(a) Factor this in order to find an expression for the number of units demanded.
(b) Use (a) to find the number of units demanded when the market price is \$38.

64. ***Power in a circuit*** Factor the following expression for the maximum power in a certain electrical circuit.

$$(R + r)^2 - 2r(R + r)$$

65. ***Revenue*** Revenue R from the sale of x units of a product is found by multiplying the price by the number of items sold.
(a) Factor the right side of $R = 300x - x^2$.
(b) What is the expression for the price of the item?

66. ***Poiseuille's law*** The expression for the speed of blood through an artery of radius r at a distance x from the artery wall is given by $r^2 - (r - x)^2$. Factor and simplify this expression.

0.7 Algebraic Fractions

Evaluating certain limits and graphing rational functions require an understanding of algebraic fractions. The fraction $6/8$ can be reduced to $3/4$ by dividing both the numerator and the denominator by 2. In the same manner, the algebraic fraction

$$\frac{(x + 2)(x + 1)}{(x + 1)(x + 3)}$$

can be reduced to

$$\frac{x + 2}{x + 3}$$

by dividing both the numerator and the denominator by $x + 1$, if $x \neq -1$.

Simplifying Fractions

We *simplify* algebraic fractions by factoring the numerator and denominator and then dividing both the numerator and the denominator by any common factors.*

● EXAMPLE 1 *Simplifying a Fraction*

Simplify $\dfrac{3x^2 - 14x + 8}{x^2 - 16}$ if $x^2 \neq 16$.

Solution

$$\begin{aligned}\frac{3x^2 - 14x + 8}{x^2 - 16} &= \frac{(3x - 2)(x - 4)}{(x - 4)(x + 4)}\\ &= \frac{(3x - 2)\overset{1}{\cancel{(x - 4)}}}{\underset{1}{\cancel{(x - 4)}}(x + 4)}\\ &= \frac{3x - 2}{x + 4}\end{aligned}$$

*We assume that all fractions are defined.

Products of Fractions

We can multiply fractions by writing the product as the product of the numerators divided by the product of the denominators. For example,

$$\frac{4}{5}\cdot\frac{10}{12}\cdot\frac{2}{5}=\frac{80}{300}$$

which reduces to $\frac{4}{15}$.

We can also find the product by reducing the fractions before we indicate the multiplication in the numerator and denominator. For example, in

$$\frac{4}{5}\cdot\frac{10}{12}\cdot\frac{2}{5}$$

we can divide the first numerator and the second denominator by 4 and divide the second numerator and the first denominator by 5, which yields

$$\frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{5}}}\cdot\frac{\overset{2}{\cancel{10}}}{\underset{3}{\cancel{12}}}\cdot\frac{2}{5}=\frac{1}{1}\cdot\frac{2}{3}\cdot\frac{2}{5}=\frac{4}{15}$$

Product of Fractions

We *multiply* algebraic fractions by writing the product of the numerators divided by the product of the denominators, and then reduce to lowest terms. We may also reduce prior to finding the product.

● EXAMPLE 2 *Multiplying Fractions*

Multiply:

(a) $\frac{4x^2}{5y}\cdot\frac{10x}{y^2}\cdot\frac{y}{8x^2}$ (b) $\frac{-4x+8}{3x+6}\cdot\frac{2x+4}{4x+12}$

Solution

(a) $\frac{4x^2}{5y}\cdot\frac{10x}{y^2}\cdot\frac{y}{8x^2}=\frac{\overset{1}{\cancel{4x^2}}}{\underset{1\cdot 1}{\cancel{5y}}}\cdot\frac{\overset{2}{\cancel{10}}x}{y^2}\cdot\frac{\overset{1}{\cancel{y}}}{\underset{2}{\cancel{8x^2}}}=\frac{1}{1}\cdot\frac{\overset{1}{\cancel{2}}x}{y^2}\cdot\frac{1}{\underset{1}{\cancel{2}}}=\frac{x}{y^2}$

(b) $$\frac{-4x+8}{3x+6}\cdot\frac{2x+4}{4x+12}=\frac{-4(x-2)}{3(x+2)}\cdot\frac{2(x+2)}{4(x+3)}$$

$$=\frac{-\overset{-1}{\cancel{4}}(x-2)}{3\underset{1}{\cancel{(x+2)}}}\cdot\frac{2\overset{1}{\cancel{(x+2)}}}{\underset{1}{\cancel{4}}(x+3)}$$

$$=\frac{-2(x-2)}{3(x+3)}$$

Quotients of Fractions

In arithmetic we learned to divide one fraction by another by inverting the divisor and multiplying. The same rule applies to division of algebraic fractions.

