Chapter 25

Electric Potential

Electric Potential

Electromagnetism has been connected to the study of forces in previous chapters.

In this chapter, electromagnetism will be linked to energy.

By using an energy approach, problems could be solved that were insoluble using forces.

The concept of potential energy is of great value in the study of electricity.

Because the electrostatic force is conservative, electrostatic phenomena can be conveniently described in terms of an electric potential energy.

This will enable the definition of *electric potential.*

Electrical Potential Energy

When a test charge is placed in an electric field, it experiences a force.

- $\vec{\mathbf{F}}_e = q_o\vec{\mathbf{E}}$
- The force is conservative.

If the test charge is moved in the field by some external agent, the work done by the field is the negative of the work done by the external agent.

$d\vec{\mathbf{s}}$ is an infinitesimal displacement vector that is oriented tangent to a path through space.

- The path may be straight or curved and the integral performed along this path is called either a *path integral* or a *line integral.*

Electric Potential Energy, cont

The work done within the charge-field system by the electric field on the charge is

$$\vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} = q_o \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

As this work is done by the field, the potential energy of the charge-field system is changed by $\Delta U = -q_o \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$

For a finite displacement of the charge from A to B, the change in potential energy of the system is

$$\Delta U = U_B - U_A = -q_o \int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Because the force is conservative, the line integral does not depend on the path taken by the charge.

Electric Potential

The potential energy per unit charge, U/q_o, is the **electric potential.**

- The potential is characteristic of the field only.
  - The potential energy is characteristic of the charge-field system.
- The potential is independent of the value of q_o.
- The potential has a value at every point in an electric field.

The electric potential is

$$V = \frac{U}{q_o}$$

Electric Potential, cont.

The potential is a scalar quantity.

- Since energy is a scalar

As a charged particle moves in an electric field, it will experience a change in potential.

$$\Delta V = \frac{\Delta U}{q_o} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

The infinitesimal displacement is interpreted as the displacement between two points in space rather than the displacement of a point charge.

Electric Potential, final

The difference in potential is the meaningful quantity.

We often take the value of the potential to be zero at some convenient point in the field.

Electric potential is a scalar characteristic of an electric field, independent of any charges that may be placed in the field.

The potential difference between two points exists solely because of a source charge and depends on the source charge distribution.

- For a potential energy to exist, there must be a system of two or more charges.
- The potential energy belongs to the system and changes only if a charge is moved relative to the rest of the system.

Work and Electric Potential

Assume a charge moves in an electric field without any change in its kinetic energy.

The work performed on the charge is

$$W = \Delta U = q \, \Delta V$$

Units: $1 \text{ V} \equiv 1 \text{ J/C}$

- V is a volt.
- It takes one joule of work to move a 1-coulomb charge through a potential difference of 1 volt.

In addition, 1 N/C = 1 V/m

- This indicates we can interpret the electric field as a measure of the rate of change of the electric potential with respect to position.

Voltage

Electric potential is described by many terms.

The most common term is *voltage.*

A voltage applied to a device or across a device is the same as the potential difference across the device.

- The voltage is not something that moves through a device.

Electron-Volts

Another unit of energy that is commonly used in atomic and nuclear physics is the electron-volt.

One ***electron-volt*** is defined as the energy a charge-field system gains or loses when a charge of magnitude *e* (an electron or a proton) is moved through a potential difference of 1 volt.

- 1 eV = 1.60×10^{-19} J

Potential Difference in a Uniform Field

The equations for electric potential between two points A and B can be simplified if the electric field is uniform:

$$V_B - V_A = \Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -E\int_A^B d\mathbf{s} = -Ed$$

The displacement points from A to B and is parallel to the field lines.

The negative sign indicates that the electric potential at point *B* is lower than at point *A.*

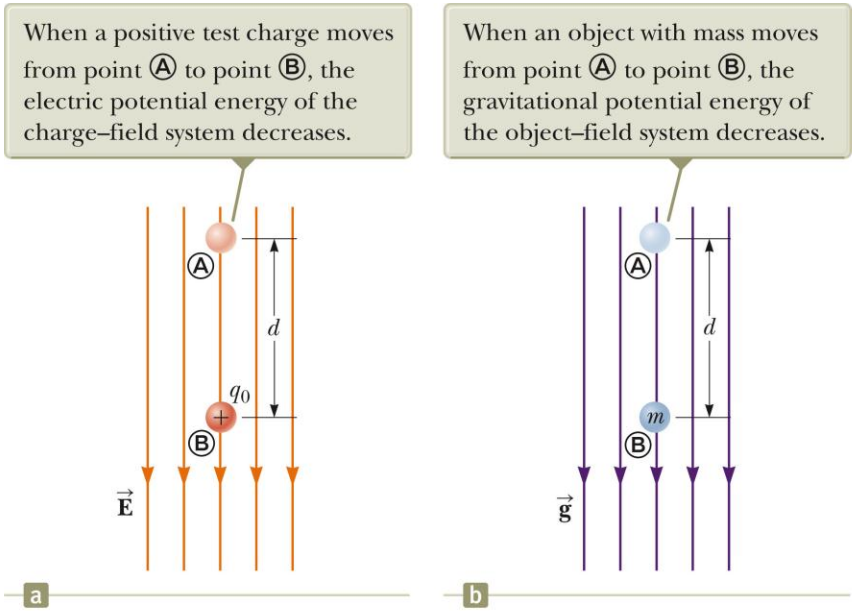
- Electric field lines always point in the direction of decreasing electric potential.

Energy and the Direction of Electric Field

When the electric field is directed downward, point *B* is at a lower potential than point *A.*

When a positive test charge moves from *A* to *B*, the charge-field system loses potential energy.

Electric field lines always point in the direction of decreasing electric potential.



More About Directions

A system consisting of a positive charge and an electric field **loses** electric potential energy when the charge moves in the direction of the field.

- An electric field does work on a positive charge when the charge moves in the direction of the electric field.

The charged particle gains kinetic energy and the potential energy of the charge-field system decreases by an equal amount.

- Another example of Conservation of Energy

Directions, cont.

If q_o is negative, then ΔU is positive.

A system consisting of a negative charge and an electric field *gains* potential energy when the charge moves in the direction of the field.

- In order for a negative charge to move in the direction of the field, an external agent must do positive work on the charge.

