

APPENDIX D: SOLAR RADIATION

The sun is the source of most energy on the earth and is a primary factor in determining the thermal environment of a locality. It is important for engineers to have a working knowledge of the earth's relationship to the sun. They should be able to make estimates of solar radiation intensity and know how to make simple solar radiation measurements. They should also understand the thermal effects of solar radiation and know how to control or utilize them.

The earth is nearly spherical with a diameter of about 7,900 miles (12.7×10^3 km). It makes one rotation about its axis every 24 hours and completes a revolution about the sun in a period of approximately 365 1/4 days. The earth revolves around the sun in a nearly circular path, with the sun located slightly off center of the circle. The earth's mean distance to the sun is about 9.3×10^7 miles (1.5×10^8 km). Around January 1, the earth is closest to the sun while on around July 1 it is most remote, about 3.3% farther away. Since the intensity of solar radiation incident upon the top of the atmosphere varies inversely with the square of the earth-sun distance, the earth receives about seven per cent more radiation in January than in July. The earth's axis of rotation is tilted 23.5 degrees with respect to its orbit about the sun. The earth's tilted position is of profound significance. Together with the earth's daily rotation and yearly revolution, it accounts for the distribution of solar radiation over the earth's surface, the changing length of hours of daylight and darkness, and the changing of the seasons.

Figure 1 shows the effect of the earth's tilted axis at various times of the year. Figure 2 shows the position of the earth relative to the sun's rays at the time of winter solstice. At the winter solstice (around December 22), the North Pole is inclined 23.5 degrees away from the sun. All points on the earth's surface north of 66.5 degrees north latitude are in total darkness while all regions within 23.5 degrees of the South Pole receive continuous sunlight. At the time of the summer solstice (around June 22), the situation is reversed. At the times of the two equinoxes (around March 22 and September 22), both poles are equidistant from the sun and all points on the earth's surface have 12 hours of daylight and 12 hours of darkness.

BASIC EARTH-SUN ANGLES

The position of a point P on the earth's surface with respect to the sun's rays is known at any instant if the latitude, l , and hour angle, h , for the point, and the sun's declination angle, d , are known. Figure 3 shows these fundamental angles. Point P represents a location on the northern hemisphere.

