25

Electric Potential

CHAPTER OUTLINE

25.1 Electric Potential and Potential Difference
25.2 Potential Difference in a Uniform Electric Field
25.3 Electric Potential and Potential Energy Due to Point Charges
25.4 Obtaining the Value of the Electric Field from the Electric Potential
25.5 Electric Potential Due to Continuous Charge Distributions
25.6 Electric Potential Due to a Charged Conductor
25.7 The Millikan Oil Drop Experiment
25.8 Application of Electrostatics

ANSWERS TO QUESTIONS

Q25.1 When one object B with electric charge is immersed in the electric field of another charge or charges A, the system possesses electric potential energy. The energy can be measured by seeing how much work the field does on the charge B as it moves to a reference location. We choose not to visualize A's effect on B as an action-at-a-distance, but as the result of a two-step process: Charge A creates electric potential throughout the surrounding space. Then the potential acts on B to inject the system with energy.

***Q25.2** (i) At points off the x axis the electric field has a nonzero y component. At points on the negative x axis the field is to the right and positive. At points to the right of $x = 500$ mm the field is to the left and nonzero. The field is zero at one point between $x = 250$ mm and $x = 500$ mm. Answer (b). (ii) The electric potential is negative at this and at all points. Answer (c). (iii) Answer (d). (iv) Answer (d).

***Q25.3** The potential is decreasing toward the bottom of the page, so the electric field is downward. Answer (f).

Q25.4 (a) The equipotential surfaces are nesting coaxial cylinders around an infinite line of charge.
(b) The equipotential surfaces are nesting concentric spheres around a uniformly charged sphere.

Q25.5 To move like charges together from an infinite separation, at which the potential energy of the system of two charges is zero, requires *work* to be done on the system by an outside agent. Hence energy is stored, and potential energy is positive. As charges with opposite signs move together from an infinite separation, energy is released, and the potential energy of the set of charges becomes negative.

***Q25.6** The same charges at the same distance away create the same contribution to the total potential. Answer (b).

***Q25.7** (i) The two spheres come to the same potential, so q/R is the same for both. With twice the radius, B has twice the charge. Answer (d).
(ii) All the charge runs away from itself to the outer surface of B. Answer (a).

Q25.8 The main factor is the radius of the dome. One often overlooked aspect is also the humidity of the air—drier air has a larger dielectric breakdown strength, resulting in a higher attainable electric potential. If other grounded objects are nearby, the maximum potential might be reduced.

***Q25.9** The change in kinetic energy is the negative of the change in electric potential energy, so we work out $-q\Delta V = -q(V_f - V_i)$ in each case.

(a) $-(-e)(60\text{ V} - 40\text{ V}) = +20\text{ eV}$ (b) $-(-e)(20\text{ V} - 40\text{ V}) = -20\text{ eV}$

(c) $-(e)(20\text{ V} - 40\text{ V}) = +20\text{ eV}$ (d) $-(e)(10\text{ V} - 40\text{ V}) = +30\text{ eV}$

(e) $-(-2e)(50\text{ V} - 40\text{ V}) = +20\text{ eV}$ (f) $-(-2e)(60\text{ V} - 40\text{ V}) = +40\text{ eV}$

With also (g) 0 and (h) +10 eV, the ranking is f > d > c = e = a > h > g > b.

SOLUTIONS TO PROBLEMS

Section 25.1 Electric Potential and Potential Difference

P25.1 (a) Energy of the proton-field system is conserved as the proton moves from high to low potential, which can be defined for this problem as moving from 120 V down to 0 V.

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f \qquad 0 + qV + 0 = \frac{1}{2}mv_p^2 + 0$$

$$\left(1.60\times10^{-19}\text{ C}\right)(120\text{ V})\left(\frac{1\text{ J}}{1\text{ V}\cdot\text{C}}\right) = \frac{1}{2}\left(1.67\times10^{-27}\text{ kg}\right)v_p^2$$

$$v_p = \boxed{1.52\times10^{5}\text{ m/s}}$$

(b) The electron will gain speed in moving the other way,

from $V_i = 0$ to $V_f = 120$ V: $\quad K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$

$$0 + 0 + 0 = \frac{1}{2}mv_e^2 + qV$$

$$0 = \frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)v_e^2 + \left(-1.60\times10^{-19}\text{ C}\right)(120\text{ J/C})$$

$$v_e = \boxed{6.49\times10^{6}\text{ m/s}}$$

P25.2 $\Delta V = -14.0$ V and $Q = -N_A e = -\left(6.02\times10^{23}\right)\left(1.60\times10^{-19}\right) = -9.63\times10^{4}$ C

$\Delta V = \dfrac{W}{Q}$, so $W = Q\Delta V = \left(-9.63\times10^{4}\text{ C}\right)\left(-14.0\text{ J/C}\right) = \boxed{1.35\text{ MJ}}$

Section 25.2 Potential Difference in a Uniform Electric Field

P25.3
$$\Delta U = -\frac{1}{2}m\left(v_f^2 - v_i^2\right) = -\frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left[\left(1.40\times10^{5}\text{ m/s}\right)^2 - \left(3.70\times10^{6}\text{ m/s}\right)^2\right]$$
$$= 6.23\times10^{-18}\text{ J}$$

$\Delta U = q\Delta V$: $\quad +6.23\times10^{-18} = \left(-1.60\times10^{-19}\right)\Delta V$

$\Delta V = \boxed{-38.9\text{ V. The origin is at highest potential.}}$

P25.4
$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_A^C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} - \int_C^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$
$$V_B - V_A = (-E\cos 180°)\int_{-0.300}^{0.500} dy - (E\cos 90.0°)\int_{-0.200}^{0.400} dx$$
$$V_B - V_A = (325)(0.800) = \boxed{+260 \text{ V}}$$

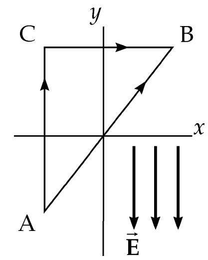

FIG. P25.4

P25.5
$$E = \frac{|\Delta V|}{d} = \frac{25.0\times 10^3 \text{ J/C}}{1.50\times 10^{-2} \text{ m}} = 1.67\times 10^6 \text{ N/C} = \boxed{1.67 \text{ MN/C}}$$

P25.6 Arbitrarily take $V = 0$ at point P. Then the potential at the original position of the charge is $-\vec{\mathbf{E}} \cdot \vec{\mathbf{s}} = -EL\cos\theta$. At the final point a, $V = -EL$. Because the table is frictionless we have $(K+U)_i = (K+U)_f$

$$0 - qEL\cos\theta = \frac{1}{2}mv^2 - qEL$$
$$v = \sqrt{\frac{2qEL(1-\cos\theta)}{m}} = \sqrt{\frac{2(2.00\times 10^{-6} \text{ C})(300 \text{ N/C})(1.50 \text{ m})(1-\cos 60.0°)}{0.010\,0 \text{ kg}}} = \boxed{0.300 \text{ m/s}}$$

P25.7 (a) Arbitrarily choose $V = 0$ at 0. Then at other points

$V = -Ex$ and $U_e = QV = -QEx$

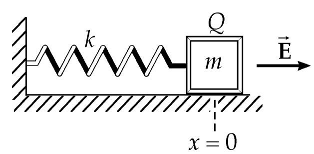

FIG. P25.7

Between the endpoints of the motion,

$$(K + U_s + U_e)_i = (K + U_s + U_e)_f$$
$$0 + 0 + 0 = 0 + \frac{1}{2}kx_{\max}^2 - QEx_{\max} \quad \text{so} \quad x_{\max} = \boxed{\frac{2QE}{k}}$$

(b) At equilibrium,

$$\sum F_x = -F_s + F_e = 0 \quad \text{or} \quad kx = QE$$

So the equilibrium position is at $x = \boxed{\frac{QE}{k}}$.

(c) The block's equation of motion is $\sum F_x = -kx + QE = m\frac{d^2x}{dt^2}$.

Let $x' = x - \frac{QE}{k}$, or $x = x' + \frac{QE}{k}$,

so the equation of motion becomes:

$$-k\left(x' + \frac{QE}{k}\right) + QE = m\frac{d^2(x + QE/k)}{dt^2}, \quad \text{or} \quad \frac{d^2x'}{dt^2} = -\left(\frac{k}{m}\right)x'$$

This is the equation for simple harmonic motion $a_{x'} = -\omega^2 x'$

with $\omega = \sqrt{\frac{k}{m}}$

The period of the motion is then $T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{m}{k}}}$

continued on next page

(d) $\left(K+U_s+U_e\right)_i+\Delta E_{\text{mech}}=\left(K+U_s+U_e\right)_f$

$$0+0+0-\mu_k mgx_{\max}=0+\frac{1}{2}kx_{\max}^2-QEx_{\max}$$

$$x_{\max}=\boxed{\frac{2\left(QE-\mu_k mg\right)}{k}}$$

P25.8 Assume the opposite. Then at some point A on some equipotential surface the electric field has a nonzero component E_p in the plane of the surface. Let a test charge start from point A and move some distance on the surface in the direction of the field component. Then $\Delta V=-\int_A^B \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$ is nonzero. The electric potential charges across the surface and it is not an equipotential surface. The contradiction shows that our assumption is false, that $E_p=0$, and that the field is perpendicular to the equipotential surface.