● EXAMPLE 3 *Dividing Fractions*

(a) Divide $\frac{a^2b}{c}$ by $\frac{ab}{c^2}$. (b) Find $\frac{6x^2-6}{x^2+3x+2}\div\frac{x-1}{x^2+4x+4}$.

Solution

(a) $\dfrac{a^2b}{c} \div \dfrac{ab}{c^2} = \dfrac{a^2b}{c} \cdot \dfrac{c^2}{ab} = \dfrac{\overset{a \cdot 1}{\cancel{a^2b}}}{\underset{1}{\cancel{c}}} \cdot \dfrac{\overset{c}{\cancel{c^2}}}{\underset{1 \cdot 1}{\cancel{ab}}} = \dfrac{ac}{1} = ac$

(b) $$\begin{aligned} \frac{6x^2 - 6}{x^2 + 3x + 2} \div \frac{x - 1}{x^2 + 4x + 4} &= \frac{6x^2 - 6}{x^2 + 3x + 2} \cdot \frac{x^2 + 4x + 4}{x - 1} \\ &= \frac{6(x - 1)(x + 1)}{(x + 2)(x + 1)} \cdot \frac{(x + 2)(x + 2)}{x - 1} \\ &= 6(x + 2) \end{aligned}$$

● ***Checkpoint***

1. Reduce: $\dfrac{2x^2 - 4x}{2x}$
2. Multiply: $\dfrac{x^2}{x^2 - 9} \cdot \dfrac{x + 3}{3x}$
3. Divide: $\dfrac{5x^2(x - 1)}{2(x + 1)} \div \dfrac{10x^2}{(x + 1)(x - 1)}$

Adding and Subtracting Fractions

If two fractions are to be added, it is convenient that both be expressed with the same denominator. If the denominators are not the same, we can write the equivalents of each of the fractions with a common denominator. We usually use the least common denominator (LCD) when we write the equivalent fractions. The **least common denominator** is the lowest-degree variable expression into which all denominators will divide. If the denominators are polynomials, then the LCD is the lowest-degree polynomial into which all denominators will divide. We can find the least common denominator as follows.

Finding the Least Common Denominator

Procedure	Example
To find the least common denominator of a set of fractions:	Find the LCD of $\dfrac{1}{x^2 - x}, \dfrac{1}{x^2 - 1}, \dfrac{1}{x^2}$.
1. Completely factor each denominator.	1. The factored denominators are $x(x - 1)$, $(x + 1)(x - 1)$, and $x \cdot x$.
2. Identify the different factors that appear.	2. The different factors are x, $x - 1$, and $x + 1$.
3. The LCD is the product of these different factors, with each factor used the maximum number of times it occurs in any one denominator.	3. x occurs a maximum of twice in one denominator, $x - 1$ occurs once, and $x + 1$ occurs once. Thus the LCD is $x \cdot x(x - 1)(x + 1) = x^2(x - 1)(x + 1)$.

The procedure for combining (adding or subtracting) two or more fractions follows.

Adding or Subtracting Fractions

Procedure	Example
To combine fractions:	Combine $\frac{y-3}{y-5} + \frac{y-23}{y^2-y-20}$.
1. Find the LCD of the fractions.	1. $y^2 - y - 20 = (y-5)(y+4)$, so the LCD is $(y-5)(y+4)$.
2. Write the equivalent of each fraction with the LCD as its denominator.	2. The sum is $\frac{(y-3)(y+4)}{(y-5)(y+4)} + \frac{y-23}{(y-5)(y+4)}$.
3. Add or subtract, as indicated, by combining like terms in the numerator over the LCD.	3. $= \frac{y^2+y-12+y-23}{(y-5)(y+4)}$ $= \frac{y^2+2y-35}{(y-5)(y+4)}$
4. Reduce the fraction, if possible.	4. $= \frac{(y+7)(y-5)}{(y-5)(y+4)} = \frac{y+7}{y+4}$, if $y \neq 5$.

● EXAMPLE 4 *Adding Fractions*

Add $\frac{3x}{a^2} + \frac{4}{ax}$.

