Palestine Technical University- Kadoorie (PTUK)

Mechanical Engineering Department

12210244: Dynamics

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This is an explanation of the Dynamics course offered at Palestine Technical University - Kadoorie

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Textbook:

Engineering Mechanics: Dynamics, 7th Edition

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Chapter Two: Kinematics of Particles

Section One: Introduction

2 Chapter Two: Kinematics of Particles

2.1 Introduction

Kinematics, a branch of dynamics, can be summarized as follows:

1. Describes the **motion** of bodies.
2. **Does not** consider the **forces** causing the motion or generated as a result of it.
3. Often referred to as the "**geometry of motion**."

2.1.1 Particle Motion

Our study of kinematics begins with the following key points:

1. We start by discussing the motions of **points or particles** in this chapter.
2. A particle is defined as a body whose **physical dimensions are negligible compared to the radius of curvature of its path**, allowing us to treat its motion as that of a point.

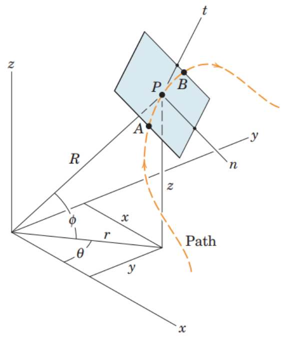
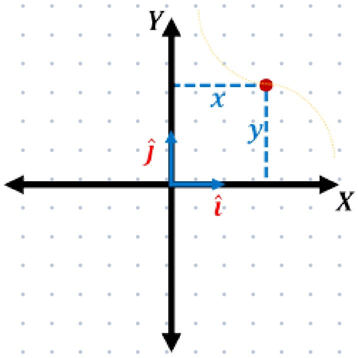


Figure 2/1

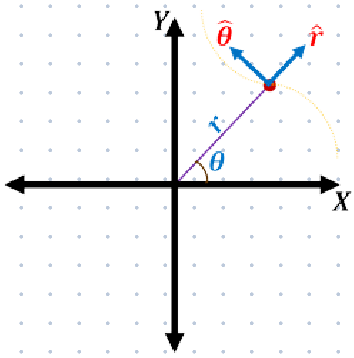
2.1.2 Choice of Coordinates

Various descriptions can be used to characterize the motion of a particle, and the most suitable choice depends **significantly on experience and the nature of the provided data**.

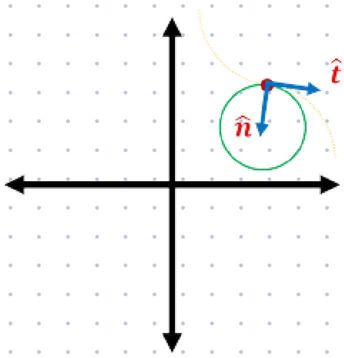
Rectangular Coordinates
Fixed or Moving



Polar Coordinates
(cylindrical coordinate)



Path Coordinates
(normal-tangential coordinate)



End of Section 2.1

Chapter Two: Kinematics of Particles

Section Two: Rectilinear Motion

2.2 Rectilinear Motion

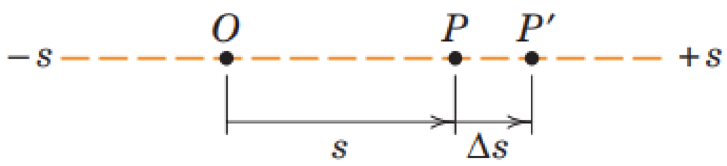
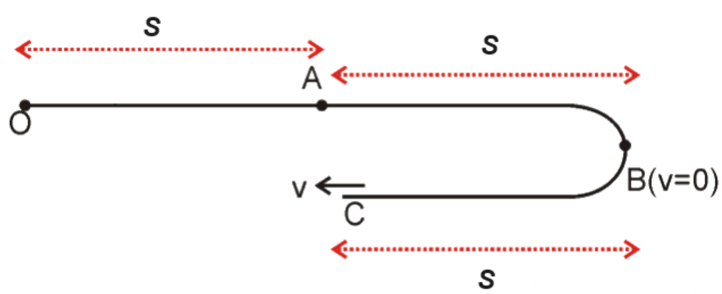


Figure 2/2

2.2.1 Position (s)

2.2.2 Displacement (Δs)

2.2.3 Distance (s_T)



2.2.4 Velocity and Acceleration

Average speed

$$\boldsymbol{v}_{\text{speed}} = \frac{s_T}{\Delta t}$$

Average velocity

$$\boldsymbol{v}_{\text{avg}} = \frac{\Delta s}{\Delta t}$$

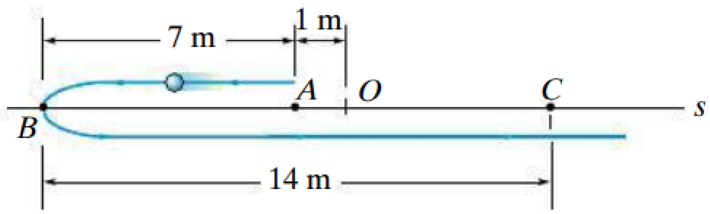
Average acceleration

$$\boldsymbol{a}_{\text{avg}} = \frac{\Delta v}{\Delta t}$$

Example 1:

When $t = 0$ sec the particle is at A. In four seconds it travels to B, then in another six seconds it travels to C. Determine the average velocity and the average speed. The origin of the coordinate is at O.

ans. $Average\ velocity = 0.7\ m/sec$ $Average\ speed = 2.1\ m/sec$



Example 2:

The position of a particle is given by the equation $s = 5t^3 + t - 1\ m$, where t is measured in seconds. Find the average velocity and the average acceleration of the particle between $t_1 = 0\ sec$ and $t_2 = 4\ sec$.
ans. $Average\ velocity = 81\ m/sec$ $Average\ acceleration = 60\ m/sec^2$

