Palestine Technical University- Kadoorie (PTUK)

Mechanical Engineering Department

12210244: Dynamics

Summer Semester, 2023/2024

This is an explanation of the Dynamics course offered at Palestine Technical University - Kadoorie

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Textbook:

Engineering Mechanics: Dynamics, 7th Edition

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Chapter Two: Kinematics of Particles

Section Three: Plane Curvilinear Motion

2 Chapter Two: Kinematics of Particles

2.3 Plane Curvilinear Motion

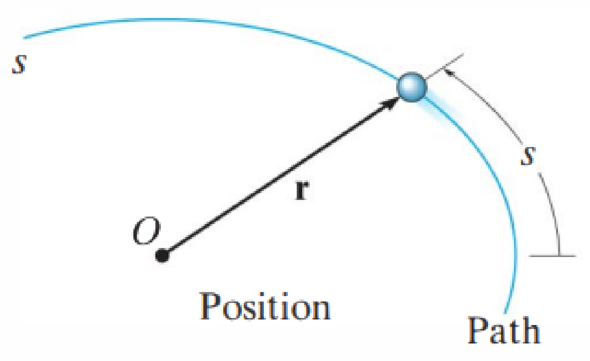
2.3.1 Position

Path function

$$s = s(t)$$

Position

$$\vec{r} = \vec{r}(t)$$

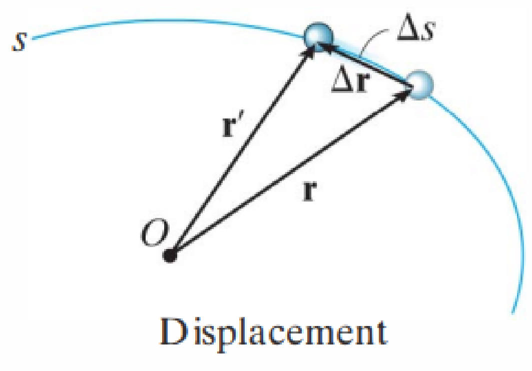


2.3.2 Displacement

During a small time interval Δt, the particle moves a distance Δs. Then, the displacement $\Delta\vec{r}$.

$$\Delta\vec{r} = \vec{r}'(t) - \vec{r}(t)$$

$$\vec{r}'(t) = \vec{r}(t) + \Delta\vec{r}$$



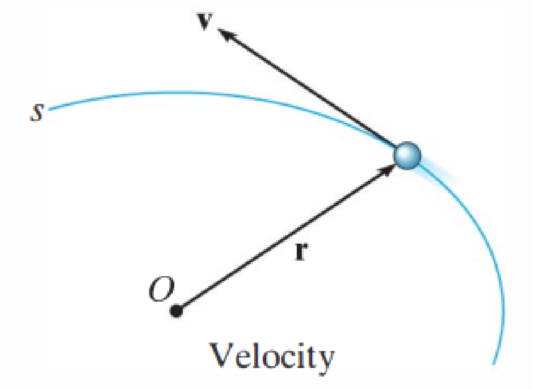
2.3.3 Velocity

Average velocity

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

Instantaneous velocity

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{\vec{r}} \quad \Rightarrow \quad \vec{v} = \vec{v}(t) \tag{2 - 4}$$



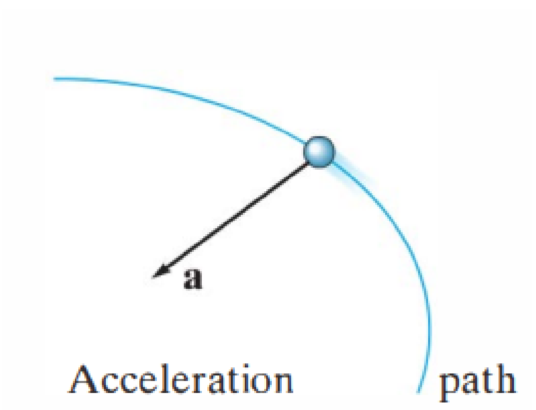
2.3.4 Acceleration

Average acceleration

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

Instantaneous acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{\vec{v}} = \frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}} \quad \Rightarrow \quad \vec{a} = \vec{a}(t) \tag{2 - 5}$$