Solution

1. The LCD is a^2x.
2. $\frac{3x}{a^2} + \frac{4}{ax} = \frac{3x}{a^2} \cdot \frac{x}{x} + \frac{4}{ax} \cdot \frac{a}{a} = \frac{3x^2}{a^2x} + \frac{4a}{a^2x}$
3. $\frac{3x^2}{a^2x} + \frac{4a}{a^2x} = \frac{3x^2+4a}{a^2x}$
4. The sum is in lowest terms.

● EXAMPLE 5 *Combining Fractions*

Combine $\frac{y-3}{(y-5)^2} - \frac{y-2}{y^2-4y-5}$.

Solution

$y^2 - 4y - 5 = (y-5)(y+1)$, so the LCD is $(y-5)^2(y+1)$. Writing the equivalent fractions and then combining them, we get

$$\frac{y-3}{(y-5)^2} - \frac{y-2}{(y-5)(y+1)} = \frac{(y-3)(y+1)}{(y-5)^2(y+1)} - \frac{(y-2)(y-5)}{(y-5)(y+1)(y-5)}$$

$$= \frac{(y^2-2y-3)-(y^2-7y+10)}{(y-5)^2(y+1)}$$

$$= \frac{y^2-2y-3-y^2+7y-10}{(y-5)^2(y+1)}$$

$$= \frac{5y-13}{(y-5)^2(y+1)}$$

Complex Fractions

A fractional expression that contains one or more fractions in its numerator or denominator is called a **complex fraction.** An example of a complex fraction is

$$\frac{\frac{1}{3}+\frac{4}{x}}{3-\frac{1}{xy}}$$

We can simplify fractions of this type using the property $\frac{a}{b}=\frac{ac}{bc}$, with c equal to the LCD of *all* the fractions contained in the numerator and denominator of the complex fraction.

For example, all fractions contained in

$$\frac{\frac{1}{3}+\frac{4}{x}}{3-\frac{1}{xy}}$$

have LCD $3xy$. We simplify this complex fraction by multiplying the numerator and denominator as follows:

$$\frac{3xy\left(\frac{1}{3}+\frac{4}{x}\right)}{3xy\left(3-\frac{1}{xy}\right)}=\frac{3xy\left(\frac{1}{3}\right)+3xy\left(\frac{4}{x}\right)}{3xy(3)-3xy\left(\frac{1}{xy}\right)}=\frac{xy+12y}{9xy-3}$$

● EXAMPLE 6 *Complex Fractions*

Simplify $\frac{x^{-3}+x^2y^{-3}}{(xy)^{-2}}$ so that only positive exponents remain.

Solution

$$\frac{x^{-3}+x^2y^{-3}}{(xy)^{-2}}=\frac{\frac{1}{x^3}+\frac{x^2}{y^3}}{\frac{1}{(xy)^2}}, \quad \text{LCD } = x^3y^3$$

$$=\frac{x^3y^3\left(\frac{1}{x^3}+\frac{x^2}{y^3}\right)}{x^3y^3\left(\frac{1}{x^2y^2}\right)}=\frac{y^3+x^5}{xy}$$

● ***Checkpoint***

4. Add or subtract:

   (a) $\frac{5x-1}{2x-5}-\frac{x+9}{2x-5}$

   (b) $\frac{x+1}{x}+\frac{x}{x-1}$

5. Simplify $\frac{\frac{y}{x}-1}{\frac{y}{x}-\frac{x}{y}}$.

Rationalizing Denominators

We can simplify algebraic fractions whose denominators contain sums and differences that involve square roots by rationalizing the denominators. Using the fact that $(x + y)(x - y) = x^2 - y^2$, we multiply the numerator and denominator of an algebraic fraction of this type by the conjugate of the denominator to simplify the fraction.

EXAMPLE 7 *Rationalizing Denominators*

Rationalize the denominators.

(a) $\dfrac{1}{\sqrt{x} - 2}$ (b) $\dfrac{3 + \sqrt{x}}{\sqrt{x} + \sqrt{5}}$

Solution

Multiplying $\sqrt{x} - 2$ by $\sqrt{x} + 2$, its conjugate, gives the difference of two squares and removes the radical from the denominator in (a). We also use the conjugate in (b).

