chapter 24

Gauss's Law

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Discussion problems
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Home work problems
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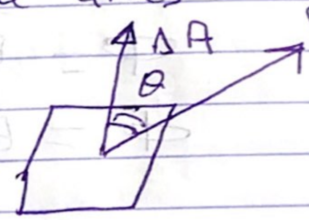
General definition of the electric flux

The electric flux (Φ_E) is proportional to the total number of the lines through this area



$\Delta\Phi_E = E \cdot \Delta A$

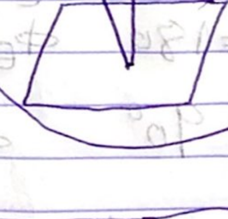
$\Delta\Phi = E\,\Delta A \cos\theta$

for the total surface

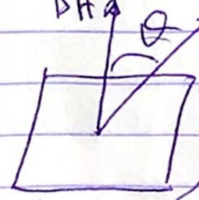
$\Phi_E = \int \vec{E} \cdot d\vec{A}$

Φ_E → Flux through closed surface

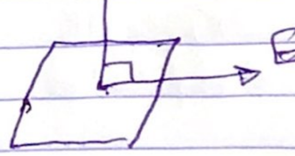
1 – Φ_E is negative when the field lines are entering the surface



2 – Φ_E is positive the lines are leaving the surface



3 – $\Delta\Phi = 0$ when E is parallel to the surface



Note

The net flux: is equal the number of lines leaving the surface minus the number of lines entering the surface.

SI units of Φ_E is

$N \cdot m^2/C \equiv$ Weber

When E is Uniform مجال كهربائي منتظم

$\Phi = \vec{E} \cdot \vec{A}$

$\Phi = EA \cos\theta$

E: Electric field (N/C)
A: surface area (m^2)
θ: angle between E and ⊥ A

$\vec{A} = A\hat{n}$

A: Area
$\hat{n}$: normal

$\hat{n}$: unit vector normal to the surface of area A

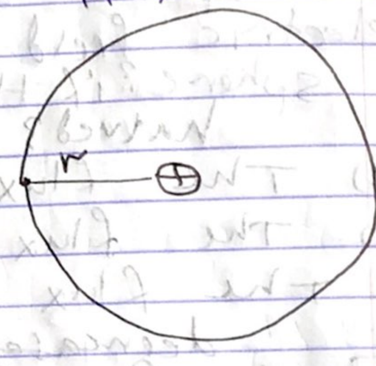
$\theta = 0 \rightarrow \Phi_E = EA$ E going out of the surface

$\theta = 180$ $\Phi_E = -EA$ E going inside the surface

$\theta = 90^\circ$ $\Phi_E = 0$

Example : Flux through a sphere with charge at center

what is the electric flux through a sphere that has a radius of 1 m and has a charge of +1 μC at its center.



$E = \frac{kq}{r^2}$

$= \frac{9\times10^9 \times 1\times10^{-6}}{1}$

$= 9\times10^3$ N/C

$\Phi = E \cdot A$

$= 9\times10^3 \times 12.56$

$= 113040 \ \frac{N}{C} \cdot m^2$

$A = 4\pi r^2$

$= 4(3.14)(1)^2$

$= 12.56$

or

$\Phi = EA$

$= \frac{kq}{r^2} 4\pi r^2$

$= 4\pi k q$

$= 4\pi \times \frac{1}{4\pi\epsilon_0} \cdot q$

$\Phi = \frac{q}{\epsilon_0}$

$= \frac{1\times10^{-6}}{8.85\times10^{-12}}$

Quiz (24-1)

