PHYS102 Previous Exam Problems

CHAPTER 24 Electric Potential

Solutions to Selected Problems

① Electric Potential Energy of *Point* Charges in *External* Electric Fields

■**21.** A proton moves in a uniform electric field of 2.5×10^{7} N/C from point *A* to point *B* by traveling a distance of 1.5 m. Find the magnitudes of the work done and the potential difference between points *A* and *B*.

21. $\Delta V = E\cdot\Delta x = 2.5\times10^{7}\times1.5 = 3.75\times10^{7}$ V (lower potential)
$W = q\Delta V = 1.6\times10^{-19}\times3.75\times10^{7} = 6\times10^{-12}$ J

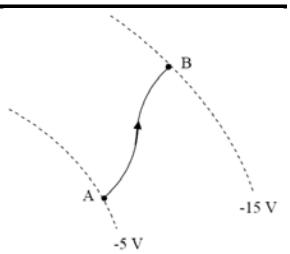
■**42.** An electron is moving parallel to the *x* axis under the influence of a uniform electric field directed along the positive *x* axis. The electron has an initial velocity of 3.0×10^{6} m/s at point *A* and its velocity is reduced to 2.0×10^{6} m/s at point *B*. Calculate the potential difference (V_B-V_A).

chapter 24

42. The electron moves in the direction of $\vec{E}$. (∵ it is decelerating)
∴ It moves towards points of lower potential.
$\Rightarrow V_B < V_A$
$\Delta K = -q\cdot\Delta V \Rightarrow \Delta V = \frac{-1}{q}\Delta K = \frac{m}{2e}(v_A^2 - v_B^2)$
$\Delta V = \frac{9.11\times10^{-31}}{2\times1.6\times10^{-19}}\times(9.0\times10^{12} - 4.0\times10^{12}) = 14$ V
$\Rightarrow V_B - V_A = -14$ V

work-K.E.: $\Delta K = W$
$\Delta K = -q\Delta V$
$\Delta V = -\frac{\Delta K}{q} = -\frac{K_f - K_i}{-e} = (K_f - K_i)/e$
$= \frac{1}{2}\frac{m}{e}(v_f^2 - v_i^2) = \frac{9.11\times10^{-31}}{2\times1.6\times10^{-19}}\times(4-9)\times10^{12}$
$= -14$ V

●**44.** Figure 22 shows two equipotential (dashed) surfaces such that V_A = -5.0 V and V_B = -15 V. What is the external work needed to move a -2.0 μC charge at constant speed from *A* to *B* along the indicated path?



44. $W = \Delta U = q\cdot\Delta V = q(V_B - V_A)$
$= (-2.0\times10^{-6})(-15+5) = +20\times10^{-6}$ J $= +20\,\mu$J

② Calculating the Electric Potential

△**1.** Two points *A* (2.0 m, 3.0 m) and *B* (5.0 m, 7.0 m) are located in a region where there is a uniform electric field that is given by $\boldsymbol{E} = 4.0\,\hat{\mathbf{i}} + 3.0\,\hat{\mathbf{j}}$ (N/C). What is potential difference (V_A-V_B)?

1. $\Delta V = -\vec{E}\cdot\Delta\vec{r}$

$V_A - V_B = -\vec{E}\cdot(\vec{r_A} - \vec{r_B}) = \vec{E}\cdot(\vec{r_B} - \vec{r_A})$

$\vec{r_B} = 5.0\hat{i} + 7.0\hat{j}$ (m)

$\vec{r_A} = 2.0\hat{i} + 3.0\hat{j}$ (m)

$\vec{r_B} - \vec{r_A} = 3.0\hat{i} + 4.0\hat{j}$ (m)

$\therefore V_A - V_B = (4.0\hat{i} + 3.0\hat{j})\cdot(3.0\hat{i} + 4.0\hat{j}) = 12 + 12 = 24\ V$

■**45.** A charge of +28 nC is placed at the origin in a uniform electric field that is directed along the positive *y* axis and has a magnitude of 4.0×10^4 V/m. What is the work done by the electric field when the charge moves to the point (3.0 m, 4.0 m)?