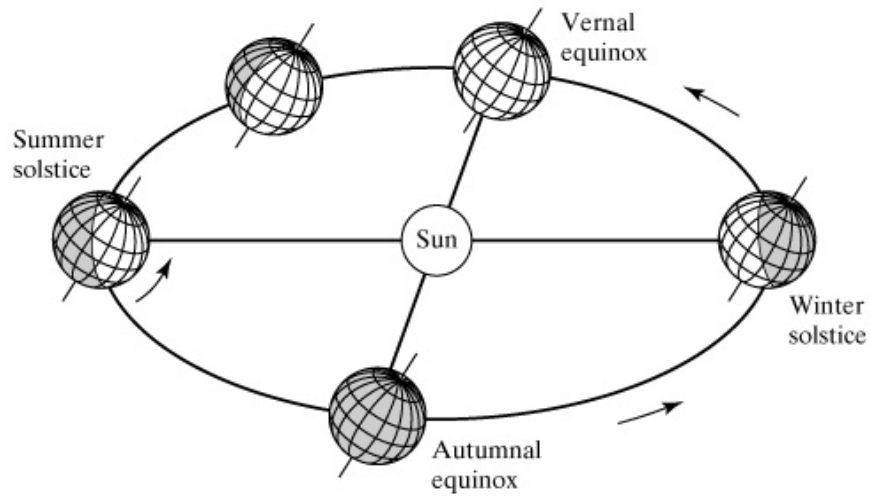


Figure 1: The earth's revolution about the sun

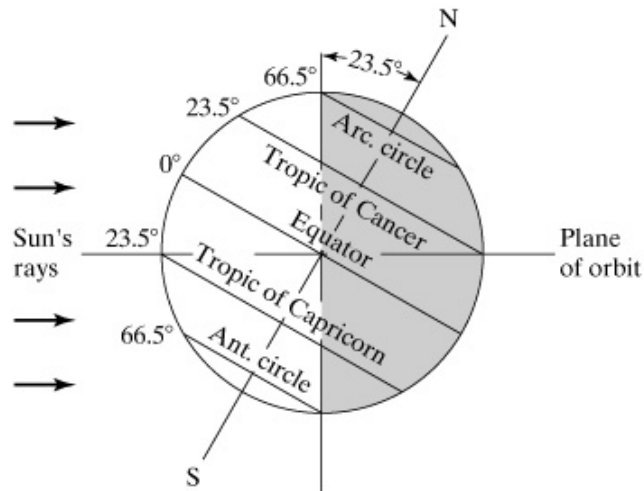


Figure 2: Position of the earth in relation to the sun's rays at time of winter solstice

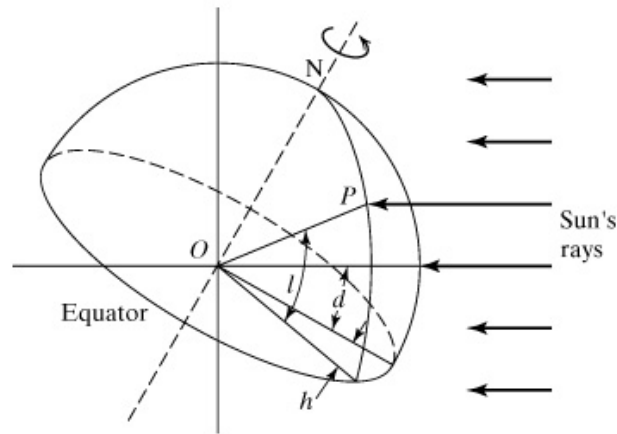


Figure 3: Latitude, hour angle, and sun's declination angle

The latitude, l , is the angular distance of the point P north (or south) of the equator. It is the angle between the line \overline{OP} and the projection of \overline{OP} on the equatorial plane. Point O represents the center of the earth. The calculation of the various solar angles, performed later in this chapter, can be simplified by the adoption of a consistent sign convention. As part of this sign convention, north latitudes are positive and south latitudes are negative.

The hour angle, h , is the angle measured in the earth's equatorial plane between the projection of \overline{OP} and the projection of a line from the center of the sun to the center of the earth. At solar noon, the hour angle is zero. The hour angle expresses the time of day with respect to solar noon. One hour of time is represented by $360 \div 24 = 15$ degrees of hour angle. As part of the convention, the hour angle is negative before solar noon and positive after solar noon.

The sun's declination angle, d , is the angular distance of a sun's rays north (or south) of the equator. It is the angle between a line extending from the center of the sun to the center of the earth and the projection of this line upon the earth's equatorial plane. The declination is positive when the sun's rays are north of the equator and negative when they are south of the equator. At the time of the winter solstice, the sun's rays are 23.5 degrees south of the earth's equator ($d = -23.5^\circ$). At the time of the summer solstice, the sun's rays are 23.5 degrees north of the earth's equator ($d = 23.5^\circ$). At the equinoxes, the sun's declination is zero.

The declination angle throughout the year can be well approximated by a sine function:

$$d = 23.45 \sin \left[\frac{360}{365} (284 + n) \right] \quad \text{[degrees]} \quad \text{Equation 1}$$

where n is the day of the year. The value of n for any day of the month "D" can be determined easily with the aid of Table 1.

