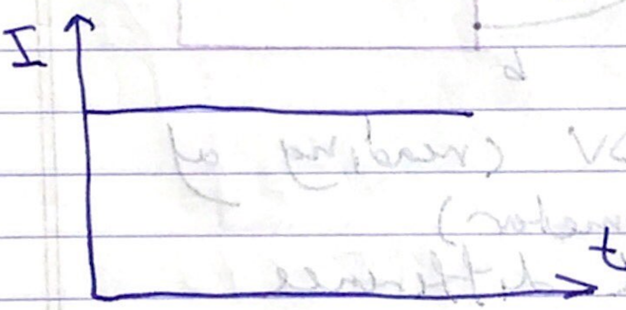
chapter 28

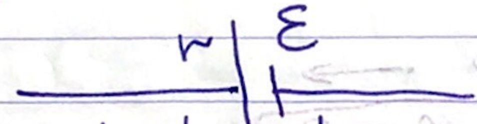
Direct-Current Circuits
دوائر التيار الكهربائي

Electromotive Force (ε)
القوة الدافعة الكهربائية

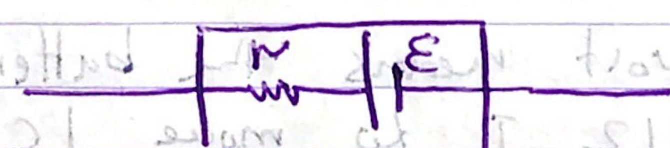
DC-current
Direct current — التيار المستمر



source of DC current is Battery
conservation of Energy in Battery
chemical Energy → Electrical Energy
Battery in circuit is



or ε: electromotive force (volt)

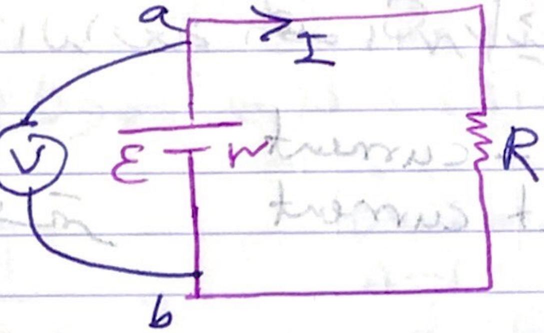


r: internal Resistance (Ω)

Electromotive force (ε): Maximum potential difference between the terminals of the battery

اقصى فرق جهد بين طرفي البطارية

unit of ε is volt $= \frac{J}{C}$



$V_a - V_b = \Delta V$ (reading of voltmeter)

$=$ potential difference

$$\boxed{\Delta V = \varepsilon - I r}$$

when $I = 0$ (open circuit)

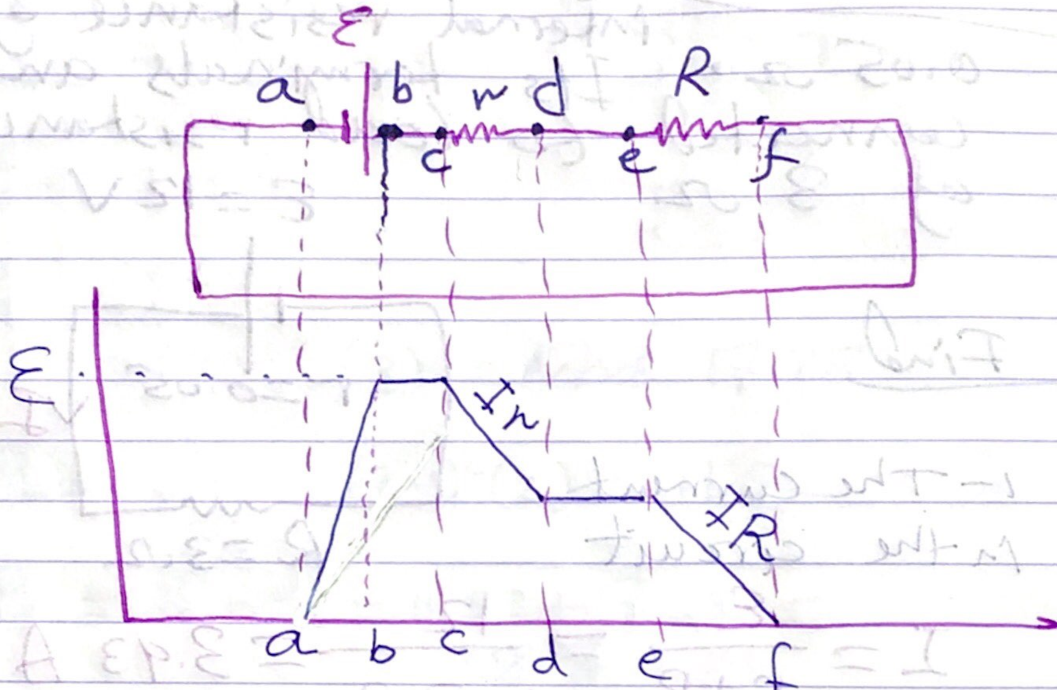
$\longrightarrow \Delta V = \varepsilon$

when $r = 0 \longrightarrow \boxed{\Delta V = \varepsilon}$

$\varepsilon = 12$ volt means the battery deliver 12 J to move 1 C from $a \rightarrow b$