(a) $\dfrac{1}{\sqrt{x} - 2} \cdot \dfrac{\sqrt{x} + 2}{\sqrt{x} + 2} = \dfrac{\sqrt{x} + 2}{(\sqrt{x})^2 - (2)^2} = \dfrac{\sqrt{x} + 2}{x - 4}$

(b) $\dfrac{3 + \sqrt{x}}{\sqrt{x} + \sqrt{5}} \cdot \dfrac{\sqrt{x} - \sqrt{5}}{\sqrt{x} - \sqrt{5}} = \dfrac{3\sqrt{x} - 3\sqrt{5} + x - \sqrt{5x}}{x - 5}$

● Checkpoint

6. Rationalize the denominator $\dfrac{\sqrt{x}}{\sqrt{x} - 3}$.

● Checkpoint Solutions

1. $\dfrac{2x^2 - 4x}{2x} = \dfrac{2x(x - 2)}{2x} = x - 2$

2. $\dfrac{x^2}{x^2 - 9} \cdot \dfrac{x + 3}{3x} = \dfrac{x^2 \cdot (x + 3)}{(x + 3)(x - 3) \cdot 3x} = \dfrac{x}{3(x - 3)} = \dfrac{x}{3x - 9}$

3. $\dfrac{5x^2(x - 1)}{2(x + 1)} \div \dfrac{10x^2}{(x + 1)(x - 1)} = \dfrac{5x^2(x - 1)}{2(x + 1)} \cdot \dfrac{(x + 1)(x - 1)}{10x^2}$

$= \dfrac{(x - 1)^2}{4}$

4. (a) $\dfrac{5x - 1}{2x - 5} - \dfrac{x + 9}{2x - 5} = \dfrac{(5x - 1) - (x + 9)}{2x - 5}$

$= \dfrac{5x - 1 - x - 9}{2x - 5}$

$= \dfrac{4x - 10}{2x - 5} = \dfrac{2(2x - 5)}{2x - 5} = 2$

(b) $\dfrac{x + 1}{x} + \dfrac{x}{x - 1}$ has LCD $= x(x - 1)$

$\dfrac{x + 1}{x} + \dfrac{x}{x - 1} = \dfrac{x + 1}{x} \cdot \dfrac{(x - 1)}{(x - 1)} + \dfrac{x}{x - 1} \cdot \dfrac{x}{x}$

$= \dfrac{x^2 - 1}{x(x - 1)} + \dfrac{x^2}{x(x - 1)} = \dfrac{x^2 - 1 + x^2}{x(x - 1)}$

$= \dfrac{2x^2 - 1}{x(x - 1)}$

$$5.\ \frac{\frac{y}{x}-1}{\frac{y}{x}-\frac{x}{y}}=\frac{\left(\frac{y}{x}-1\right)}{\left(\frac{y}{x}-\frac{x}{y}\right)}\cdot\frac{xy}{xy}=\frac{\frac{y}{x}\cdot xy-1\cdot xy}{\frac{y}{x}\cdot xy-\frac{x}{y}\cdot xy}$$

$$=\frac{y^2-xy}{y^2-x^2}=\frac{y(y-x)}{(y+x)(y-x)}=\frac{y}{y+x}$$

$$6.\ \frac{\sqrt{x}}{\sqrt{x}-3}=\frac{\sqrt{x}}{\sqrt{x}-3}\cdot\frac{\sqrt{x}+3}{\sqrt{x}+3}=\frac{x+3\sqrt{x}}{x-9}$$

0.7 Exercises

Simplify the following fractions.

1. $\frac{18x^3y^3}{9x^3z}$
2. $\frac{15a^4b^5}{30a^3b}$
3. $\frac{x-3y}{3x-9y}$
4. $\frac{x^2-6x+8}{x^2-16}$
5. $\frac{x^2-2x+1}{x^2-4x+3}$
6. $\frac{x^2-5x+6}{9-x^2}$
7. $\frac{6x^3y^3-15x^2y}{3x^2y^2+9x^2y}$
8. $\frac{x^2y^2-4x^3y}{x^2y-2x^2y^2}$

In Problems 9–36, perform the indicated operations and simplify.

