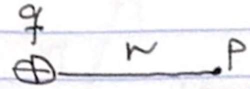
Electric Field

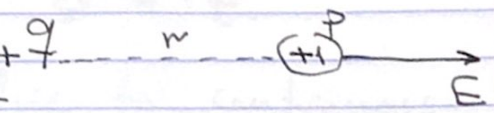
Electric Field due to point charge q

$$\boxed{E = \frac{kq}{r^2}}$$

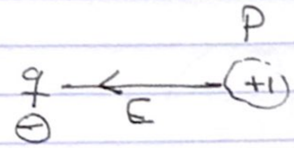


direction of $\vec{E}$ due to q

(1) q is positive
E away from it

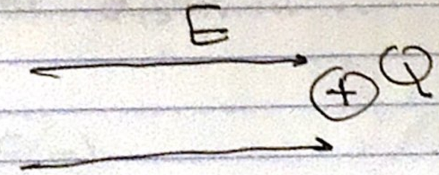


(2) q is negative $-q$
E towards it



Electric Force and Electric Field

$$\boxed{\vec{F}_Q = Q \cdot \vec{E}}$$



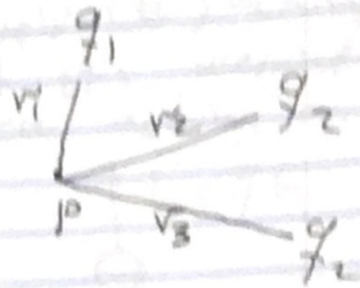
direction of F and E

(1) F و E بنفس الاتجاه إذا كانت Q موجبة

(2) F و E باتجاهين متعاكسين إذا كانت Q سالبة

وحدة E هي N/C

Electric Field due to many charges at point P



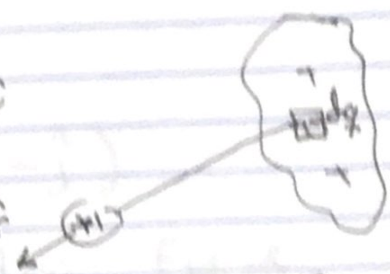
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}$$

$$\vec{E} = \sum \vec{E}_i$$

vector sum

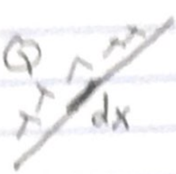
Electric Field due to continuous charge Distributions

$$\vec{E} = k \int \frac{dq}{r^2} \hat{r}$$



dq has 3 cases

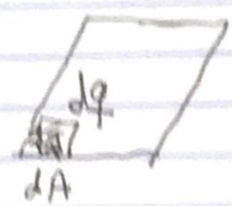
1 Linear charge distribution (λ)



$$dq = \lambda \, dx$$

$$\lambda = \frac{Q}{L} \quad (C/m)$$

2 surface charge distribution



$$dq = \sigma \, dA$$

$$\sigma = \frac{Q}{A} \quad (C/m^2)$$

3 volume charge distribution

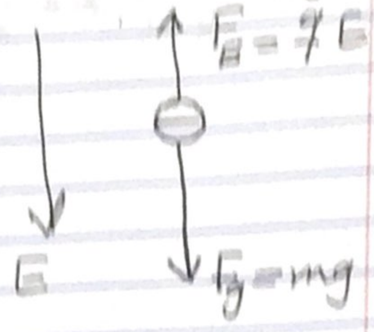
$$\rho = \frac{Q}{V}$$

$$dq = \rho \, dV$$

$$\rho : C/m^3$$

Examples

(1) Calculate the Electric Field (E) acting on an electron so that it's weight equal the Electric Force acting on it.



$$F_E = F_g$$

$$qE = mg$$

$$E = \frac{mg}{q}$$

$$= \frac{9.11 \times 10^{-31} \times 10}{1.6 \times 10^{-19}}$$

$$= 6 \times 10^{-11}\ N/C\ (-\hat{j})$$

Calculate the Electric Force acting on the electron $F_E = ?$

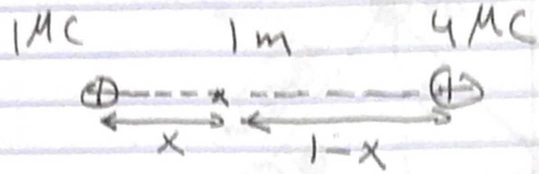
$$F_E = qE$$

$$= 1.6 \times 10^{-19} \times 6 \times 10^{-11}$$

$$= 9.6 \times 10^{-30}\ N\ (+\hat{j})$$

Ex Find the point where the Electric Field equal zero.