Simple current - circuit

معادلة الدارة البسيطة



$$\varepsilon = Ir + IR$$

$$I = \frac{\varepsilon}{r + R}$$ simple circuit equation

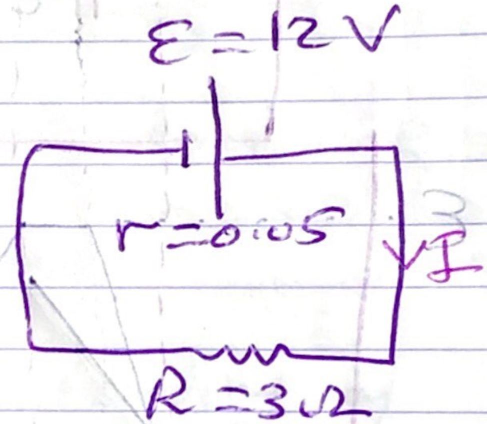
$$I(\varepsilon = Ir + IR)$$

$$I\varepsilon = I^2 r + I^2 R$$ conservation of energy

$P_\varepsilon = I\varepsilon$	power from battery
$P_r = I^2 r$	power delivered to internal resistance
$P_R = I^2 R$	" " load resistance

EX 28.1 / 834

A battery has emf of $\mathcal{E} = 12$ volt and an internal resistance of 0.05 Ω. Its terminals are connected to load resistance of 3 Ω



Find

1- The current in the circuit

$$I = \frac{\mathcal{E}}{r+R} = \frac{12}{0.05+3} = 3.93 \text{ A}$$

2- The terminal voltage of the battery

$$\Delta V = \mathcal{E} - Ir$$
$$= 12 - 3.93 * 0.05 = 11.8 \text{ volt}$$

3- Power delivered to load resistance

$$P_R = I^2 R = (3.93)^2 (3) = 46.3 \text{ watt}$$

4- Power delivered to internal resistance

$$P_r = I^2 r = (3.93)^2 (0.05) = 0.772 \text{ W}$$

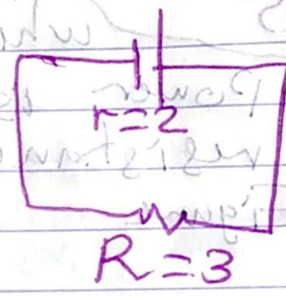
5- Power delivered by the battery

$$P_{\mathcal{E}} = I\mathcal{E} = 3.93 * 12 = 47.16$$

or $P_{\mathcal{E}} = P_r + P_R =$

قَدِيمَة

suppose the battery ages its internal resistance $r = 2\,\Omega$ $\mathcal{E} = 12$



①

$$I = \frac{\mathcal{E}}{r+R} = \frac{12}{2+3}$$

$$= \frac{12}{5} = 2.4\ A$$

② $P_R = I^2 R$

$$= (2.4)^2 (3) = 17.28$$

$$P_r = I^2 r = (2.4)^2 (2) = 11.52$$

$$P_\mathcal{E} = P_r + P_R = 11.52 + 17.28$$

$$= 28.8 \text{ watt}$$

or $P_\mathcal{E} = I\mathcal{E}$

$$= (2.4)(12) = 28.8\ W$$

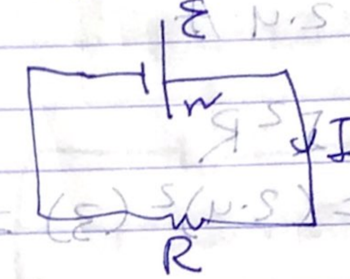
terminal voltage of the battery

$$\Delta V = \mathcal{E} - Ir$$

$$= 12 - 2.4(2)$$

$$= 7.2 \text{ volt}$$

EX (28.2) 835 Find the Load Resistance R for which the maximum power is delivered to the load resistance is maximum in Figure



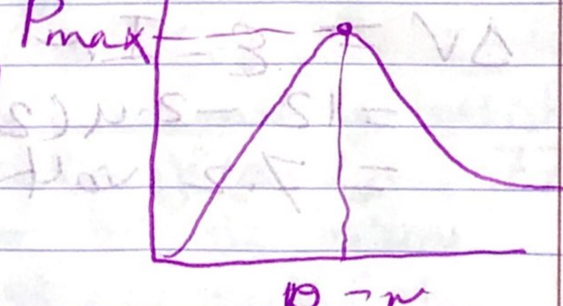
$$P = I^2 R$$

$$= \frac{\varepsilon^2 R}{(r+R)^2}$$

$$\frac{dP}{dR} = \frac{d}{dR}\left(\frac{\varepsilon^2 R}{(\varepsilon+r)^2}\right)$$

