chapter 27

current and Resistance

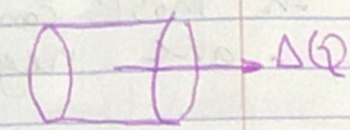
chapter outline — مواضيع الوحدة

- Electric current — التيار الكهربائي
- Current Density — كثافة التيار
- Resistivity and Ohm's Law — المقاومة وقانون أوم
- Temperature Dependence of Resistivity — اعتماد المقاومية على درجة الحرارة
- Superconductors — المواد فائقة الموصلية
- Electrical power — القدرة الكهربائية

Definition of Electric current

average current: I_{av} — معدل التيار

amount of charge ΔQ that passes through area during time Δt



$$I_{av} = \frac{\Delta Q}{\Delta t}$$

Instantaneous current: I

$$I = \frac{dQ}{dt}$$

unit of current is Ampere (A)

$$A = \frac{C}{s}$$

$1\,mA = 10^{-3}\,A$

$1\,\mu A = 10^{-6}\,A$

current is a scalar quantity (not a vector)

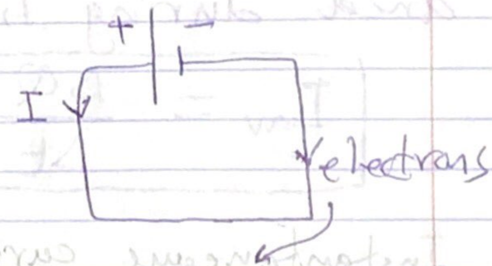
Notes

- 100 watt light bulb : ≃ 1A
- car starter motor : 200 A
- TV, computer, phone (nA ⟶ mA)
- current is a scalar (not vector)
- has a sign associated with it
- conventional (اصطلاحي) current is a flow of positive charge

⟶ I : positive charge flows right or negative charge flows left.

In most conductors, charge carriers are negative electrons

conventional current (التيار الاصطلاحي)



(التيار الإلكتروني) electronic current

an electron flowing from − to +
a proton flowing from + to −

EX 3.8×10^{21} electrons pass through an area in a wire in 4 minutes.

① what is the charge ?

② what is the average current?

(a) $\Delta Q = N_e \, q_e$

$= 3.8 \times 10^{21} \times 1.6 \times 10^{-19}$

$\Delta Q = 608 \text{ C}$

(b) $I = \frac{\Delta Q}{\Delta t} = \frac{608}{4 \times 60}$

$= 2.53 \text{ A}$

Microscopic Model of current النموذج المجهري

consider a cylindrical conductor

A: cross sectional area

v_d: drift speed

n: number of electrons per unit volume

: volume charge density

q_e: charge of electron

$$\boxed{I_{av} = n \cdot q_e \cdot v_d \cdot A}$$

I_{av}: average current

J current density كثافة تيار

$$\boxed{J = \frac{I}{A}}$$

J: A/m^2

EX (27.1) Drift speed in Copper wire

The 12-gauge copper wire in a typical residential building has a cross-sectional area of $3.31 \times 10^{-6} m^2$. It carries a constant current of 10 A. what is the drift speed of the electrons in the wire? Assume each copper atom contributes one free electron to the current. The density of copper is $8.92\ g/cm^3$.

$\rho_d = 8.92\ g/cm^3$

each Cu atom $\longrightarrow$ 1 free electron

$A = 3.31 \times 10^{-6}\ m^2$

$q = 1.6 \times 10^{-19} C$ $\quad v_d = ??$

$I = 10\ A$, molar mass $= 63.5\ g/mole$

Sol

$n = ?$ (number of electrons per unit volume)

$$\rho_d = \frac{8.92\ g}{1\ cm^3}$$

$$= \frac{8.92 \times 10^{-3}}{1 \times 10^{-6}}$$

$$\rho_d = 8.92 \times 10^{3}\ Kg/m^3$$

$$N_{moles} = \frac{\text{Mass}}{\text{Mass per unit volume}}$$

↙ Number of moles

$$= \frac{\text{Mass}}{\text{Molar Mass}}$$

$$= \frac{8.92}{63.5} = 0.14 \text{ mole}$$

$1 \text{ mole} = 6.02 \times 10^{23} \text{ atom/mole}$

$N_{atom} = N_{moles} \times 6.02 \times 10^{23}$

$= 0.14 \times 6.02 \times 10^{23}$

$N_{atom} = 0.8456 \times 10^{23} \text{ atoms}$

$N_{electrons} = N_{atoms} \times 1$ (each atom has 1 free electron)