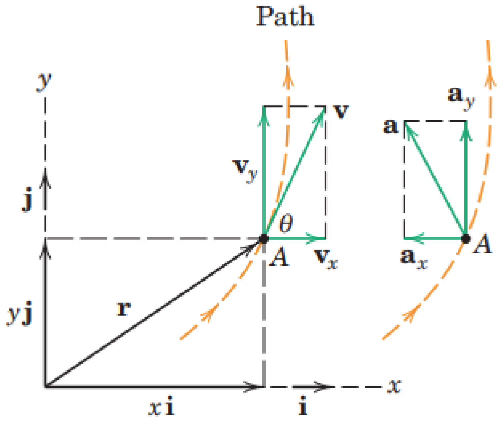


End of Section 2.3

Chapter Two: Kinematics of Particles Section Four: Rectangular Coordinates (x-y)

2.4 Rectangular Coordinates (x-y)

2.4.1 Vector Representation



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2.4.2 Position ($\vec{r}(t)$)

$$\vec{r} = x\hat{i} + y\hat{j}$$

At any given moment, the magnitude of the position:

$$r = \sqrt{(x)^2 + (y)^2}$$

At any given moment, the angle which the position vector make with the x-axis:

$$\theta = tan^{-1}\left(\frac{y}{x}\right)$$

2.4.3 Velocity ($\vec{v}$)

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \dot{\vec{r}}(t) = \dot{x}(t)\hat{i} + \dot{y}(t)\hat{j} = v_x(t)\hat{i} + v_y(t)\hat{j}$$

At any given moment, the magnitude of the velocity:

$$v = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{(\dot{x})^2 + (\dot{y})^2}$$

At any given moment, the angle which the velocity vector make with the x-axis:

$$\theta = tan^{-1}\left(\frac{v_y}{v_x}\right)$$

2.4.4 Acceleration ($\vec{a}$)

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \dot{\vec{v}}(t) = \frac{d^2\vec{r}(t)}{dt^2} = \ddot{\vec{r}}(t) = \ddot{x}(t)\hat{i} + \ddot{y}(t)\hat{j} = a_x(t)\hat{i} + a_y(t)\hat{j}$$

At any given moment, the magnitude of the acceleration:

$$a = \sqrt{(a_x)^2 + (a_y)^2} = \sqrt{(\ddot{x})^2 + (\ddot{y})^2}$$

At any given moment, the angle which the acceleration vector make with the x-axis:

$$\theta = tan^{-1}\left(\frac{a_y}{a_x}\right)$$

Example 1:

At time $t = 0$ sec, the position vector of a particle moving in the x-y plane is $\vec{r}_1 = 5\hat{i}$ m. By time $t = 0.02$ sec, its position vector has become $\vec{r}_2 = 5.1\hat{i} + 0.4\hat{j}$ m. Determine the magnitude v_{avg} of its average velocity during this interval and the angle made by the average velocity with the x-axis.

ans. $\vec{v}_{avg} = 5\hat{i} + 20\hat{j}$ m/sec and $v_{avg} = 20.6$ m/sec and $\theta = 75.6^o$

Example 2:

A particle moving in the x-y plane has a velocity at time $t = 6$ *sec* given by $\vec{v_1} = 4\hat{i} + 5\hat{j}$ and at $t = 6.1$ *sec* its velocity has become $\vec{v_2} = 4.3\hat{i} + 5.4\hat{j}$. Calculate the magnitude a_{avg} of its average acceleration during the 0.1 *sec* interval and the angle it makes with the x-axis.

ans. $\vec{a}_{avg} = 3\hat{i} + 4\hat{j}\ m/sec^2$ and $a_{avg} = 5\ m/sec^2$ and $\theta = 53.1^o$

Example 3:

The curvilinear motion of a particle is defined by $v_x = 50 - 16t$ and $y = 100 - 4t^2$, where v_x is in meters per second, y is in meters, and t is in seconds. It is also known that $x = 0$ when $t = 0$. Determine its velocity and acceleration when the position $y = 0$ is reached.