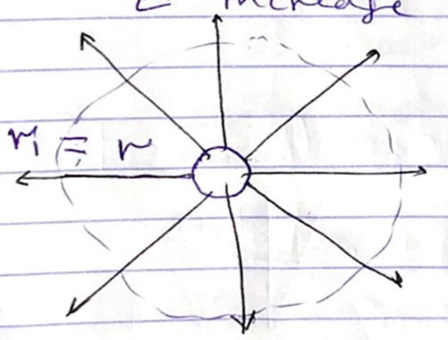
In the last example

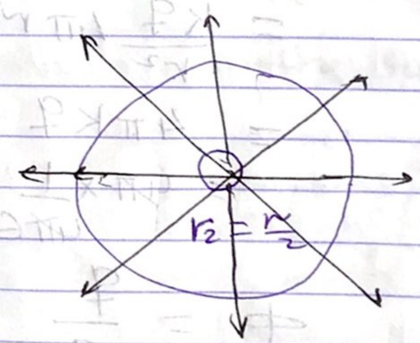
what happens to the flux through the sphere and the magnitude of the electric field at the surface of the sphere? if the radius of the sphere is halved?

(a) The flux and field both increase
(b) The flux and field both decrease
(c) The flux increase and the field decreases
(d) The flux remains the same and the electric field increases.

flux remains the same because the number of lines the same

E increase because r decrease





$E_1 = \frac{kq}{r^2}$

$E_2 = \frac{kq}{(\frac{r}{2})^2}$

$= 4\frac{kq}{r^2}$

$\phi_r = \phi$

$E_1 = E$

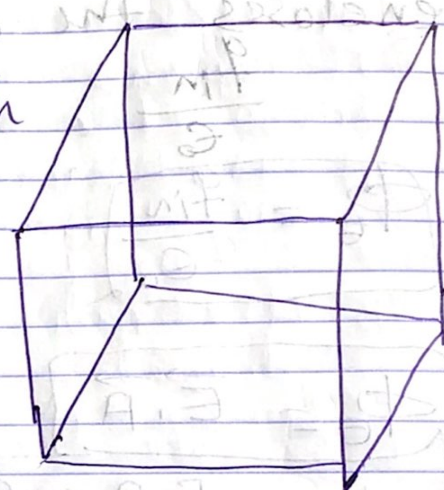
$\phi_2 = \phi$

$E_2 = 4E$

Example 24.1 / 728

Flux through a Cube

Consider a Uniform electric field E oriented in the x-direction. Find the net electric flux through the surface of a cube of edge l ($A = l^2$)



