57. ***Blood types*** Blood types are determined by the presence or absence of three antigens: A antigen, B antigen, and an antigen called the Rh factor. The resulting blood types are classified as follows:

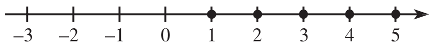
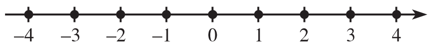
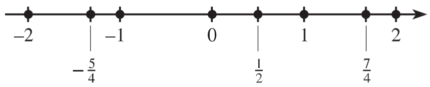
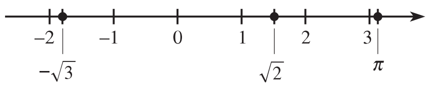
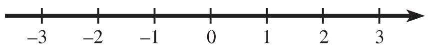
    type A if the A antigen is present
    type B if the B antigen is present
    type AB if both the A and B antigens are present
    type O if neither the A nor the B antigen is present

    These types are further classified as *Rh-positive* if the Rh-factor antigen is present and *Rh-negative* otherwise.
    (a) Draw a Venn diagram that illustrates this classification scheme.
    (b) Identify the blood type determined by each region of the Venn diagram (such as A^+ to indicate type A, Rh-positive).
    (c) Use a library or another source to find what percentage of the U.S. population has each blood type.

0.2 The Real Numbers

This text uses the set of **real numbers** as the universal set. We can represent the real numbers along a line called the **real number line.** This number line is a picture, or graph, of the real numbers. Each point on the real number line corresponds to exactly one real number, and each real number can be located at exactly one point on the real number line. Thus, two real numbers are said to be equal whenever they are represented by the same point on the real number line. The equation $a = b$ (a equals b) means that the symbols a and b represent the same real number. Thus, $3 + 4 = 7$ means that $3 + 4$ and 7 represent the same number. Table 0.1 lists special subsets of the real numbers.

TABLE 0.1 Subsets of the Set of Real Numbers

	Description	Example (some elements shown)
Natural numbers	{1, 2, 3, . . .} The counting numbers.	
Integers	{. . . , −2, −1, 0, 1, 2, . . .} The natural numbers, 0, and the negatives of the natural numbers.	
Rational numbers	All numbers that can be written as the ratio of two integers, a/b, with $b \neq 0$. These numbers have decimal representations that either terminate or repeat.	
Irrational numbers	Those real numbers that *cannot* be written as the ratio of two integers. Irrational numbers have decimal representations that neither terminate nor repeat.	
Real numbers	The set containing all rational and irrational numbers (the entire number line).	

The properties of the real numbers are fundamental to the study of algebra. These properties follow.

Properties of the Real Numbers

Let a, b, and c denote real numbers.

1. Addition and multiplication are commutative.

$$a + b = b + a \qquad ab = ba$$

2. Addition and multiplication are associative.

$$(a + b) + c = a + (b + c) \qquad (ab)c = a(bc)$$

3. The additive identity is 0.

$$a + 0 = 0 + a = a$$

4. The multiplicative identity is 1.

$$a \cdot 1 = 1 \cdot a = a$$

5. Each element a has an additive inverse, denoted by $-a$.

$$a + (-a) = -a + a = 0$$

   Note that there is a difference between a negative number and the negative of a number.
6. Each nonzero element a has a multiplicative inverse, denoted by a^{-1}.

$$a \cdot a^{-1} = a^{-1} \cdot a = 1$$

   Note that $a^{-1} = 1/a$.
7. Multiplication is distributive over addition.

$$a(b + c) = ab + ac$$

Note that Property 5 provides the means to subtract by defining $a - b = a + (-b)$ and Property 6 provides a means to divide by defining $a \div b = a \cdot (1/b)$. The number 0 has no multiplicative inverse, so division by 0 is undefined.

Inequalities and Intervals

We say that a is less than b (written $a < b$) if the point representing a is to the left of the point representing b on the real number line. For example, $4 < 7$ because 4 is to the left of 7 on the real number line. We may also say that 7 is greater than 4 (written $7 > 4$). We may indicate that the number x is less than or equal to another number y by writing $x \leq y$. We may also indicate that p is greater than or equal to 4 by writing $p \geq 4$.

● EXAMPLE 1 *Inequalities*

Use $<$ or $>$ notation to write

(a) 6 is greater than 5.
(b) 10 is less than 15.
(c) 3 is to the left of 8 on the real number line.
(d) x is at most 12.