P25.9 Arbitrarily take $V=0$ at the initial point. Then at distance d downfield, where L is the rod length, $V=-Ed$ and $U_e=-\lambda LEd$.

(a) $(K+U)_i=(K+U)_f$

$$0+0=\frac{1}{2}\mu Lv^2-\lambda LEd$$

$$v=\sqrt{\frac{2\lambda Ed}{\mu}}=\sqrt{\frac{2\left(40.0\times10^{-6}\text{ C/m}\right)\left(100\text{ N/C}\right)\left(2.00\text{ m}\right)}{\left(0.100\text{ kg/m}\right)}}=\boxed{0.400\text{ m/s}}$$

(b) $\boxed{\text{The same.}}$ Each bit of the rod feels a force of the same size as before.

Section 25.3 Electric Potential and Potential Energy Due to Point Charges

P25.10 (a) $E_x=\dfrac{k_e q_1}{x^2}+\dfrac{k_e q_2}{(x-2.00)^2}=0$ becomes $E_x=k_e\left(\dfrac{+q}{x^2}+\dfrac{-2q}{(x-2.00)^2}\right)=0$

Dividing by k_e, $2qx^2=q(x-2.00)^2$ $x^2+4.00x-4.00=0$

Therefore $E=0$ when $x=\dfrac{-4.00\pm\sqrt{16.0+16.0}}{2}=\boxed{-4.83\text{ m}}$

(Note that the positive root does not correspond to a physically valid situation.)

(b) $V=\dfrac{k_e q_1}{x}+\dfrac{k_e q_2}{2.00-x}=0$ or $V=k_e\left(\dfrac{+q}{x}-\dfrac{2q}{2.00-x}\right)=0$

Again solving for x, $2qx=q(2.00-x)$

For $0\le x\le 2.00$ $V=0$ when $x=\boxed{0.667\text{ m}}$

and $\dfrac{q}{|x|}=\dfrac{-2q}{|2-x|}$ For $x<0$ $x=\boxed{-2.00\text{ m}}$

P25.11 (a) The potential at 1.00 cm is

$$V_1 = k_e \frac{q}{r} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)}{1.00\times10^{-2}\ \text{m}} = \boxed{1.44\times10^{-7}\ \text{V}}$$

(b) The potential at 2.00 cm is

$$V_2 = k_e \frac{q}{r} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)}{2.00\times10^{-2}\ \text{m}} = 0.719\times10^{-7}\ \text{V}$$

Thus, the difference in potential between the two points is $\Delta V = V_2 - V_1 = \boxed{-7.19\times10^{-8}\ \text{V}}$.

(c) The approach is the same as above except the charge is $-1.60\times10^{-19}\ \text{C}$. This changes the sign of each answer, with its magnitude remaining the same.

That is, the potential at 1.00 cm is $\boxed{-1.44\times10^{-7}\ \text{V}}$.

The potential at 2.00 cm is $-0.719\times10^{-7}\ \text{V}$, so $\Delta V = V_2 - V_1 = \boxed{7.19\times10^{-8}\ \text{V}}$

P25.12 (a) Since the charges are equal and placed symmetrically, $\boxed{F = 0}$.

(b) Since $F = qE = 0$, $\boxed{E = 0}$.

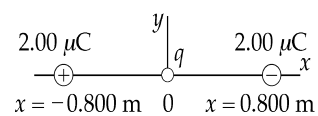

FIG. P25.12

(c) $$V = 2k_e \frac{q}{r} = 2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{2.00\times10^{-6}\ \text{C}}{0.800\ \text{m}}\right)$$

$$V = 4.50\times10^{4}\ \text{V} = \boxed{45.0\ \text{kV}}$$

P25.13 (a) $$E = \frac{|Q|}{4\pi \epsilon_0 r^2}$$

$$V = \frac{Q}{4\pi \epsilon_0 r}$$

$$r = \frac{|V|}{|E|} = \frac{3\,000\ \text{V}}{500\ \text{V/m}} = \boxed{6.00\ \text{m}}$$

(b) $$V = -3\,000\ \text{V} = \frac{Q}{4\pi \epsilon_0 (6.00\ \text{m})}$$

$$Q = \frac{-3\,000\ \text{V}}{\left(8.99\times10^{9}\ \text{V}\cdot\text{m/C}\right)}(6.00\ \text{m}) = \boxed{-2.00\ \mu\text{C}}$$

P25.14 (a) $U = \dfrac{qQ}{4\pi \in_0 r} = \dfrac{\left(5.00\times10^{-9}\text{ C}\right)\left(-3.00\times10^{-9}\text{ C}\right)\left(8.99\times10^{9}\text{ V}\cdot\text{m/C}\right)}{(0.350\text{ m})} = \boxed{-3.86\times10^{-7}\text{ J}}$

The minus sign means it takes 3.86×10^{-7} J to pull the two charges apart from 35 cm to a much larger separation.

(b) $V = \dfrac{Q_1}{4\pi \in_0 r_1} + \dfrac{Q_2}{4\pi \in_0 r_2}$

$$= \frac{\left(5.00\times10^{-9}\text{ C}\right)\left(8.99\times10^{9}\text{ V}\cdot\text{m/C}\right)}{0.175\text{ m}} + \frac{\left(-3.00\times10^{-9}\text{ C}\right)\left(8.99\times10^{9}\text{ V}\cdot\text{m/C}\right)}{0.175\text{ m}}$$

$V = \boxed{103\text{ V}}$

P25.15 $V = \sum_i k\dfrac{q_i}{r_i}$

$$V = \left(8.99\times10^{9}\right)\left(7.00\times10^{-6}\right)\left[\frac{-1}{0.010\,0} - \frac{1}{0.010\,0} + \frac{1}{0.038\,7}\right]$$

$V = \boxed{-1.10\times10^{7}\text{ V} = -11.0\text{ MV}}$

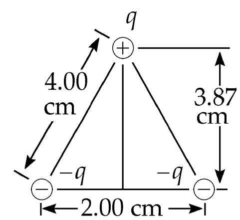

FIG. P25.15

P25.16 (a) $V = \dfrac{k_e q_1}{r_1} + \dfrac{k_e q_2}{r_2} = 2\left(\dfrac{k_e q}{r}\right)$

$$V = 2\left(\frac{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(2.00\times10^{-6}\text{ C}\right)}{\sqrt{(1.00\text{ m})^2 + (0.500\text{ m})^2}}\right)$$

$V = 3.22\times10^{4}\text{ V} = \boxed{32.2\text{ kV}}$

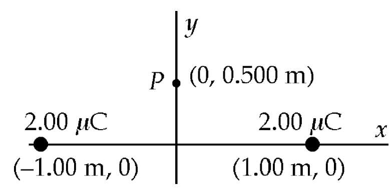

FIG. P25.16

(b) $U = qV = \left(-3.00\times10^{-6}\text{ C}\right)\left(3.22\times10^{4}\text{ J/C}\right) = \boxed{-9.65\times10^{-2}\text{ J}}$

P25.17 $U_e = q_4 V_1 + q_4 V_2 + q_4 V_3 = q_4\left(\dfrac{1}{4\pi \in_0}\right)\left(\dfrac{q_1}{r_1} + \dfrac{q_2}{r_2} + \dfrac{q_3}{r_3}\right)$

$$U_e = \left(10.0\times10^{-6}\text{ C}\right)^2\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{0.600\text{ m}} + \frac{1}{0.150\text{ m}} + \frac{1}{\sqrt{(0.600\text{ m})^2 + (0.150\text{ m})^2}}\right)$$

$U_e = \boxed{8.95\text{ J}}$

P25.18 Each charge creates equal potential at the center. The total potential is:

$$V = 5\left[\frac{k_e(-q)}{R}\right] = \boxed{-\frac{5k_e q}{R}}$$

P25.19 $U = U_1 + U_2 + U_3 + U_4$

$$U = 0 + U_{12} + (U_{13} + U_{23}) + (U_{14} + U_{24} + U_{34})$$

$$U = 0 + \frac{k_e Q^2}{s} + \frac{k_e Q^2}{s}\left(\frac{1}{\sqrt{2}} + 1\right) + \frac{k_e Q^2}{s}\left(1 + \frac{1}{\sqrt{2}} + 1\right)$$

$$U = \frac{k_e Q^2}{s}\left(4 + \frac{2}{\sqrt{2}}\right) = \boxed{5.41\frac{k_e Q^2}{s}}$$

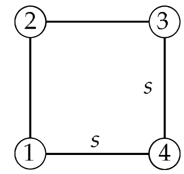

FIG. P25.19

We can visualize the term $\left(4 + \frac{2}{\sqrt{2}}\right)$ as arising directly from the 4 side pairs and 2 face diagonal pairs.