case 1



$$E_1 = E$$

$$\frac{\cancel{K} q_1}{r_1^2} = \frac{\cancel{K} q_2}{r_2^2}$$

$$\frac{1}{x^2} = \frac{4}{(1-x)^2}$$

$$\sqrt{(1-x)^2} = \sqrt{4x^2}$$

$$1-x = 2x$$

$$3x = 1$$

$$x = \frac{1}{3}\ m$$

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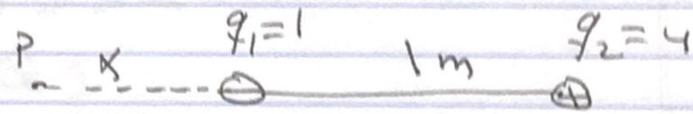
case 2



$$E_1 = E_2$$

$$\frac{\cancel{K} q_1}{x^2} = \frac{\cancel{K} q_2}{(1+x)^2}$$

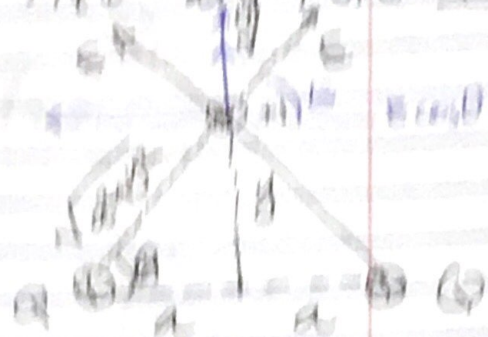
$$\sqrt{\frac{1}{x^2}} = \sqrt{\frac{4}{(1+x)^2}}$$

$$\frac{1}{x} = \frac{2}{1+x}$$

$$2x = 1+x$$

$$x = 1\ m$$

Q4 Find the Electric Field at point P



$$E = k\frac{q}{r^2}$$

$$E = k\frac{Q}{a^2+y^2}$$

$$E_p = 2E\sin\theta$$

$$\sin\theta = \frac{y}{\sqrt{a^2+y^2}}$$

$$= \frac{2kQ}{a^2+y^2}\cdot\frac{y}{\sqrt{a^2+y^2}}$$

$$\vec{E}_p = \frac{2kQy}{(a^2+y^2)^{3/2}}\,\hat{j}$$

when $y \gg a$

$$\vec{E}_p = \frac{2kQy}{y^3\left(1+\frac{a^2}{y^2}\right)^{3/2}}$$

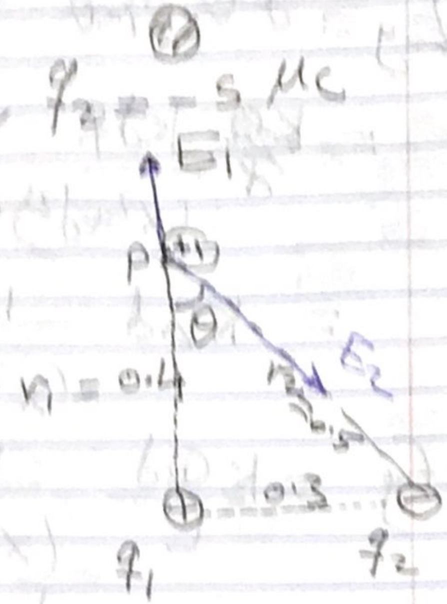
$$= \frac{2kQy}{y^3} = \frac{2kQ}{y^2}$$

هذا يا شباب لان الشحنتان تؤثران
كشحنة واحدة شدتها 2Q

Ex

given $q_1 = 7\mu c$, $q_2 = -5\mu c$

Calculate the Electric Field at point P due to q_1, q_2



$$E_1 = \frac{Kq_1}{r_1^2} = \frac{9\times10^9 \times 7\times10^{-6}}{(0.4)^2}$$

$$= 39.375\times10^4$$

$$E_2 = \frac{Kq_2}{r_2^2} = \frac{9\times10^9 \times 5\times10^{-6}}{(0.5)^2} = 18\times10^4$$

$$E_x = E_2\sin\theta$$

$$= (18\times10^4)(\cancel{0.8}\ 0.6) = \cancel{14.4\times10^4}\ 10.8\times10^4$$

