



Course: Sustainable Energy Technology 1  
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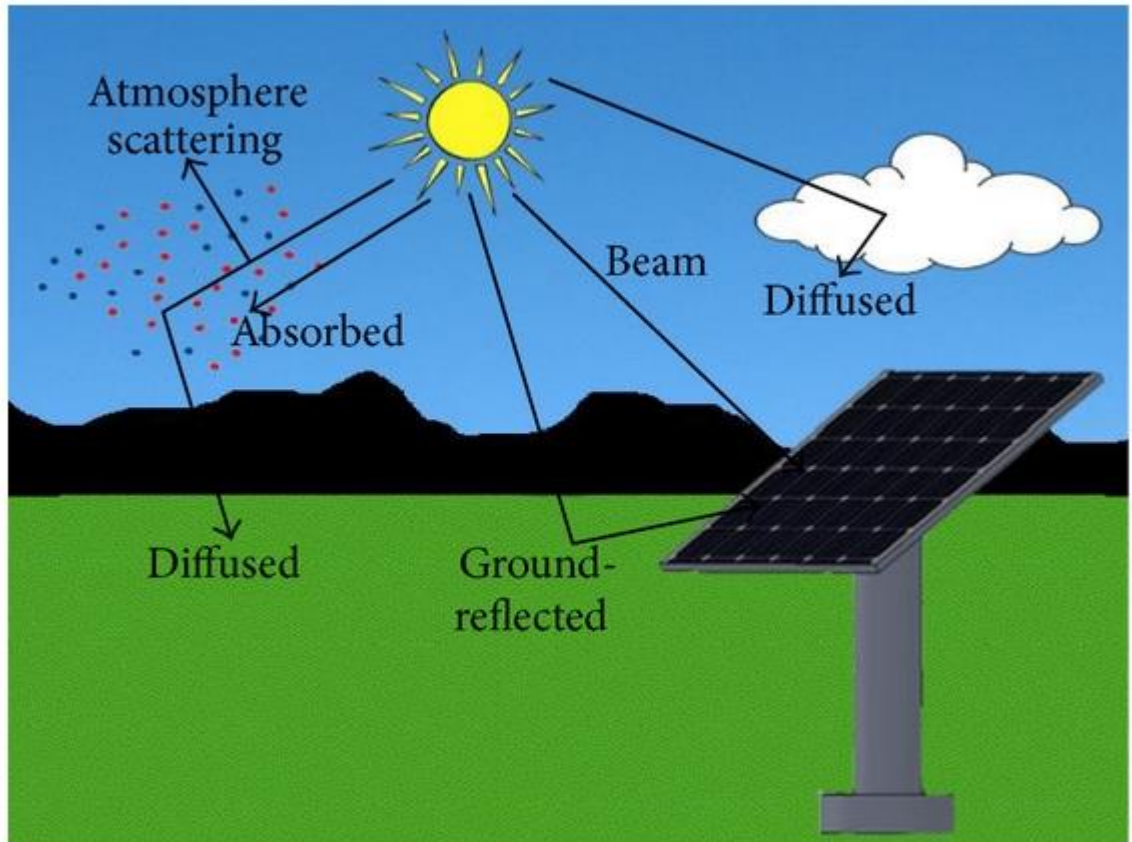
**Title: Solar Energy-L3**