***P25.20** (a) The first expression, with distances squared, describes an electric field. The second expression describes an electric potential. Then a $\boxed{\text{positive 7 nC charge}}$ is 7 cm from the origin. To create field that is to the left and downward, it must be in the first quadrant, with position vector $\boxed{7 \text{ cm at } 70°}$. A $\boxed{\text{negative 8 nC charge}}$ 3 cm from the origin creates an upward electric field at the origin, so it must be at $\boxed{3 \text{ cm at } 90°}$. We evaluate the given expressions:

$$\vec{\mathbf{E}} = -4.39 \text{ kN/C}\,\hat{\mathbf{i}} + 67.8 \text{ kN/C}\,\hat{\mathbf{j}}$$
$$V = -1.50 \text{ kV}$$

(b) $\vec{\mathbf{F}} = q\vec{\mathbf{E}} = -16 \times 10^{-9} \text{ C}\left(-4.39\hat{\mathbf{i}} + 67.8\hat{\mathbf{j}}\right)10^3 \text{ N/C} = \boxed{\left(7.03\hat{\mathbf{i}} - 109\hat{\mathbf{j}}\right) \times 10^{-5} \text{ N}}$

(c) $U_e = qV = -16 \times 10^{-9} \text{ C}\left(-1.50 \times 10^3 \text{ J/C}\right) = \boxed{+2.40 \times 10^{-5} \text{ J}}$

P25.21 (a) Each charge separately creates positive potential everywhere. The total potential produced by the three charges together is then the sum of three positive terms. There is $\boxed{\text{no point}}$, at a finite distance from the charges, at which this total potential is zero.

(b) $V = \frac{k_e q}{a} + \frac{k_e q}{a} = \boxed{\frac{2k_e q}{a}}$

P25.22 (a) $V(x) = \dfrac{k_e Q_1}{r_1} + \dfrac{k_e Q_2}{r_2} = \dfrac{k_e(+Q)}{\sqrt{x^2+a^2}} + \dfrac{k_e(+Q)}{\sqrt{x^2+(-a)^2}}$

$$V(x) = \frac{2k_e Q}{\sqrt{x^2+a^2}} = \frac{k_e Q}{a}\left(\frac{2}{\sqrt{(x/a)^2+1}}\right)$$

$$\frac{V(x)}{(k_e Q/a)} = \boxed{\frac{2}{\sqrt{(x/a)^2+1}}}$$

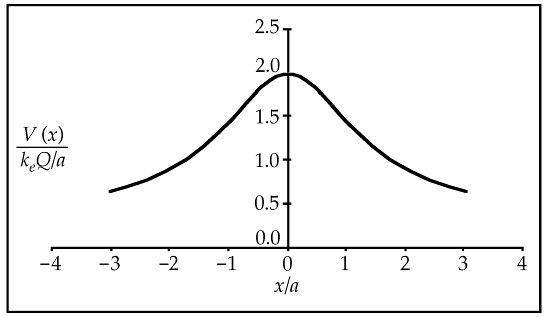

FIG. P25.22(a)

(b) $V(y) = \dfrac{k_e Q_1}{r_1} + \dfrac{k_e Q_2}{r_2} = \dfrac{k_e(+Q)}{|y-a|} + \dfrac{k_e(-Q)}{|y+a|}$

$$V(y) = \frac{k_e Q}{a}\left(\frac{1}{|y/a-1|} - \frac{1}{|y/a+1|}\right)$$

$$\frac{V(y)}{(k_e Q/a)} = \boxed{\left(\frac{1}{|y/a-1|} - \frac{1}{|y/a+1|}\right)}$$

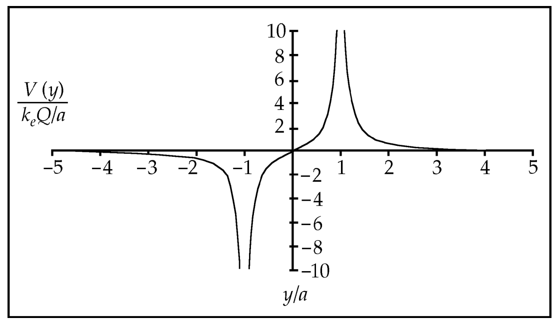

FIG. P25.22(b)

P25.23 Consider the two spheres as a system.

(a) Conservation of momentum: $0 = m_1 v_1 \hat{\mathbf{i}} + m_2 v_2 (-\hat{\mathbf{i}})$ or $v_2 = \dfrac{m_1 v_1}{m_2}$

By conservation of energy, $0 = \dfrac{k_e(-q_1)q_2}{d} = \dfrac{1}{2}m_1 v_1^2 + \dfrac{1}{2}m_2 v_2^2 + \dfrac{k_e(-q_1)q_2}{r_1+r_2}$

and $\dfrac{k_e q_1 q_2}{r_1+r_2} - \dfrac{k_e q_1 q_2}{d} = \dfrac{1}{2}m_1 v_1^2 + \dfrac{1}{2}\dfrac{m_1^2 v_1^2}{m_2}$

$$v_1 = \sqrt{\frac{2m_2 k_e q_1 q_2}{m_1(m_1+m_2)}\left(\frac{1}{r_1+r_2} - \frac{1}{d}\right)}$$

$$v_1 = \sqrt{\frac{2(0.700\text{ kg})(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(2\times10^{-6}\text{ C})(3\times10^{-6}\text{ C})}{(0.100\text{ kg})(0.800\text{ kg})}\left(\frac{1}{8\times10^{-3}\text{ m}} - \frac{1}{1.00\text{ m}}\right)}$$

$$= \boxed{10.8\text{ m/s}}$$

$$v_2 = \frac{m_1 v_1}{m_2} = \frac{0.100\text{ kg}(10.8\text{ m/s})}{0.700\text{ kg}} = \boxed{1.55\text{ m/s}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

P25.24 Consider the two spheres as a system.

(a) Conservation of momentum: $0 = m_1 v_1 \hat{\mathbf{i}} + m_2 v_2 \left(-\hat{\mathbf{i}}\right)$

or $v_2 = \dfrac{m_1 v_1}{m_2}$.

By conservation of energy, $0 = \dfrac{k_e(-q_1)q_2}{d} = \dfrac{1}{2}m_1 v_1^2 + \dfrac{1}{2}m_2 v_2^2 + \dfrac{k_e(-q_1)q_2}{r_1 + r_2}$

and $\dfrac{k_e q_1 q_2}{r_1 + r_2} - \dfrac{k_e q_1 q_2}{d} = \dfrac{1}{2}m_1 v_1^2 + \dfrac{1}{2}\dfrac{m_1^2 v_1^2}{m_2}$.

$$v_1 = \boxed{\sqrt{\frac{2m_2 k_e q_1 q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}}$$

$$v_2 = \left(\frac{m_1}{m_2}\right)v_1 = \boxed{\sqrt{\frac{2m_1 k_e q_1 q_2}{m_2(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

P25.25 The original electrical potential energy is

$$U_e = qV = q\frac{k_e q}{d}$$

In the final configuration we have mechanical equilibrium. The spring and electrostatic forces on each charge are $-k(2d) + q\dfrac{k_e q}{(3d)^2} = 0$. Then $k = \dfrac{k_e q^2}{18d^3}$. In the final configuration the total potential energy is $\dfrac{1}{2}kx^2 + qV = \dfrac{1}{2}\dfrac{k_e q^2}{18d^3}(2d)^2 + q\dfrac{k_e q}{3d} = \dfrac{4}{9}\dfrac{k_e q^2}{d}$. The missing energy must have become internal energy, as the system is isolated: $\dfrac{k_e q^2}{d} = \dfrac{4k_e q^2}{9d} + \Delta E_{\text{int}}$.

$$\boxed{\Delta E_{\text{int}} = \frac{5}{9}\frac{k_e q^2}{d}}$$

P25.26 Using conservation of energy for the alpha particle-nucleus system,

we have $K_f + U_f = K_i + U_i$

But $U_i = \dfrac{k_e q_\alpha q_{\text{gold}}}{r_i}$ and $r_i \approx \infty$ Thus, $U_i = 0$