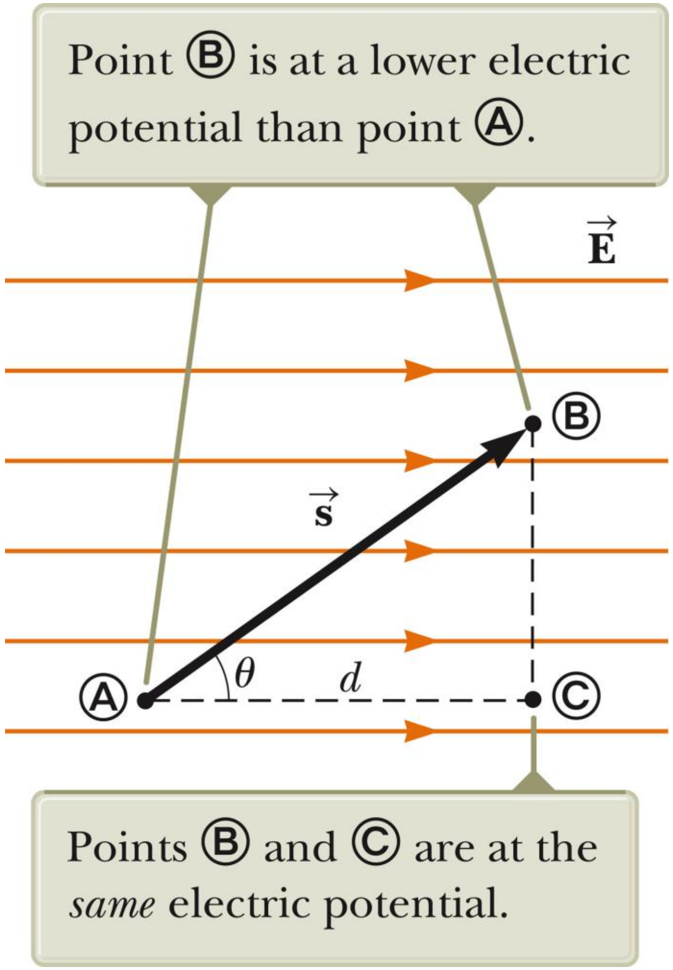
Equipotentials

Point *B* is at a lower potential than point *A.*

Points *A* and *C* are at the same potential.

- All points in a plane perpendicular to a uniform electric field are at the same electric potential.

The name **equipotential surface** is given to any surface consisting of a continuous distribution of points having the same electric potential.



Charged Particle in a Uniform Field, Example

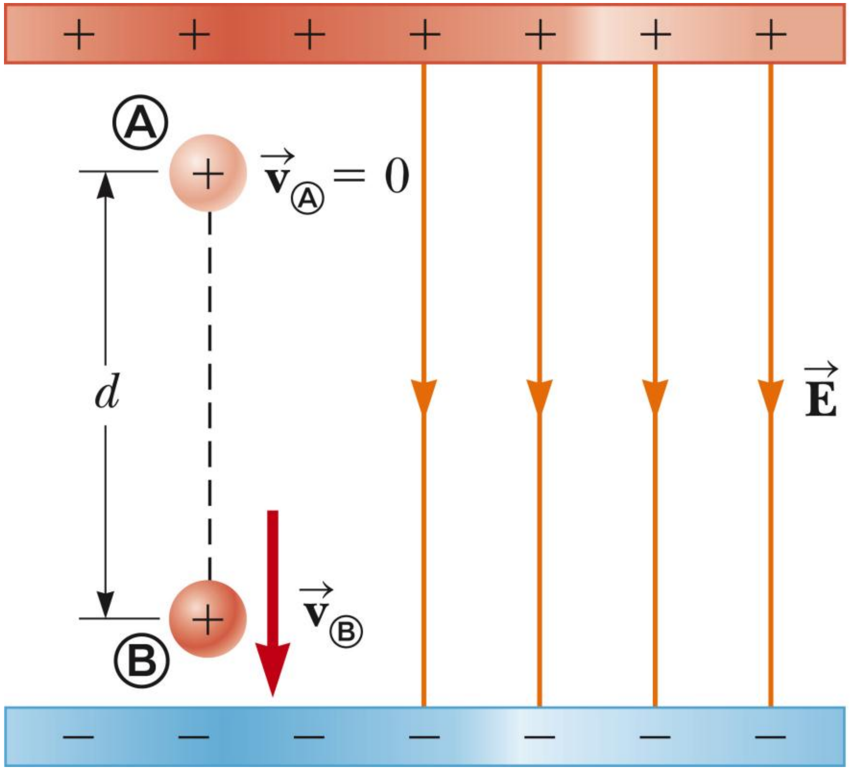
A positive charge is released from rest and moves in the direction of the electric field.

The change in potential is negative.

The change in potential energy is negative.

The force and acceleration are in the direction of the field.

Conservation of Energy can be used to find its speed.

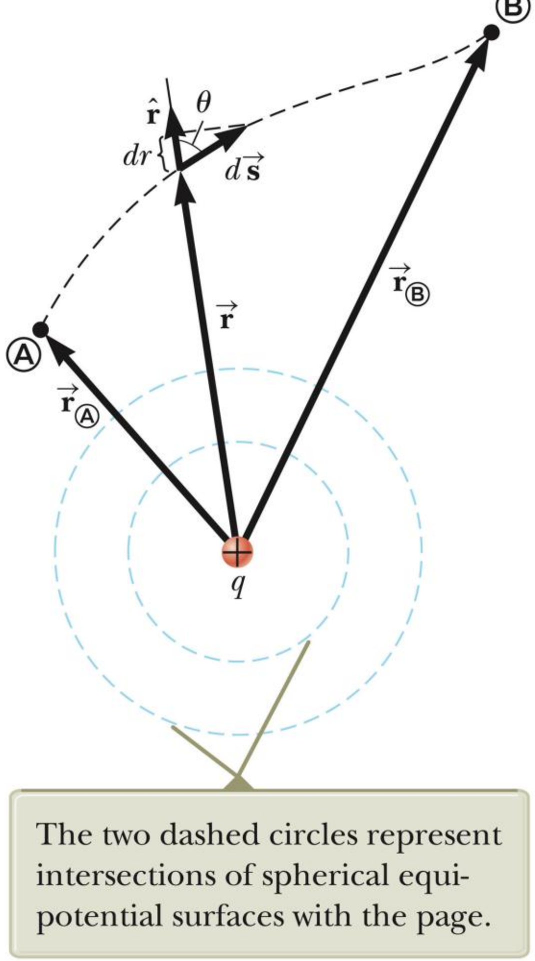


Potential and Point Charges

An isolated positive point charge produces a field directed radially outward.

The potential difference between points *A* and *B* will be

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$



Potential and Point Charges, cont.

The electric potential is independent of the path between points *A* and *B.*

It is customary to choose a reference potential of $V = 0$ at $r_A = \infty$.

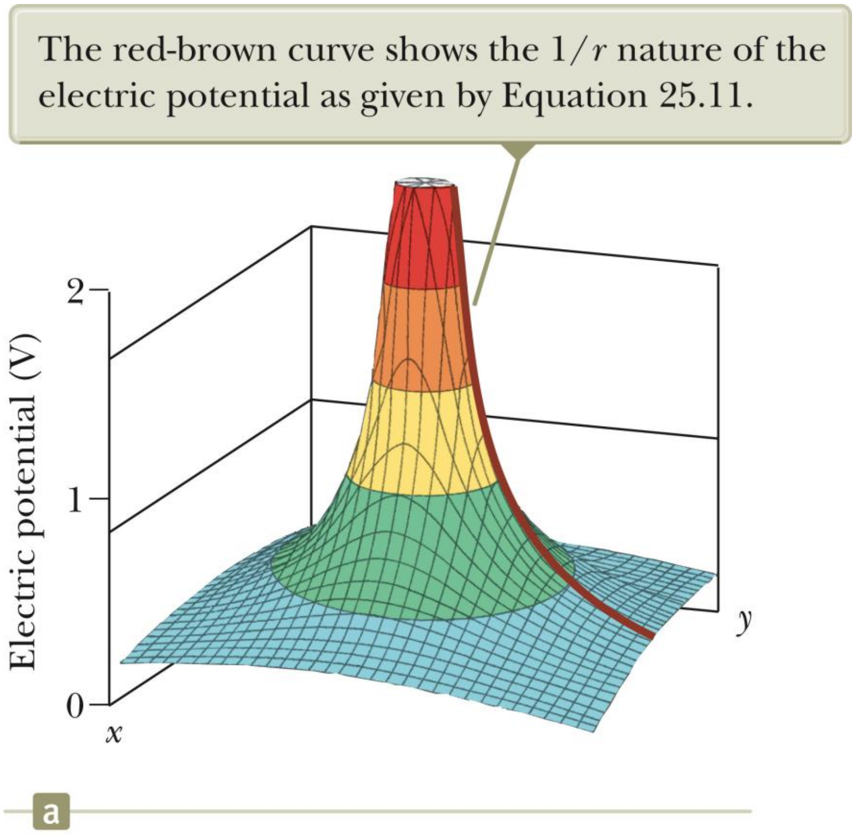
Then the potential due to a point charge at some point *r* is

$$V = k_e \frac{q}{r}$$

Electric Potential of a Point Charge

The electric potential in the plane around a single point charge is shown.