ans. $t = 5\ sec$ and $\vec{v} = -30\hat{i} - 40\hat{j}\ m/sec$ and $\vec{a} = -16\hat{i} - 8\hat{j}\ m/sec^2$

Example 4:

The x-coordinate of a particle in curvilinear motion is given by $x = 3t^2 - 3t$ where x is in feet and t is in seconds. The y-component of acceleration in feet per second squared is given by $a_y = 4t$. If the particle has y-components $y = 0$ and $\dot{y} = 4$ ft/sec when $t = 0$, find the magnitudes of the velocity v and acceleration a when $t = 2\ sec$.

ans. $v = 15\ ft/sec$ and $a = 10\ ft/sec^2$

Example 5:

The y-coordinate of a particle in curvilinear motion is given by $y = 4t^3 - 3t$, where y is in inches and t is in seconds. Also, the particle has an acceleration in the x-direction given by $a_x = 12t\ in/sec^2$. If the velocity of the particle in the x-direction is 4 in/sec, when $t = 0\ sec$, calculate the magnitudes of the velocity v and acceleration a of the particle when $t = 1\ sec$. Construct $\vec{v}$ and $\vec{a}$ in your solution.

ans. $v = 13.5\ in/sec$ and $\theta_v = 42^o$ and $a = 26.8\ in/sec^2$ and $\theta_a = 63.4^o$

2.4.5 Projectile Motion

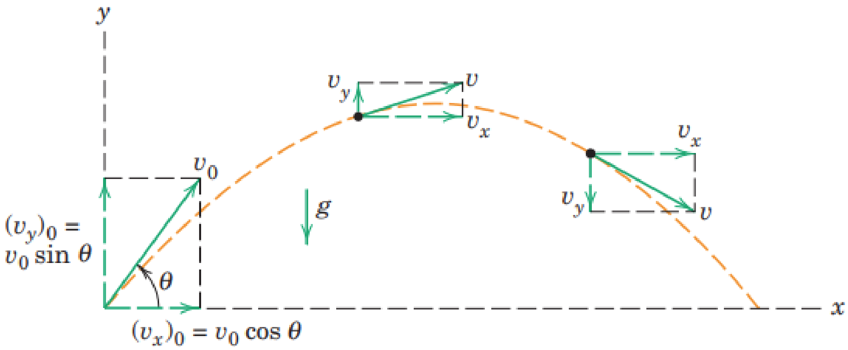


Figure 2/8

$$\vec{a} = 0\hat{i} + g\hat{j} \quad \Rightarrow \quad a_x = 0 \quad (x - direction) \quad and \quad a_y = -g \quad (y - direction)$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} \quad \Rightarrow \quad v_x = vcos(\theta) \quad (x - direction) \quad and \quad v_y = vsin(\theta) \quad (y - direction)$$

X - coordinate:

$$dv_x = a_x dt \quad \rightarrow \quad \int_{(v_x)_0}^{v_x} dv_x = \int_{t_0}^{t} a_x \, dt \quad \rightarrow \quad v_x = (v_x)_0 + a_x(t - t_0)$$

$$v_x = (v_x)_0$$

$$dx = v_x dt \quad \rightarrow \quad \int_{x_0}^{x} dx = \int_{t_0}^{t} ((v_x)_0 + a_x t) \, dt \quad \rightarrow \quad x - x_0 = (v_x)_0(t - t_0) + \frac{1}{2}a_x(t^2 - t_0^2)$$

$$x = x_0 + (v_x)_0(t - t_0)$$

$$v_x dv_x = a_x dx \quad \rightarrow \quad \int_{(v_x)_0}^{v_x} v_x \, dv_x = \int_{x_0}^{x} a_x \, dx \quad \rightarrow \quad v_x^2 = (v_x)_0^2 + 2a_x(x - x_0)$$