45. $\vec{r_i} = 0$, $\vec{r_f} = 3\hat{i} + 4\hat{j}$ (m) $\Rightarrow \Delta\vec{r} = \vec{r_f} - \vec{r_i} = 3\hat{i} + 4\hat{j}$ (m)

$W = -\Delta U = -q\Delta V = (-q)(-\vec{E}\cdot\Delta\vec{r})$

$= (q)(\vec{E}\cdot\Delta\vec{r}) = 28\times10^{-9} \times (4.0\times10^4\hat{j})\cdot(3\hat{i} + 4\hat{j})$

$= +\ 28\times10^{-9}\times4\times10^4\times4 = +\ 4.5\ mJ$

$\Delta V = -\int\vec{E}\cdot d\vec{s} = -\vec{E}\cdot\Delta\vec{s} = -\vec{E}\cdot\Delta\vec{r}$

△**63.** If the electric field has magnitude of 200 V/m and makes an angle of 30° with the positive x-axis, what is the potential difference V_B-V_A between point A (0, 0) and point B (3.0 m, 0 m)?

63.

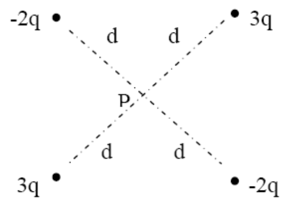
$\Delta\vec{r} = \vec{r_B} - \vec{r_A} = (3\hat{i}) - 0 = 3\hat{i}$ (m)

$\vec{E} = E_x\hat{i} + E_y\hat{j} = (200\times\cos30°)\hat{i} + (200\times\sin30\hat{j}) = 173\hat{i} + 100\hat{j}$ (V/m)

$\Delta V = -\vec{E}\cdot\Delta\vec{r} = -(3\hat{i})\cdot(173\hat{i} + 100\hat{j}) = -520\ V$

③ Electric Potential due to Point Charges

□33. In figure 15, what is the net electric potential at point *P* due to the four point charges if $V = 0$ at infinity? [Take $d = 2$ cm, $q = 1.0$ μC]



$$33.\quad V_p = \frac{k}{d}(-2q+3q-2q+3q) = \frac{2kq}{d} = \frac{2\times9\times10^{9}\times10^{-6}}{0.02} = 9\times10^{5}\ V$$

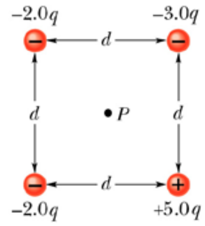
□51. A non-conducting solid sphere of radius $R = 10.0$ cm has a uniformly distributed charge $Q = +1.50$ μC. Find the magnitude of the potential difference between a point at $r = 50.0$ cm and a point on the surface of the sphere.

51. $V_r = \frac{kQ}{r}$

$V_s = kQ/R$

$\Delta V = V_s - V_r = kQ\left(\frac{1}{R} - \frac{1}{r}\right) = 9\times10^{9}\times1.5\times10^{-6}\times\left(\frac{1}{0.1} - \frac{1}{0.5}\right) = 108\ kV$

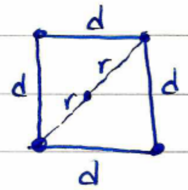
□55. In figure 30, point *P* is at the center of the square. Find the net electric potential at point *P*. Assume $V = 0$ at infinity.