The red line shows the 1/*r* nature of the potential.



Electric Potential with Multiple Charges

The electric potential due to several point charges is the sum of the potentials due to each individual charge.

- This is another example of the superposition principle.
- The sum is the algebraic sum

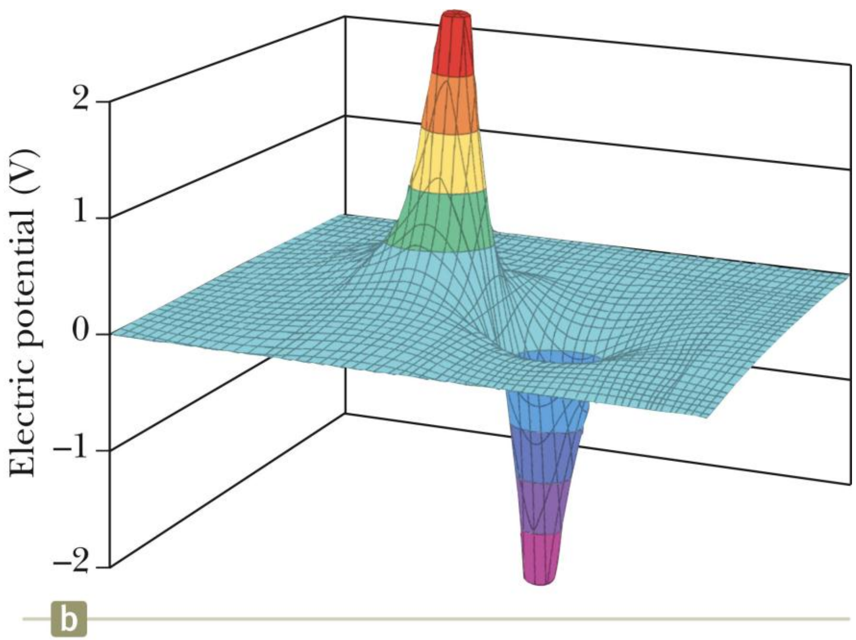
$$V = k_e \sum_i \frac{q_i}{r_i}$$

  - $V = 0$ at $r = \infty$

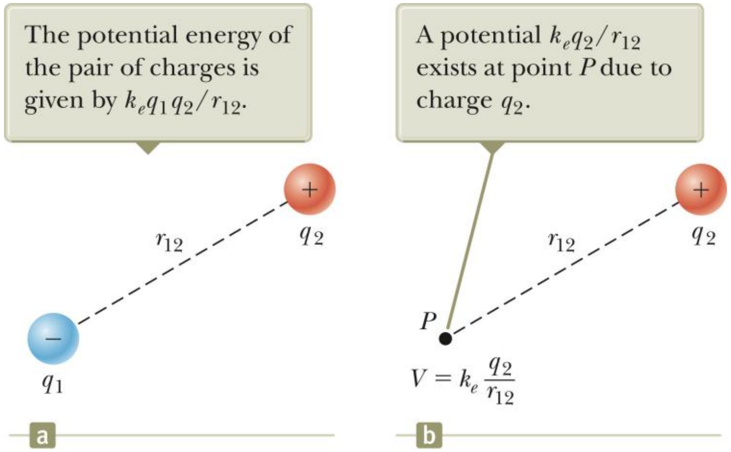
Electric Potential of a Dipole

The graph shows the potential (y-axis) of an electric dipole.

The steep slope between the charges represents the strong electric field in this region.



Potential Energy of Multiple Charges



The potential energy of the system is $U = k_e \dfrac{q_1 q_2}{r_{12}}$.

If the two charges are the same sign, U is positive and work must be done to bring the charges together.

If the two charges have opposite signs, U is negative and work is done to keep the charges apart.

U with Multiple Charges, final

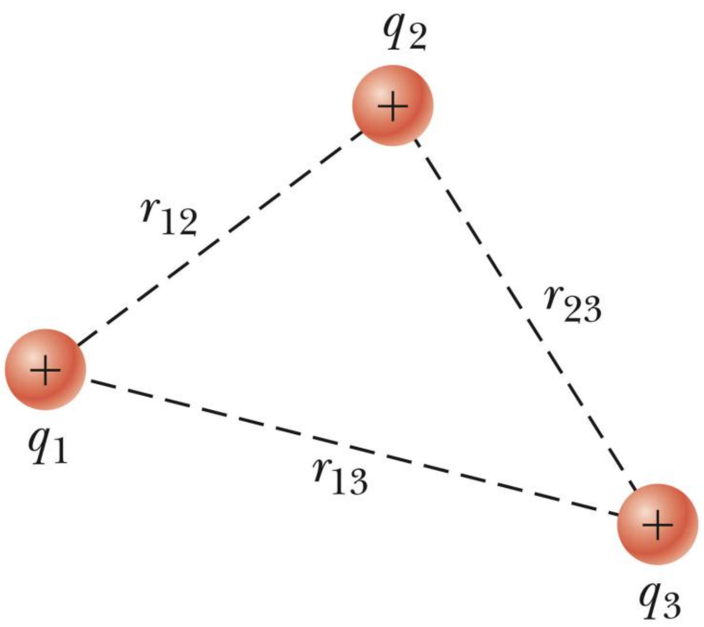
If there are more than two charges, then find *U* for each pair of charges and add them.

For three charges:

$$U = k_e\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$

- The result is independent of the order of the charges.

The potential energy of this system of charges is given by Equation 25.14.



Finding E From *V*

Assume, to start, that the field has only an *x* component.

$$E_x = -\frac{dV}{dx}$$

Similar statements would apply to the *y* and *z* components.