$$E_y = E_1 - E_2\cos\theta$$

$$= (39.375 - 18\times0.8)\times10^4$$

$$= \cancel{28.575\times10^4}\ 24.975\times10^4$$

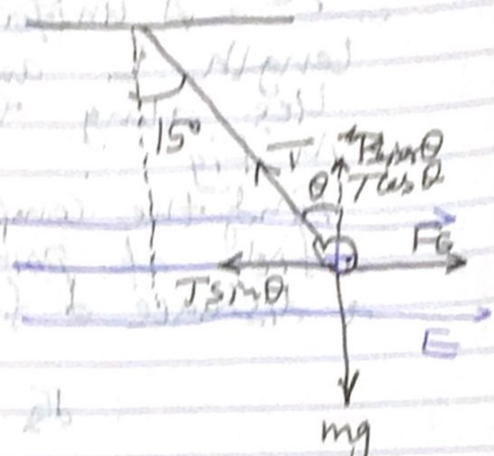
$$\vec{E} = (10.8\hat{i} + 24.975\hat{j})\times10^4$$

$$|E| = \sqrt{E_x^2+E_y^2} = 10^4\sqrt{(10.8)^2+(24.975)^2}$$

$$= 27.2 \ N/c$$

direction $\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = 66.6^\circ$

Ex: In the Fig. if the charge Q is in equilibrum in a uniform Electric Field $E = 500$ N/C. If the length of the wire is 20 cm, $m = 2$ gm, $\theta = 15^\circ$. Find Q



since Q is equilibrum

$$\Sigma F_x = 0 \longrightarrow F_E = T\sin\theta \quad (1)$$

$$\Sigma F_y = 0 \longrightarrow mg = T\cos\theta$$

$$\frac{QE}{mg} = \frac{T\sin\theta}{T\cos\theta}$$

$$\cancel{mg = QE\tan\theta}$$

$$\cancel{Q = \frac{mg}{E\tan\theta} = \frac{2\times10^{-3}\times10}{500\times\tan15}}$$

$$\cancel{Q = 1.49\times10^{-4}\ C}$$

$$QE = mg\tan\theta$$

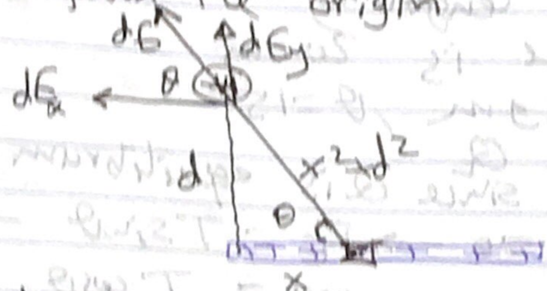
$$Q = \frac{mg\tan\theta}{E}$$

$$= \frac{2\times10^{-3}\times10\times\tan15}{500} = 1.07\times10^{-5}\ C$$

$$= 10.7\times10^{-6}$$

$$= 10.7\ \mu C$$

42 A uniformly charged rod of length L and total charge Q lies along the x axis as shown in Fig. Find the components of the Electric field at point P on the y axis a distance d from the origin.



$$dE_x = \frac{k\,dq}{} \; dE \cos\theta = \frac{k\,dq}{r^2}\cos\theta$$

$$\frac{k\,dq}{(x^2+d^2)} \cdot \frac{x}{(x^2+d^2)} \qquad dq = \lambda dx$$

$$= k\lambda dx \frac{x}{(x^2+d^2)^{3/2}} \qquad = \frac{Q}{L}dx$$

$$dE_x = \frac{kQ}{L}\frac{x\,dx}{(x^2+d^2)^{3/2}}$$

$$E_x = \frac{kQ}{L}\int_0^L \frac{x\,dX}{(x^2+d^2)} \qquad I = \int \frac{x\,dx}{(x^2+d^2)^{3/2}}$$

$$u = x^2+d^2$$

$$du = 2x\,dx$$

$$\frac{du}{2} = x\,dx$$

$$= \frac{kQ}{L}\left[\frac{-1}{\sqrt{x^2+d^2}}\right]_0^L \qquad I = \frac{1}{2}\int\frac{du}{u^{3/2}}$$

$$E_x = \frac{kQ}{L}\left[\frac{1}{d} - \frac{1}{\sqrt{L^2+d^2}}\right]$$

$$= \frac{1}{2}\frac{u^{-\frac{1}{2}}}{-\frac{1}{2}} = \frac{-1}{\sqrt{u}}$$

$$dE_y = dE \sin\theta$$

$$= \frac{K\,dq}{(x^2+d^2)} \cdot \frac{d}{(x^2+d^2)^{1/2}}$$

$$= K\lambda d \frac{1}{(x^2+d^2)^{3/2}}$$

$$dE_y = \frac{KQd}{L} \frac{1}{(x^2+d^2)^{3/2}}$$

$$E_y = \frac{KQ}{L} \int_0^L \frac{1}{(x^2+d^2)^{3/2}}$$

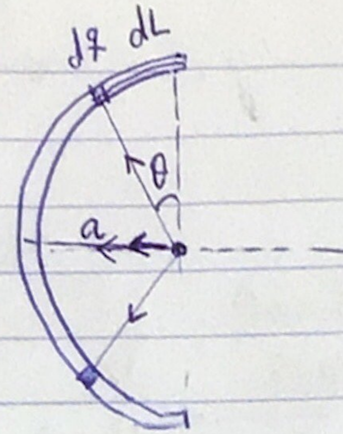
$$= \frac{KQ}{\cancel{L}} \left[\frac{\cancel{L}}{d^2\sqrt{L^2+d^2}} \right]$$