9. $\frac{6x^3}{8y^3}\cdot\frac{16x}{9y^2}\cdot\frac{15y^4}{x^3}$
10. $\frac{25ac^2}{15a^2c}\cdot\frac{4ad^4}{15abc^3}$
11. $\frac{8x-16}{x-3}\cdot\frac{4x-12}{3x-6}$
12. $(x^2-4)\cdot\frac{2x-3}{x+2}$
13. $\frac{x^2+7x+12}{3x^2+13x+4}\cdot(9x+3)$
14. $\frac{4x+4}{x-4}\cdot\frac{x^2-6x+8}{8x^2+8x}$
15. $\frac{x^2-x-2}{2x^2-8}\cdot\frac{18-2x^2}{x^2-5x+4}\cdot\frac{x^2-2x-8}{x^2-6x+9}$
16. $\frac{x^2-5x-6}{x^2-5x+4}\cdot\frac{x^2-x-12}{x^3-6x^2}\cdot\frac{x-x^3}{x^2-2x+1}$
17. $\frac{15ac^2}{7bd}\div\frac{4a}{14b^2d}$
18. $\frac{16}{x-2}\div\frac{4}{3x-6}$
19. $\frac{y^2-2y+1}{7y^2-7y}\div\frac{y^2-4y+3}{35y^2}$
20. $\frac{6x^2}{4x^2y-12xy}\div\frac{3x^2+12x}{x^2+x-12}$
21. $(x^2-x-6)\div\frac{9-x^2}{x^2-3x}$
22. $\frac{2x^2+7x+3}{4x^2-1}\div(x+3)$
23. $\frac{2x}{x^2-x-2}-\frac{x+2}{x^2-x-2}$
24. $\frac{4}{9-x^2}-\frac{x+1}{9-x^2}$
25. $\frac{a}{a-2}-\frac{a-2}{a}$
26. $x-\frac{2}{x-1}$
27. $\frac{x}{x+1}-x+1$
28. $\frac{x-1}{x+1}-\frac{2}{x^2+x}$
29. $\frac{4a}{3x+6}+\frac{5a^2}{4x+8}$
30. $\frac{b-1}{b^2+2b}+\frac{b}{3b+6}$
31. $\frac{3x-1}{2x-4}+\frac{4x}{3x-6}-\frac{x-4}{5x-10}$
32. $\frac{2x+1}{4x-2}+\frac{5}{2x}-\frac{x+4}{2x^2-x}$
33. $\frac{x}{x^2-4}+\frac{4}{x^2-x-2}-\frac{x-2}{x^2+3x+2}$
34. $\frac{3x^2}{x^2-4}+\frac{2}{x^2-4x+4}-3$
35. $\frac{-x^3+x}{\sqrt{3-x^2}}+2x\sqrt{3-x^2}$
36. $\frac{3x^2(x+1)}{\sqrt{x^3+1}}+\sqrt{x^3+1}$

In Problems 37–46, simplify each complex fraction.

37. $\frac{3-\frac{2}{3}}{14}$
38. $\frac{4}{\frac{1}{4}+\frac{1}{4}}$
39. $\frac{x+y}{\frac{1}{x}+\frac{1}{y}}$
40. $\frac{\frac{5}{2y}+\frac{3}{y}}{\frac{1}{4}+\frac{1}{3y}}$
41. $\frac{2-\frac{1}{x}}{2x-\frac{3x}{x+1}}$
42. $\frac{1-\frac{2}{x-2}}{x-6+\frac{10}{x+1}}$
43. $\frac{\sqrt{a}-\frac{b}{\sqrt{a}}}{a-b}$
44. $\frac{\sqrt{x-1}+\frac{1}{\sqrt{x-1}}}{x}$

45. $$\frac{\sqrt{x^2+9} - \frac{13}{\sqrt{x^2+9}}}{x^2 - x - 6}$$

46. $$\frac{\sqrt{x^2+3} - \frac{x+5}{\sqrt{x^2+3}}}{x^2 + 5x + 4}$$

In Problems 47–50, rewrite each of the following so that only positive exponents remain, and simplify.

47. (a) $(2^{-2} - 3^{-1})^{-1}$ (b) $(2^{-1} + 3^{-1})^2$

48. (a) $(3^2 + 4^2)^{-1/2}$ (b) $(2^2 + 3^2)^{-1}$

49. $\dfrac{2a^{-1} - b^{-2}}{(ab^2)^{-1}}$ 50. $\dfrac{x^{-2} + xy^{-2}}{(x^2y)^{-2}}$

In Problems 51 and 52, rationalize the denominator of each fraction and simplify.

51. $\dfrac{1 - \sqrt{x}}{1 + \sqrt{x}}$ 52. $\dfrac{x - 3}{x - \sqrt{3}}$

In Problems 53 and 54, rationalize the numerator of each fraction and simplify.