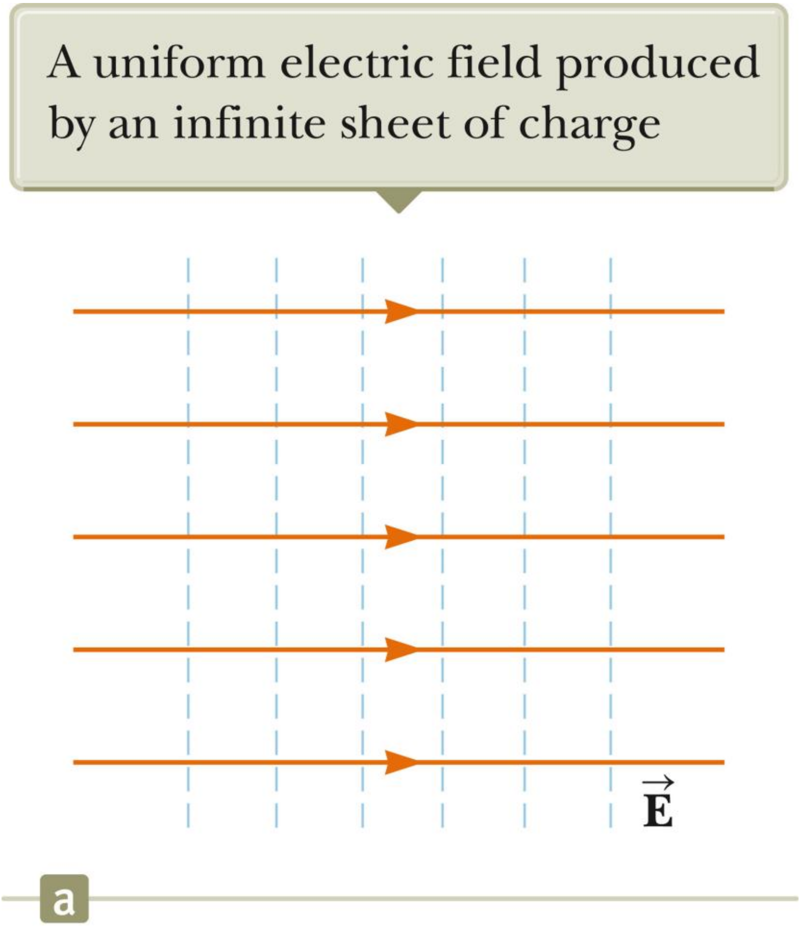
Equipotential surfaces must always be perpendicular to the electric field lines passing through them.

E and *V* for an Infinite Sheet of Charge

The equipotential lines are the dashed blue lines.

The electric field lines are the brown lines.

The equipotential lines are everywhere perpendicular to the field lines.



E and V for a Point Charge

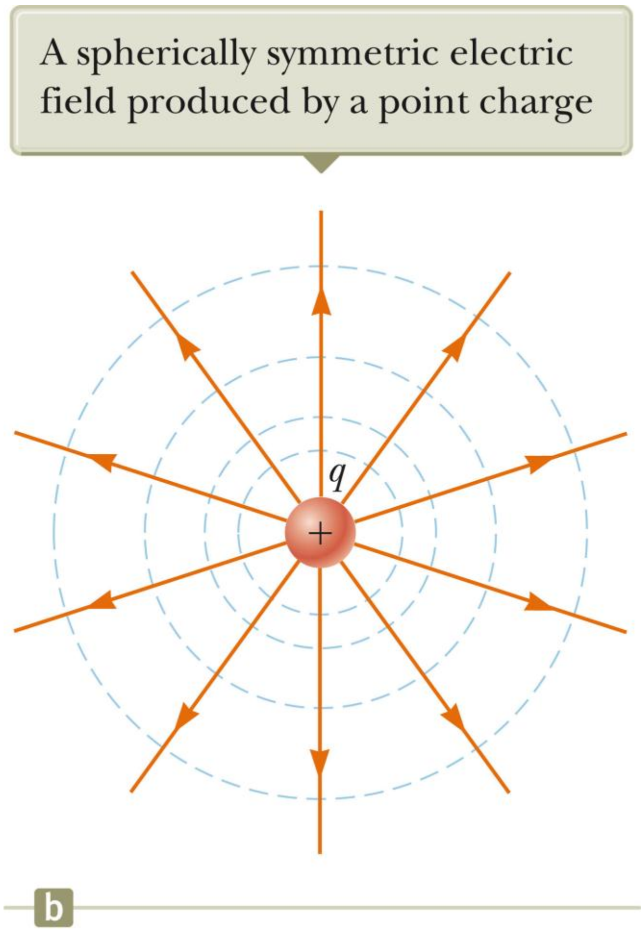
The equipotential lines are the dashed blue lines.

The electric field lines are the brown lines.

The electric field is radial.

$E_r = - dV / dr$

The equipotential lines are everywhere perpendicular to the field lines.



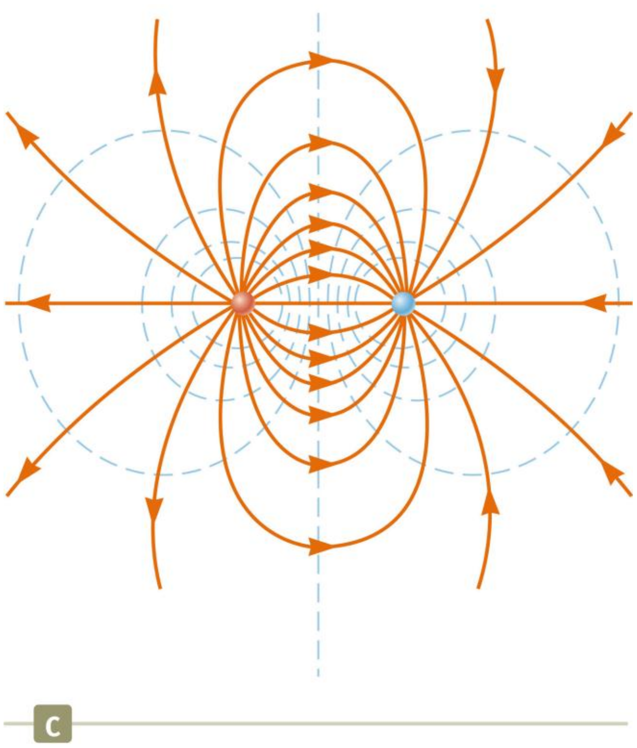
E and *V* for a Dipole

The equipotential lines are the dashed blue lines.

The electric field lines are the brown lines.

The equipotential lines are everywhere perpendicular to the field lines.

An electric field produced by an electric dipole



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Electric Field from Potential, General

In general, the electric potential is a function of all three dimensions.

Given $V(x, y, z)$ you can find E_x, E_y and E_z as partial derivatives:

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

Electric Potential for a Continuous Charge Distribution

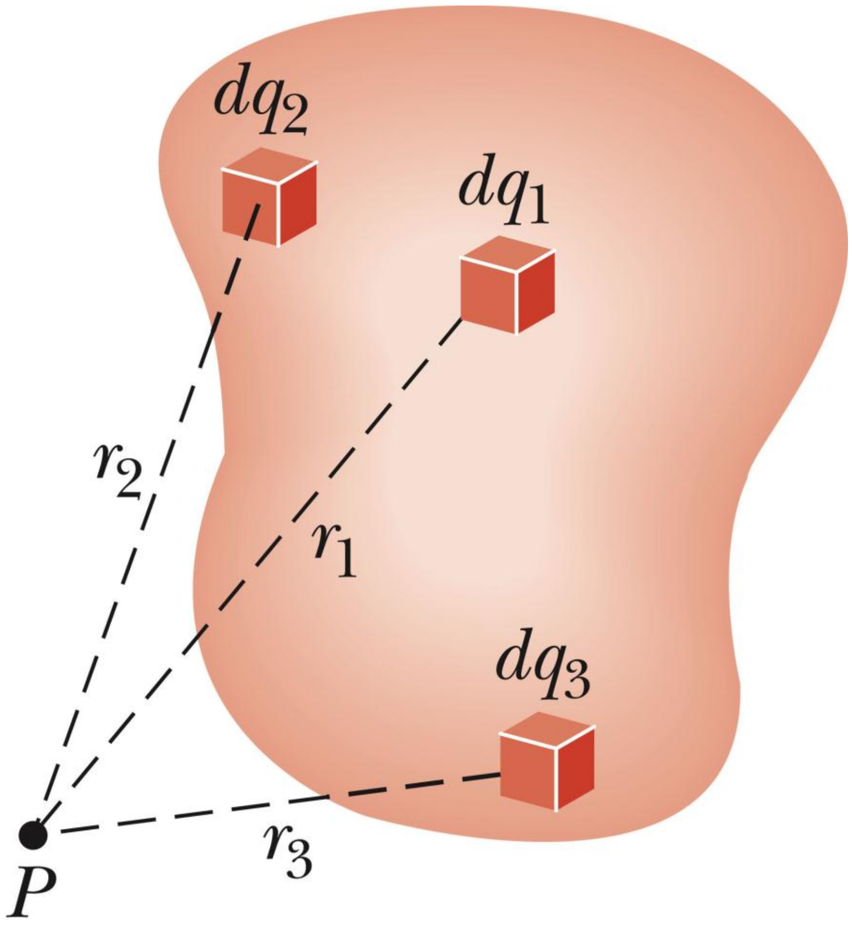
Method 1: The charge distribution is known.

