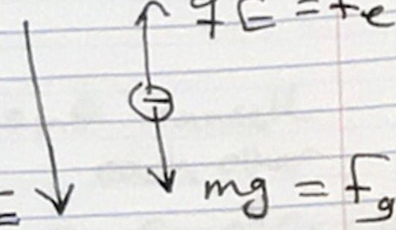
23 chapter تكملة أسئلة

Ex 23.5 A Suspended Water Droplet

A water droplet of mass 3×10^{-12} kg is located in the air near the ground during a stormy day. An atmospheric electric Field of magnitude 6×10^{3} N/C points vertically downward. The droplet remains suspended at rest in the air. what is the electric charge of the droplet.



$$F_e = F_g$$

$$qE = mg$$

$$q = \frac{mg}{E}$$

$$= \frac{3\times10^{-12}\times10}{6\times10^{3}}$$

$$= 5\times10^{-15}\ C$$

charge must be negative

E (−y)

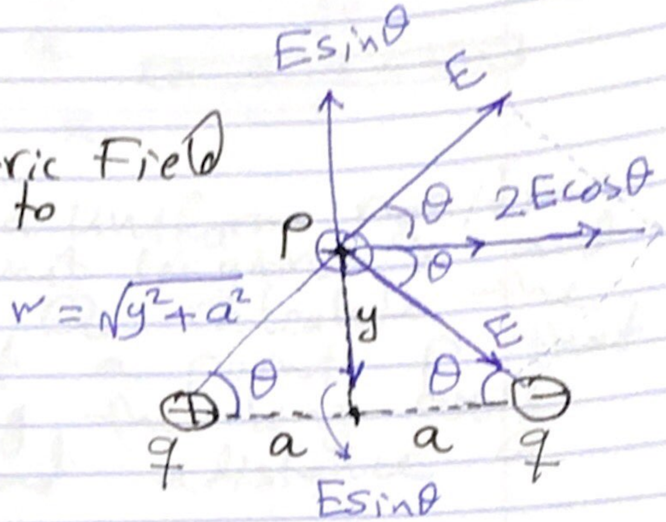
F_e (+y)

F_g (−y)

Example 23.6

702

Calculate the Electric Field at point P due to charge +q, −q



$E = \frac{kq}{r^2}$

$E = \frac{kq}{y^2+a^2}$

~~$E_P = 2E\cos\theta$~~

~~$= 2\frac{kq}{y^2+a^2} \cdot \frac{a}{(y^2+a^2)^{1/2}}$~~

E sinθ cancell each other

$E_y = E\sin\theta - E\sin\theta$

$= 0$

$E_x = E\cos\theta + E\cos\theta$

$= 2E\cos\theta$

$= 2\frac{kq}{y^2+a^2} \cdot \frac{a}{(y^2+a^2)}$

$E_x = \frac{(2aq)K}{(y^2+a^2)^{3/2}}$

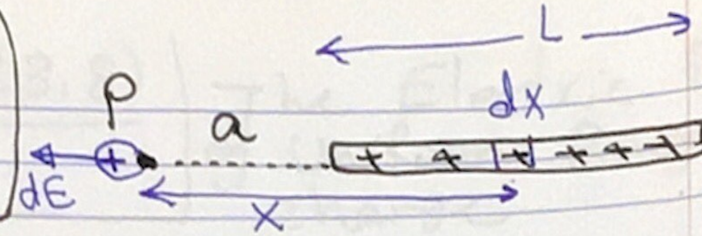
if $y >> a \rightarrow E_x = \frac{2aq\ K}{y^3(1+\frac{a^2}{y^2})^{3/2}}$

$E_x = \frac{2aq}{y^3} K$

(16)

Ex 23.7

705



A rod of length L has a uniform positive charge per unit length λ and a total charge Q. Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end.