$\Phi_{net} = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6$

$\Phi_1 = E A \cos 90 = 0$

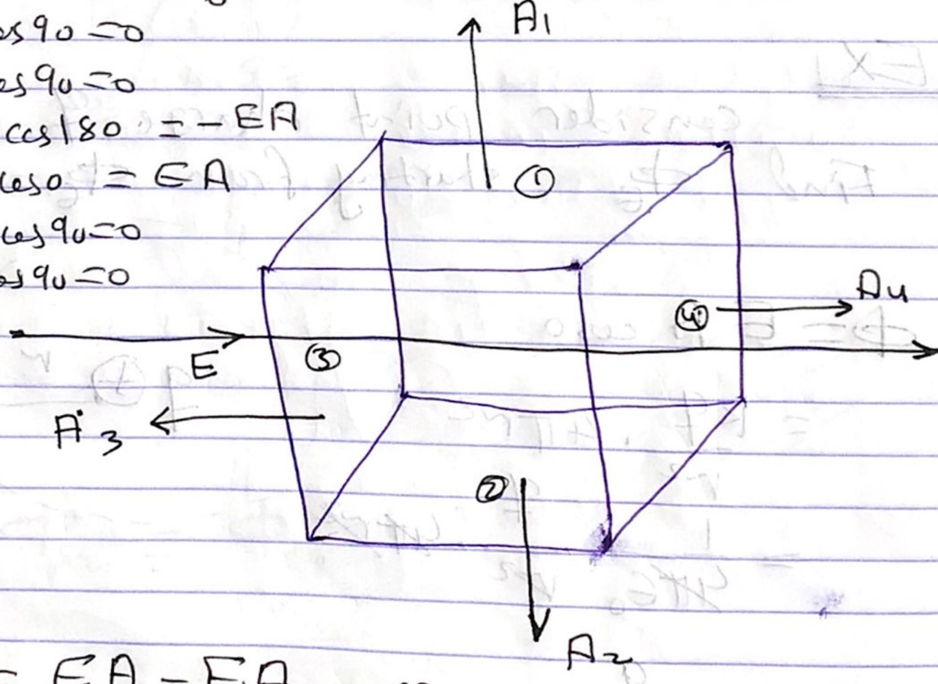
$\Phi_2 = E A \cos 90 = 0$

$\Phi_3 = E A \cos 180 = -EA$

$\Phi_4 = E A \cos 0 = EA$

$\Phi_5 = E A \cos 90 = 0$

$\Phi_6 = E A \cos 90 = 0$



$\Phi_{net} = EA - EA = 0$

The total flux through the cube is zero

The total flux through any closed surface is uniform Electric field is zero.

Gauss's Law

The net flux Φ_E through a Gaussian surface (a surface that encloses the charge) is equal $\frac{q_{in}}{\epsilon_0}$

$$\boxed{\Phi_E = \frac{q_{in}}{\epsilon_0}}$$

q_{in} : q inside

$\epsilon_0 = 8.85 \times 10^{-12}$

$$\boxed{\Phi_E = \vec{E} \cdot \vec{A}}$$

$$\boxed{\Phi_E = EA\cos\theta}$$

Ex

consider point charge q

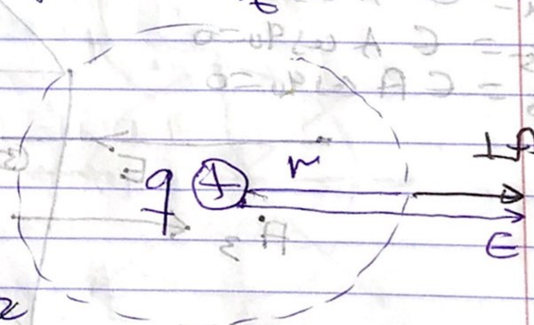
Find Φ_E starting from $\Phi_E = EA\cos\theta$

$$\Phi = EA\cos 0$$

$$= \frac{kq}{r^2} \cdot 4\pi r^2$$

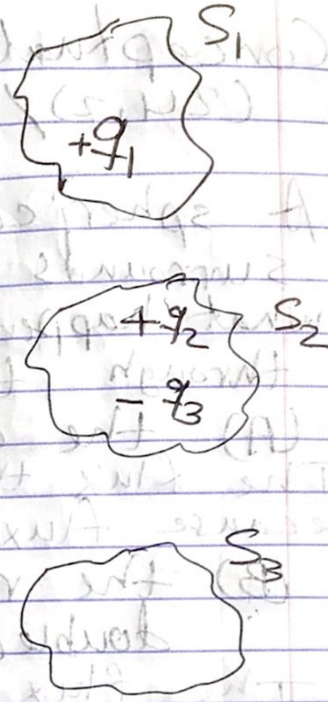
$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot 4\pi r^2$$