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# Solar Energy

## The Sun Radiation angles and Sun Radiation on tilted surfaces:

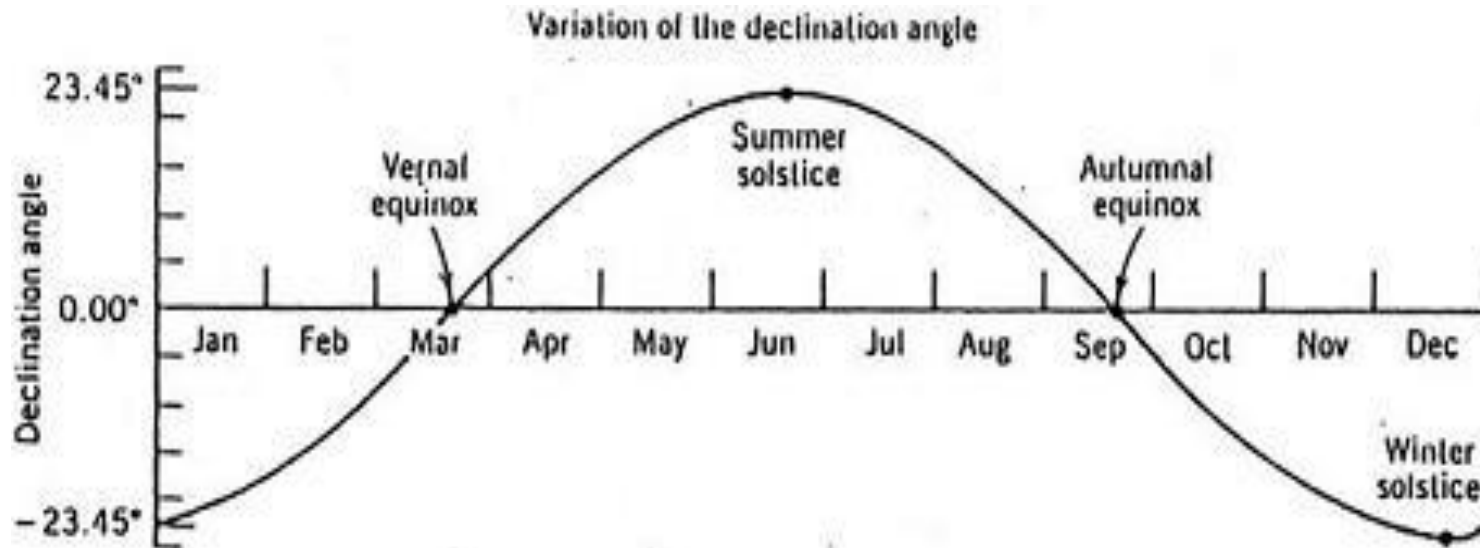
The global solar radiation incident on a tilted surface ( $G_T$ ) is composed of three components, including beam radiation, diffuse radiation and radiation reflected from various surfaces facing the tilted surface.



# Solar Energy

**The Sun Radiation angles:  
The declination angle values**

$$\delta_s = 23.45 * \sin[360(N + 284)/365]$$



# Solar Energy

## The Sun Radiation angles and Sun Radiation on tilted surfaces:

This global radiation ( $G_T$ ) can be written as

$$G_T = G_{Tb} + G_{Td} + G_{Tr}$$

where  $G_{Tb}$ ,  $G_{Td}$ ,  $G_{Tr}$  are the beam radiation, diffuse radiation and reflected radiation, respectively, incident on the tilted surface. In terms of solar radiation components incident on a horizontal surface,  $G_T$  can be written as

$$G_T = G_{bn} * R_b + G_d * R_d + G_H * \rho_g * R_r$$

where  $R_b$  is the ratio between the beam (direct) solar radiation incident on tilted surface to that incident on horizontal surface. Its value can be determined using Eq. (1).  $R_d$  is the ratio between diffuse solar radiation on the tilted surface to that on the horizontal surface, and it can be determined using Eq. (2).  $R_r$  is the view factor of the tilted surface to the ground and its value is calculated using Eq. (3), while  $\rho_g$  is the reflectance factor of the surroundings. It is 0.2 for dry bare ground; 0.3 for dry grass land; 0.4 for desert sand; 0.5 – 0.8 for snow; 0.1 for water; 0.2 for vegetation; 0.1 for dark soil; 0.3 for pale soil.