Solution

(a) $6 > 5$ (b) $10 < 15$ (c) $3 < 8$
(d) "x is at most 12" means it must be less than or equal to 12. Thus, $x \leq 12$.

The subset of the real numbers consisting of all real numbers x that lie between a and b, excluding a and b, can be denoted by the *double inequality* $a < x < b$ or by the **open interval** (a, b). It is called an open interval because neither of the endpoints is included in the interval. The **closed interval** $[a, b]$ represents the set of all real numbers x satisfying $a \leq x \leq b$. Intervals containing one endpoint, such as $(a, b]$ and $[a, b)$, are called **half-open intervals.**

We can use $[a, +\infty)$ to represent the inequality $x \geq a$ and $(-\infty, a)$ to represent $x < a$. In each of these cases, the symbols $+\infty$ and $-\infty$ are not real numbers but represent the fact that x increases without bound ($+\infty$) or decreases without bound ($-\infty$). Table 0.2 summarizes three types of intervals.

TABLE 0.2 Intervals

Type of Interval	Inequality Notation	Interval Notation	Graph
Open interval	$x > a$	(a, ∞)	
	$x < b$	$(-\infty, b)$	
	$a < x < b$	(a, b)	
Half-open interval	$x \geq a$	$[a, \infty)$	
	$x \leq b$	$(-\infty, b]$	
	$a \leq x < b$	$[a, b)$	
	$a < x \leq b$	$(a, b]$	
Closed interval	$a \leq x \leq b$	$[a, b]$	

● ***Checkpoint***

1. Evaluate the following, if possible. For any that are meaningless, so state.
   (a) $\frac{4}{0}$ (b) $\frac{0}{4}$ (c) $\frac{4}{4}$ (d) $\frac{4-4}{4-4}$
2. For (a)–(d), write the inequality corresponding to the given interval and sketch its graph on a real number line.
   (a) $(1, 3)$ (b) $(0, 3]$ (c) $[-1, \infty)$ (d) $(-\infty, 2)$
3. Express each of the following inequalities in interval notation and name the type of interval.
   (a) $3 \leq x \leq 6$ (b) $-6 \leq x < 4$

Absolute Value

Sometimes we are interested in the *distance* a number is from the origin (0) of the real number line, without regard to direction. The distance a number a is from 0 on the number line is the **absolute value** of a, denoted $|a|$. The absolute value of any nonzero number is positive, and the absolute value of 0 is 0.

● EXAMPLE 2 *Absolute Value*

Evaluate the following.

(a) $|-4|$ (b) $|+2|$ (c) $|0|$ (d) $|-5 - |-3||$

Solution

(a) $|-4| = +4 = 4$ (b) $|+2| = +2 = 2$
(c) $|0| = 0$ (d) $|-5 - |-3|| = |-5 - 3| = |-8| = 8$

Note that if a is a nonnegative number, then $|a| = a$, *but if a is negative, then $|a|$ is the positive number* $(-a)$. Thus

Absolute Value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

In performing computations with real numbers, it is important to remember the rules for computations.

Operations with Real (Signed) Numbers

Procedure	Example
1. (a) To add two real numbers with the same sign, add their absolute values and affix their common sign.	1. (a) $(+5) + (+6) = +11$ $\left(-\frac{1}{6}\right) + \left(-\frac{2}{6}\right) = -\frac{3}{6} = -\frac{1}{2}$
(b) To add two real numbers with unlike signs, find the difference of their absolute values and affix the sign of the number with the larger absolute value.	(b) $(-4) + (+3) = -1$ $(+5) + (-3) = +2$ $\left(-\frac{11}{7}\right) + (+1) = -\frac{4}{7}$
2. To subtract one real number from another, change the sign of the number being subtracted and proceed as in addition.	2. $(-9) - (-8) = (-9) + (+8) = -1$ $16 - (8) = 16 + (-8) = +8$
3. (a) The product of two real numbers with like signs is positive.	3. (a) $(-3)(-4) = +12$ $\left(+\frac{3}{4}\right)(+4) = +3$
(b) The product of two real numbers with unlike signs is negative.	(b) $5(-3) = -15$ $(-3)(+4) = -12$
4. (a) The quotient of two real numbers with like signs is positive.	4. (a) $(-14) \div (-2) = +7$ $+36/4 = +9$
(b) The quotient of two real numbers with unlike signs is negative.	(b) $(-28)/4 = -7$ $45 \div (-5) = -9$

When two or more operations with real numbers are indicated in an evaluation, it is important that everyone agree upon the order in which the operations are performed so that a unique result is guaranteed. The following **order of operations** is universally accepted.