$$= \frac{q}{\epsilon_0}$$



EX

Calculate Φ_E for each surface



$$\Phi_{S_1} = \frac{q_1}{\epsilon_0}$$

$$\Phi_{S_2} = \frac{q_2 - q_3}{\epsilon_0}$$

$$\Phi_{S_3} = 0$$

when there is no charge inside the net flux is zero

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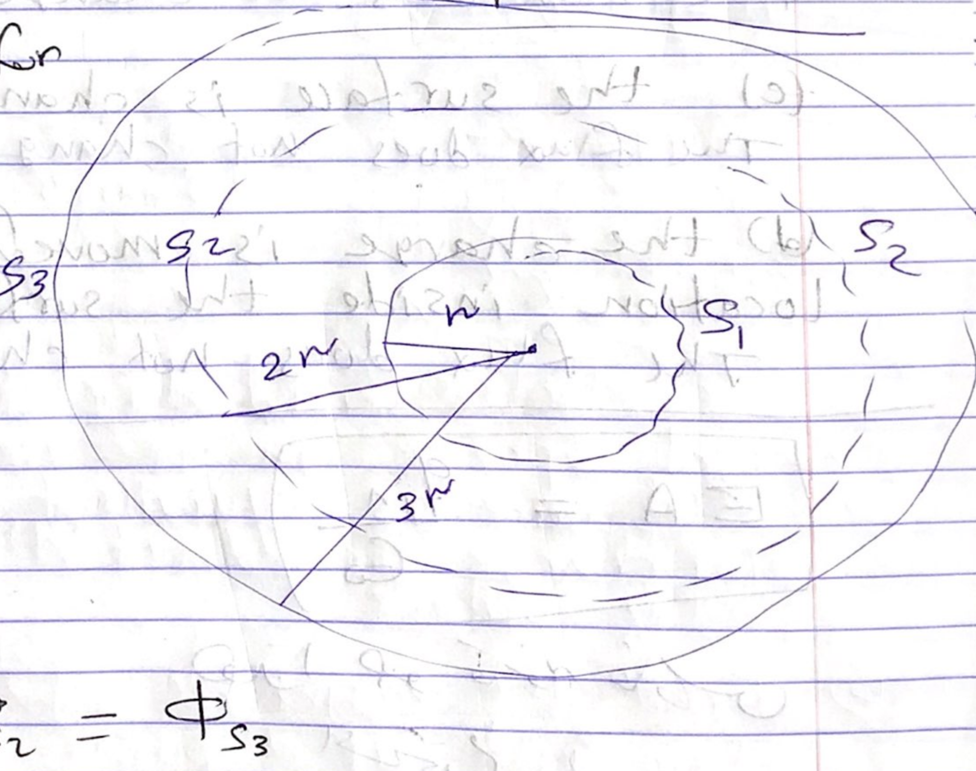
find Φ_E for

S_1

S_2

S_3



$$\Phi_{S_1} = \Phi_{S_2} = \Phi_{S_3}$$

Conceptual Example
(24,2)/730

A spherical gaussian surface surrounds a point charge q. Describe what happens to the total flux through the surface if

(A) the charge is tripled

The flux through the surface is tripled because flux is $\propto$ with q_{in}

(B) the radius of the sphere is doubled

The flux does not change

(c) the surface is changed to cube

The flux does not change.

(d) the charge is moved to another location inside the surface.

The flux does not change

$$EA = \frac{q_{in}}{\epsilon_0}$$

هذا هو قانون غاوس

المتغير هو E

24.3 Application of Gauss's Law to various charge Distributions

Gauss's Law is used to determine the electric field when the charge distribution has high degree of symmetry

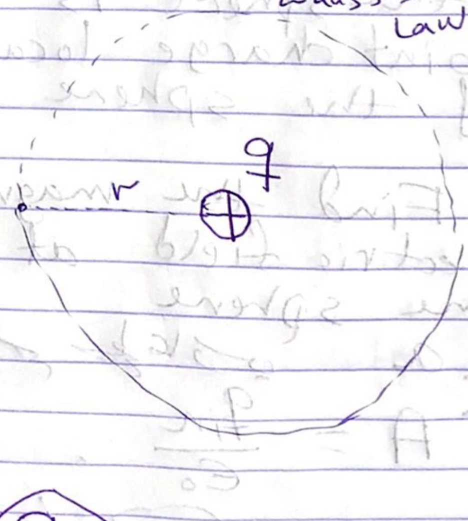
We Draw Gaussian surface to satisfy

1- E has constant value on the surface
2- E is parallel to ⊥A
3- E is perpendicular to A

The Electric Field Due to a point charge

Calculate the Electric Field due to point charge (q) at Point (P) located distance r from the charge