Also $K_f = 0$ ($v_f = 0$ at turning point),

so $U_f = K_i$

or $\dfrac{k_e q_\alpha q_{\text{gold}}}{r_{\min}} = \dfrac{1}{2}m_\alpha v_\alpha^2$

$$r_{\min} = \frac{2k_e q_\alpha q_{\text{gold}}}{m_\alpha v_\alpha^2} = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(6.64\times10^{-27}\ \text{kg}\right)\left(2.00\times10^7\ \text{m/s}\right)^2} = 2.74\times10^{-14}\ \text{m}$$

$$= \boxed{27.4\ \text{fm}}$$

Section 25.4 **Obtaining the Value of the Electric Field from the Electric Potential**

P25.27 $V = 5x - 3x^2y + 2yz^2$

Evaluate E at $(1,\ 0,\ -2)$.

$$E_x = -\frac{\partial V}{\partial x} = \boxed{-5+6xy} = -5+6(1)(0) = -5$$

$$E_y = -\frac{\partial V}{\partial y} = \boxed{+3x^2-2z^2} = 3(1)^2 - 2(-2)^2 = -5$$

$$E_z = -\frac{\partial V}{\partial z} = \boxed{-4yz} = -4(0)(-2) = 0$$

$$E = \sqrt{E_x^2+E_y^2+E_z^2} = \sqrt{(-5)^2+(-5)^2+0^2} = \boxed{7.07\ \text{N/C}}$$

P25.28 (a) For $r < R$ $\quad V = \dfrac{k_eQ}{R}$

$$E_r = -\frac{dV}{dr} = \boxed{0}$$

(b) For $r \ge R$ $\quad V = \dfrac{k_eQ}{r}$

$$E_r = -\frac{dV}{dr} = -\left(-\frac{k_eQ}{r^2}\right) = \boxed{\frac{k_eQ}{r^2}}$$

P25.29 $V = a + bx = 10.0\ \text{V} + (-7.00\ \text{V/m})x$

(a) At $x = 0$, $\quad V = \boxed{10.0\ \text{V}}$

At $x = 3.00$ m, $\quad V = \boxed{-11.0\ \text{V}}$

At $x = 6.00$ m, $\quad V = \boxed{-32.0\ \text{V}}$

(b) $E = -\dfrac{dV}{dx} = -b = -(-7.00\ \text{V/m}) = \boxed{7.00\ \text{N/C in the } +x \text{ direction}}$

P25.30 (a) $\boxed{E_A > E_B \text{ since } E = \dfrac{\Delta V}{\Delta s}}$

(b) $E_B = -\dfrac{\Delta V}{\Delta s} = -\dfrac{(6-2)\ \text{V}}{2\ \text{cm}} = \boxed{200\ \text{N/C}}$ down

(c) The figure is shown to the right, with sample field lines sketched in.

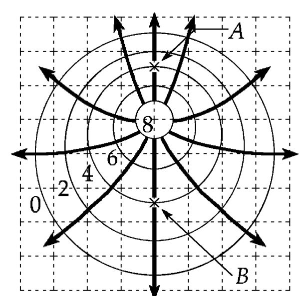

FIG. P25.30

P25.31 $E_y = -\dfrac{\partial V}{\partial y} = -\dfrac{\partial}{\partial y}\left[\dfrac{k_eQ}{\ell}\ln\left(\dfrac{\ell+\sqrt{\ell^2+y^2}}{y}\right)\right]$

$$E_y = \frac{k_eQ}{\ell y}\left[1 - \frac{y^2}{\ell^2+y^2+\ell\sqrt{\ell^2+y^2}}\right] = \boxed{\frac{k_eQ}{y\sqrt{\ell^2+y^2}}}$$

Section 25.5 **Electric Potential Due to Continuous Charge Distributions**

P25.32 $$\Delta V = V_{2R} - V_0 = \frac{k_e Q}{\sqrt{R^2 + (2R)^2}} - \frac{k_e Q}{R} = \frac{k_e Q}{R}\left(\frac{1}{\sqrt{5}} - 1\right) = \boxed{-0.553\frac{k_e Q}{R}}$$

P25.33 (a) $$[\alpha] = \left[\frac{\lambda}{x}\right] = \frac{\mathrm{C}}{\mathrm{m}} \cdot \left(\frac{1}{\mathrm{m}}\right) = \boxed{\frac{\mathrm{C}}{\mathrm{m}^2}}$$

(b) $$V = k_e \int \frac{dq}{r} = k_e \int \frac{\lambda dx}{r} = k_e \alpha \int_0^L \frac{x dx}{d + x} = \boxed{k_e \alpha \left[L - d \ln\left(1 + \frac{L}{d}\right)\right]}$$

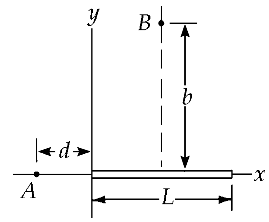

FIG. P25.33

P25.34 $$V = \int \frac{k_e dq}{r} = k_e \int \frac{\alpha x dx}{\sqrt{b^2 + (L/2 - x)^2}}$$

Let $z = \frac{L}{2} - x.$

Then $x = \frac{L}{2} - z,$ and $dx = -dz$

$$V = k_e \alpha \int \frac{(L/2 - z)(-dz)}{\sqrt{b^2 + z^2}} = -\frac{k_e \alpha L}{2} \int \frac{dz}{\sqrt{b^2 + z^2}} + k_e \alpha \int \frac{z dz}{\sqrt{b^2 + z^2}}$$

$$= -\frac{k_e \alpha L}{2} \ln\left(z + \sqrt{z^2 + b^2}\right) + k_e \alpha \sqrt{z^2 + b^2}$$

$$V = -\frac{k_e \alpha L}{2} \ln\left[\left(\frac{L}{2} - x\right) + \sqrt{\left(\frac{L}{2} - x\right)^2 + b^2}\right]\Bigg|_0^L + k_e \alpha \sqrt{\left(\frac{L}{2} - x\right)^2 + b^2}\Bigg|_0^L$$

$$V = -\frac{k_e \alpha L}{2} \ln\left[\frac{L/2 - L + \sqrt{(L/2)^2 + b^2}}{L/2 + \sqrt{(L/2)^2 + b^2}}\right] + k_e \alpha \left[\sqrt{\left(\frac{L}{2} - L\right)^2 + b^2} - \sqrt{\left(\frac{L}{2}\right)^2 + b^2}\right]$$

$$V = \boxed{-\frac{k_e \alpha L}{2} \ln\left[\frac{\sqrt{b^2 + (L^2/4)} - L/2}{\sqrt{b^2 + (L^2/4)} + L/2}\right]}$$

P25.35 $$V = \int dV = \frac{1}{4\pi \in_0} \int \frac{dq}{r}$$

All bits of charge are at the same distance from O.

So $V = \frac{1}{4\pi \in_0}\left(\frac{Q}{R}\right) = \left(8.99 \times 10^9 \ \mathrm{N \cdot m^2/C^2}\right)\left(\frac{-7.50 \times 10^{-6} \ \mathrm{C}}{0.140 \ \mathrm{m}/\pi}\right) = \boxed{-1.51 \ \mathrm{MV}}.$

P25.36 $$V = k_e \int\limits_{\text{all charge}} \frac{dq}{r} = k_e \int_{-3R}^{-R} \frac{\lambda dx}{-x} + k_e \int\limits_{\text{semicircle}} \frac{\lambda ds}{R} + k_e \int_{R}^{3R} \frac{\lambda dx}{x}$$

$$V = -k_e \lambda \ln(-x)\Big|_{-3R}^{-R} + \frac{k_e \lambda}{R} \pi R + k_e \lambda \ln x\Big|_{R}^{3R}$$

$$V = k_e \ln \frac{3R}{R} + k_e \lambda \pi + k_e \ln 3 = \boxed{k_e \lambda (\pi + 2\ln 3)}$$

Section 25.6 **Electric Potential Due to a Charged Conductor**

P25.37 The electric field on the surface of a conductor varies inversely with the radius of curvature of the surface. Thus, the field is most intense where the radius of curvature is smallest and vice-versa. The local charge density and the electric field intensity are related by

$$E = \frac{\sigma}{\epsilon_0} \qquad \text{or} \qquad \sigma = \epsilon_0 E$$

(a) Where the radius of curvature is the greatest,

$$\sigma = \epsilon_0 E_{\min} = \left(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right)\left(2.80 \times 10^4 \text{ N/C}\right) = \boxed{248 \text{ nC/m}^2}$$

(b) Where the radius of curvature is the smallest,

$$\sigma = \epsilon_0 E_{\max} = \left(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right)\left(5.60 \times 10^4 \text{ N/C}\right) = \boxed{496 \text{ nC/m}^2}$$

P25.38 Substituting given values into $V = \frac{k_e q}{r}$

$$7.50 \times 10^3 \text{ V} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) q}{0.300 \text{ m}}$$