<i>Month</i>	<i>n for the Day of the Month, D</i>	<i>Month</i>	<i>n for the Day of the Month, D</i>
January	<i>D</i>	July	181 + <i>D</i>
February	31 + <i>D</i>	August	212 + <i>D</i>
March	59 + <i>D</i>	September	243 + <i>D</i>
April	90 + <i>D</i>	October	273 + <i>D</i>
May	120 + <i>D</i>	November	304 + <i>D</i>
June	151 + <i>D</i>	December	334 + <i>D</i>

Table 1: Variation in "n" throughout the year for use in Equation 1

RELATIONSHIP BETWEEN CLOCK TIME AND SOLAR TIME

Solar radiation calculations must be made in terms of solar time. In a discussion of time, numerous designations may be used. We will consider here only a brief description to allow us to convert local clock time to solar time for engineering calculations.

Time reckoned from midnight at the Greenwich meridian (zero longitude) is known as Greenwich Civil Time or Universal Time. Such time is expressed on an hour scale from zero to 24. Thus, midnight is 0^h and noon is 12^h. Local Civil Time is reckoned from the precise longitude of the observer. On any particular meridian, Local Civil Time is more advanced at the same instant than on any meridian further west and less advanced than on any meridian further east. The difference amounts to 1/15 hour (4 minutes) of time for each degree difference in longitude.

At a given locality, clock time generally differs from civil time. Clocks are usually set for the same reading throughout an entire zone covering about 15 degrees of longitude. The United States is divided into four time zones. The time kept in each zone is the Local Civil Time of a selected meridian near the center of the zone. Such time is called Standard Time. The four standard meridians in the United States are at west longitudes of 75 degrees (Eastern Standard Time, EST), 90 degrees (Central Standard Time, CST), 105 degrees (Mountain Standard Time, MST), and 120 degrees (Pacific Standard Time, PST). In many localities, clocks are advanced

one hour beyond Standard Time in summer. In the United States, such time is called Daylight Savings Time.

Time as measured by the apparent diurnal motion of the sun is called Apparent Solar Time, Local Solar Time, or Solar Time. Whereas a civil day is precisely 24 hours, a solar day is slightly different due to irregularities of the earth's rotation, obliquity of the earth's orbit and other factors. The difference between Local Solar Time, LST and Local Civil Time, LCT is called the Equation of Time, E.

The factors described above can be included into a single equation, which relates solar time and clock time:

$$LST = CT + \left(\frac{1}{15}\right)(L_{std} - L_{loc}) + E - DT \quad [\text{hr}] \quad \text{Equation 2}$$

Where:

LST = Local Solar Time [hr]

CT = Clock Time [hr]

L_{std} = Standard Meridian for the local time zone [degrees west]

L_{loc} = Longitude of actual location [degrees west]

E = Equation of Time [hr]

DT = Daylight Savings Time correction (DT = 0 if not on Daylight Savings Time, otherwise DT is equal to the number of hours that the time is advanced for Daylight Savings Time, usually 1hr)

In using Equation 2, all of the times must first be converted to decimal format from zero to 24, (e.g., a clock time of 3:45 p.m. is expressed as CT = 15.75 hr).

Values of the Equation of Time, E, are calculated by:

$$E = 0.165 \sin 2B - 0.126 \cos B - 0.025 \sin B \quad [\text{hr}] \quad \text{Equation 3a}$$

where:
$$B = \frac{360(n - 81)}{364} \quad \text{[degrees]} \quad \text{Equation 3b}$$

and n is the day of the year.

Once Local Solar Time is established, the solar hour angle, h can be calculated. By recalling that the hour angle varies at the rate of 15 degrees per hour, that $h = 0$ at solar noon, and that the sign convention is $h < 0$ before solar noon, the equation for the hour angle is determined by:

$$h = 15(LST - 12) \quad \text{[degrees]} \quad \text{Equation 4}$$

DERIVED SOLAR ANGLES

Besides the three basic angles (latitude, hour angle, and sun's declination), several other angles are useful in solar radiation calculations. Such angles include the sun's zenith angle θ_H , altitude angle β , and azimuth angle ϕ . For a particular surface orientation, the sun's incidence angle θ , and surface-solar azimuth angle γ , may be defined. All of these additional angles may be expressed in terms of the three basic angles.

