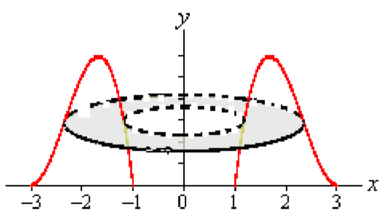
Note: $\boxed{A_{shell} = 2\pi(\text{shell.radius})(\text{shell.height})}$

ex. Determine the volume of the solid obtained by rotating the region bounded by $y = (x-1)(x-3)^2$ and the $x-$axis about the y-axis. $(\frac{12\pi}{5})$

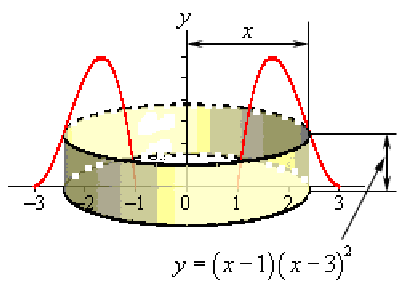




Solid



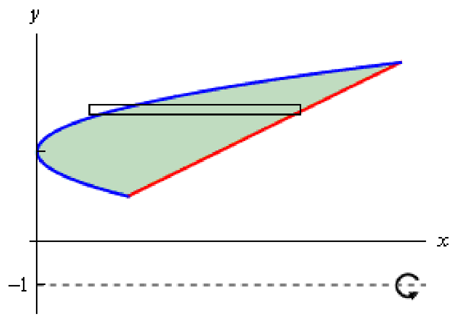
DISK/WASHER METHOD

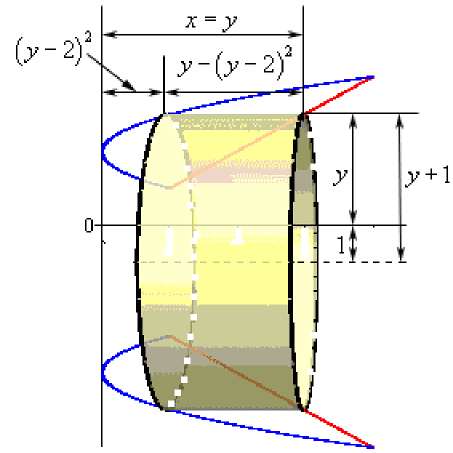


SHELL METHOD

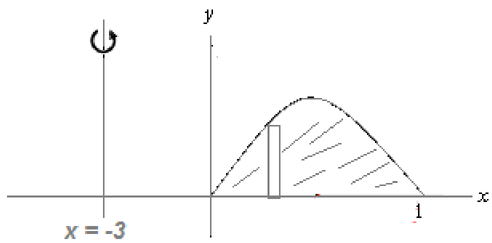
The remaining examples will have axis of rotation about axis other than the $x-$, and $y-$axis:

ex. Determine the volume of the solid obtained by rotating the region bounded by $x = (y-2)^2$ and $y = x$ about the line $y = -1$. $(\frac{63\pi}{2})$





ex. Determine the volume of the solid obtained by rotating the region bounded by $y = -x^2 + x$ and $y = 0$ about the line $x = -3$. $(\frac{7\pi}{6})$



Volume of Solid not generated as solids of revolution

The **Volume** $\boxed{V = \int_a^b A(x)dx, \quad V = \int_c^d A(y)dy}$

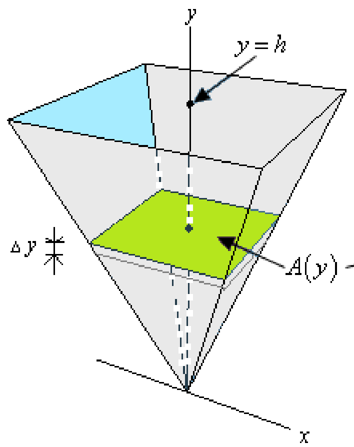
where $A(x), A(y)$ are cross-sectional area.

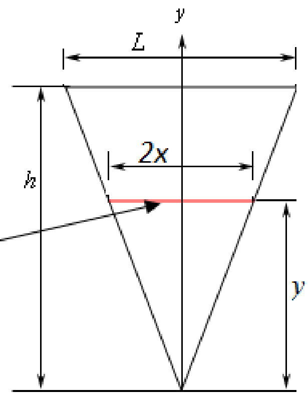
Question: How to find the cross-sectional area of such solids?