$$v_x = (v_x)_0$$

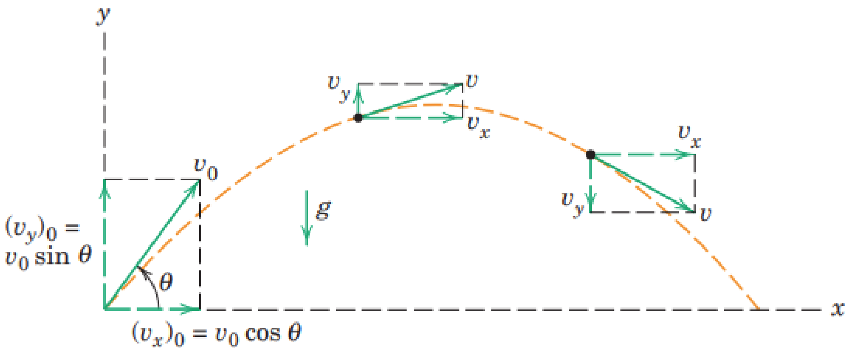


Figure 2/8

$$\vec{a} = 0\hat{i} + g\hat{j} \quad \Rightarrow \quad a_x = 0 \quad \textcolor{red}{(x - direction)} \quad and \quad a_y = -g \quad \textcolor{red}{(y - direction)}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} \quad \Rightarrow \quad v_x = vcos(\theta) \quad \textcolor{red}{(x - direction)} \quad and \quad v_y = vsin(\theta) \quad \textcolor{red}{(y - direction)}$$

Y - coordinate:

$$dv_y = a_y dt \quad \rightarrow \quad \int_{(v_y)_0}^{v_y} dv_y = \int_{t_0}^{t} a_y \, dt \quad \rightarrow \quad v_y = (v_y)_0 + a_y(t - t_0)$$

$$v_y = (v_y)_0 - g(t - t_0)$$

$$dy = v_y dt \quad \rightarrow \quad \int_{y_0}^{y} dy = \int_{t_0}^{t} ((v_y)_0 + a_y t) \, dt \quad \rightarrow \quad y - y_0 = (v_y)_0(t - t_0) + \frac{1}{2}a_y(t^2 - t_0^2)$$

$$y = y_0 + (v_y)_0(t - t_0) - \frac{1}{2}g(t^2 - t_0^2)$$

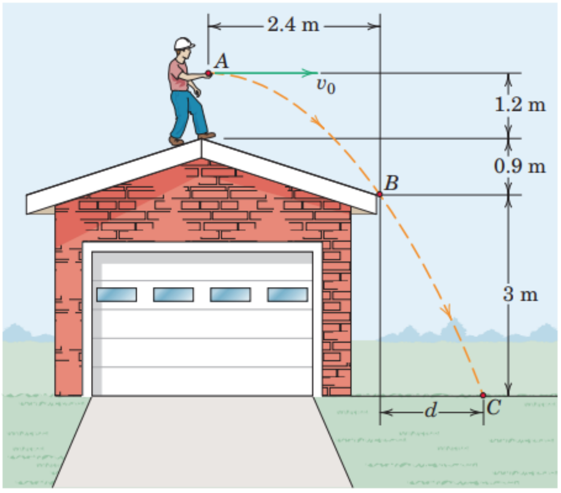
$$v_y dv_y = a_y dy \quad \rightarrow \quad \int_{(v_y)_0}^{v_y} v_y \, dv_y = \int_{y_0}^{y} a_y \, dy \quad \rightarrow \quad v_y^2 = (v_y)_0^2 + 2a_y(y - y_0)$$

$$v_y^2 = (v_y)_0^2 - 2g(y - y_0)$$

Example 6:

A roofer tosses a small tool to the ground. What minimum magnitude of horizontal velocity is required to just miss the roof corner B? Also determine the distance d.

ans. $v_{ox} = 3.67\ m/sec$ and $d = 1.34\ m$



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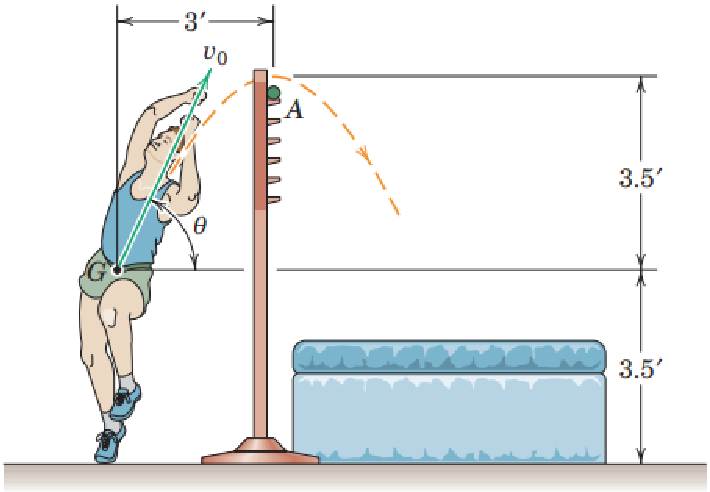
Ans.