To an observer on the earth, the sun appears to move across the sky following the path of a circular arc from horizon to horizon. Figure 4 schematically shows one apparent solar path and defines the sun's zenith, altitude, and azimuth angles. Point P represents the position of the observer, point O is the center of the earth, and I_{DN} is a vector representing the sun's rays. The zenith angle θ_H is the angle between the sun's rays and local vertical, i.e. a line perpendicular to the horizontal plane at P. The altitude angle β is the angle in a vertical plane between the sun's rays and the projection of the sun's rays on the horizontal plane. It follows that $\beta + \theta_H = \pi/2 = 90^\circ$. The azimuth angle ϕ is the angle in the horizontal plane measured from south to the horizontal projection of the sun's rays.

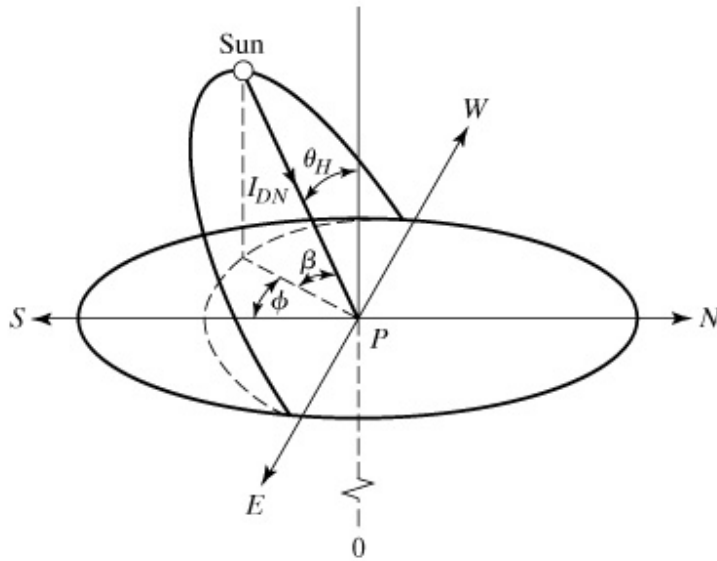


Figure 4: Definition of sun's zenith, altitude, and azimuth angles

Thus the following equations can be derived:

$$\cos \theta_H = \cos l \cos h \cos d + \sin l \sin d \quad \text{Equation 5a}$$

Since $\beta = 90 - \theta_H$, [degrees] Equation 5b

$$\sin \beta = \cos l \cos h \cos d + \sin l \sin d \quad \text{Equation 6}$$

The sun's azimuth, ϕ , is given by the relation:

$$\cos \phi = \frac{1}{\cos \beta} (\cos d \sin l \cos h - \sin d \cos l) \quad \text{Equation 7}$$

The sign convention used for the azimuth angle, ϕ , is negative east of south and positive west of south. Notice that this sign convention results in the hour angle, h , and the sun's azimuth angle, ϕ , always having the same sign. Since the cosine is an even function, calculating the right hand side of Equation 7 and taking the inverse cosine will not provide the information needed for the sign convention. **The user must assign the appropriate sign.**

Equations 5 - 7 allow calculation of the sun's zenith, altitude and azimuth angles if the declination, hour angle, and latitude are known. In applying these equations, attention must be given to correct signs. A summary of the sign convention is:

- l :** north latitudes are positive, south latitudes are negative
- d :** the declination is positive when the sun's rays are north of the equator, i.e. for the summer period in the northern hemisphere, March 22 to September 22 approximately, and negative when the sun's rays are south of the equator.
- h :** the hour angle is negative before solar noon and positive after solar noon
- ϕ :** the sun's azimuth angle is negative east of south and positive west of south

In calculations involving other than horizontal surfaces, it is convenient to express the sun's position relative to the surface in terms of the incidence angle, θ . The sun's angle of incidence is the angle between the solar rays and the surface normal. (Notice that for a horizontal surface, the surface normal is the local vertical and the incidence angle is equal to the zenith angle, θ_H .)