Substituting $q = 2.50 \times 10^{-7}$ C,

$$N = \frac{2.50 \times 10^{-7} \text{ C}}{1.60 \times 10^{-19} \text{ C}/e^-} = \boxed{1.56 \times 10^{12} \text{ electrons}}$$

P25.39 (a) $E = \boxed{0}$;

$$V = \frac{k_e q}{R} = \frac{\left(8.99 \times 10^9\right)\left(26.0 \times 10^{-6}\right)}{0.140} = \boxed{1.67 \text{ MV}}$$

(b) $$E = \frac{k_e q}{r^2} = \frac{\left(8.99 \times 10^9\right)\left(26.0 \times 10^{-6}\right)}{(0.200)^2} = \boxed{5.84 \text{ MN/C}} \text{ away}$$

$$V = \frac{k_e q}{R} = \frac{\left(8.99 \times 10^9\right)\left(26.0 \times 10^{-6}\right)}{0.200} = \boxed{1.17 \text{ MV}}$$

(c) $$E = \frac{k_e q}{R^2} = \frac{\left(8.99 \times 10^9\right)\left(26.0 \times 10^{-6}\right)}{(0.140)^2} = \boxed{11.9 \text{ MN/C}} \text{ away}$$

$$V = \frac{k_e q}{R} = \boxed{1.67 \text{ MV}}$$

P25.40 (a) Both spheres must be at the same potential according to $\frac{k_e q_1}{r_1} = \frac{k_e q_2}{r_2}$

where also $q_1 + q_2 = 1.20 \times 10^{-6}$ C

Then $q_1 = \frac{q_2 r_1}{r_2}$

$$\frac{q_2 r_1}{r_2} + q_2 = 1.20 \times 10^{-6}\ \text{C}$$

$$q_2 = \frac{1.20 \times 10^{-6}\ \text{C}}{1 + 6\ \text{cm}/2\ \text{cm}} = 0.300 \times 10^{-6}\ \text{C on the smaller sphere}$$

$$q_1 = 1.20 \times 10^{-6}\ \text{C} - 0.300 \times 10^{-6}\ \text{C} = 0.900 \times 10^{-6}\ \text{C}$$

$$V = \frac{k_e q_1}{r_1} = \frac{(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)(0.900 \times 10^{-6}\ \text{C})}{6 \times 10^{-2}\ \text{m}} = \boxed{1.35 \times 10^5\ \text{V}}$$

(b) Outside the larger sphere,

$$\vec{\mathbf{E}}_1 = \frac{k_e q_1}{r_1^2}\hat{\mathbf{r}} = \frac{V_1}{r_1}\hat{\mathbf{r}} = \frac{1.35 \times 10^5\ \text{V}}{0.06\ \text{m}}\hat{\mathbf{r}} = \boxed{2.25 \times 10^6\ \text{V/m away}}$$

Outside the smaller sphere,

$$\vec{\mathbf{E}}_2 = \frac{1.35 \times 10^5\ \text{V}}{0.02\ \text{m}}\hat{\mathbf{r}} = \boxed{6.74 \times 10^6\ \text{V/m away}}$$

The smaller sphere carries less charge but creates a much stronger electric field than the larger sphere.

Section 25.7 **The Millikan Oil Drop Experiment**

Section 25.8 **Application of Electrostatics**

P25.41 (a) $E_{\text{max}} = 3.00 \times 10^6\ \text{V/m} = \frac{k_e Q}{r^2} = \frac{k_e Q}{r}\left(\frac{1}{r}\right) = V_{\text{max}}\left(\frac{1}{r}\right)$

$$V_{\text{max}} = E_{\text{max}} r = 3.00 \times 10^6 (0.150) = \boxed{450\ \text{kV}}$$

(b) $\frac{k_e Q_{\text{max}}}{r^2} = E_{\text{max}}$ $\left\{\text{or } \frac{k_e Q_{\text{max}}}{r} = V_{\text{max}}\right\}$ $Q_{\text{max}} = \frac{E_{\text{max}} r^2}{k_e} = \frac{3.00 \times 10^6 (0.150)^2}{8.99 \times 10^9} = \boxed{7.51\ \mu\text{C}}$

P25.42 (a) $V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ and the field at distance r from a uniformly charged rod (where $r >$ radius of charged rod) is

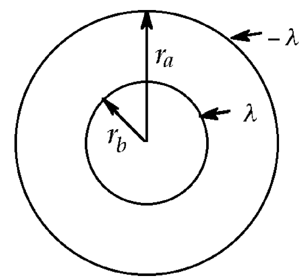

FIG. P25.42

$$E = \frac{\lambda}{2\pi \in_0 r} = \frac{2k_e\lambda}{r}$$

In this case, the field between the central wire and the coaxial cylinder is directed perpendicular to the line of charge so that

$$V_B - V_A = -\int_{r_a}^{r_b} \frac{2k_e\lambda}{r} dr = 2k_e\lambda \ln\left(\frac{r_a}{r_b}\right)$$

or $\boxed{\Delta V = 2k_e\lambda \ln\left(\frac{r_a}{r_b}\right)}$

(b) From part (a), when the outer cylinder is considered to be at zero potential, the potential at a distance r from the axis is

$$V = 2k_e\lambda \ln\left(\frac{r_a}{r}\right)$$

The field at r is given by

$$E = -\frac{\partial V}{\partial r} = -2k_e\lambda\left(\frac{r}{r_a}\right)\left(-\frac{r_a}{r^2}\right) = \frac{2k_e\lambda}{r}$$

But, from part (a), $2k_e\lambda = \frac{\Delta V}{\ln(r_a/r_b)}$.

Therefore, $\boxed{E = \frac{\Delta V}{\ln(r_a/r_b)}\left(\frac{1}{r}\right)}$.

P25.43 (a) From the previous problem,

$$E=\frac{\Delta V}{\ln\left(r_a/r_b\right)}\frac{1}{r}$$

We require just outside the central wire

$$5.50\times10^{6}\ \text{V/m}=\frac{50.0\times10^{3}\ \text{V}}{\ln\left(0.850\ \text{m}/r_b\right)}\left(\frac{1}{r_b}\right)$$

or $$\left(110\ \text{m}^{-1}\right)r_b\ln\left(\frac{0.850\ \text{m}}{r_b}\right)=1$$

We solve by homing in on the required value

$r_b\,(\text{m})$	0.0100	0.00100	0.00150	0.00145	0.00143	0.00142
$\left(110\ \text{m}^{-1}\right)r_b\ln\left(\frac{0.850\ \text{m}}{r_b}\right)$	4.89	0.740	1.05	1.017	1.005	0.999

Thus, to three significant figures,

$$r_b=\boxed{1.42\ \text{mm}}$$

(b) At r_a,

$$E=\frac{50.0\ \text{kV}}{\ln\left(0.850\ \text{m}/0.001\,42\ \text{m}\right)}\left(\frac{1}{0.850\ \text{m}}\right)=\boxed{9.20\ \text{kV/m}}$$

Additional Problems

***P25.44** (a) The field within the conducting Earth is zero. $E=\sigma/\epsilon_0$

$\sigma=E\epsilon_0=(-120\ \text{N/C})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)=\boxed{-1.06\ \text{nC/m}^2}$

(b) $QE=\sigma A=\sigma 4\pi r^2=(-1.06\times10^{-9}\ \text{C/m}^2)\,4\pi\,(6.37\times10^{6}\,\text{m})^2=\boxed{-542\ \text{kC}}$

(c) $$V=\frac{k_eQ}{R}=\frac{8.99\times10^{9}\ \text{C}^2(-5.42\times10^{5}\ \text{C})}{\text{N}\cdot\text{m}^2(6.37\times10^{6}\ \text{m})}=\boxed{-764\ \text{MV}}$$

(d) $V_{head}-V_{feet}=Ed=(120\ \text{N/C})1.75\ \text{m}=\boxed{210\ \text{V}}$

(e) $$F=\frac{k_eq_1q_2}{r^2}=\frac{8.99\times10^{9}\ \text{N}\cdot\text{m}^2(5.42\times10^{5}\ \text{C})^2(0.273)}{\text{C}^2(3.84\times10^{8}\ \text{m})^2}=\boxed{4.88\times10^{3}\ \text{N away from Earth}}$$

(f) The gravitational force is

$$F=\frac{GM_E\,M_M}{r^2}=\frac{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2(5.98\times10^{24}\ \text{kg})(7.36\times10^{22}\ \text{kg})}{\text{kg}^2(3.84\times10^{8}\ \text{m})^2}=1.99\times10^{20}\,\text{N}$$

toward the Earth

The gravitational force is larger by $1.99\times10^{20}/4.88\times10^{3}=4.08\times10^{16}$ times and in the opposite direction.