55. r = distace of any charge from the center

$V = \frac{k}{r}(-2q-2q+5q-3q) = -\frac{2kq}{r}$

$= -\frac{2kq}{d/\sqrt{2}} = -2\sqrt{2}\,\frac{kq}{d} = -2.8\ kq/d$

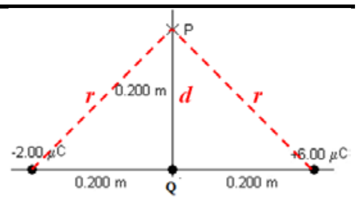


$4r^2 = 2d^2$

$\Rightarrow r^2 = d^2/2$

$\Rightarrow r = d/\sqrt{2}$

□64. Three point charges –2.00 μC, Q, and + 6.00 μC are fixed along the x-axis as shown in figure 34. If the net electric potential at point P due to these charges is zero, what is the charge Q?



64.

$\overset{\to 0}{\cancel{V_p}} = k\left(-\frac{2}{r} + \frac{Q}{d} + \frac{6}{r}\right)$

$\Rightarrow \frac{Q}{d} = \frac{2}{r} - \frac{6}{r} = -\frac{4}{r} \Rightarrow Q = -\frac{4d}{r}$

$\therefore Q = -\frac{4\times0.2}{\sqrt{2}\times0.2} = -2.83\ \mu C$

④ Calculating the Electric Field

○6. Over a certain region of space, the electric potential is give by: $V(x,y) = x^2 + y^2 + 2xy$, where V is in volts and x and y are in meters. Find the magnitude of the electric field at the point P (1.0, 2.0).

6. $E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(x^2+y^2+2xy) = -2x - 2y$

$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}(x^2+y^2+2xy) = -2y - 2x$

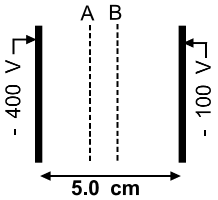
At the requested point:

$E_x = -2.0 - 4.0 = -6.0$ V/m

$E_y = -2.0 - 4.0 = -6.0$ V/m

$\Rightarrow E = E_x^2 + E_y^2 = 6\sqrt{2} = 8.5$ V/m

✪15. Consider the parallel conducting plates shown in figure 4. The distance between the equipotential surfaces A and B is 1.00 cm, and the electric potential on surface A is -280 V. What is the electric potential on the equipotential surface B?



15. For the whole region: $E = \frac{\Delta V}{\Delta x} = \frac{300}{0.05} = 6$ kV/m $\rightarrow$ uniform

Now, consider the A-B plates:

$\Delta V = E \cdot \Delta x = 6\times10^3 \times 1.00\times10^{-2} = 60$ V

From the configuration given: $V_B > V_A$

$\Rightarrow V_B - V_A = 60$ V

$\Rightarrow V_B = V_A + 60 = -280 + 60 = -220$ V

○19. In a certain region of the xy plane, the electric potential is given by $V(x,y) = 2xy - 3x^2 + 5y$, where V is in volts, and x and y are in meters. At which point is the electric field equal to zero?

19. $E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(2xy - 3x^2 + 5y) = -2y + 6x$

$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}(2xy - 3x^2 + 5y) = -2x - 5$

$E_y = 0 \Rightarrow 2x = -5 \Rightarrow x = -2.5$ m

$E_x = 0 \Rightarrow -2y + 6x = 0 \Rightarrow 2y = 6x = -15 \Rightarrow y = -7.5$ m

○22. The electric potential at point in an xy plane is given by $V = 3x^2 - 4y^2$. What are the magnitude and direction of the electric field at the point (4.0, 2.0) m?

22. $E_x = -\partial V/\partial x = -6x$

$E_y = -\partial V/\partial y = +4y$

At the point (4.0, 2.0): $\vec{E} = -24\hat{i} + 16\hat{j}$ (V/m)

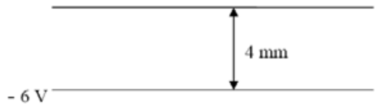
$\Rightarrow E = (24^2 + 16^2)^{1/2} = 28.8 \rightarrow 29$ V/m

$\vec{E}$ is in the 2^{nd} quad.