53. $\dfrac{\sqrt{x+h} - \sqrt{x}}{h}$ 54. $\dfrac{\sqrt{9 + 2h} - 3}{h}$

APPLICATIONS

55. ***Time study*** Workers A, B, and C can complete a job in a, b, and c hours, respectively. Working together, they can complete

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$$

of the job in 1 hour. Add these fractions over a common denominator to obtain an expression for what they can do in 1 hour, working together.

56. ***Focal length*** Two thin lenses with focal lengths p and q and separated by a distance d have their combined focal length given by the reciprocal of

$$\frac{1}{p} + \frac{1}{q} - \frac{d}{pq}$$

(a) Combine these fractions.
(b) Use the reciprocal of your answer in (a) to find the combined focal length.

Average cost **A company's average cost per unit when x units are produced is defined to be**

$$\text{Average cost} = \frac{\text{Total cost}}{x}$$

Use this equation in Problems 57 and 58.

57. Suppose a company's average costs are given by

$$\text{Average cost} = \frac{4000}{x} + 55 + 0.1x$$

(a) Express the average-cost formula as a single fraction.
(b) Write the expression that gives the company's total costs.

58. Suppose a company's average costs are given by

$$\text{Average cost} = \frac{40{,}500}{x} + 190 + 0.2x$$

(a) Express the average-cost formula as a single fraction.
(b) Write the expression that gives the company's total costs.

59. ***Advertising and sales*** Suppose that a company's daily sales volume attributed to an advertising campaign is given by

$$\text{Sales volume} = 1 + \frac{3}{t+3} - \frac{18}{(t+3)^2}$$

where t is the number of days since the campaign started. Express the sales volume as a single fraction.

60. ***Annuity*** The formula for the future value of an annuity due involves the expression

$$\frac{(1+i)^{n+1} - 1}{i} - 1$$

Write this expression over a common denominator and factor the numerator to simplify.

Key Terms and Formulas

Section	Key Terms	Formulas
0.1	Sets and set membership	
	Natural numbers	$N = \{1, 2, 3, 4 \dots\}$
	Empty set	$\varnothing$
	Set equality	
	Subset	$A \subseteq B$

(continued)

Section	Key Terms	Formulas
	Universal set	U
	Venn diagrams	
	Set intersection	$A \cap B$
	Disjoint sets	$A \cap B = \varnothing$
	Set union	$A \cup B$
	Set complement	A'
0.2	Real numbers	
	Subsets and properties	
	Real number line	
	Inequalities	
	Intervals and interval notation	
	Closed interval	$a \le x \le b$ or $[a, b]$
	Open interval	$a < x < b$ or (a, b)
	Absolute value	
	Order of operations	
0.3	Exponent and base	a^n has base a, exponent n
	Zero exponent	$a^0 = 1, a \neq 0$
	Negative exponent	$a^{-n} = \dfrac{1}{a^n}$
	Rules of exponents	
0.4	Radical	$\sqrt[n]{a}$
	Radicand, index	radicand $= a$; index $= n$
	Principal nth root	$\sqrt[n]{a} = b$ only if $b^n = a$ and $a \ge 0$ and $b \ge 0$ when n is even
	Fractional exponents	$a^{1/n} = \sqrt[n]{a}$
		$a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
	Properties of radicals	
	Rationalizing the denominator	
0.5	Algebraic expression	
	Variable	
	Constant term	
	Coefficient; leading coefficient	
	Term	
	Polynomial	$a_nx^n + \cdots + a_1x + a_0$
	Degree	
	Monomial	
	Binomial	
	Trinomial	
	Like terms	
	Distributive Law	$a(b + c) = ab + ac$
	Binomial products	
	Division of polynomials	

Section	Key Terms	Formulas
0.6	Factor	
	Common factor	
	Factoring by grouping	
	Special factorizations	
	Difference of squares	$a^2 - b^2 = (a + b)(a - b)$
	Perfect squares	$a^2 + 2ab + b^2 = (a + b)^2$
		$a^2 - 2ab + b^2 = (a - b)^2$
	Conjugates	$a + b; a - b$
	Quadratic polynomials	$ax^2 + bx + c$
	Factoring completely	
0.7	Algebraic fractions	
	Numerator	
	Denominator	
	Reduce	
	Product of fractions	
	Quotient of fractions	
	Common denominator	
	Least common denominator (LCD)	
	Addition and subtraction of fractions	
	Complex fraction	
	Rationalize the denominator	

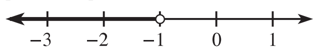
Review Exercises

1. Is $A \subseteq B$, if $A = \{1, 2, 5, 7\}$ and $B = \{x: x \text{ is a positive integer}, x \le 8\}$?
2. Is it true that $3 \in \{x: x > 3\}$?
3. Are $A = \{1, 2, 3, 4\}$ and $B = \{x: x \le 1\}$ disjoint?