Electrical forces are negligible in accounting for planetary motion.

continued on next page

(g) We require $m(-g) + qE = 0$

$$6 \times 10^{-6}\ \text{kg}(-9.8\ \text{m/s}^2) + q(-120\ \text{N/C}) = 0$$

$$q = 5.88 \times 10^{-5}\ \text{N}/(-120\ \text{N/C}) = \boxed{-490\ \text{nC}}$$

(h) Less charge to be suspended at the equator. The gravitational force is weaker at a greater distance from the Earth's center. The suspended particle is not quite in equilibrium, but accelerating downward to participate in the daily rotation. At uniform potential, the planet's surface creates a stronger electric field at the equator, where its radius of curvature is smaller.

P25.45 (a) $$U = \frac{k_e q_1 q_2}{r} = \frac{-\left(8.99\times10^{9}\right)\left(1.60\times10^{-19}\right)^2}{0.052\,9\times10^{-9}} = -4.35\times10^{-18}\ \text{J} = \boxed{-27.2\ \text{eV}}$$

(b) $$U = \frac{k_e q_1 q_2}{r} = \frac{-\left(8.99\times10^{9}\right)\left(1.60\times10^{-19}\right)^2}{2^2\left(0.052\,9\times10^{-9}\right)} = \boxed{-6.80\ \text{eV}}$$

(c) $$U = \frac{k_e q_1 q_2}{r} = \frac{-k_e e^2}{\infty} = \boxed{0}$$

***P25.46** (a) The two particles exert forces of repulsion on each other. As the projectile approaches the target particle, the projectile slows. The target starts to move in the x direction. As long as the projectile is moving faster than the second particle, the two will be approaching. Kinetic energy will be being converted into electric potential energy. When both particles move with equal speeds, the distance between them will momentarily not be changing: this is the instant of closest approach. Thereafter, the target particle, still feeling a forward force, will move faster than the projectile. The particles will separate again. The particles exert forces on each other but never touch. The particles will eventually be very far apart, with zero electric potential energy. All of the U_e they had at closest approach is converted back into kinetic energy. The whole process is an elastic collision. Compare this problem with Problem 9.49 in Chapter 9.

(b) Momentum is constant throughout the process. We equate it at the large-separation initial point and the point b of closest approach.

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1b} + m_2\vec{\mathbf{v}}_{2b}$$

$$(2\ \text{g})\left(21\hat{\mathbf{i}}\ \text{m/s}\right) + 0 = (2\ \text{g} + 5\ \text{g})\vec{\mathbf{v}}_b$$

$$\vec{\mathbf{v}}_b = \boxed{6.00\hat{\mathbf{i}}\ \text{m/s}}$$

(c) Energy conservation between the same two points:

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 + 0 = \frac{1}{2}\left(m_1 + m_2\right)v_b^2 + \frac{k_e q_1 q_2}{r_b}$$

$$\frac{1}{2}0.002\ \text{kg}\left(21\ \text{m/s}\right)^2 + \frac{1}{2}0.005\ \text{kg}\left(0\right)^2 + 0 = \frac{1}{2}0.007\ \text{kg}\left(6\ \text{m/s}\right)^2$$
$$+\frac{8.99\times10^{9}\ \text{Nm}^2}{\text{C}^2}\ \frac{15\times10^{-6}\ \text{C}\ 8.5\times10^{-6}\ \text{C}}{r_b}$$

$$0.441\ \text{J} - 0.126\ \text{J} = \frac{1.15\ \text{J}\cdot\text{m}}{r_b}$$

$$r_b = \frac{1.15\ \text{m}}{0.315} = \boxed{3.64\ \text{m}}$$

continued on next page

(d) The overall elastic collision is described by conservation of momentum:

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1d} + m_2\vec{\mathbf{v}}_{2d}$$
$$(2\text{ g})\left(21\hat{\mathbf{i}}\text{ m/s}\right) + 0 = 2\text{ g}\vec{\mathbf{v}}_{1d}\hat{\mathbf{i}} + 5\text{ g}\vec{\mathbf{v}}_{2d}\hat{\mathbf{i}}$$

and by the relative velocity equation:

$$v_{1i} - v_{2i} = v_{2d} - v_{1d}$$
$$21\text{ m/s} - 0 = v_{2d} - v_{1d}$$

we substitute

$$v_{2d} = 21\text{ m/s} + v_{1d}$$
$$42\text{ g}\cdot\text{m/s} = 2\text{ g}v_{1d} + 5\text{ g}\left(21\text{ m/s} + v_{1d}\right) = 2\text{ g}v_{1d} + 105\text{ g}\cdot\text{m/s} + 5\text{ g}v_{1d}$$
$$-63\text{ g}\cdot\text{m/s} = 7\text{ g}v_{1d}$$
$$v_{1d} = -9.00\text{ m/s}$$
$$\boxed{\vec{\mathbf{v}}_{1d} = -9.00\hat{\mathbf{i}}\text{ m/s}}$$
$$v_{2d} = 21\text{ m/s} - 9\text{ m/s} = 12.0\text{ m/s}$$
$$\boxed{\vec{\mathbf{v}}_{2d} = 12.0\hat{\mathbf{i}}\text{ m/s}}$$

P25.47 $U = qV = k_e\dfrac{q_1q_2}{r_{12}} = \left(8.99\times10^9\right)\dfrac{(38)(54)\left(1.60\times10^{-19}\right)^2}{(5.50+6.20)\times10^{-15}} = 4.04\times10^{-11}\text{ J} = \boxed{253\text{ MeV}}$

P25.48 (a) Take the origin at the point where we will find the potential. One ring, of width dx, has charge $\dfrac{Qdx}{h}$ and, according to Example 25.5, creates potential

$$dV = \frac{k_eQdx}{h\sqrt{x^2+R^2}}$$

The whole stack of rings creates potential

$$V = \int_{\text{all charge}} dV = \int_d^{d+h}\frac{k_eQdx}{h\sqrt{x^2+R^2}} = \frac{k_eQ}{h}\ln\left(x+\sqrt{x^2+R^2}\right)\Bigg|_d^{d+h}$$
$$= \boxed{\frac{k_eQ}{h}\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)}$$

(b) A disk of thickness dx has charge $\dfrac{Qdx}{h}$ and charge-per-area $\dfrac{Qdx}{\pi R^2h}$. According to Example 25.6, it creates potential

$$dV = 2\pi k_e\frac{Qdx}{\pi R^2h}\left(\sqrt{x^2+R^2}-x\right)$$

Integrating,

$$V = \int_d^{d+h}\frac{2k_eQ}{R^2h}\left(\sqrt{x^2+R^2}\,dx - xdx\right) = \frac{2k_eQ}{R^2h}\left[\frac{1}{2}x\sqrt{x^2+R^2} + \frac{R^2}{2}\ln\left(x+\sqrt{x^2+R^2}\right) - \frac{x^2}{2}\right]_d^{d+h}$$
$$V = \boxed{\frac{k_eQ}{R^2h}\left[(d+h)\sqrt{(d+h)^2+R^2} - d\sqrt{d^2+R^2} - 2dh - h^2 + R^2\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)\right]}$$

***P25.49** For a charge at ($x = -1$ m, $y = 0$), the radial distance away is given by $\sqrt{(x+1)^2+y^2}$. So the first term will be the potential it creates if

$(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)Q_1 = 36\ \text{V}\cdot\text{m}$ $\quad Q_1 = 4.00$ nC

The second term is the potential of a charge at ($x = 0$, $y = 2$ m) with

$(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)Q_2 = -45\ \text{V}\cdot\text{m}$ $\quad Q_2 = -5.01$ nC

Thus we have $\boxed{4.00\text{ nC at }(-1.00\text{ m}, 0)\text{ and }-5.01\text{ nC at }(0, 2.00\text{ m})}$.

P25.50 The plates create uniform electric field to the right in the picture, with magnitude $\frac{V_0-(-V_0)}{d}=\frac{2V_0}{d}$. Assume the ball swings a small distance x to the right. It moves to a place where the voltage created by the plates is lower by $-Ex=-\frac{2V_0}{d}x$. Its ground connection maintains it at $V=0$ by allowing charge q to flow from ground onto the ball, where $-\frac{2V_0x}{d}+\frac{k_eq}{R}=0 \quad q=\frac{2V_0xR}{k_ed}$. Then the ball feels electric force $F=qE=\frac{4V_0^2xR}{k_ed^2}$ to the right. For equilibrium this must be balanced by the horizontal component of string tension according to $T\cos\theta=mg \quad T\sin\theta=\frac{4V_0^2xR}{k_ed^2} \quad \tan\theta=\frac{4V_0^2xR}{k_ed^2mg}=\frac{x}{L}$ for small x.

Then $V_0=\left(\frac{k_ed^2mg}{4RL}\right)^{1/2}$. If V_0 is less than this value, the only equilibrium position of the ball is hanging straight down. If V_0 exceeds this value the ball will swing over to one plate or the other.