$\phi = \tan^{-1}(16/24) = 33.7°$

$\Rightarrow \theta = 180 - \phi = 146°$ from the (+) x axis

○27. In figure 12, two large horizontal metal plates are separated by 4 mm. The lower plate is at a potential of -6.0 V. What potential should be applied to the upper plate to create an electric field of strength 4000 V/m upwards in the space between the plates?



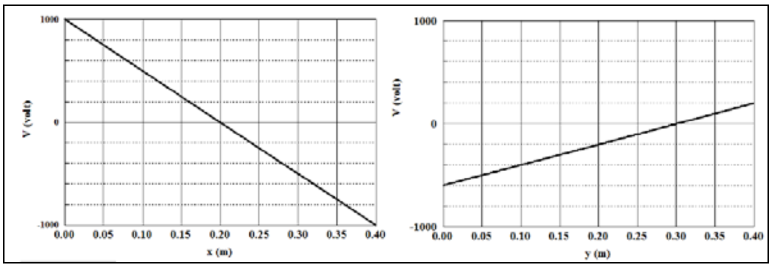
27. What is the magnitude of ΔV?

$\Delta V = E.\Delta x = 4000 \times 4 \times 10^{-3} = 16\ V$

If $\vec{E}$ points upward, then the upper plate is at the lower potential

$V_u = V_l - 16 = -6 - 16 = -22\ V$

○46. An electron is placed in an *xy* plane where the electric potential depends on *x* and *y* as shown in figure 23 (the potential does not depend on *z*). What is the electric field (in units of kV/m)?



46. The $\vec{E}$ field is the (−) of the slop of V vs. x

$E_x = -\frac{dV_x}{dx} = -\frac{-2000}{0.40} = +5000\ V/m = +5\ kV/m$

$E_y = -\frac{dV_y}{dy} = -\frac{800}{0.40} = -2000\ V/m = -2\ kV/m$

$\Rightarrow \vec{E} = 5\hat{i} - 2\hat{j}\ (kV/m)$

⑤ Work and Potential Energy for a System of Charges

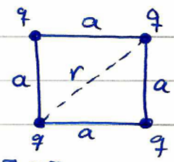
●9. What is the external work required to bring four 3.0×10^{-9} C positive point charges from infinity and place them at the corners of a square of side 0.12 m?

9. $U = kq^2\left(\frac{1}{a} + \frac{1}{r} + \frac{1}{a} + \frac{1}{a} + \frac{1}{r} + \frac{1}{a}\right)$

$= kq^2\left(\frac{4}{a} + \frac{2}{r}\right) = 2kq^2\left(\frac{2}{a} + \frac{1}{r}\right)$

$= 2kq^2\left(\frac{2}{a} + \frac{1}{\sqrt{2}a}\right) = \frac{2kq^2}{a}\left(2 + \frac{1}{\sqrt{2}}\right)$

$= \frac{2\times9\times10^9\times9\times10^{-18}}{0.12}\times\left(2+\frac{1}{\sqrt{2}}\right) = 3.65\times10^{-6}\ J = +3.7\ \mu J$



●10. A point charge $q_1 = +2.4\ \mu C$ is held stationary at the origin. A second point charge $q_2 = -4.3\ \mu C$ moves from $x_1 = 0.15$ m, $y_1 = 0$ to a point $x_2 = 0.25$ m, $y_2 = 0.25$ m. How much work is done by the electric force on q_2?