# Solar Energy

The Sun Radiation angles and Sun Radiation on tilted surfaces:

$$R_b = \frac{\cos \theta}{\sin \alpha} \quad (1)$$

$$R_d = \cos^2 \left( \frac{\beta}{2} \right) \quad (2)$$

$$R_r = \sin^2 \left( \frac{\beta}{2} \right) \quad (3)$$

where  $\beta$  is the tilt angle of the inclined surface, and  $\theta_i$  is the incidence angle. The incidence angle represents the angle between the beam radiation on a certain surface and the normal to that surface, given by

$$\theta_i = \cos^{-1}[\cos(\gamma_s - \gamma) * \cos \alpha * \sin \beta + \cos \beta * \sin \alpha]$$

where  $\gamma$  is the surface azimuth angle, which is the angle between the projection of the normal to a certain tilted surface and the true south, while  $\gamma_s$  is the solar azimuth angle representing the angle between the projection of the beam radiation with the true south. **These two angles are positive to east of south and negative to west of south.**

This solar azimuth angle can be calculated using the following equation:

$$\gamma_s = \sin^{-1}[-1 * \sin(\omega) * \cos(\delta_s) / \cos(\alpha)]$$

# Solar Energy

**The Sun Radiation angles and Sun Radiation on tilted surfaces:**

The global radiation ( $G_T$ ) on tilted surface is

$$G_T = G_{bn} * R_b + G_d * R_d + G_H * \rho_g * R_r$$

For estimation of the beam and diffused radiation, depending on global radiation measured on horizontal surface, different correlations are suggested. Orgill and Holands (1977) used data from Canadian stations to obtain a correlation between ( $G_d/G_H$ ), and the hourly clearness index ( $K_T$ )

The Orgill and Holands correlation is represented by the following equations:

$$\frac{G_d}{G_H} = \begin{cases} 1.0 - 0.249 * K_T & \text{for } 0 \leq K_T \leq 0.35 \\ 1.557 - 1.84 * K_T & \text{for } 0.35 < K_T < 0.75 \\ 0.177 & \text{for } K_T > 0.75 \end{cases}$$

# Solar Energy

## Sun Radiation on tilted surfaces :

The hourly clearness index  $K_T$  is the ratio between the global radiation measured on the horizontal surface, and the extraterrestrial radiation and given by

$$K_T = \frac{G_H}{G_{H0}}$$

where  $G_{H0}$  is the solar radiation incident on a horizontal plane outside of the atmosphere and given by

$$G_{H0} = 1367 * \left( 1 + 0.033 * \cos \left( N * \frac{360}{365} \right) \right) * \sin(\alpha)$$

# Solar Energy

## Examples on Sun Radiation angles and Sun Radiation Calculations:

**Example 1:** Find the equation of calculating ts for the city of Hebron (31.32 N; 35:05 E)

Longitude is 35:05 which is equivalent for 35.0938 since 5 minutes are equivalent to  $5/60 = 0.0938$

$$ts = t_{loc} - 4 ( L_{SM} - L_{loc} ) + EOT \text{ since Hebron is east of Greenwich}$$

$$ts = t_{loc} - 4 ( 30 - 35.093 ) + EOT$$

$$ts = t_{loc} + EOT + 20.36 \text{ (in minutes)}$$

**Example 2:** Find the solar time on 10 March (N=69) at 2:30 for a city located at  $23^{\circ}:40'$

$$B = 360/365 (N-81) = -11.87$$

$$EOT = 9.87 \sin 2B - 7.35 \cos B - 1.5 \sin B = -11 \text{ minutes}$$

$$L_{loc} = 23^{\circ}:40' = 23.66$$

$$L_{SM} = 30^{\circ}$$

$$ts = t_{loc} - 4 ( L_{SM} - L_{loc} ) + EOT \text{ since the city is east of Greenwich}$$

$$ts = 14:30 - 4(30-23.66) - 0:11 = 14:30 - 0:25 \text{ (min)} - 0:11 \text{ (min)}$$

$$ts = 13:54 = 13.9$$



# Solar Energy

## Examples on Sun Radiation angles and Sun Radiation Calculations:

**Example 3:** Find the maximum and minimum solar noon altitude angle for a location at 40° N.

At Solar noon,  $t_s = 12:00$

The maximum  $\alpha$  is at Summer Solstice ( $\delta_s = 23.45$ )

$$\alpha = \sin^{-1}[\sin \delta_s * \sin \varphi + \cos \delta_s * \cos \varphi * \cos \omega] = 73.5^\circ$$