In order to evaluate the angle of incidence we need to specify the direction of the surface normal. This is done in terms of the surface tilt angle, Σ , and the surface azimuth angle, Ψ . These angles are defined in Figure 5. The surface tilt angle is the angle between the surface normal and vertical. The surface azimuth angle is the angle between south and the horizontal projection of the surface normal. The same sign convention is used for the surface azimuth angle as is used for the solar azimuth angle, i.e. Ψ is negative for a surface that faces east of south and positive for a surface that faces west of south. The azimuth angle for a horizontal surface is undefined.

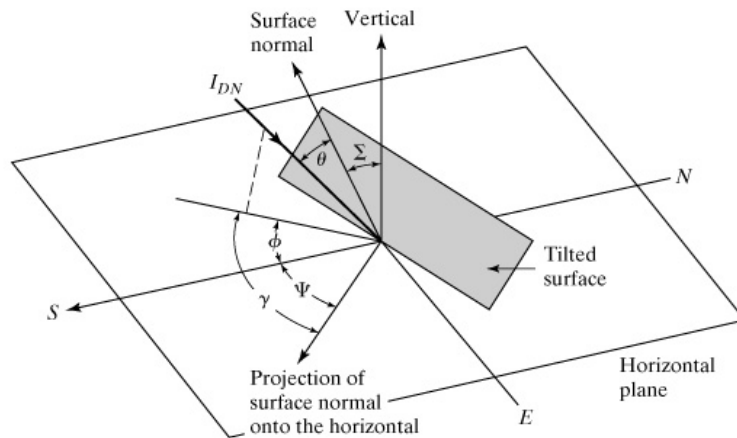


Figure 5: Definitions of surface azimuth, surface tilt, and surface-solar azimuth angles and the relation of the sun's rays to a tilted surface

It is convenient to define one additional angle, the surface-solar azimuth angle, γ . As shown in Figure 5, the surface-solar azimuth angle is defined as the angle between the horizontal projection of the solar rays and the horizontal projection of the surface normal. Examination of Figure 5 reveals that as long as one adheres to the sign conventions for the azimuth angles ϕ and Ψ , the surface-solar azimuth angle is given by the simple relation:

$$\gamma = |(\phi - \Psi)| \quad \text{[degrees]} \quad \text{Equation 8}$$

In Figure 5, for the tilted surface:

$$\cos \theta = \cos \beta \cos \gamma \sin \Sigma + \sin \beta \cos \Sigma \quad \text{Equation 9}$$

If the surface is vertical ($\Sigma=90^\circ$), then:

$$\cos \theta = \cos \beta \cos \gamma \quad \text{Equation 10}$$

If the surface is horizontal ($\Sigma=0$), then:

$$\cos \theta = \sin \beta = \cos \theta_H \quad \text{Equation 11}$$

Thus, as previously described, it is seen that the incidence angle for a horizontal surface is equal to the zenith angle.

ESTIMATION OF INTENSITY OF SOLAR RADIATION DURING AVERAGE CLEAR DAYS

In order to predict the solar contributions to building cooling loads it is desirable to estimate the solar intensity on typical or average clear days. An estimate of the direct normal solar flux at the earth's surface for an average clear day is:

$$I_{DN} = A e^{-B/\sin \beta} \quad \text{Equation 12}$$

where the coefficients A and B are empirically determined from measurements of I_{DN} made on typical clear days. The coefficients can be interpreted as:

A = Apparent direct normal solar flux at the outer edge of the earth's atmosphere

B = Apparent atmospheric extinction coefficient

The numerical values of A and B vary throughout the year because of seasonal changes in the dust and water vapor content of the atmosphere and because of the changing earth-sun distance. The ASHRAE Handbook of Fundamentals lists recommended values for the coefficients A and B for the twenty-first day of each month. These values are presented in Table 2. Also included in the table are the values of the declination angle and Equation of Time that ASHRAE lists in conjunction with the clear day coefficients. These values are for the base year 1964. A comparison of the declination angles listed for 1964 and those predicted by Equation 1 reveals that they are in good agreement. Similarly, the calculated values for the Equation of Time agree to within 0.01 hour with those for the 1964 base year. Thus, for the purpose of making HVAC calculations the approximations of Equations 1 and 3 provide acceptable values for the declination angle and Equation of Time.