Order of Operations

1. Perform operations within parentheses.
2. Find indicated powers ($2^3 = 2 \cdot 2 \cdot 2 = 8$).
3. Perform multiplications and divisions from left to right.
4. Perform additions and subtractions from left to right.

● EXAMPLE 3 *Order of Operations*

Evaluate the following.

(a) $-4 + 3$ (b) $-4^2 + 3$ (c) $(-4 + 3)^2 + 3$ (d) $6 \div 2(2 + 1)$

Solution

(a) -1

(b) Note that with -4^2 the power 2 is applied only to 4, not to -4 (which would be written $(-4)^2$). Thus $-4^2 + 3 = -(4^2) + 3 = -16 + 3 = -13$

(c) $(-1)^2 + 3 = 1 + 3 = 4$ (d) $6 \div 2(3) = (6 \div 2)(3) = 3 \cdot 3 = 9$

● Checkpoint

True or false:

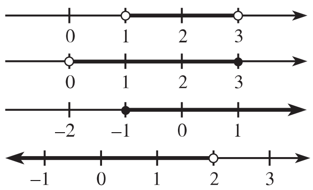
4. $-(-5)^2 = 25$
5. $|4 - 6| = |4| - |6|$
6. $9 - 2(2)(-10) = 7(2)(-10) = -140$

The text assumes that you have a scientific or graphing calculator. Discussions of some of the capabilities of graphing calculators and graphing utilities will be found throughout the text.

Most scientific and graphing calculators use standard algebraic order when evaluating arithmetic expressions. Working outward from inner parentheses, calculations are performed from left to right. Powers and roots are evaluated first, followed by multiplications and divisions and then additions and subtractions.

● Checkpoint Solutions

1. (a) Meaningless. A denominator of zero means division by zero, which is undefined.
   (b) $\frac{0}{4} = 0$. A numerator of zero (when the denominator is not zero) means the fraction has value 0.
   (c) $\dfrac{4}{4} = 1$
   (d) Meaningless. The denominator is zero.
2. (a) $1 < x < 3$
   (b) $0 < x \le 3$
   (c) $-1 \le x < \infty$ or $x \ge -1$
   (d) $-\infty < x < 2$ or $x < 2$



3. (a) $[3, 6]$; closed interval
   (b) $[-6, 4)$; half-open interval
4. False. $-(-5)^2 = (-1)(-5)^2 = (-1)(25) = -25$. Exponentiation has priority and applies only to -5.
5. False. $|4 - 6| = |-2| = 2$ and $|4| - |6| = 4 - 6 = -2$.
6. False. Without parentheses, multiplication has priority over subtraction.
   $9 - 2(2)(-10) = 9 - 4(-10) = 9 + 40 = 49$.

0.2 Exercises

In Problems 1–2, indicate whether the given expression is one or more of the following types of numbers: rational, irrational, integer, natural. If the expression is meaningless, so state.

1. (a) $\dfrac{-\pi}{10}$ (b) -9 (c) $\dfrac{9}{3}$ (d) $\dfrac{4}{0}$
2. (a) $\dfrac{0}{6}$ (b) -1.2916 (c) 1.414 (d) $\dfrac{9}{6}$

Which property of real numbers is illustrated in each part of Problems 3–6?

3. (a) $8 + 6 = 6 + 8$ (b) $5(3 + 7) = 5(3) + 5(7)$
4. (a) $6(4 \cdot 5) = (6 \cdot 4)(5)$ (b) $-15 + 0 = -15$
5. (a) $-e \cdot 1 = -e$ (b) $4 + (-4) = 0$
6. (a) $\left(\dfrac{3}{2}\right)\left(\dfrac{2}{3}\right) = 1$ (b) $(12)\left(\dfrac{3}{4}\right) = \left(\dfrac{3}{4}\right)(12)$

Insert the proper sign $<$, $=$, or $>$ to replace $\square$ in Problems 7–12.

7. $-14 \square -3$
8. $\pi \square 3.14$
9. $0.333 \square \dfrac{1}{3}$
10. $\dfrac{1}{3} + \dfrac{1}{2} \square \dfrac{5}{6}$
11. $|-3| + |5| \square |-3 + 5|$
12. $|-9 - 3| \square |-9| + |3|$

In Problems 13–24, evaluate each expression.