Using Gauss's law



$$EA = \frac{q_{in}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

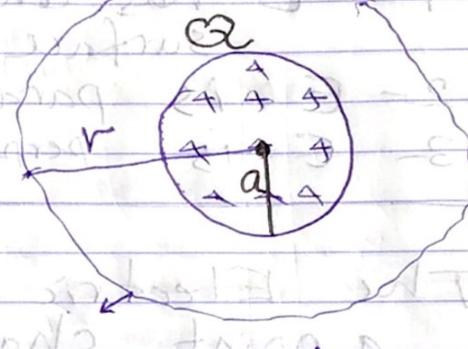
$$E = \frac{Kq}{r^2}$$

Ex (24.3) 731 | An insulating solid sphere of radius a has a Uniform volume charge density ρ and carries a total positive charge Q

(A) Calculate the magnitude of the electric field at a point outside the sphere using Gauss's law

Gauss's law

$$E\,A = \frac{q_{in}}{\epsilon_0}$$



$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi \epsilon_0 r^2}$$

Gauss's surface

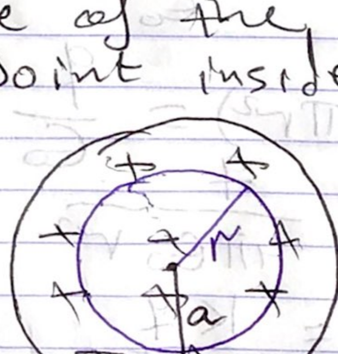
$$\boxed{E_{out} = \frac{KQ}{r^2}} \qquad K = \frac{1}{4\pi \epsilon_0}$$

↳ the electric field due to Uniformly charged sphere in the region external to the sphere is equivilent to that of a point charge located at the center of the sphere

B) Find the magnitude of the electric field at a point inside the sphere

$r < a$ نرسم سطح غاوسي

$$E\,A = \frac{q_{in}}{\epsilon_0}$$



$A = 4\pi r^2$

$q_{in} = \rho \cdot V_{in} = \rho\left(\frac{4}{3}\pi r^3\right)$

$EA = \frac{q_{in}}{\epsilon_0}$

$E \cdot 4\pi r^2 = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\epsilon_0}$

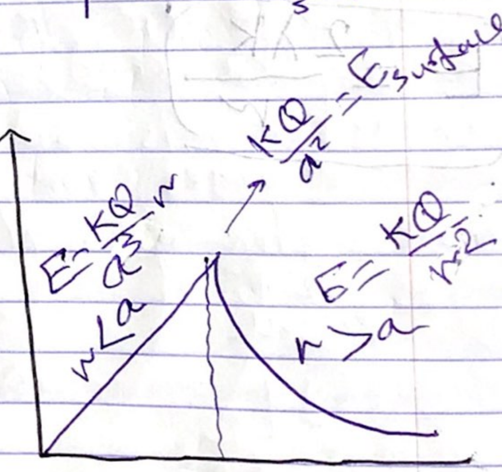
$E_{in} = \frac{\frac{4}{3} r \pi \rho}{\epsilon_0 4\pi} = \frac{\rho}{3\epsilon_0} r$

$$\boxed{E_{in} = \frac{KQ}{a^3} \cdot r} \quad \text{for } r < a$$

$\rho = \frac{Q}{V_{sphere}} = \frac{Q}{\frac{4}{3}\pi a^3}$

for insulating sphere

$$E = \begin{cases} \frac{KQ}{a^3} \cdot r, & r < a \\ \frac{KQ}{a^2}, & r = a \\ \frac{KQ}{r^2}, & r > a \end{cases}$$



plot of E versus r for a Uniformly charged insulating sphene

E insid linearly increasing with r

Eout seems as point charge.