$N_{electrons} = 8.456 \times 10^{22} \text{ elect}$

$$n = \frac{N}{V} = \frac{8.456 \times 10^{22}}{1 \times 10^{-6}}$$ $(V = 1 \text{ cm}^3 = 1 \times 10^{-6} \text{ m}^3)$

$n = 8.456 \times 10^{28} \text{ e/m}^3$

→ $I = n \cdot q_e \cdot V_d \cdot A$

$10 = 8.456 \times 10^{28} \times 1.6 \times 10^{-19} \times 3.31 \times 10^{-6} \times V_d$

$V_d = 0.223 \times 10^{-3} \text{ m/s}$

$= 0.223 \text{ mm/s}$

Current Density (J) كثافة التيار

Current density (J) is current per unit area.

$$J = \frac{I}{A}$$

unit

J: A/m^2

$$J = n q V_d$$

Ex

In the Ex 27.1 Find (J)

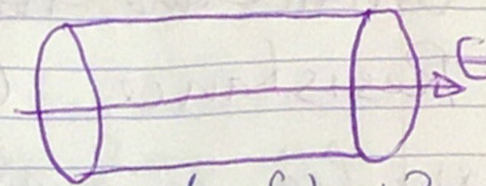
$$J = \frac{I}{A}$$

$$= \frac{10}{3.31 \times 10^{-6}}$$

$$= 3.02 \times 10^{6} \ A/m^2$$

Resistivity (ρ)

المقاومية



why does electric current flow?

- electric field creates force acting on charge carriers (eletrons)

<u>ohom's law</u>

- In many materials current density proportional to electric field which is called ohom's law

$$\boxed{J = \sigma E}$$ ohom's law

$$\boxed{J = \frac{E}{\rho}}$$

$$\boxed{\sigma = \frac{1}{\rho}}$$

E: Electric Field (V/m)

ρ: Resistivity (ohm.m)

σ: conductivity

ρ : المقاومية

σ : الموصلية

E : المجال الكهربائي

(للو) ثابت ↑

unit of ρ

$$\rho = \frac{E}{J} = \frac{\frac{V}{m}}{\frac{A}{m^2}}$$

$$= \frac{V}{m} * \frac{m^2}{A} = \frac{V}{A} \cdot m$$

تذكر $\rho = \Omega \cdot m$ Ω : أوم (ohom)

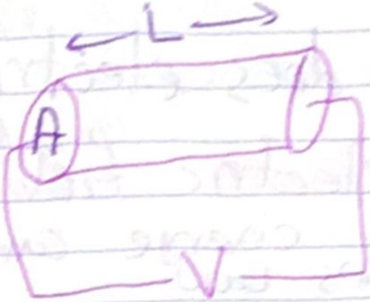
<u>Caution</u> ρ is not volume density

σ " " surface "

~~ohmic and non-ohmic~~

Resistance (R) المقاومة

The fig is a wire of



Length : L

Area : A

potential diff : V

resistivity ρ

I : current

$$J = \sigma E$$

$$J = \frac{1}{\rho} E$$

$$E = \rho J$$

$$E = \rho \frac{I}{A}$$

$$\frac{V}{L} = \rho \frac{I}{A}$$

$$V = \left(\frac{\rho L}{A}\right) . I$$

$$E = \frac{V}{L}$$

$$R = \frac{\rho L}{A}$$

$$V = R\,I$$ ohm's law

$$R = \frac{V}{I}$$ $V/A \equiv \Omega$ (ohm)