The minimum  $\alpha$  is at Winter Solstice ( $\delta_s = -23.45$ )

$$\alpha = \sin^{-1}[\sin \delta_s * \sin \varphi + \cos \delta_s * \cos \varphi * \cos \omega] = 26.5^\circ$$

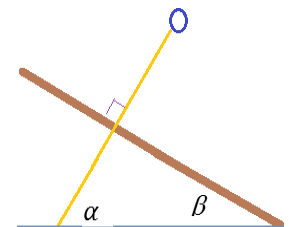
**Example 4:** Find the optimum tilt angle for a south facing PV module located at Tulkarm (32.1 N) at solar noon on 1 March ( $N=60$ )

$$\delta_s = 23.45 * \sin[360(N + 284)/365] = -8.3^\circ ; \omega = 0 \text{ since } t_s = 12:00 \text{ at solar noon}$$

$$\alpha = \sin^{-1}[\sin \delta_s * \sin \varphi + \cos \delta_s * \cos \varphi * \cos \omega] = 49.6^\circ$$

*The optimum tilt angle is when  $\theta_i = 0$ . The direct beam is perpendicular to the tilted surface, so  $\alpha + \beta = 90^\circ$*

$$\text{So, } \beta = 90 - \alpha = 40.4^\circ$$



# Solar Energy

## Examples on Sun Radiation angles and Sun Radiation Calculations:

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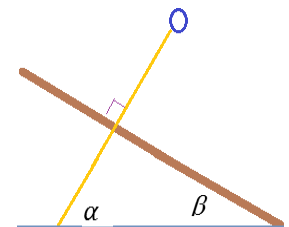
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$$\delta_s = 23.45 * \sin[360(N + 284)/365] = -8.3^\circ ; \omega = 0 \text{ since } t_s = 12:00 \text{ at solar noon}$$

$$\alpha = \sin^{-1}[\sin \delta_s * \sin \varphi + \cos \delta_s * \cos \varphi * \cos \omega] = 49.6^\circ$$

*The optimum tilt angle is when  $\theta_i = 0$ . The direct beam is perpendicular to the tilted surface, so  $\alpha + \beta = 90^\circ$*

$$\text{So, } \beta = 90 - \alpha = 40.4^\circ$$



# Solar Energy

## Examples on Sun Radiation angles and Sun Radiation Calculations:

**Example 5:** Find the altitude angle and the azimuth angle for the sun at 3:00 PM solar time at  $40^\circ$  N on summer solstice.

At Summer Solstice  $\delta_s = 23.45$ ;  $\omega = 15 (15-12) = 45^\circ$

$$\alpha = \sin^{-1}[\sin \delta_s * \sin \varphi + \cos \delta_s * \cos \varphi * \cos \omega] = 48.8^\circ \text{ (Altitude angle)}$$

$$\gamma_s = \sin^{-1}[-1 * \sin(\omega) * \cos(\delta_s) / \cos(\alpha)] = -80^\circ \text{ (Solar azimuth angle)}$$

**Example 6:** Find the clock time at solar noon on 1 July at a location of  $71.1^\circ$  W.

$$B = 360/365 (N-81) = -99.89^\circ ; \text{EOT} = 9.87 \sin 2B - 7.35 \cos B - 1.5 \sin B = -3.5 \text{ min}$$

$$L_{SM} = 75^\circ$$

$$t_s = t_{loc} + 4 (L_{SM} - L_{loc}) + \text{EOT} \text{ since city is west of Greenwich}$$

$$t_{loc} = t_s - 4 (L_{SM} - L_{loc}) - \text{EOT} = 11:40:54$$

# Solar Energy

## Examples on Sun Radiation angles and Sun Radiation Calculations:

**Example 7:** Find the incidence angle at 12:00 noon on a solar collector surface tilted by 30 degrees and oriented toward west of south by 10 degrees on 1 Feb (N= 32) at a location with the coordinates 29.68 N, 82.27 W.