In Problems 4–7, use sets $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3, 9\}$, and $B = \{1, 3, 5, 6, 7, 8, 10\}$ to find the elements of the sets described.

4. $A \cup B'$
5. $A' \cap B$
6. $(A' \cap B)'$
7. Does $(A' \cup B')' = A \cap B$?
8. State the property of the real numbers that is illustrated in each case.
   (a) $6 + \frac{1}{3} = \frac{1}{3} + 6$ (b) $2(3 \cdot 4) = (2 \cdot 3)4$
   (c) $\frac{1}{3}(6 + 9) = 2 + 3$
9. Indicate whether each given expression is one or more of the following: rational, irrational, integer, natural, or meaningless.
   (a) π (b) $0/6$ (c) $6/0$
10. Insert the proper sign ($<$, $=$, or $>$) to replace each $\square$.
   (a) $\pi \,\square\, 3.14$ (b) $-100 \,\square\, 0.1$ (c) $-3 \,\square\, -12$

For Problems 11–18, evaluate each expression. Use a calculator when necessary.

11. $|5 - 11|$
12. $44 \div 2 \cdot 11 - 10^2$
13. $(-3)^2 - (-1)^3$
14. $\dfrac{(3)(2)(15) - (5)(8)}{(4)(10)}$
15. $2 - [3 - (2 - |-3|)] + 11$
16. $-4^2 - (-4)^2 + 3$
17. $\dfrac{4 + 3^2}{4}$
18. $\dfrac{(-2.91)^5}{\sqrt{3.29^5}}$
19. Write each inequality in interval notation, name the type of interval, and graph it on a real number line.
   (a) $0 \le x \le 5$
   (b) $x \ge -3$ and $x < 7$
   (c) $(-4, \infty) \cap (-\infty, 0)$
20. Write an inequality that represents each of the following.
   (a) $(-1, 16)$
   (b) $[-12, 8]$
   (c) 

21. Evaluate each of the following without a calculator.
(a) $\left(\frac{3}{8}\right)^0$ (b) $2^3 \cdot 2^{-5}$
(c) $\frac{4^9}{4^3}$ (d) $\left(\frac{1}{7}\right)^3\left(\frac{1}{7}\right)^{-4}$

22. Use the rules of exponents to simplify each of the following with positive exponents. Assume all variables are nonzero.
(a) $x^5 \cdot x^{-7}$ (b) x^8/x^{-2} (c) $(x^3)^3$
(d) $(y^4)^{-2}$ (e) $(-y^{-3})^{-2}$

For Problems 23–28, rewrite each expression so that only positive exponents remain. Assume all variables are nonzero.

23. $\frac{-(2xy^2)^{-2}}{(3x^{-2}y^{-3})^2}$ 24. $\left(\frac{2}{3}x^2y^{-4}\right)^{-2}$

25. $\left(\frac{x^{-2}}{2y^{-1}}\right)^2$ 26. $\frac{(-x^4y^{-2}z^2)^0}{-(x^4y^{-2}z^2)^{-2}}$

27. $\left(\frac{x^{-3}y^4z^{-2}}{3x^{-2}y^{-3}z^{-3}}\right)^{-1}$ 28. $\left(\frac{x}{2y}\right)\left(\frac{y}{x^2}\right)^{-2}$

29. Find the following roots.
(a) $-\sqrt[3]{-64}$ (b) $\sqrt{4/49}$ (c) $\sqrt[7]{1.9487171}$

30. Write each of the following with an exponent and with the variable in the numerator.
(a) $\sqrt{x}$ (b) $\sqrt[3]{x^2}$ (c) $1/\sqrt[4]{x}$

31. Write each of the following in radical form.
(a) $x^{2/3}$ (b) $x^{-1/2}$ (c) $-x^{3/2}$

32. Rationalize each of the following denominators and simplify.
(a) $\frac{5xy}{\sqrt{2x}}$ (b) $\frac{y}{x\sqrt[3]{xy^2}}$

In Problems 33–38, use the properties of exponents to simplify so that only positive exponents remain. Assume all variables are positive.