13. $-3^2 + 10 \cdot 2$
14. $(-3)^2 + 10 \cdot 2$
15. $\dfrac{4 + 2^2}{2}$
16. $\dfrac{(4 + 2)^2}{2}$
17. $\dfrac{16 - (-4)}{8 - (-2)}$
18. $\dfrac{(-5)(-3) - (-2)(3)}{-9 + 2}$
19. $\dfrac{|5 - 2| - |-7|}{|5 - 2|}$
20. $\dfrac{|3 - |4 - 11||}{-|5^2 - 3^2|}$

21. $\dfrac{(-3)^2 - 2 \cdot 3 + 6}{4 - 2^2 + 3}$ 22. $\dfrac{6^2 - 4(-3)(-2)}{6 - 6^2 \div 4}$

23. $\dfrac{-4^2 + 5 - 2 \cdot 3}{5 - 4^2}$ 24. $\dfrac{3 - 2(5 - 2)}{(-2)^2 - 2^2 + 3}$

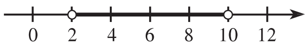
25. What part of the real number line corresponds to the interval $(-\infty, \infty)$?

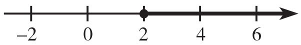
26. Write the interval corresponding to $x \geq 0$.

In Problems 27–30, express each inequality or graph using interval notation, and name the type of interval.

27. $1 < x \leq 3$

28. $-4 \leq x \leq 3$

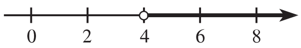
29. 

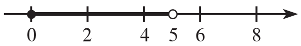
30. 

In Problems 31–34, write an inequality that describes each interval or graph.

31. $[-3, 5)$

32. $(-2, \infty)$

33. 

34. 

In Problems 35–42, graph the subset of the real numbers that is represented by each of the following and write your answer in interval notation.

35. $(-\infty, 4) \cap (-3, \infty)$ 36. $[-4, 17) \cap [-20, 10]$

37. $x > 4$ and $x \geq 0$ 38. $x < 10$ and $x < -1$

39. $[0, \infty) \cup [-1, 5]$ 40. $(-\infty, 4) \cup (0, 2)$

41. $x > 7$ or $x < 0$ 42. $x > 4$ and $x < 0$

In Problems 43–48, use your calculator to evaluate each of the following. List all the digits on your display in the answer.

43. $\dfrac{-1}{25916.8}$ 44. $\dfrac{51.412}{127.01}$

45. $(3.679)^7$ 46. $(1.28)^{10}$

47. $\dfrac{2500}{(1.1)^6 - 1}$ 48. $100\left[\dfrac{(1.05)^{12} - 1}{0.05}\right]$

APPLICATIONS

49. ***Take-home pay*** A sales representative's take-home pay is found by subtracting all taxes and retirement contributions from gross pay (which consists of salary plus commission). Given the following information, complete (a)–(c).

Salary = \$300.00 Commission = \$788.91

Retirement = 5% of gross pay

Taxes: State = 5% of gross pay,
Local = 1% of gross pay
Federal withholding = 25% of (gross pay less retirement)
Federal social security and Medicare = 7.65% of gross pay

(a) Find the gross pay.
(b) Find the amount of federal withholding.
(c) Find the take-home pay.

50. ***Public health expenditures*** The expenditures E for government public health activities (in billions of dollars) can be approximated by

$$E = 0.04t^2 - 0.56t + 1.29$$

where t is the number of years past 1960. (*Source:* Centers for Medicare and Medicaid Services)
(a) What t-value represents the year 1995?
(b) Actual expenditures for 1995 were \$31.0 billion. What does the formula give as the 1995 approximation?
(c) Predict the expenditures for 2012.

51. ***Health insurance coverage*** The percentage P of the U.S. population with no health insurance can be approximated quite accurately either by
(1) $P = 0.3179t + 13.85$ or by
(2) $P = 0.0194t^3 - 0.1952t^2 + 0.8282t + 13.63$
where t is the number of years past 2000. (*Source:* U.S. Census Bureau)
(a) Both (1) and (2) closely approximate the data, but which is more accurate for 2006, when 15.8% of the population had no health insurance?
(b) Use both formulas to predict the percentage of the U.S. population not covered in 2012. Which equation's result seems more accurate?

52. ***Health statistics*** From data adapted from the National Center for Health Statistics, the height H in inches and age A in years for boys between 4 and 16 years of age are related according to

$$H = 2.31A + 31.26$$

To account for normal variability among boys, normal height for a given age is ±5% of the height obtained from the equation.
(a) Find the normal height range for a boy who is 10.5 years old, and write it as an inequality.
(b) Find the normal height range for a boy who is 5.75 years old, and write it as an inequality.