ملاحظات : نقع ال rod على x axis

نختار مقطع صغير طوله dx

شحنة المقطع الصغير dx تساوي dq

حيث $dq = \lambda dx$

$$dq = \lambda dx$$

$$dE = k\frac{dq}{x^2} = k\frac{\lambda dx}{x^2}$$

$$\lambda = \frac{Q}{L}$$

$$dE = k\frac{Q}{L}\frac{dx}{x^2}$$

$$E = k\int\frac{dq}{r^2}\hat{r}$$

$$\hat{r} = -\hat{i}$$

$$E = \int_a^{a+L}\frac{kQ}{L}\frac{dx}{x^2}(-\hat{i})$$

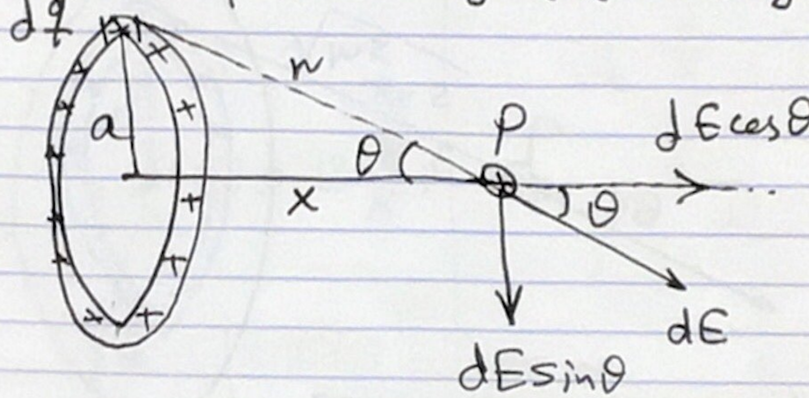
$$= \frac{kQ}{L}(-\hat{i})\int_a^{a+L}\frac{1}{x^2}dx$$

$$= \frac{kQ}{L}(-\hat{i})\left(\frac{-1}{x}\right)\Big|_a^{a+L}$$

$$\vec{E}_p = \frac{kQ}{L}\left(\frac{1}{a} - \frac{1}{a+L}\right) = \frac{kQ}{a(a+L)}(-\hat{i})$$

Example (23.8) 706 | The Electric Field of Uniform Ring of charge

A ring of radius a carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring.



$$dE_x = dE\cos\theta \qquad r^2 = x^2 + a^2$$

$$= \frac{k\,dq}{r^2}\cos\theta$$

$$= k\frac{dq}{x^2+a^2}\cos\theta \qquad \cos\theta = \frac{x}{r}$$

$dE\sin\theta$: يلغو بعض

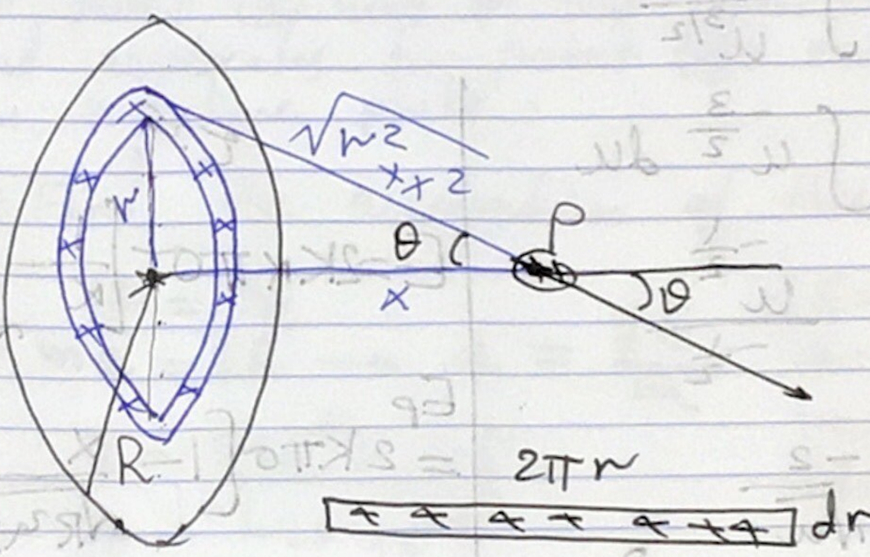
$$dE_x = \frac{k\,dq}{(x^2+a^2)} \cdot \frac{x}{(x^2+a^2)^{1/2}}$$

$$E_x = \frac{kx}{(x^2+a^2)^{3/2}}\int dq = \frac{kxQ}{(a^2+x^2)^{3/2}}$$

Example 23.9

707

A disk of radius R has a Uniform surface charge density σ. Calculate the electric field at a point P that lies along the central perpendicular axis of the disk and a distance x from the center of the disk.



$$dq = \sigma\, dA$$
$$= \sigma\,(2\pi r\, dr) = 2\pi\sigma\, r\, dr$$

dE_y : cancell each other

$$dE_x = dE\cos\theta = \frac{K\,dq}{r^2+x^2}\cdot\frac{x}{(r^2+x^2)^{1/2}}$$

$$E_x = Kx\pi\sigma\int_0^R \frac{2r\,dr}{(r^2+x^2)^{3/2}}$$ تكامل بالتعويض

$$E_x = 2\pi K\sigma\left[1-\frac{x}{(R^2+x^2)^{1/2}}\right]$$

R تصنيع التكامل

$$I = \int_0^R \frac{2r\,dr}{(r^2+x^2)^{3/2}}$$

let $u = r^2+x^2$

$du = 2r\,dr$

$$I = \int \frac{du}{u^{3/2}}$$

$$= \int u^{-\frac{3}{2}}\,du$$

$$= \frac{u^{-\frac{1}{2}}}{-\frac{1}{2}}$$

$$= \frac{-2}{\sqrt{u}}$$

$$= \frac{-2}{\sqrt{r^2+x^2}}\Bigg|_0^R$$

$$E = 2K \times \pi\sigma \cdot \left[\frac{1}{x} - \frac{1}{\sqrt{R^2+x^2}}\right]$$