↳ Resistance of the wire

Ex ~~Suppose you want~~

copper wire

Suppose you want to fabricate a wire of length 20m and a resistance 0.1 Ω. What should be its diameter d

$$R = \frac{\rho L}{A}$$

$$0.1 = \frac{1.7\times10^{-8}\times20}{A} \qquad \left\{ \rho_{cu} = 1.7\times10^{-8}\ \Omega.m \right.$$

$$A = 3.4\times10^{-6}\ m^2$$

$$\pi r^2 = 3.4\times10^{-6}$$

$$3.14\left(\frac{d}{2}\right)^2 = 3.4\times10^{-6}$$

$$d = 2.08\times10^{-3}\ m$$

$$d = 2.08\ mm$$

$$\approx 2.1\ mm$$

(b) if the current flows is 4 A what is the potential difference (V)

$$V = IR$$

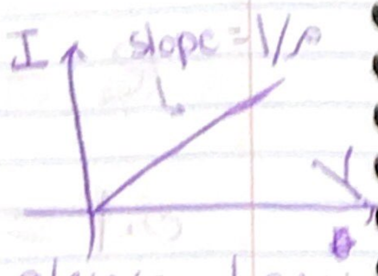
$$= 4\times0.1$$

$$V = 0.4\ \text{volt}$$

ohmic and non-ohmic materials

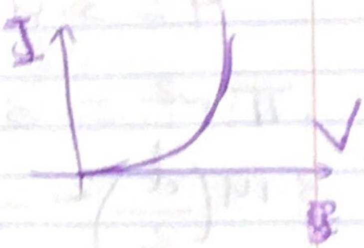
- materials that follow ohm's law are called "ohmic" materials
- resistivity ρ is constant
- Liner I vs. V graph

I vs V



- materials that do not follow ohm's law are called "non-ohmic" materials
- nonlinear I vs. V graph

I vs V



Resistors in circuits

1) symbol of Resistor is

R ——/\/\/\——

2) all lamps, batteries are resistors
3) wires are resistors.
4)

Temperature Dependence of Resistivity

Many materials have resistivities that depend on temperature as in the equation

$$\rho = \rho_0 \left[1 + \alpha (T - T_0)\right]$$

where ρ_0 is the resistivity at temperature T_0.

α : is the temperature coefficient of resistivity

T_0 : reference temp ($T_0 = 20$) or $T_0 = 0$

T : temperature

ρ : resistivity at T

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when $R \propto \rho$

$\longrightarrow$

$$R = R_0 \left[1 + \alpha (T - T_0)\right]$$

Metals : increasing T will increase ρ, R

semiconductors : increasing T will increase ρ and R

Ex If a certain silver wire has a resistance of $6\,\Omega$ at $T_0 = 20°C$ what resistance will it have at $T = 34°C$ given $\alpha = 3.8 \times 10^{-3}\ C^{-1}$

$$R = R_0[1 + \alpha(T - T_0)]$$

$$= 6[1 + 3.8 \times 10^{-3}(34 - 20)]$$

$$= 6.3\,\Omega$$

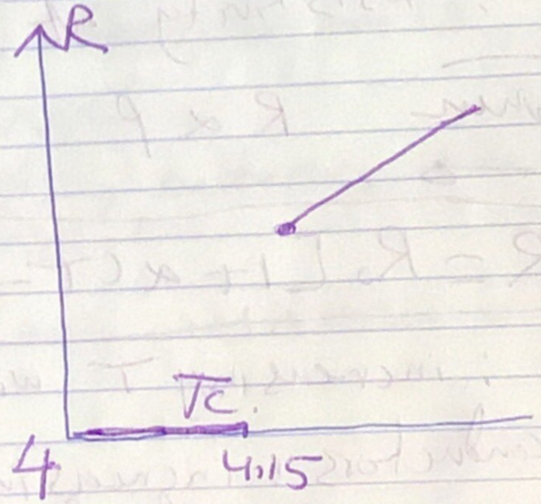
B1 | **Superconductors** [illegible]

metals and compounds whose resistance decrease to zero when they are below a certain temp. T_c known as the critical temperature

The resistance drops to zero at T_c

R vs T for Hg at $T_c = 4.15$

$R = 0$



31 | A 34.5 m Length of copper
827 wire at 20°C has a radius of 0.25 mm. If a potential difference of 9 V is applied across the length of the wire determine the current in the wire. (b) If the wire is heated to 30°C while the 9 volt is maintained what is the current?