10. $r_i = 0.15\ m,\quad r_f = \sqrt{2}\times0.25 = 0.35\ m$

$W = -\Delta U = -(U_f - U_i) = U_i - U_f = \frac{kq_1q_2}{r_i} - \frac{kq_1q_2}{r_f}$

$= kq_1q_2\left(\frac{1}{r_i} - \frac{1}{r_f}\right) = 9\times10^9\times(2.4\times10^{-6})(-4.3\times10^{-6})\left(\frac{1}{0.15} - \frac{1}{0.35}\right)$

$= -0.36\ J$

●**16.** A point charge of 5.0×10^{-9} C is transferred, by an external agent, from infinity to the surface of a ball of radius 5.0 cm. If the ball has a charge density of 5.0×10^{-4} C/m^2, what is the amount of work done, by the external agent, in the process? [assume $V = 0$ at infinity]

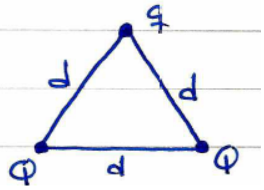
16. V_f at the surface of the sphere:

$$V_f = \frac{kQ}{R} = \frac{k}{R}\cdot\sigma A = \frac{k}{R}\cdot\sigma\cdot 4\pi R^2 = 4\pi k\sigma R = 4\pi\times9\times10^9\times5\times10^{-4}\times5.0\times10^{-2}$$

$$= 2.83\times10^6\ V$$

$$W = q\cdot\Delta V = q(V_f - \overset{0}{V_i}) = 5.0\times10^{-9}\times2.83\times10^6 = 1.4\times10^{-2}\ J$$

●**30.** Three point charges are initially infinitely far apart. Two of the point charges are identical and have charge Q. If zero net work is required to assemble the three charges at the corners of an equilateral triangle of side d, what is the value of the third charge?



30. $W = \Delta U = U_f = \frac{kQ^2}{d} + \frac{kqQ}{d} + \frac{kqQ}{d}$

$$0 = \frac{kQ^2}{d} + 2\frac{kqQ}{d}$$

$$\Rightarrow 2\cancel{k}\frac{q\cancel{Q}}{\cancel{d}} = -\frac{\cancel{k}Q^{\cancel{2}}}{\cancel{d}} \Rightarrow q = -\frac{Q}{2}$$

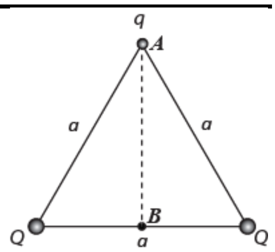
●**34.** Two balls with charges 5.0 μC and 10 μC are at a distance of 1.0 m from each other. In order to reduce the distance between them to 0.5 m, what amount of work needs to be performed?

34. $W = \Delta U = U_f - U_i$

$$= \frac{kq_1q_2}{r_f} - \frac{kq_1q_2}{r_i} = kq_1q_2\left(\frac{1}{0.5} - \frac{1}{1}\right) = kq_1q_2$$

$$= 9\times10^9\times5\times10\times10^{-12} = 0.45\ J$$

●**59.** As shown in figure 32, two particle with charge $Q = 10$ μC each are fixed at the vertices of an equilateral triangle with sides of length $a = 0.30$ m. How much work is required to move a particle with a charge $q = 1$ μC from point ***A*** at the other vertex to point ***B*** at the center of the line joining the fixed charges?



59. The potentials due to charges Q:

$$V_A = \frac{kQ}{a} + \frac{kQ}{a} = 2kQ/a$$

$$V_B = \frac{kQ}{a/2} + \frac{kQ}{a/2} = 4kQ/a$$

The work:

$$W = q\Delta V = q(V_B - V_A) = 2k\frac{qQ}{a} = 2\times9\times10^9\times\frac{10^{-6}\times10^{-5}}{0.3} = 0.6\ J$$

⑥ Conservation of Energy

◆**4.** Two identical and isolated 8.0-μC point charges are positioned on the *x* axis, one is at *x* = +1.0 m and the other is at *x* = -1.0 m. They are released from rest simultaneously. What is the kinetic energy of either of the charges after it has moved 2.0 m along the *x* axis?