Consider a small charge element *dq*

- Treat it as a point charge.

The potential at some point due to this charge element is

$$dV = k_e \frac{dq}{r}$$



V for a Continuous Charge Distribution, cont.

To find the total potential, you need to integrate to include the contributions from all the elements.

$$V = k_e \int \frac{dq}{r}$$

- This value for V uses the reference of $V = 0$ when P is infinitely far away from the charge distributions.

V for a Continuous Charge Distribution, final

If the electric field is already known from other considerations, the potential can be calculated using the original approach:

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

- If the charge distribution has sufficient symmetry, first find the field from Gauss' Law and then find the potential difference between any two points,
  - Choose V = 0 at some convenient point

Problem-Solving Strategies

Conceptualize

- Think about the individual charges or the charge distribution.
- Imagine the type of potential that would be created.
- Appeal to any symmetry in the arrangement of the charges.

Categorize

- Group of individual charges or a continuous distribution?
- The answer will determine the procedure to follow in the analysis step.

Problem-Solving Strategies, 2

Analyze

- General
  - Scalar quantity, so no components
  - Use algebraic sum in the superposition principle
    - Keep track of signs
  - Only changes in electric potential are significant
  - Define $V = 0$ at a point infinitely far away from the charges.
    - If the charge distribution extends to infinity, then choose some other arbitrary point as a reference point.

Problem-Solving Strategies, 3

Analyze, cont

- If a group of individual charges is given
  - Use the superposition principle and the algebraic sum.
- If a continuous charge distribution is given
  - Use integrals for evaluating the total potential at some point.
  - Each element of the charge distribution is treated as a point charge.
- If the electric field is given
  - Start with the definition of the electric potential.
  - Find the field from Gauss' Law (or some other process) if needed.

Problem-Solving Strategies, final

Finalize

- Check to see if the expression for the electric potential is consistent with your mental representation.
- Does the final expression reflect any symmetry?
- Image varying parameters to see if the mathematical results change in a reasonable way.

V for a Uniformly Charged Ring

P is located on the perpendicular central axis of the uniformly charged ring .

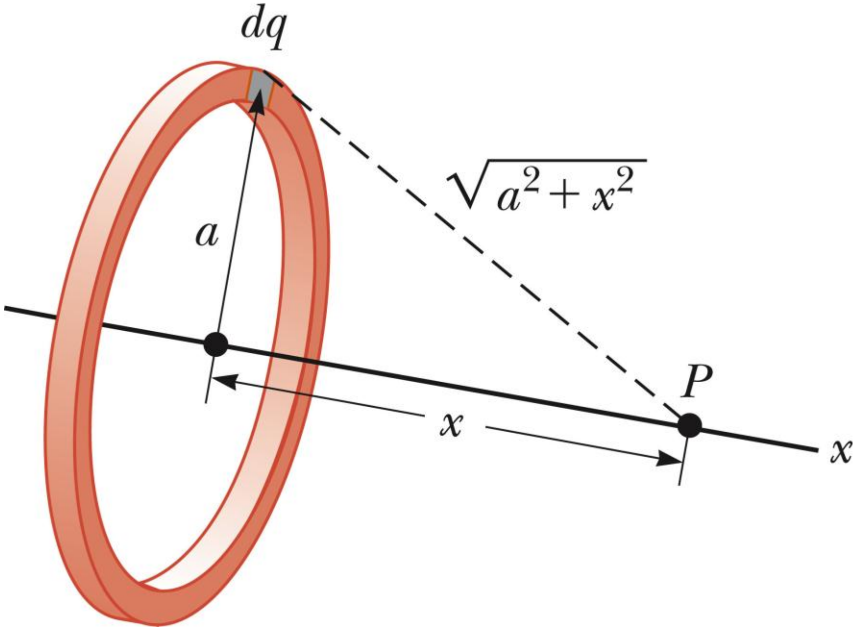
The symmetry of the situation means that all the charges on the ring are the same distance from point P.

- The ring has a radius *a* and a total charge *Q.*

The potential and the field are given by

$$V = k_e \int \frac{dq}{r} = \frac{k_e Q}{\sqrt{a^2 + x^2}}$$

$$E_x = \frac{k_e x}{\left(a^2 + x^2\right)^{3/2}} Q$$



V for a Uniformly Charged Disk

The ring has a radius *R* and surface charge density of *σ.*

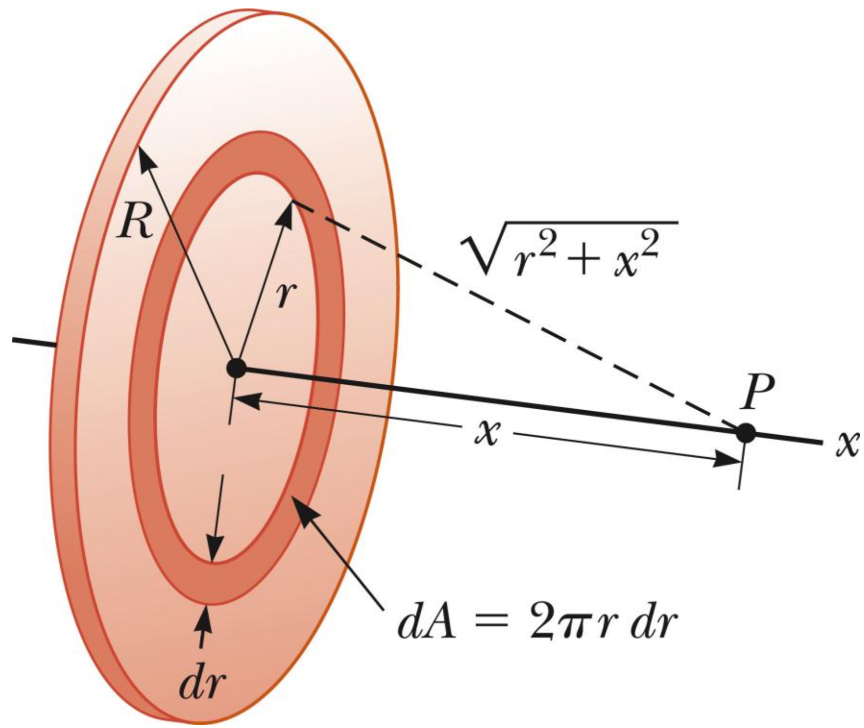
P is along the perpendicular central axis of the disk.