A few suggestions to consider:

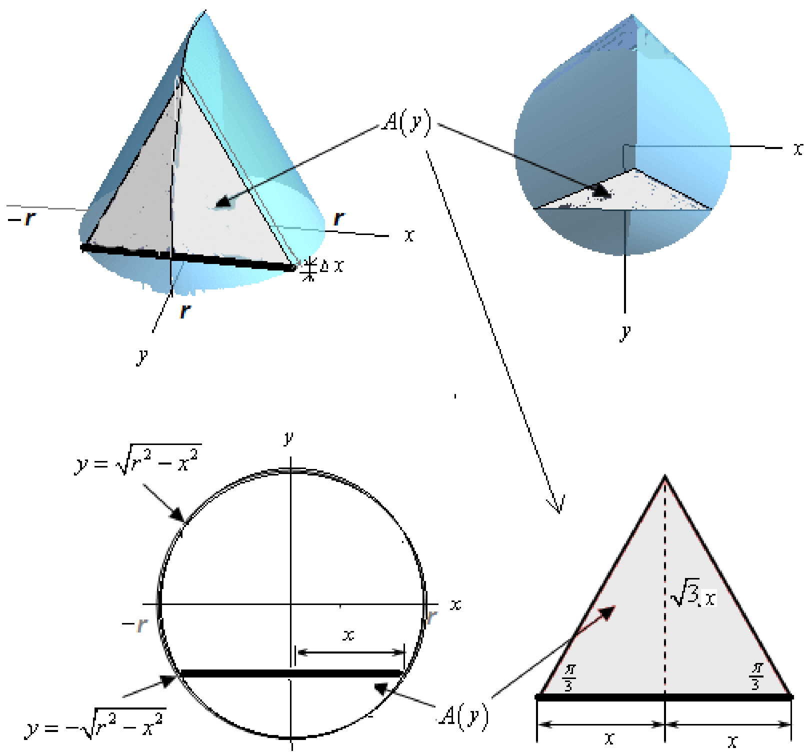
1. Sketch the solid
2. Determine the cross-sectional area (This is usually the hardest part of the problem. You may need to use Pythagorean Thm, Similar triangle proportion, 30-60-90 triangle, or the equation of a line $\cdots$)
3. Determine the limit of integration
4. Integrate the area

ex. Find the volume of a pyramid whose base is a square with sides of length L and whose height is h.





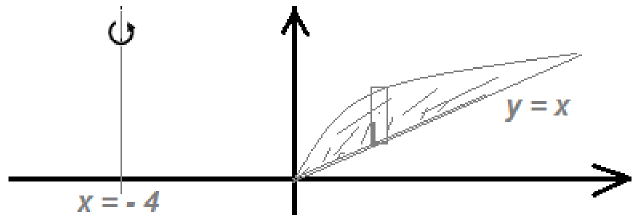
ex. Find the volume of the solid whose base is a disk of radius r and whose cross-sections are equilateral triangles.



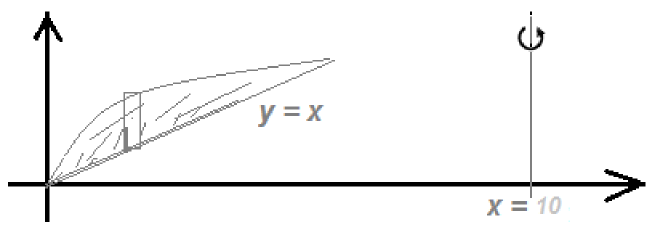
NYTI:

1. Try this last problem with different kinds of cross-sectional shapes, like a semi-circle, or a square.

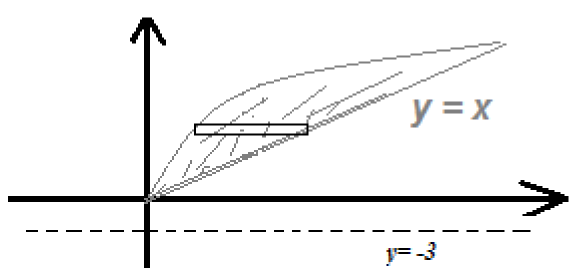
2. • Determine the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$ and $y = x$ about the line $x = -4$. $(\frac{22\pi}{15})$



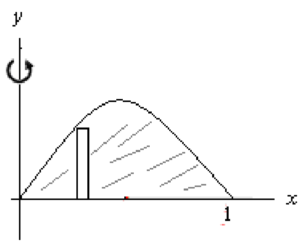
• Determine the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$ and $y = x$ about the line $x = 10$. $(\frac{116\pi}{5})$



• Determine the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$ and $y = x$ about the line $y = -3$. $(\frac{7\pi}{6})$



3. Determine the volume of the solid obtained by rotating the region bounded by $y = -x^2 + x$ and $y = 0$ about the line $y-$axis. $(\frac{\pi}{6})$



Using an alternative method, cylindrical shell method, sometimes is necessary especially when it is difficult to evaluate the base area with disk or washer methods.