	A		B	C	Declination, deg	Equation of Time, hr
	Btu hr · ft ²	W m ²	Dimensionless Ratios			
January	390	1230	0.142	0.058	-20.0	-0.19
February	385	1215	0.144	0.060	-10.8	-0.23
March	376	1186	0.156	0.071	0.0	-0.13
April	360	1136	0.180	0.097	11.6	0.02
May	350	1104	0.196	0.121	20.0	0.06
June	345	1088	0.205	0.134	23.45	-0.02
July	344	1085	0.207	0.136	20.6	-0.10
August	351	1107	0.201	0.122	12.3	-0.04
September	365	1151	0.177	0.092	0	0.13
October	378	1192	0.160	0.073	-10.5	0.26
November	387	1221	0.149	0.063	-19.8	0.23
December	391	1233	0.142	0.057	-23.45	0.03

Table 2: Coefficients for average clear day solar radiation calculations for the twenty-first day of each month, base year 1964

The use of Equation 12 with the coefficients presented in Table 2 is commonly referred to as the ASHRAE Clear Day Solar Flux Model. The model also approximates the average clear day diffuse solar flux from the sky that strikes a horizontal surface, I_{dH} , by the relation:

$$I_{dH} = CI_{DN} \tag{Equation 13}$$

The recommended values of the dimensionless coefficient C are listed in Table 2.

The ASHRAE Clear Day model does not give the maximum values for I_{DN} that can occur during the month, but rather are representative of conditions on average, cloudless days. For very clear atmospheres, ASHRAE points out that values of I_{DN} can be 15% higher than those calculated using Equation 12 and Table 2.

SOLAR RADIATION STRIKING A SURFACE

The solar radiation striking a surface generally consists of three components, **direct, diffuse and reflected**. The direct, or beam, solar radiation is that received from the sun without having been scattered by the atmosphere. The direct solar flux (energy/area-time) striking a surface is denoted by I_D . If the surface is perpendicular to the solar rays, the incident solar flux is equal to the Direct Normal flux, I_{DN} . From Figure 5 it can be seen that if the solar flux strikes a surface at an angle of incidence θ , the direct solar flux striking the surface is given by:

$$I_D = I_{DN} \cos \theta \quad \text{Equation 14}$$

In the case of a horizontal surface, an additional subscript, H, is used. Thus, I_{DH} is the direct solar flux striking the horizontal. Notice that for a horizontal surface the incidence angle is equal to the zenith angle, θ_H , and therefore:

$$I_{DH} = I_{DN} \cos \theta_H = I_{DN} \sin \beta \quad \text{Equation 15}$$

The diffuse solar radiation is that received from the sun after its direction has been changed by scattering by the atmosphere. The diffuse solar flux striking a surface is denoted by I_d for the general case and by I_{dH} for the special case of a horizontal surface. Diffuse radiation is typically of rather short wavelength, since short-wavelength radiation is scattered more by the atmosphere. Although diffuse solar radiation on clear days is usually small compared to direct radiation, it cannot be ignored in engineering calculations. During extremely cloudy days, only diffuse solar radiation may reach the ground. Because of its non-directional nature, diffuse solar radiation is more difficult to analyze than direct solar radiation, consequently less is known about it. A common approximation is that the sky is a uniform radiator of diffuse radiation. Assuming the sky to be a diffuse source, the ratio of the diffuse solar flux striking the surface to that striking the horizontal is given by:

$$\frac{I_d}{I_{dH}} = (1 + \cos \Sigma)/2 \quad \text{Equation 16}$$

The reflected solar radiation is that which strikes a surface after the radiation is reflected from surrounding surfaces. In general, the solar radiation reflected upon a surface depends on the particular location, orientation and solar reflectance characteristics of the surrounding surfaces. One commonly occurring situation is where the solar radiation is reflected from the ground.