P is on the cental axis of the disk, symmetry indicates that all points in a given ring are the same distance from *P.*

The potential and the field are given by

$$V = 2\pi k_e \sigma \left[\left(R^2 + x^2 \right)^{1/2} - x \right]$$

$$E_x = 2\pi k_e \sigma \left[1 - \frac{x}{\left(R^2 + x^2 \right)^{1/2}} \right]$$

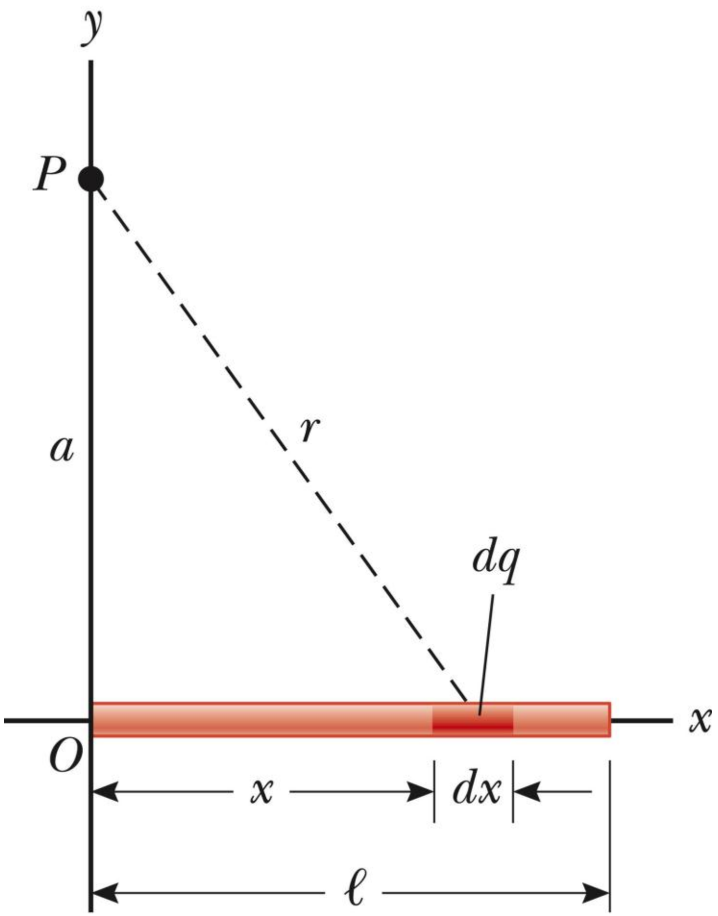


V for a Finite Line of Charge

A rod of line ℓ has a total charge of *Q* and a linear charge density of *λ.*

- There is no symmetry to use, but the geometry is simple.

$$V = \frac{k_e Q}{\ell} \ln\left(\frac{\ell + \sqrt{a^2 + \ell^2}}{a}\right)$$



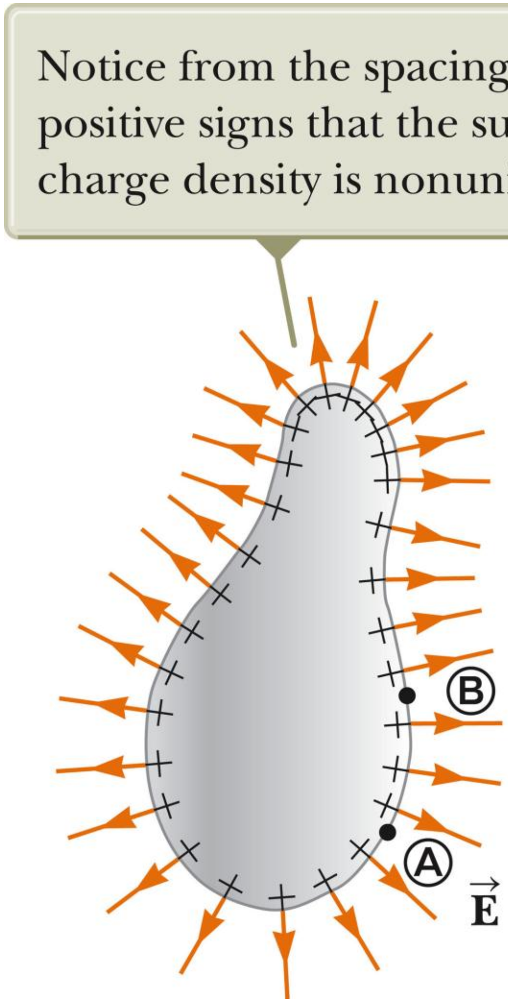
V Due to a Charged Conductor

Consider two points on the surface of the charged conductor as shown.

$\vec{\mathbf{E}}$ is always perpendicular to the displacement $d\vec{\mathbf{s}}$.

Therefore, $\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$

Therefore, the potential difference between A and B is also zero.



V Due to a Charged Conductor, cont.

V is constant everywhere on the surface of a charged conductor in equilibrium.

- $\Delta V = 0$ between any two points on the surface

The surface of any charged conductor in electrostatic equilibrium is an equipotential surface.

Every point on the surface of a charge conductor in equilibrium is at the same electric potential.

Because the electric field is zero inside the conductor, we conclude that the electric potential is constant everywhere inside the conductor and equal to the value at the surface.

Irregularly Shaped Objects

The charge density is high where the radius of curvature is small.

- And low where the radius of curvature is large

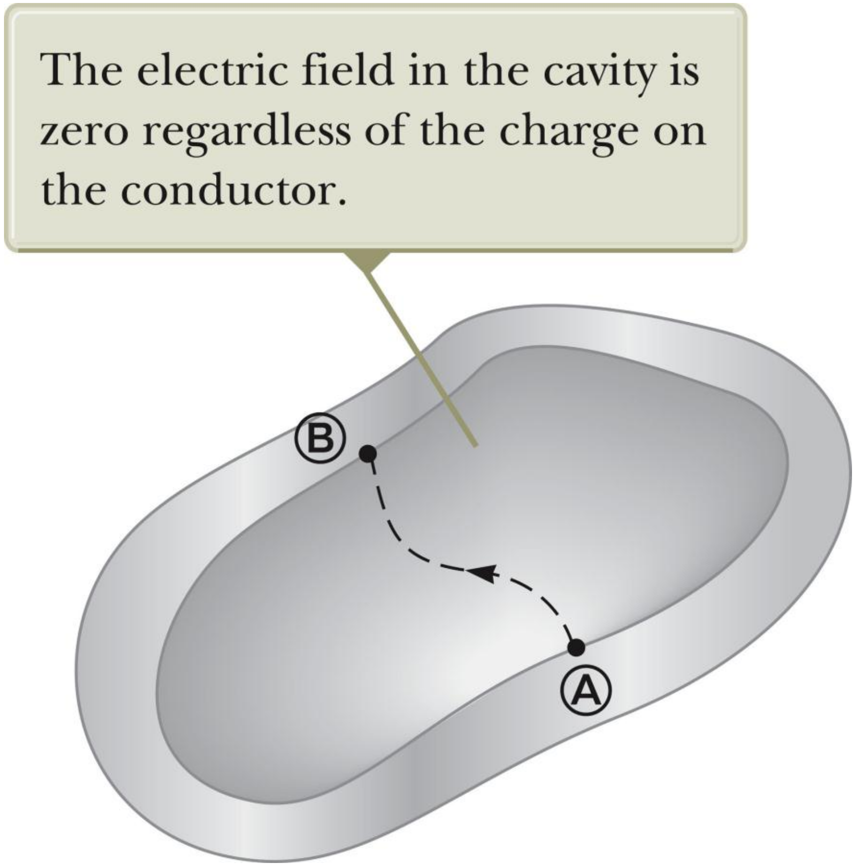
The electric field is large near the convex points having small radii of curvature and reaches very high values at sharp points.