$$E_y = \frac{KQ}{d^2\sqrt{L^2+d^2}}$$

36/ $\sum dE_y = 0$

$dE_x = dE \sin\theta$

$dE = \frac{k\,dq}{r^2} \sin\theta$



$= \int \frac{k \lambda a\, d\theta \sin\theta}{a^2}$

$= k \lambda a$

$= \frac{k\lambda}{a} \int_0^{\pi} \sin\theta$

$L = 14.0$ cm

$q = -7.50\,\mu C$

$dq = \lambda\, dL$

$dq = \lambda a\, d\theta$

$= \frac{k\lambda}{a} \left(-\cos\theta \Big|_0^{\pi} \right)$

$= \frac{-k\lambda}{a} (-1-1)$

$= \frac{2k\lambda}{a}$

$\lambda = \frac{q}{L}$

$= \frac{+7.50 \times 10^{-6}}{14.0 \times 10^{-2}}$

$= \frac{2k \frac{q}{L}}{\frac{L}{\pi}}$

$= \frac{2kq}{L} \times \frac{\pi}{L}$

$\pi a = 14$

$\pi a = L$

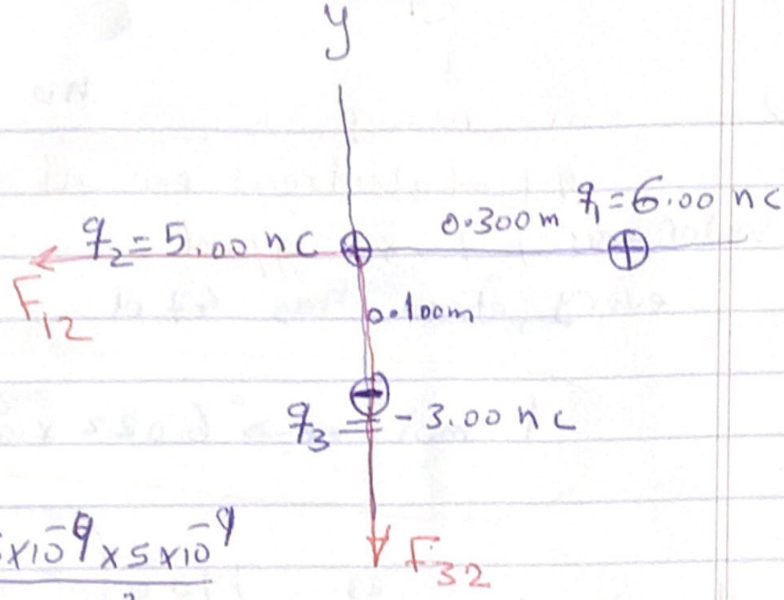
$a = \frac{L}{\pi}$

$= \frac{2 \times 9 \times 10^{9} \times 7.5 \times 10^{-6} \times 3.14}{(14)^2 \times 10^{-4}}$

$= 21.6 \,(-\hat{i})$ MN/C

9/

$F_2 = ??$



$$F_{12} = \frac{k q_1 q_2}{r_{12}^2}$$

$$= \frac{9\times10^{9} \times 6\times10^{-9} \times 5\times10^{-9}}{(0.300)^2}$$

$$= 3\times10^{-6}\ N\,(-\hat{i})$$

$$= 30\times10^{-7}\ N\,(-\hat{i})$$

$$F_{32} = \frac{9\times10^{9} \times 3\times10^{-9} \times 5\times10^{-9}}{(0.1)^2}$$

$$= 135\times10^{-7}\ N\,(-\hat{j})$$

$$\vec{F} = 30\times10^{-7}(-\hat{i}) + 135\times10^{-7}(-\hat{j})$$

$$|F| = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(30)^2 + (135)^2}\ \times10^{-7}$$

$$= 138.3\times10^{-7}\ N$$

$$\tan\phi = \frac{F_y}{F_x}$$

$$= \frac{135}{30}$$

$$= 4.5$$

$$\phi = 77.5^\circ$$