Ex (24.4) / 733 For the electric field of A cylindrically symmetric charge Distribution

توزيع شحنات اسطوانى منتظم الشكل

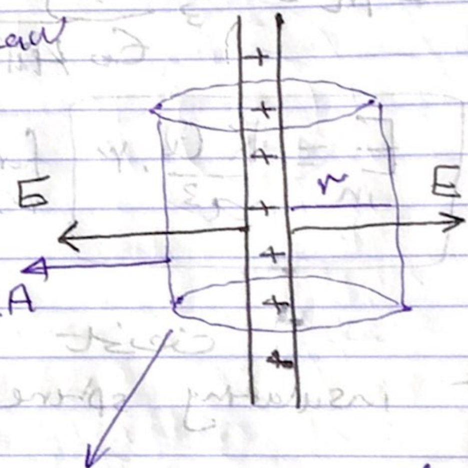
Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ as in Fig.

$$EA = \frac{q_{in}}{\epsilon_0} \quad \text{gauss's Law}$$

$$E(2\pi r L) = \frac{\lambda \cdot L}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi \epsilon_0 r}$$

$$E = \frac{2\lambda K}{r}$$



Gaussian surface

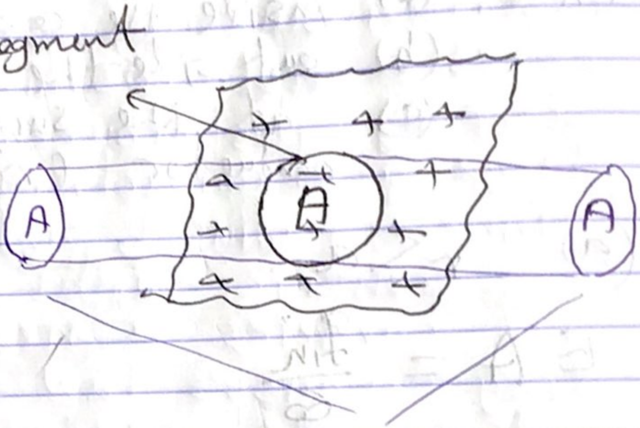
$$A = 2\pi r L$$

$$q_{in} = \lambda \cdot L$$

كثافة على شكل مستوى → A plane of charge

Ex (24.5) / 734 — Find the electric field due to an infinit plane of positive charge with uniform surface charge density σ.

كثافة سطحية



Gaussian surface

$$E\,A_G = \frac{q_{in}}{\epsilon_0}$$

$$E(2A) = \frac{\sigma A}{\epsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

$q_{in} = \sigma A$

$A_G = 2A$

→ Electric field due to infinit plane charge distribution

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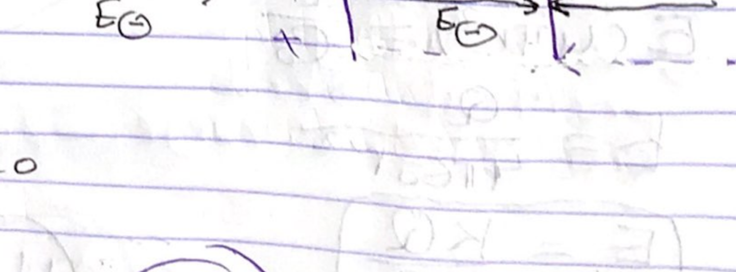
two infinit plane with opposite charges



$$E_{in} = E_{(+)} + E_{(-)} = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$

$$\boxed{E_{in} = \frac{\sigma}{\epsilon_0}}$$

$$E_{out} = \frac{\sigma}{\epsilon_0} - \frac{\sigma}{\epsilon_0} = 0$$

$$\boxed{E_{out} = 0}$$

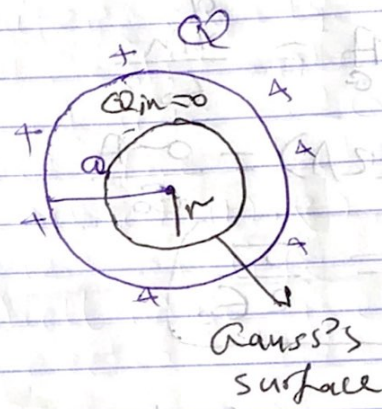
Spherical conductor

Spherical conductor of radius a carries a charge Q find the Electric Field (a) inside the conductor ($r < a$)

(b) out side the conductor ($r > a$)

(c) at the surface of the conductor ($r = a$)