Cavity in a Conductor

Assume an irregularly shaped cavity is inside a conductor.

Assume no charges are inside the cavity.

The electric field inside the conductor must be zero.



Cavity in a Conductor, cont

The electric field inside does not depend on the charge distribution on the outside surface of the conductor.

For all paths between *A* and *B*,

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$

A cavity surrounded by conducting walls is a field-free region as long as no charges are inside the cavity.

Corona Discharge

If the electric field near a conductor is sufficiently strong, electrons resulting from random ionizations of air molecules near the conductor accelerate away from their parent molecules.

These electrons can ionize additional molecules near the conductor.

This creates more free electrons.

The **corona discharge** is the glow that results from the recombination of these free electrons with the ionized air molecules.

The ionization and corona discharge are most likely to occur near very sharp points.

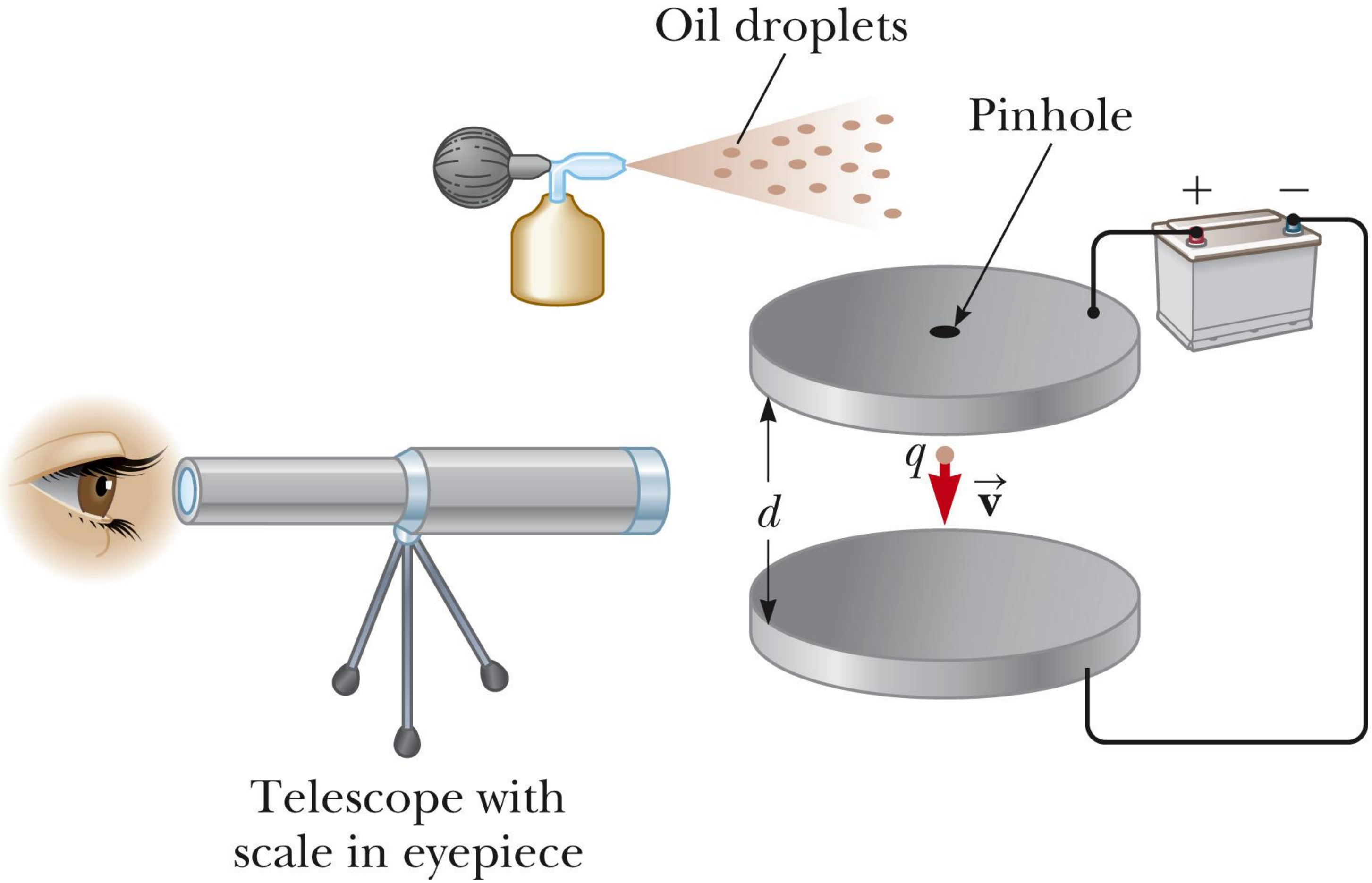
Millikan Oil-Drop Experiment

Robert Millikan measured *e*, the magnitude of the elementary charge on the electron.

He also demonstrated the quantized nature of this charge.

Oil droplets pass through a small hole and are illuminated by a light.

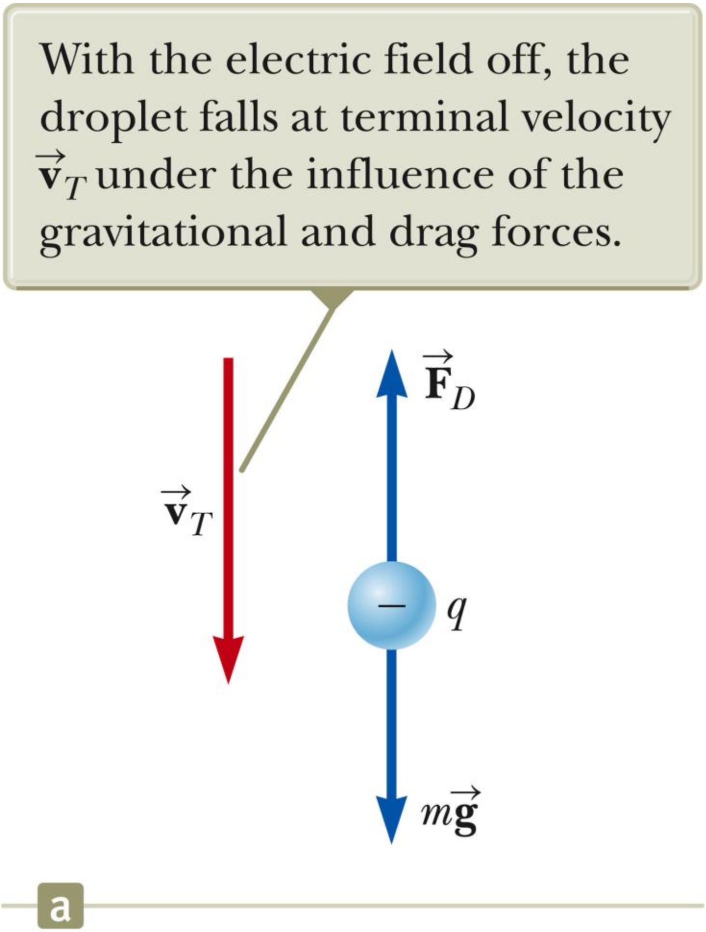
Millikan Oil-Drop Experiment – Experimental Set-Up



Oil-Drop Experiment, 2

With no electric field between the plates, the gravitational force and the drag force (viscous) act on the electron.