$$E_p = 2K\pi\sigma\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]$$

$$I = 2\left[\frac{1}{x} - \frac{1}{\sqrt{R^2+x^2}}\right]$$

when $x << R$

$$E = 2\pi K\sigma = 2\pi \cdot \frac{1}{4\pi\epsilon_0} \cdot \sigma$$

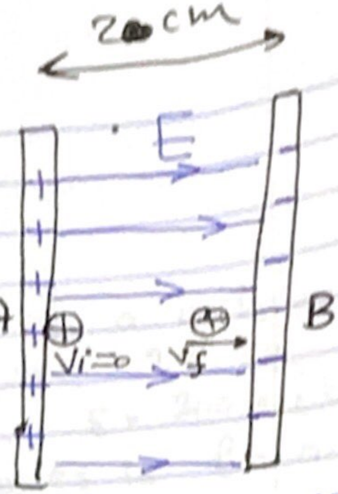
$$E = \frac{\sigma}{2\epsilon_0}$$

A

EX 23.10
711

Uniform Electric field ($\vec{E}$) is directed along the x-axis between Parallel plates of charge seperated by a distance (d) as shown in Fig.

A positive point charge (q) of mass m is released from west at point (A) next to the positive plate and accelerates to point (B) next to the negative plate.



(A) Find the acceleration of the charge

$E = 1\times10^{4}$ N/C
$q = 5\times10^{-6}$ C
$m = 1.25\times10^{-8}$
$d = 20$ cm

$$F = F_e$$
$$ma = qE \longrightarrow a = \frac{qE}{m}$$
$$a = \frac{5\times10^{-6}\times1\times10^{4}}{1.25\times10^{-8}}$$
$$a = 4\times10^{10}\ m/s^2.$$

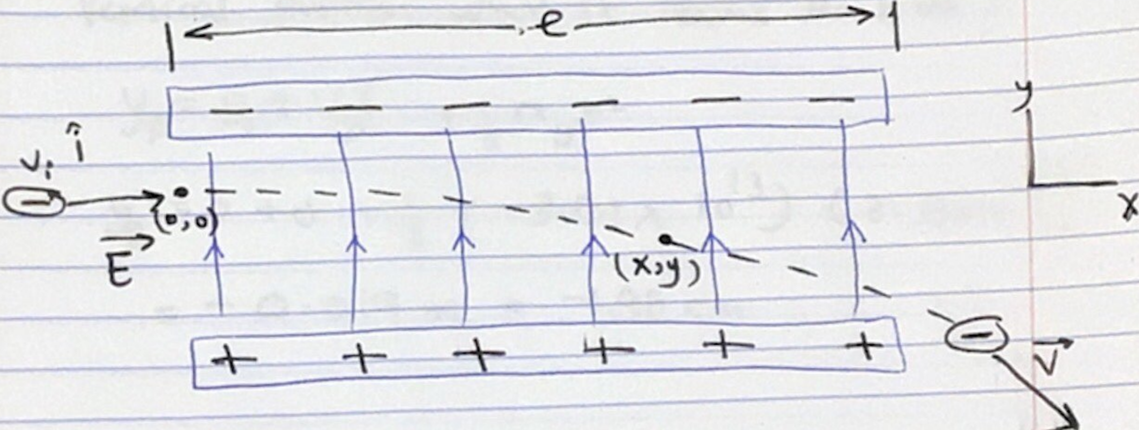
(B) Find the final velocity at B

$$V_B^2 = V_A^2 + 2ad$$
$$= 0 + 2\times4\times10^{10}\times2\times10^{-2}$$
$$V_B = 4\times10^{4}\ m/s.$$

Example 23.11 / 7/2

An electron enters the region of a uniform electric field as shown in Figure 23.24, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $\ell = 0.100$ m.

A) Find the acceleration of the electron while it is in the electric field



B) Assuming the electron enters the field at time $t=0$, find the time at which it leaves the field

---

A)

$$a_y = \frac{eE}{m_e}$$

$$= -\frac{(1.6 \times 10^{-19})(200)}{9.11 \times 10^{-31}} = -3.51 \times 10^{13} \text{ m/s}^2$$

B) $x_f = x_p + v_x t \rightarrow t = \frac{x_f - x_p}{v_x}$

$t = \frac{\ell - 0}{v_x} = \frac{0.100}{3.00 \times 10^6}$

$= 3.33 \times 10^{-8}$ s

C) Assuming the vertical position of the electron as it enters the field is $y_i = 0$, what is its vertical position when it leaves the field?

$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$

$y_f = 0 + 0 + \frac{1}{2}(-3.51 \times 10^{13})(3.33 \times 10^{-8})^2$

$= -0.019$ m $= -1.95$ cm