(a) $r < a$



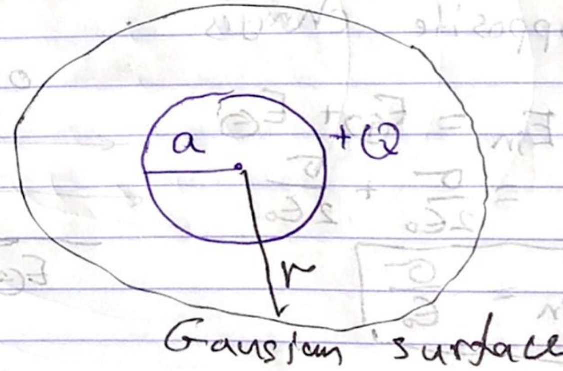
$$EA = \frac{q_{in}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{0}{\epsilon_0}$$

$$\boxed{E_{in} = 0}$$

The Electric Field is zero every where inside the conductor wheather the conductor is solid or hollow. because the charges stay far away at the surface so $q_{in} = 0$.

(b) $r > a$



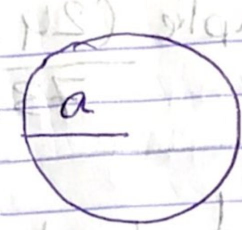
$$EA = \frac{q_{in}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

$$\boxed{E_{out} = \frac{KQ}{r^2}}$$

$$E_{out} = \frac{Q}{4\pi\epsilon_0 r^2}$$



(c) at the surface $r = a$

$$E_{surface} = \frac{Q}{4\pi\epsilon_0 a^2} = \frac{KQ}{a^2}$$

$$Q = \sigma A = \sigma(4\pi a^2)$$

$$E_{surface} = \frac{\sigma(4\pi a^2)}{4\pi\epsilon_0 a^2}$$

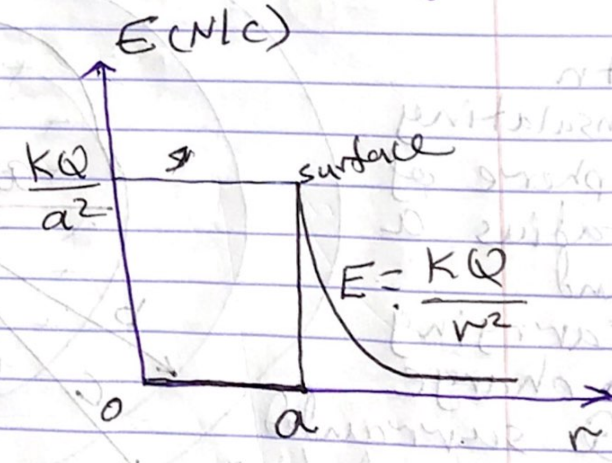
$$E_{surface} = \frac{\sigma}{\epsilon_0} \quad \text{or} \quad E = \frac{KQ}{a^2} \quad \text{or} \quad E = \frac{Q}{4\pi\epsilon_0 a^2}$$