Example 7:

The center of mass G of a high jumper follows the trajectory shown. Determine the component , measured in the vertical plane of the figure, of his takeoff velocity and angle if the apex of the trajectory just clears the bar at A. (In general, must the mass center G of the jumper clear the bar during a successful jump?).

ans. $v_o = 16.3\ m/sec$ and $\theta = 66.8^o$



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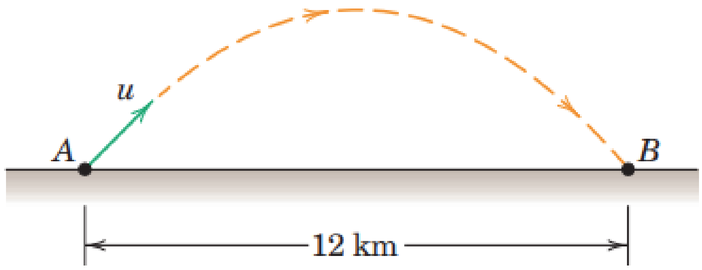
Ans.

Example 8:

Calculate the minimum possible magnitude u of the muzzle velocity which a projectile must have when fired from point A to reach a target B on the same horizontal plane **12 km** away.

ans. $u = 343\ m/sec$



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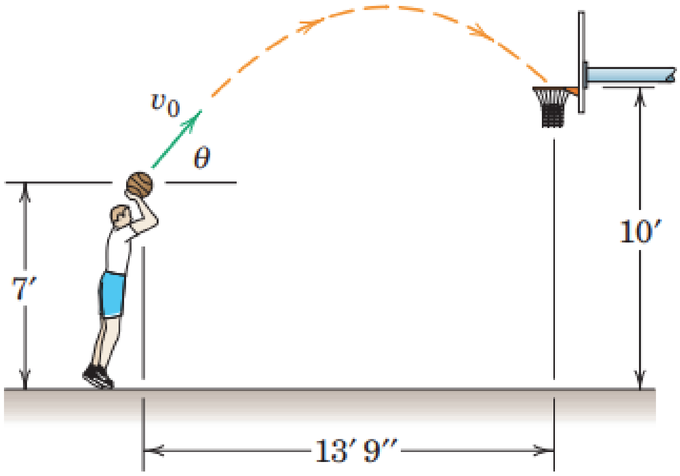
Ans.

Example 9:

The basketball player likes to release his foul shots with an initial speed $v_o = 25$ ft/sec. What value(s) of the initial angle θ will cause the ball to pass through the center of the rim? Neglect clearance considerations as the ball passes over the front portion of the rim.

ans. $\theta = 63.7^o$ or $\theta = 38.58^o$



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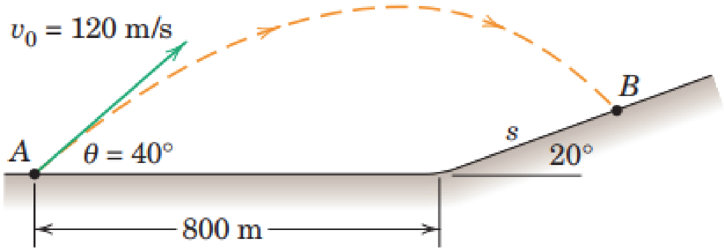
Ans.

Example 10:

A projectile is launched from point A with the initial conditions shown in the figure. Determine the slant distance s which locates the point B of impact. Calculate the time of flight t.

ans. $s = 455\ m$ or $t = 13.3\ sec$



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Ans.

End of Section 2.4