33. $x^{1/2} \cdot x^{1/3}$ 34. $y^{-3/4}/y^{-7/4}$ 35. $x^4 \cdot x^{1/4}$
36. $1/(x^{-4/3} \cdot x^{-7/3})$ 37. $(x^{4/5})^{1/2}$ 38. $(x^{1/2}y^2)^4$

In Problems 39–44, simplify each expression. Assume all variables are positive.

39. $\sqrt{12x^3y^5}$ 40. $\sqrt{1250x^6y^9}$
41. $\sqrt[3]{24x^4y^4} \cdot \sqrt[3]{45x^4y^{10}}$ 42. $\sqrt{16a^2b^3} \cdot \sqrt{8a^3b^5}$
43. $\frac{\sqrt{52x^3y^6}}{\sqrt{13xy^4}}$ 44. $\frac{\sqrt{32x^4y^3}}{\sqrt{6xy^{10}}}$

In Problems 45–62, perform the indicated operations and simplify.

45. $(3x + 5) - (4x + 7)$
46. $x(1 - x) + x[x - (2 + x)]$
47. $(3x^3 - 4xy - 3) + (5xy + x^3 + 4y - 1)$
48. $(4xy^3)(6x^4y^2)$
49. $(3x - 4)(x - 1)$ 50. $(3x - 1)(x + 2)$
51. $(4x + 1)(x - 2)$ 52. $(3x - 7)(2x + 1)$
53. $(2x - 3)^2$ 54. $(4x + 3)(4x - 3)$
55. $(2x^2 + 1)(x^2 + x - 3)$ 56. $(2x - 1)^3$
57. $(x - y)(x^2 + xy + y^2)$ 58. $\frac{4x^2y - 3x^3y^3 - 6x^4y^2}{2x^2y^2}$
59. $(3x^4 + 2x^3 - x + 4) \div (x^2 + 1)$
60. $(x^4 - 4x^3 + 5x^2 + x) \div (x - 3)$
61. $x^{4/3}(x^{2/3} - x^{-1/3})$
62. $(\sqrt{x} + \sqrt{a - x})(\sqrt{x} - \sqrt{a - x})$

In Problems 63–73, factor each expression completely.

63. $2x^4 - x^3$ 64. $4(x^2 + 1)^2 - 2(x^2 + 1)^3$
65. $4x^2 - 4x + 1$ 66. $16 - 9x^2$ 67. $2x^4 - 8x^2$
68. $x^2 - 4x - 21$ 69. $3x^2 - x - 2$
70. $x^2 - 5x + 6$ 71. $x^2 - 10x - 24$
72. $12x^2 - 23x - 24$ 73. $16x^4 - 72x^2 + 81$
74. Factor as indicated: $x^{-2/3} + x^{-4/3} = x^{-4/3}(?)$
75. Reduce each of the following to lowest terms.
(a) $\frac{2x}{2x + 4}$ (b) $\frac{4x^2y^3 - 6x^3y^4}{2x^2y^2 - 3xy^3}$

In Problems 76–82, perform the indicated operations and simplify.

76. $\frac{x^2 - 4x}{x^2 + 4} \cdot \frac{x^4 - 16}{x^4 - 16x^2}$
77. $\frac{x^2 + 6x + 9}{x^2 - 7x + 12} \div \frac{x^2 + 4x + 3}{x^2 - 3x - 4}$
78. $\frac{x^4 - 2x^3}{3x^2 - x - 2} \div \frac{x^3 - 4x}{9x^2 - 4}$ 79. $1 + \frac{3}{2x} - \frac{1}{6x^2}$
80. $\frac{1}{x - 2} - \frac{x - 2}{4}$ 81. $\frac{x + 2}{x^2 - x} - \frac{x^2 + 4}{x^2 - 2x + 1} + 1$
82. $\frac{x - 1}{x^2 - x - 2} - \frac{x}{x^2 - 2x - 3} + \frac{1}{x - 2}$

In Problems 83 and 84, simplify each complex fraction.

83. $\frac{x - 1 - \frac{x - 1}{x}}{\frac{1}{x - 1} + 1}$ 84. $\frac{x^{-2} - x^{-1}}{x^{-2} + x^{-1}}$

85. Rationalize the denominator of $\frac{3x - 3}{\sqrt{x} - 1}$ and simplify.
86. Rationalize the numerator of $\frac{\sqrt{x} - \sqrt{x - 4}}{2}$ and simplify.