If the position (s) is a function of time (t) $\longrightarrow$ s(t) **Instantaneous velocity**

$$v = \frac{ds}{dt} = \dot{s} \quad \rightarrow \quad v = v(t) \tag{1}$$

Instantaneous acceleration

$$a = \frac{dv}{dt} = \dot{v} = \frac{d^2s}{dt^2} = \ddot{s} \quad \rightarrow \quad a = a(t) \tag{2}$$

By eliminating the time ***dt*** between **Eq.1** and **Eq.2**.

$$vdv = ads \quad \rightarrow \quad \dot{s}d\dot{s} = \ddot{s}ds \tag{3}$$

Example 3:

If $s = 2t^3\ m$, where t is in seconds, determinethe displacement s, the velocity v and the acceleration a when $t = 2\ sec$.
ans. $s = 16\ m$ and $v = 24\ m/sec$ and $a = 24\ m/sec^2$

Example 4:

If $v = 4t^2 + 5\ m/sec$, where t is in seconds, determine the velocity v and the acceleration a when $t = 2\ sec$.
ans. $v = 21\ m/sec$ and $a = 16\ m/sec^2$

If the acceleration (a) is a function of time (t) $\rightarrow$ a(t)

$$dv = a(t)dt \rightarrow \int_{v_0}^{v} dv = \int_{t_0}^{t} a(t)\,dt \rightarrow v = v_0 + \int_{t_0}^{t} a(t)\,dt \rightarrow v = v(t)$$

$$ds = v(t)dt \rightarrow \int_{s_0}^{s} ds = \int_{t_0}^{t} v(t)\,dt \rightarrow s = s_0 + \int_{t_0}^{t} v(t)\,dt \rightarrow s = s(t)$$

Example 5:

Given the acceleration $a = 40t^3 + 2\ m/s^2$, where t is in seconds, find the acceleration a, the velocity v, and the displacement s at $t = 2\ sec$, assuming that both the displacement and velocity are zero when $t = 0\ sec$.

ans. $s = 68\ m$ and $v = 164\ m/sec$ and $a = 322\ m/sec^2$

If the acceleration (a) is cosntant

$$dv = a dt \quad \rightarrow \quad \int_{v_0}^{v} dv = \int_{t_0}^{t} a \, dt \quad \rightarrow \quad v = v_0 + a(t - t_0)$$

$$ds = v dt \quad \rightarrow \quad \int_{s_0}^{s} ds = \int_{t_0}^{t} (v_0 + at) \, dt \quad \rightarrow \quad s - s_0 = v_0(t - t_0) + \frac{1}{2} a(t^2 - t_0^2)$$

$$\int_{v_0}^{v} v \, dv = \int_{s_0}^{s} a \, ds \quad \rightarrow \quad v^2 = v_0^2 + 2a(s - s_0)$$

Example 6:

If $a = 2 \ m/sec^2$, determine the velocity v and displacement s when $t = 2 \ sec$ if $v = 0 \ m/sec$ and $s = 0 \ m$ when $t = 0 \ sec$.

$ans.$ $s = 4 \ m$ and $v = 4 \ m/sec$

Example 7:

If $a = 2 \ m/sec^2$, determine the velocity v and the time t at $s = 4 \ m$ if at $t = 0 \ sec$ the velocity is $v = 3 \ m/sec$ and the displacemnt is $s = 0 \ m$.

$ans.$ $t = 1.5 \ sec$ and $v = 4 \ m/sec$

If the acceleration (a) is a function of Velocity (v) $\rightarrow$ **a(v)**

$$\boldsymbol{a(v) = \frac{dv}{dt} \quad \rightarrow \quad dt = \frac{dv}{a(v)}}$$

$$\boldsymbol{\int_{t_0}^{t} dt = \int_{v_0}^{v} \frac{1}{a(v)}\, dv \quad \rightarrow \quad t - t_0 = \int_{v_0}^{v} \frac{1}{a(v)}\, dv \quad \rightarrow \quad t = t(v)}$$

$$\boldsymbol{vdv = a(v)ds \quad \rightarrow \quad \frac{v}{a(v)} dv = ds}$$

$$\boldsymbol{\int_{s_0}^{s} ds = \int_{v_0}^{v} \frac{v}{a(v)}\, dv \quad \rightarrow \quad s - s_0 = \int_{v_0}^{v} \frac{v}{a(v)}\, dv \quad \rightarrow \quad s = s(v)}$$

Example 8:

If $a = -v^2\ m/sec^2$, where v is in m/sec, determine the displacement s and the time t when $v = 5\ m/sec$ if $t = 0\ sec$ and $s = 4\ m$ at $v = 8\ m/sec$.
$ans.$ $t = 0.075\ sec$ and $s = 4.47\ m$

If the acceleration (a) is a function of position (s) $\rightarrow$ **a(s)**

$$v\,dv = a(s)\,ds$$

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} a(s)\,ds \quad \rightarrow \quad v = v(s)$$

$$v(s) = \frac{ds}{dt} \quad \rightarrow \quad dt = \frac{1}{v(s)}ds$$

$$\int_{t_0}^{t} dt = \int_{s_0}^{s} \frac{1}{v(s)}\,ds \quad \rightarrow \quad t - t_0 = \int_{s_0}^{s} \frac{1}{v(s)}\,ds \quad \rightarrow \quad t = t(s)$$

Example 9:

If $a = s\ m/sec^2$, where s is in meters, determine the velocity v and the time t when $s = 5\ m$ if $t = 0\ sec$ and $v = 0$ at $s = 4\ m$.

ans. $t = 0.693\ sec$ and $v = 3\ m/sec$

Ans.

End of Section 2.2