4. $U_i = \frac{kq^2}{r_i} = \frac{9\times10^9 \times 64\times10^{-12}}{2.0} = 288 \text{ mJ}$, $K_i = 0$

$U_f = \frac{kq^2}{r_f} = \frac{9\times10^9\times 64\times10^{-12}}{6.0} = 96 \text{ mJ}$, $K_f = ?$

$U_i + \cancel{K_i}^{0} = U_f + K_f$

$K_f = U_i - U_f = 288 - 96 = 192 \text{ mJ}$

$\therefore$ The K.E. of each $= \frac{192}{2} = 96 \text{ mJ}$

◆**8.** A particle, with a mass of 9.0×10^{-9} kg and a charge of +8 nC , has a kinetic energy of 36 μJ at point *A* and moves to point *B* where the potential is 3.0×10^{3} V greater than that at point A. What is the particle's kinetic energy at point B?

8. $V_B = V_A + 3.0\times10^3 \Rightarrow V_B - V_A = +3.0\times10^3 \text{ V}$

$K_A + U_A = K_B + U_B$

$K_B = K_A + (U_A - U_B) = K_A + q(V_A - V_B)$

$= (36\times10^{-6}) + (8.0\times10^{-9})(-3.0\times10^3)$

$= 36\times10^{-6} - 24\times10^{-6} = 12\ \mu\text{J}$

◆**11.** An electron is accelerated from a speed of 3×10^{6} m/s to 8×10^{6} m/s. Calculate the electric potential through which electron has to pass to gain this acceleration?

11. $K_i + U_i = K_f + U_f$

$K_i + qV_i = K_f + qV_f$

$K_i - K_f = q(V_f - V_i)$

$\frac{1}{2}m(v_i^2 - v_f^2) = q(V_f - V_i)$

$\Delta V = V_f - V_i = \frac{m(v_i^2 - v_f^2)}{2q} = \frac{9.11\times10^{-31}\times(9\times10^{12} - 64\times10^{12})}{(2)(-1.6\times10^{-19})} = 157 \text{ V}$

◆**18.** Two electrons are initially far away. Each electron is initially moving toward the other one with a speed of 500 m/s. Find the closest distance they can get to each other.

18. The force of repulsion will stop them

$U_i = 0 \rightarrow$ far away

$K_i = (2)(\frac{1}{2}mv_i^2) = mv_i^2$

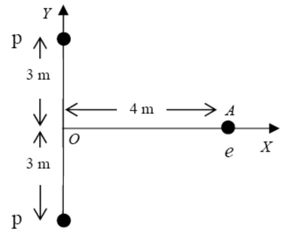
$K_f = 0 \rightarrow$ they will stop

$U_f = \frac{ke^2}{r_f}$

$\cancel{U_i}^{0} + K_i = U_f + \cancel{K_f}^{0}$

$\Rightarrow \frac{ke^2}{r_f} = mv_i^2 \Rightarrow r_f = \frac{k}{m}\cdot\left(\frac{e}{v_i}\right)^2 = \frac{9\times10^9}{9.1\times10^{-31}}\cdot\left(\frac{1.6\times10^{-19}}{500}\right)^2 = 1.01 \text{ mm}$

◆**25.** Two protons, *p*, are fixed 6.0 m apart, as shown in figure 10. An electron, *e*, is released from point *A*. Find its speed at point *O*, midway between the protons.



25. $\mathcal{U}_i = \frac{ke^2}{6} - \frac{ke^2}{5} - \frac{ke^2}{5}$, $K_i = 0$

$\mathcal{U}_f = \frac{ke^2}{6} - \frac{ke^2}{3} - \frac{ke^2}{3}$, $K_f = ?$

$\mathcal{U}_i + K_i = \mathcal{U}_f + K_f$

$K_f = \mathcal{U}_i - \mathcal{U}_f = -\frac{2ke^2}{5} + \frac{2ke^2}{3} = \frac{4ke^2}{15}$

$\frac{1}{2} m v_f^2 = \frac{4}{15} ke^2 \Rightarrow v_f = \left(\frac{8k}{15m}\right)^{1/2}. e = \left(\frac{8\times9\times10^9}{15\times9\times10^{-31}}\right)^{1/2}\times1.6\times10^{-19} = 11.6\,m/s$

◆**48.** An electron is projected with an initial kinetic energy of 3.6×10^{-24} J toward a fixed proton. If the electron is initially infinitely far from the proton, at what distance from the proton is its speed equal to twice its initial speed?