plot of E versus r for conducting sphere

$E_{in} = 0$

$E_{surface} = \frac{KQ}{a^2}$

$E_{r>a} = \frac{KQ}{r^2}$



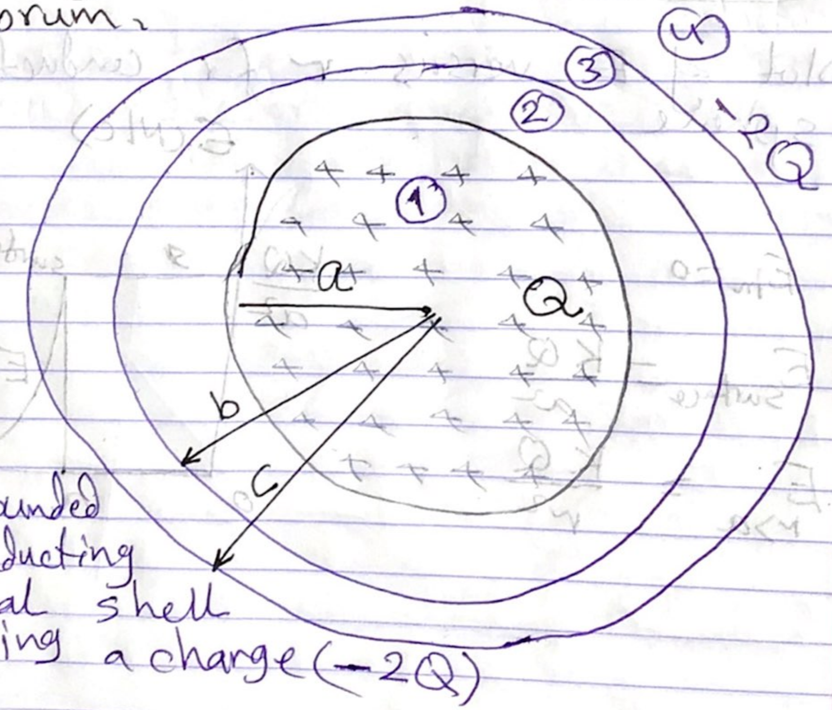
كرة عازلة

Example (24.7) / 736 | A sphere Inside a spherical shell

غلاف كروي موصل

A solid insulating sphere of radius a carries a net positive charge Q uniformly distributed throughout its volume. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and carries a net charge −2Q. Using Gauss's Law find the electric field in the regions labled (1), (2), (3) and (4) in Fig and the charge distribution on the shell when the entire system is in electrostatic equlibrum.

An insulating sphere of radius a and carrying a charge Q surrounded by a conducting spherical shell carrying a charge (−2Q)



① $r < a$

$$EA = \frac{q_{in}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{\left(\frac{4}{3}\pi r^3\right)\rho}{\epsilon_0}$$

$$\boxed{E_{①} = K\frac{Q}{a^3}\cdot r} \quad r < a$$

② $a < r < b$

$$\boxed{E_{②} = \frac{KQ}{r^2}} \qquad EA = \frac{q_{in}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

③ $b < r < c$

inside the shell

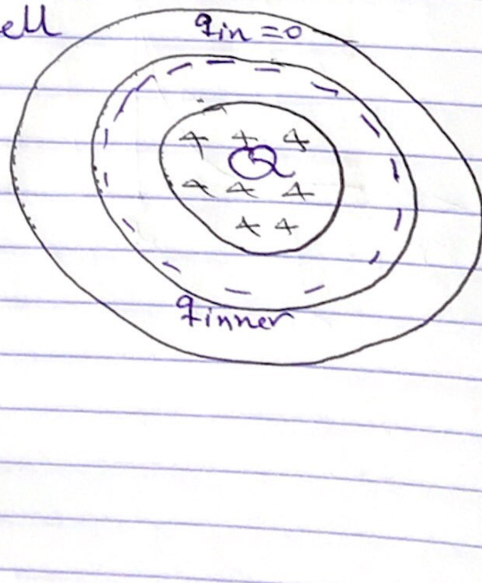
$\boxed{E_{③} = 0}$ $\quad q_{in} = 0$ inside conductor

Note $\quad q_{in} = q_{sphere} + q_{inner}$

$$0 = Q + q_{inner}$$

$$q_{inner} = -Q$$

the amount of charge q_{inner} on the inner surface of the shell



④ $r > C$

$$EA = \frac{q_{in}}{\epsilon_0}$$

$$E_{④}(4\pi r^2) = \frac{Q - 2Q}{\epsilon_0}$$

$$\boxed{E_{④} = -\frac{KQ}{r^2}} \quad r > C$$

How would the results of this problem differ if the sphere were conducting instead of insulating?

Answer

The only change would be in region ①, where $r < a$. Because there can be no charge inside a conductor in electrostatic equilibrium

$q_{in} = 0$

$E_{①} = 0$

$E_{②} = \frac{KQ}{r^2}$

$E_{③} = 0$

$E_{④} = \frac{-KQ}{r^2}$