$$\beta = 30^\circ; \gamma = -10^\circ$$

$$\delta_s = 23.45 * \sin[360(N + 284)/365] = -17.5^\circ$$

$$\text{EOT} = 9.87 \sin 2B - 7.35 \cos B - 1.5 \sin B = -13.7 \text{ min}$$

$$t_s = t_{\text{loc}} + 4 (L_{\text{SM}} - L_{\text{loc}}) + \text{EOT} \text{ since city is west of Greenwich}$$

$$T_s = 12:00 + 4(75 - 82.27) - 13.7 = 11:17.2 = 11.29$$

$$\omega = 15 * (t_s - 12) = -10.7^\circ (\text{before noon})$$

$$\alpha = \sin^{-1}[\sin \delta_s * \sin \varphi + \cos \delta_s * \cos \varphi * \cos \omega] = 41.7^\circ$$

$$\gamma_s = \sin^{-1}[-1 * \sin(\omega) * \cos(\delta_s) / \cos(\alpha)] = 13.7^\circ (\text{east of south})$$

$$\theta_i = \cos^{-1}[\cos(\gamma_s - \gamma) * \cos \alpha * \sin \beta + \cos \beta * \sin \alpha] = 23.4^\circ$$

# Solar Energy

## Examples on Sun Radiation angles and Sun Radiation Calculations:

**Example 8:** The global solar radiation on a horizontal surface at a site (30° N) at 10:00 AM solar time on 20 April (N=110) was 750 W/m<sup>2</sup>.

Find the global solar radiation on a surface of a collector tilted by 20° and directed by an angle 45° to the east of south. The reflection index of the ground surface at this location is 0.38.

$$\beta = 30^\circ; \gamma = 45^\circ; \varphi = 30^\circ; G_H = 750 \text{ W/m}^2$$

$$\delta_s = 23.45 * \sin[360(N + 284)/365] = 11.1^\circ$$

$$\omega = 15 * (t_s - 12) = -30^\circ$$

$$\alpha = \sin^{-1}[\sin \delta_s * \sin \varphi + \cos \delta_s * \cos \varphi * \cos \omega] = 56.3^\circ$$

$$\gamma_s = \sin^{-1}[-1 * \sin(\omega) * \cos(\delta_s) / \cos(\alpha)] = 62.2^\circ \text{ (east of south)}$$

$$\theta_i = \cos^{-1}[\cos(\gamma_s - \gamma) * \cos \alpha * \sin \beta + \cos \beta * \sin \alpha] = 15.6^\circ$$

$$G_{H0} = 1367 * \left(1 + 0.033 * \cos\left(N * \frac{360}{365}\right)\right) * \sin(\alpha) = 1125.4 \text{ W/m}^2$$

$$K_T = \frac{G_H}{G_{H0}} = 0.666$$

$$\frac{G_d}{G_H} = \begin{cases} 1.0 - 0.249 * K_T & \text{for } 0 \leq K_T \leq 0.35 \\ 1.557 - 1.84 * K_T & \text{for } 0.35 < K_T < 0.75 \\ 0.177 & \text{for } K_T > 0.75 \end{cases}$$

$$\frac{G_d}{G_H} = 1.557 - 1.84 * K_T \quad ; \quad G_d = 248.1 \text{ W/m}^2$$

$$G_{bn} = G_H - G_d = 501.9 \text{ W/m}^2$$

# Solar Energy

## Examples on Sun Radiation angles and Sun Radiation Calculations:

**Complete Example 8:** The global solar radiation on a horizontal surface at a site (30° N) at 10:00 AM solar time on 20 April (N=110) was 750 W/m<sup>2</sup>.

Find the global solar radiation on a surface of a collector tilted by 20° and directed by an angle 45° to the east of south. The reflection index of the ground surface at this location is 0.38.

$$G_T = G_{bn} * R_b + G_d * R_d + G_H * \rho_g * R_r$$

$$R_b = \frac{\cos \theta}{\sin \alpha} = 1.1577$$

$$R_d = \cos^2 \left( \frac{\beta}{2} \right) = 0.97$$

$$R_r = \sin^2 \left( \frac{\beta}{2} \right) = 0.03$$

$$G_T = 830.3 \frac{W}{m^2}$$