$L = 34.5$ m
copper at 20°C
$r = 0.25$ mm

a) $\Delta V = 9$ volt
Find I

$\Delta V = IR$

$R = \frac{\rho L}{A}$

$= \frac{\rho_{Cu} \cdot L}{A}$

$= \frac{1.7 \times 10^{-8} \times 34.5}{\pi \times (0.25 \times 10^{-3})^2}$

$= 3 \Omega$

$I = \frac{9}{3}$

$= 3$ A

(b) $T_F = 30°C$

$R = R_0[1 + \alpha(T - T_0)]$

$= 3[1 + 3.9 \times 10^{-3}(30 - 20)]$

$R = 3.117 \Omega$

$\Delta V = IR$

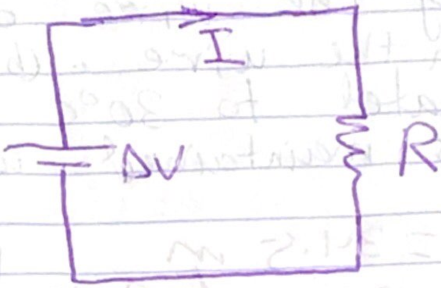
$9 = I \times 3.117$

$I = 2.89$ A

Electrical Power

القدرة الكهربائية

Rate of energy transfer (P)



$$P = I\Delta V$$

$$P = I^2 R$$

$$P = \frac{(\Delta V)^2}{R}$$

Unit of power is Watt (W)

$W = J/S$

Electric Energy (E)

$$E = Pt$$

P: power (W)

t: time (sec)

E: Energy (J)

Ex 27.4

An electric heater is constructed by applying a potential diff of 120 V across a wire that has resistance $R = 8\Omega$

Find (a) current (b) power (c) Energy in 5 min

(a) $I = \frac{\Delta V}{R} = \frac{120}{8} = 15 A$

(b) $P = I^2 R$

$= (15)^2 (8)$

$= 1800$ watt

$= 1.8$ kW

(c) $E = P \cdot t$

$= (1800)(5)(60)$

$= 540000$ J

Ex Calculate the magnitude of charge passing through conductor having current of 2 A in 5 min

$\Delta Q = I \Delta t$

$= 2(5)(60)$

$= 600\ C$

Find no of electrons

$N = \frac{\Delta Q}{q_e} = \frac{600}{1.6 \times 10^{-19}}$

$= 3.75 \times 10^{21}$

Ex If $q = 2t^2 + 3t + 1$

Find I at $t = 1.5$ sec

$I = \frac{dq}{dt} = 4t + 3$

$= 4(1.5) + 3 = 9\ A$

If $I = 3t^2$

Find q at $t = 0 \rightarrow t = 2$

$q = \int I dt = \int 3t^2 dt = t^3 \Big|_0^2$

$= (2)^3 = 8\ C.$

Ex Carbonic Resistor of $A = 10\,mm^2$ and $I = 5 \times 10^{-3}\,A$ and $V = 15\,V$

Find 1) R ; 2) L

given $\rho = 1.4 \times 10^{-5}$

$$R = \frac{V}{I} = \frac{15}{5 \times 10^{-3}} = 3000\,\Omega$$

$$R = \frac{\rho L}{A}$$

$$3 \times 10^{3} = \frac{1.4 \times 10^{-5} \times L}{10 \times 10^{-6}\,m^2}$$

$$L = 2.14 \times 10^{3}\,m$$

3) Find σ

$$\sigma = \frac{1}{\rho} = \frac{1}{1.4 \times 10^{-5}} = 71.4 \times 10^{3}\,(\Omega \cdot m)^{-1}$$

4) Find E

$$E = \frac{\Delta V}{L} = \frac{15}{2.14 \times 10^{3}} = 7 \times 10^{-3}\,V/m$$

5) $J = ?$

$$J = \frac{I}{A} = \frac{5 \times 10^{-3}}{10 \times 10^{-6}} = 500\,A/m^2$$