If the ground is horizontal and if the reflection is diffuse, an approximation for the reflected solar flux striking a surface, I_R , is given by:

$$I_R = \rho_g I_H (1 - \cos \Sigma)/2 \quad \text{Equation 17}$$

where:

ρ_g = solar reflectance of the ground

I_H = Total solar flux striking the horizontal ground

The reflectance of the ground varies with the type of ground cover. The reflectance of browned grass is about 0.2, while that of bare soil is about 0.1. The reflectance of fresh snow cover may be as high as 0.87, with the value decreasing to less than 0.5 as the snow becomes dirty.

The total solar flux striking a surface at any instant is the sum of the three components:

$$I = I_D + I_d + I_R \quad \text{Equation 18}$$

Notice that if any two of the solar flux values that appear in the above set of equations are known, either by **prediction or measurement**, the rest of the quantities can be evaluated. In the case of the use of the ASHRAE Clear Day model, the quantities I_{DN} and I_{dH} are predicted using Equations 12 and 13 respectively, along with the coefficients presented in Table 2.

SOLAR RADIATION MEASUREMENT

Experimental determination of the energy transferred to a surface by solar radiation requires instruments that will measure the heating effect of direct solar radiation and diffuse solar radiation. There are two general classes of solar radiation measuring devices. The instrument

used to measure direct normal or beam radiation is referred to as a pyrheliometer. The other instrument, called a pyranometer, is able to measure total radiation within its hemispherical field of view. A pyranometer can also be used to measure diffuse radiation alone by shading the sensing element from the sun's direct rays.

Figure 6 is a photograph of a pyrheliometer and the required tracking device on which it is mounted. The pyrheliometer consists of a long collimating tube with a detector at the far end. The aperture angle of the instrument is 5.7° , so the detector receives radiation from the sun and from an area of the sky two orders of magnitude larger than the sun. The detector consists of a multi-junction thermopile with a blackened surface. Temperature compensation is provided to minimize sensitivity to variations in the ambient temperature. The temperature of the blackened junction of the thermopile is a measure of the solar flux striking it. The output measured is a dc voltage, typically in the millivolt range, and the manufacturer, Eppley Laboratories, provides a calibration constant to convert dc voltage to solar flux.



<http://www.eppleylab.com>



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Figure 6: An Eppley normal incidence pyrheliometer and a tracking mount

Figure 7 is a photograph of a pyranometer manufactured by Eppley Laboratories. It uses a thermopile detector, two concentric hemispherical optically ground covers to ensure a proper cosine effect and temperature compensation. This type of pyranometer measures the total solar flux coming from the hemisphere above the detector (the detector is the black disk in the center of the pyranometer). The units are usually mounted horizontally so they measure the total solar flux striking the horizontal. However, they can be mounted in any orientation. In our experiment involving solar collectors, a pyranometer is mounted with the detector in the plane of the collectors, thus measuring the total solar flux striking the collector surface.



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Figure 7: An Eppley Precision pyranometer

A pyranometer can be fitted with a shading ring to block the beam or direct solar radiation. In this case, the device measures approximately the diffuse plus reflected solar flux from the hemisphere above the detector. Figure 8 is a photograph of a pyranometer with a shading ring. The design of the ring allows continuous recording of the diffuse plus reflected flux throughout the day. The position of the ring is adjusted for the changing declination angle only and can be done every few days.



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Figure 8: Pyranometer with shading ring to eliminate direct solar radiation