P25.51 From an Example in the chapter text, the potential at the center of the ring is $V_i=\frac{k_eQ}{R}$ and the potential at an infinite distance from the ring is $V_f=0$. Thus, the initial and final potential energies of the point charge-ring system are:

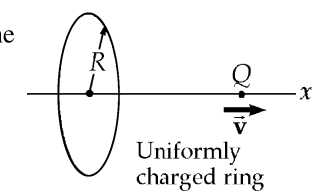

FIG. P25.51

$$U_i=QV_i=\frac{k_eQ^2}{R}$$

and $\quad U_f=QV_f=0$

From conservation of energy,

$$K_f+U_f=K_i+U_i$$

or $\quad \frac{1}{2}Mv_f^2+0=0+\frac{k_eQ^2}{R}$

giving $v_f=\boxed{\sqrt{\frac{2k_eQ^2}{MR}}}$

P25.52 Take the illustration presented with the problem as an initial picture. No external horizontal forces act on the set of four balls, so its center of mass stays fixed at the location of the center of the square. As the charged balls 1 and 2 swing out and away from each other, balls 3 and 4 move up with equal y-components of velocity. The maximum-kinetic-energy point is illustrated. System energy is conserved:

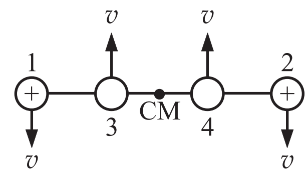

FIG. P25.52

$$\frac{k_e q^2}{a} = \frac{k_e q^2}{3a} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$\frac{2k_e q^2}{3a} = 2mv^2 \qquad \boxed{v = \sqrt{\frac{k_e q^2}{3am}}}$$

P25.53 $$V_2 - V_1 = -\int_{r_1}^{r_2} \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = -\int_{r_1}^{r_2} \frac{\lambda}{2\pi \in_0 r} dr$$

$$V_2 - V_1 = \boxed{\frac{-\lambda}{2\pi \in_0} \ln\left(\frac{r_2}{r_1}\right)}$$

P25.54 $$V = k_e \int_a^{a+L} \frac{\lambda dx}{\sqrt{x^2+b^2}} = k_e \lambda \ln\left[x + \sqrt{\left(x^2+b^2\right)}\right]_a^{a+L} = \boxed{k_e \lambda \ln\left[\frac{a+L+\sqrt{(a+L)^2+b^2}}{a+\sqrt{a^2+b^2}}\right]}$$

P25.55 For an element of area which is a ring of radius r and width dr, $dV = \dfrac{k_e dq}{\sqrt{r^2+x^2}}$.

$dq = \sigma dA = Cr\left(2\pi r dr\right)$ and

$$V = C\left(2\pi k_e\right)\int_0^R \frac{r^2 dr}{\sqrt{r^2+x^2}} = \boxed{C\left(\pi k_e\right)\left[R\sqrt{R^2+x^2} + x^2 \ln\left(\frac{x}{R+\sqrt{R^2+x^2}}\right)\right]}$$

P25.56 $dU = Vdq$ where the potential $V = \dfrac{k_e q}{r}$.

The element of charge in a shell is $dq = \rho$ (volume element) or $dq = \rho\left(4\pi r^2 dr\right)$ and the charge q in a sphere of radius r is

$$q = 4\pi\rho \int_0^r r^2 dr = \rho\left(\frac{4\pi r^3}{3}\right)$$

Substituting this into the expression for dU, we have

$$dU = \left(\frac{k_e q}{r}\right)dq = k_e \rho\left(\frac{4\pi r^3}{3}\right)\left(\frac{1}{r}\right)\rho\left(4\pi r^2 dr\right) = k_e\left(\frac{16\pi^2}{3}\right)\rho^2 r^4 dr$$

$$U = \int dU = k_e\left(\frac{16\pi^2}{3}\right)\rho^2 \int_0^R r^4 dr = k_e\left(\frac{16\pi^2}{15}\right)\rho^2 R^5$$

But the *total* charge, $Q = \rho \dfrac{4}{3}\pi R^3$. Therefore, $\boxed{U = \dfrac{3}{5}\dfrac{k_e Q^2}{R}}$.

P25.57 (a) $V = \frac{k_e q}{r_1} - \frac{k_e q}{r_2} = \frac{k_e q}{r_1 r_2}(r_2 - r_1)$

From the figure, for $r \gg a$, $r_2 - r_1 \approx 2a\cos\theta$

Then $v \approx \frac{k_e q}{r_1 r_2} 2a\cos\theta \approx \frac{k_e p\cos\theta}{r^2}$

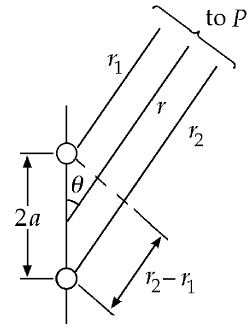

FIG. P25.57

(b) $E_r = -\frac{\partial V}{\partial r} = \boxed{\frac{2k_e p\cos\theta}{r^3}}$

In spherical coordinates, the θ component of the gradient is $-\frac{1}{r}\left(\frac{\partial}{\partial\theta}\right)$.

Therefore, $E_\theta = -\frac{1}{r}\left(\frac{\partial V}{\partial\theta}\right) = \boxed{\frac{k_e p\sin\theta}{r^3}}$

For $r \gg a$ $E_r(0°) = \frac{2k_e p}{r^3}$

and $E_r(90°) = 0$,

$E_\theta(0°) = 0$

and $E_\theta(90°) = \frac{k_e p}{r^3}$

These results are $\boxed{\text{reasonable for } r \gg a}$. Their directions are as shown in Figure 25.13 (c). However, for $\boxed{r \to 0,\ E(0) \to \infty. \text{ This is unreasonable,}}$ since r is not much greater than a if it is 0.

(c) $\boxed{V = \frac{k_e p y}{(x^2 + y^2)^{3/2}}}$

and $\boxed{E_x = -\frac{\partial V}{\partial x} = \frac{3k_e p x y}{(x^2 + y^2)^{5/2}}}$

$\boxed{E_y = -\frac{\partial V}{\partial y} = \frac{k_e p(2y^2 - x^2)}{(x^2 + y^2)^{5/2}}}$

***P25.58** (a) $k_eQ/r = 8.99 \times 10^9\ (1.6 \times 10^{-9})\ \text{V}/2 = \boxed{7.19\ \text{V}}$

(b) $$\frac{8.99\ \ 1.6}{2(1+\frac{1}{2})} + \frac{8.99\ \ 1.6}{2(1+\frac{3}{2})} = \boxed{7.67\ \text{V}}$$

(c) $$\frac{8.99\ \ 1.6}{4}\left(\frac{1}{1+\frac{1}{4}} + \frac{1}{1+\frac{3}{4}} + \frac{1}{1+\frac{5}{4}} + \frac{1}{1+\frac{7}{4}}\right) = \boxed{7.84\ \text{V}}$$

(d) We find $$\frac{8.99\ \ 1.6}{32}\left(\frac{1}{1+\frac{1}{32}} + \frac{1}{1+\frac{3}{32}} + \cdots + \frac{1}{1+\frac{63}{32}}\right) = \boxed{7.900\ 2\ \text{V}}$$

(e) We find $$\frac{8.99\ \ 1.6}{64}\left(\frac{1}{1+\frac{1}{64}} + \frac{1}{1+\frac{3}{64}} + \cdots + \frac{1}{1+\frac{127}{64}}\right) = \boxed{7.901\ 0\ \text{V}}$$

(f) We represent the exact result as

$$V = \frac{k_eQ}{\ell}\ln\left(\frac{\ell+a}{a}\right) = \frac{8.99\ \ 1.6\ \text{V}}{2}\ln\left(\frac{3}{1}\right) = 7.901\ 2\ \text{V}$$

Modeling the line as a set of points works nicely. The exact result, represented as 7.901 2 V, is approximated to within 0.8% by the four-particle version. The 16-particle approximation gives a result accurate to three digits, to within 0.05%. The 64-charge approximation gives a result accurate to four digits, differing by only 0.003% from the exact result.

P25.59 The positive plate by itself creates a field $E = \frac{\sigma}{2\epsilon_0} = \frac{36.0\times10^{-9}\ \text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 2.03\ \text{kN/C}$ away from the + plate. The negative plate by itself creates the same size field and between the plates it is in the same direction. Together the plates create a uniform field $4.07\ \text{kN/C}$ in the space between.

(a) Take $V = 0$ at the negative plate. The potential at the positive plate is then

$$V - 0 = -\int_0^{12.0\ \text{cm}} (-4.07\ \text{kN/C})\,dx$$

The potential difference between the plates is $V = \left(4.07\times10^3\ \text{N/C}\right)(0.120\ \text{m}) = \boxed{488\ \text{V}}$.