48. $\mathcal{U}_i = 0$

$\mathcal{U}_f = -\frac{ke^2}{r}$

$K_f = \frac{1}{2} m V_f^2 = \frac{1}{2} m (2V_i)^2 = 4K_i$

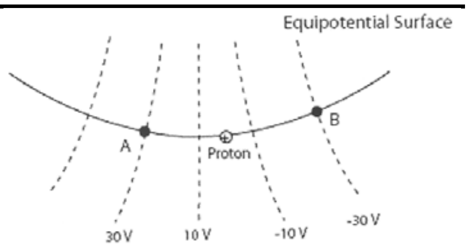
$\mathcal{U}_i^{\,0} + K_i = \mathcal{U}_f + K_f$

$\mathcal{U}_f = K_i - K_f = K_i - 4K_i = -3K_i$

$\cancel{-}\frac{ke^2}{r} = \cancel{-}3K_i \Rightarrow r = \frac{ke^2}{3K_i} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{3\times3.6\times10^{-24}} = 2.13\times10^{-5}\,m$

$= 21\,\mu m$

◆**53.** In figure 29, a proton's speed as it passes point *A* is 5.0×10^4 m/s. It follows the trajectory shown in the figure. What is the proton's speed at point *B*? [mass of the proton = 1.67×10^{-27} kg]



53. $\Delta V = V_B - V_A = -30 - 30 = -60\,V$

$\Delta\mathcal{U} = q\,\Delta V = (1.6\times10^{-19})(-60) = -9.6\times10^{-18}\,J$

$\Delta K + \Delta\mathcal{U} = 0$

$\Rightarrow \Delta K = -\Delta\mathcal{U}$

$K_B - K_A = -\Delta\mathcal{U} \Rightarrow K_B = K_A - \Delta\mathcal{U}$

$\frac{1}{2} m v_B^2 = \frac{1}{2} m v_A^2 - \Delta\mathcal{U}$

$v_B^2 = v_A^2 - \frac{2\Delta\mathcal{U}}{m} = 25\times10^8 + \frac{2\times9.6\times10^{-18}}{1.67\times10^{-27}} = 1.4\times10^{10}\,(m/s)^2$

$\Rightarrow v_B = 118\times10^3 = 1.2\times10^5\,(m/s)$

◆**60.** A metallic sphere, of radius 8 cm, is charged to a potential of – 500 V (take V = 0 at infinity). An electron is initially 15 cm from the center of the sphere. What must be the initial speed of the electron to barely hit the sphere ($v_f = 0$)?

60. $K_i + U_i = \cancel{K_f}^{\,0} + U_f$

$K_i = U_f - U_i$

consider the sphere as a particle:

$\frac{1}{2} m v_i^2 = (kQ)(-e)\left(\frac{1}{R} - \frac{1}{r_i}\right)$

$\frac{1}{2} m v_i^2 = -RVe\left(\frac{1}{R} - \frac{1}{r_i}\right)$

$v_i^2 = -\frac{2RVe}{m}\left(\frac{1}{R} - \frac{1}{r_i}\right)$

$\left.\begin{array}{l} V = kQ/R \\ \Rightarrow kQ = VR \end{array}\right\}$

$= \frac{-2 \times 0.08 \times (-500) \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}} \times \left(\frac{1}{0.08} - \frac{1}{0.15}\right) \Rightarrow v_i = 9.1 \times 10^{6}$ m/s

⑦ Conductors

We did problems 62 and 66 from the textbook in the lecture.