The drop reaches terminal velocity with
$\vec{\mathbf{F}}_D = m\vec{\mathbf{g}}$

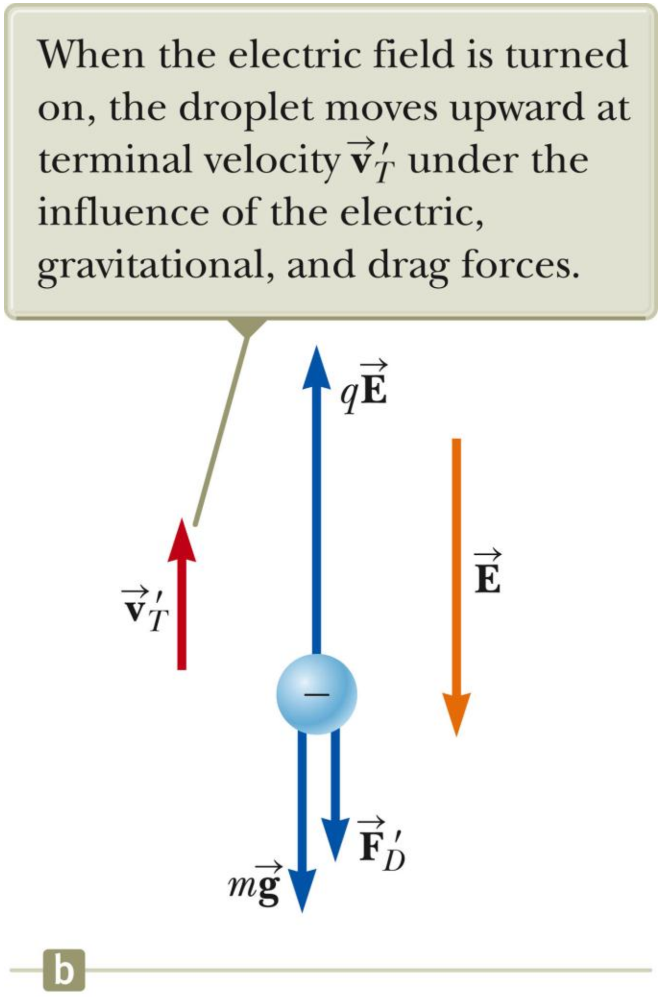


Oil-Drop Experiment, 3

An electric field is set up between the plates.

- The upper plate has a higher potential.

The drop reaches a new terminal velocity when the electrical force equals the sum of the drag force and gravity.



Oil-Drop Experiment, final

The drop can be raised and allowed to fall numerous times by turning the electric field on and off.

After many experiments, Millikan determined:

- $q = ne$ where $n = 0, -1, -2, -3, \ldots$
- $e = 1.60 \times 10^{-19}$ C

This yields conclusive evidence that charge is quantized.

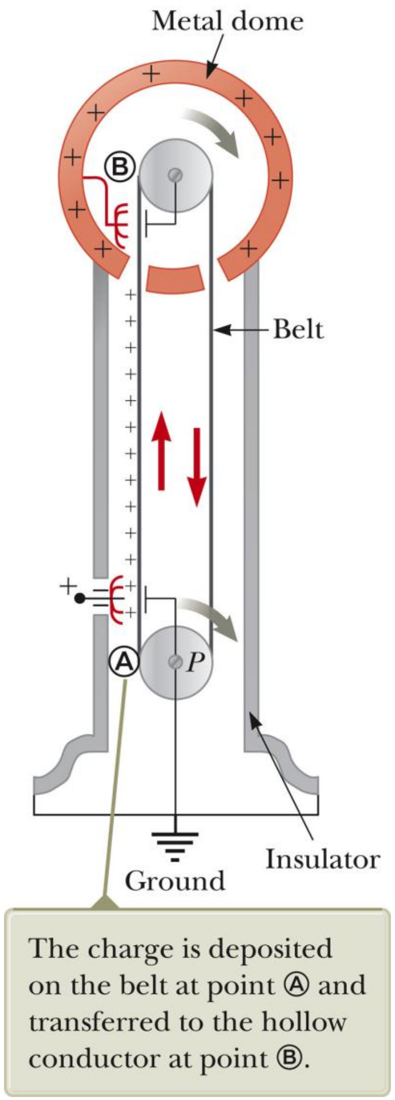
Van de Graaff Generator

Charge is delivered continuously to a high-potential electrode by means of a moving belt of insulating material.

The high-voltage electrode is a hollow metal dome mounted on an insulated column.

Large potentials can be developed by repeated trips of the belt.

Protons accelerated through such large potentials receive enough energy to initiate nuclear reactions.



Electrostatic Precipitator

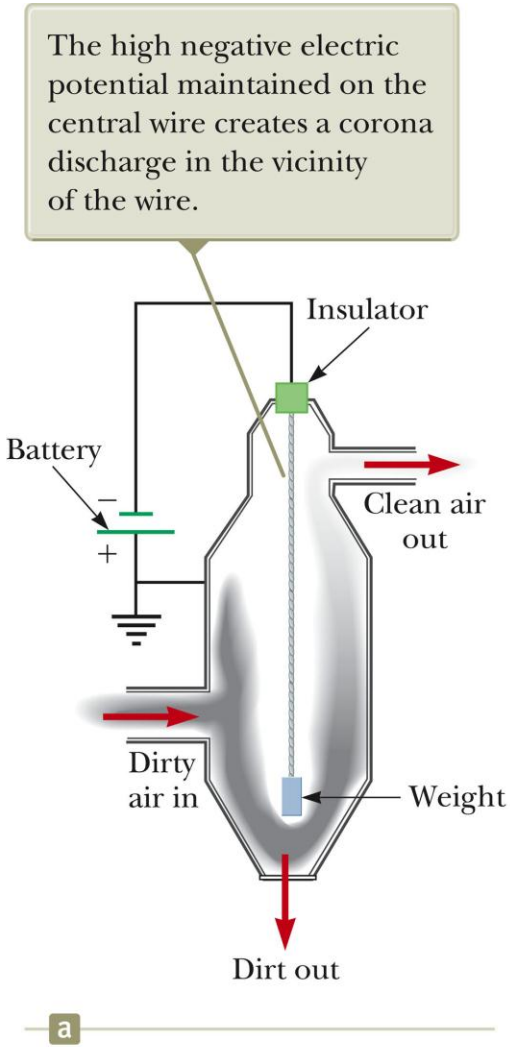
An application of electrical discharge in gases is the electrostatic precipitator.

It removes particulate matter from combustible gases.

The air to be cleaned enters the duct and moves near the wire.

As the electrons and negative ions created by the discharge are accelerated toward the outer wall by the electric field, the dirt particles become charged.

Most of the dirt particles are negatively charged and are drawn to the walls by the electric field.



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