(b) $$\left(\frac{1}{2}mv^2 + qV\right)_i = \left(\frac{1}{2}mv^2 + qV\right)_f$$

$$qV = \left(1.60\times10^{-19}\ \text{C}\right)(488\ \text{V}) = \frac{1}{2}mv_f^2 = \boxed{7.81\times10^{-17}\ \text{J}}$$

(c) $v_f = \boxed{306\ \text{km/s}}$

(d) $$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$$

$$\left(3.06\times10^5\ \text{m/s}\right)^2 = 0 + 2a(0.120\ \text{m})$$

$a = \boxed{3.90\times10^{11}\ \text{m/s}^2}$ toward the negative plate

(e) $\sum F = ma = \left(1.67\times10^{-27}\ \text{kg}\right)\left(3.90\times10^{11}\ \text{m/s}^2\right) = \boxed{6.51\times10^{-16}\ \text{N}}$ toward the negative plate

(f) $$E = \frac{F}{q} = \frac{6.51\times10^{-16}\ \text{N}}{1.60\times10^{-19}\ \text{C}} = \boxed{4.07\ \text{kN/C}}$$

P25.60 For the given charge distribution, $V(x, y, z) = \frac{k_e(q)}{r_1} + \frac{k_e(-2q)}{r_2}$

where $r_1 = \sqrt{(x+R)^2 + y^2 + z^2}$ and $r_2 = \sqrt{x^2 + y^2 + z^2}$

The surface on which $V(x, y, z) = 0$

is given by $k_e q\left(\frac{1}{r_1} - \frac{2}{r_2}\right) = 0$, or $2r_1 = r_2$

This gives: $4(x+R)^2 + 4y^2 + 4z^2 = x^2 + y^2 + z^2$

which may be written in the form: $x^2 + y^2 + z^2 + \left(\frac{8}{3}R\right)x + (0)y + (0)z + \left(\frac{4}{3}R^2\right) = 0$ [1]

The general equation for a sphere of radius a centered at (x_0, y_0, z_0) is:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 - a^2 = 0$$

or $x^2 + y^2 + z^2 + (-2x_0)x + (-2y_0)y + (-2z_0)z + (x_0^2 + y_0^2 + z_0^2 - a^2) = 0$ [2]

Comparing equations [1] and [2], it is seen that the equipotential surface for which $V = 0$ is indeed a sphere and that:

$$-2x_0 = \frac{8}{3}R;\ -2y_0 = 0;\ -2z_0 = 0;\quad x_0^2 + y_0^2 + z_0^2 - a^2 = \frac{4}{3}R^2$$

Thus, $x_0 = -\frac{4}{3}R$, $y_0 = z_0 = 0$, and $a^2 = \left(\frac{16}{9} - \frac{4}{3}\right)R^2 = \frac{4}{9}R^2$

The equipotential surface is therefore a sphere centered at $\boxed{\left(-\frac{4}{3}R,\ 0,\ 0\right)}$, having a radius $\boxed{\frac{2}{3}R}$.

P25.61 Inside the sphere, $\boxed{E_x = E_y = E_z = 0}$.

Outside, $E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(V_0 - E_0 z + E_0 a^3 z\left(x^2 + y^2 + z^2\right)^{-3/2}\right)$

So $E_x = -\left[0 + 0 + E_0 a^3 z\left(-\frac{3}{2}\right)\left(x^2 + y^2 + z^2\right)^{-5/2}(2x)\right] = \boxed{3E_0 a^3 xz\left(x^2 + y^2 + z^2\right)^{-5/2}}$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left(V_0 - E_0 z + E_0 a^3 z\left(x^2 + y^2 + z^2\right)^{-3/2}\right)$$

$$E_y = -E_0 a^3 z\left(-\frac{3}{2}\right)\left(x^2 + y^2 + z^2\right)^{-5/2} 2y = \boxed{3E_0 a^3 yz\left(x^2 + y^2 + z^2\right)^{-5/2}}$$

$$E_z = -\frac{\partial V}{\partial z} = E_0 - E_0 a^3 z\left(-\frac{3}{2}\right)\left(x^2 + y^2 + z^2\right)^{-5/2}(2z) - E_0 a^3\left(x^2 + y^2 + z^2\right)^{-3/2}$$

$$E_z = \boxed{E_0 + E_0 a^3\left(2z^2 - x^2 - y^2\right)\left(x^2 + y^2 + z^2\right)^{-5/2}}$$

ANSWERS TO EVEN PROBLEMS

P25.2 1.35 MJ

P25.4 +260 V

P25.6 $0.300\ \text{m/s}$

P25.8 See the solution.

P25.10 (a) -4.83 m (b) $0.667\ \text{m}$ and $-2.00\ \text{m}$

P25.12 (a) 0 (b) 0 (c) 45.0 kV

P25.14 (a) -386 nJ (b) 103 V

P25.16 (a) 32.2 kV (b) -96.5 mJ

P25.18 $-\dfrac{5k_e q}{R}$

P25.20 (a) +7.00 nC with position vector 7.00 cm at 70.0° and -8.00 nC with position vector 3.00 cm at 90.0° (b) $(0.070\,3\ \hat{\mathbf{i}} - 1.09\ \hat{\mathbf{j}})$mN (c) +24.0 μJ

P25.22 See the solution.

P25.24 (a) $v_1 = \sqrt{\dfrac{2m_2 k_e q_1 q_2}{m_1(m_1+m_2)}\left(\dfrac{1}{r_1+r_2}-\dfrac{1}{d}\right)}$ $\quad v_2 = \sqrt{\dfrac{2m_1 k_e q_1 q_2}{m_2(m_1+m_2)}\left(\dfrac{1}{r_1+r_2}-\dfrac{1}{d}\right)}$ (b) Faster than calculated in (a)

P25.26 27.4 fm

P25.28 (a) 0 (b) $\dfrac{k_e Q}{r^2}$ radially outward

P25.30 (a) larger at *A* (b) $200\ \text{N/C}$ down (c) See the solution.

P25.32 $-0.553\dfrac{k_e Q}{R}$

P25.34 $-\dfrac{k_e \alpha L}{2}\ln\left[\dfrac{\sqrt{b^2+(L^2/4)}-L/2}{\sqrt{b^2+(L^2/4)}+L/2}\right]$

P25.36 $k_e\lambda(\pi + 2\ln 3)$

P25.38 1.56×10^{12} electrons

P25.40 (a) 135 kV (b) $2.25\ \text{MV/m}$ away from the large sphere and $6.74\ \text{MV/m}$ away from the small sphere

P25.42 See the solution.

P25.44 (a) negative 1.06 nC/m^2 (b) −542 kC (c) 764 MV (d) His head is higher in potential by 210 V. (e) 4.88 kN away from the Earth (f) The gravitational force is in the opposite direction and 4.08×10^{16} times larger. Electrical forces are negligible in accounting for planetary motion. (g) −490 nC (h) Less charge to be suspended at the equator. The gravitational force is weaker at a greater distance from the Earth's center. The suspended particle is not quite in equilibrium, but accelerating downward to participate in the daily rotation. At uniform potential, the planet's surface creates a stronger electric field at the equator, where its radius of curvature is smaller.

P25.46 (a) The velocity of one particle relative to the other is first a velocity of approach, then zero at closest approach, and then a velocity of recession. (b) 6.00 $\hat{\mathbf{i}}$ m/s (c) 3.64 m (d) −9.00 $\hat{\mathbf{i}}$ m/s for the incident particle and 12.0 $\hat{\mathbf{i}}$ m/s for the target particle.

P25.48 (a) $\dfrac{k_e Q}{h}\ln\left(\dfrac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)$

(b) $\dfrac{k_e Q}{R^2 h}\left[(d+h)\sqrt{(d+h)^2+R^2}-d\sqrt{d^2+R^2}-2dh-h^2+R^2\ln\left(\dfrac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)\right]$

P25.50 See the solution.

P25.52 $\left(\dfrac{k_e q^2}{3am}\right)^{1/2}$

P25.54 $k_e\lambda\ln\left[\dfrac{a+L+\sqrt{(a+L)^2+b^2}}{a+\sqrt{a^2+b^2}}\right]$

P25.56 $\dfrac{3}{5}\dfrac{k_e Q^2}{R}$

P25.58 (a) 7.19 V (b) 7.67 V (c) 7.84 V (d) 7.900 2 V (e) 7.901 0 V (f) Modeling the line as a set of points works nicely. The exact result, represented as 7.901 2 V, is approximated to within 0.8% by the four-particle version. The 16-particle approximation gives a result accurate to three digits, to within 0.05%. The 64-charge approximation gives a result accurate to four digits, differing by only 0.003% from the exact result.

P25.60 See the solution.