$$= \frac{d}{dR}\left(\varepsilon^2 R (R+r)^{-2}\right) = 0$$

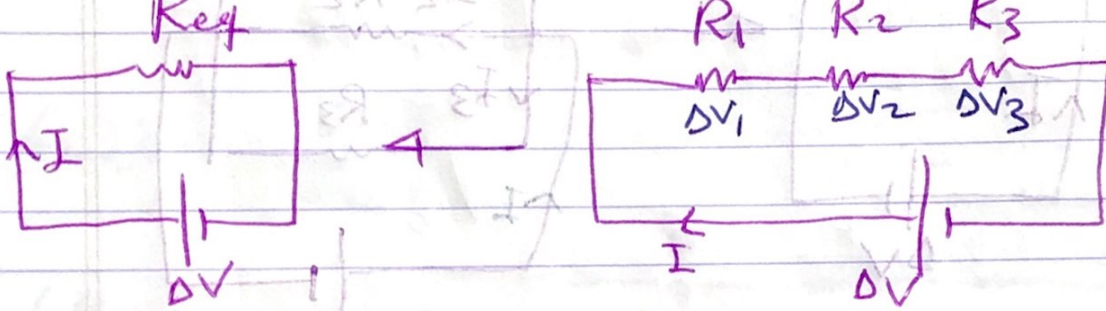
$\longrightarrow$ $\boxed{R = r}$



$$\boxed{P_{max} = \frac{\varepsilon^2}{4r}}$$

28.2 Resistors in series and parallel

→ Series combination



I : the same

ΔV : divided

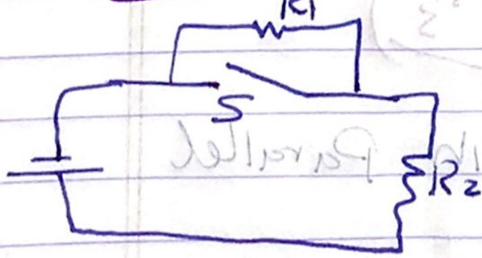
$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$

$R_{eq} = ??$

$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$

$I R_{eq} = I R_1 + I R_2 + I R_3$

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$



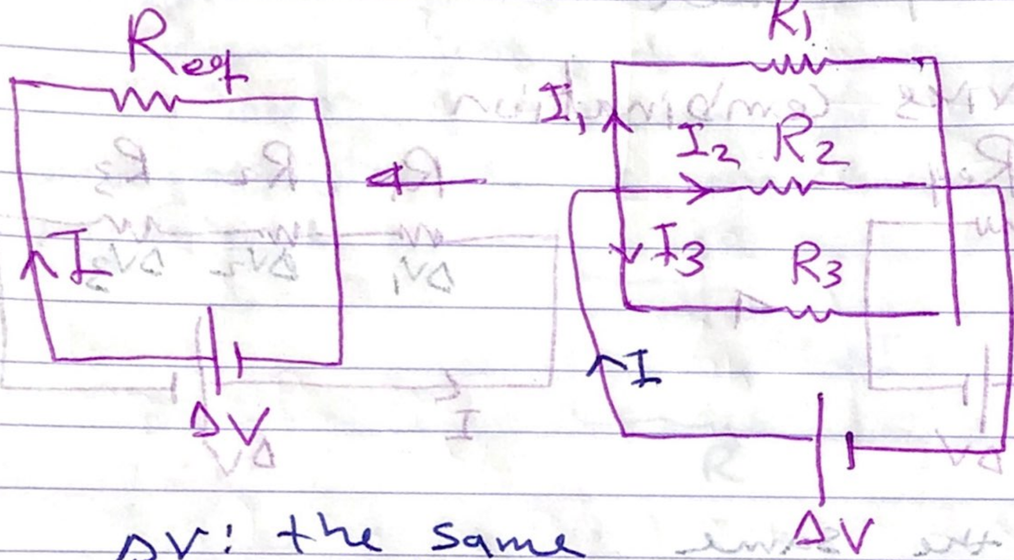
S : open

$R_{eq} = R_2 + R_1$

S : closed

$R_{eq} = R_2$ only

Resistors In Parallel



ΔV: the same
I: divided

$$I = I_1 + I_2 + I_3$$

but $I = \frac{\Delta V}{R}$

$$\frac{\Delta V}{R_{eq}} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} + \frac{\Delta V}{R_3}$$

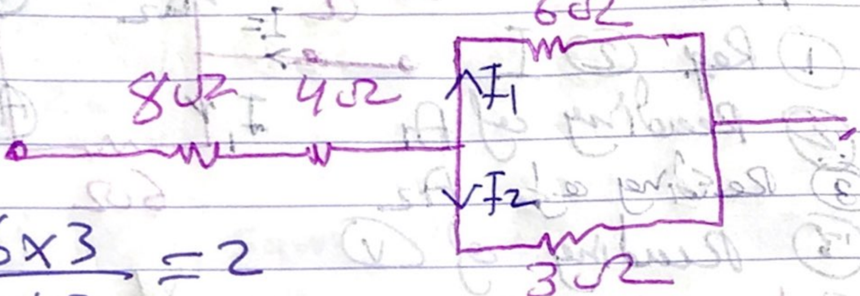
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

For R_1, R_2 only in Parallel

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

(1)

Ex (28.4) 841 Find the equivalent Resistance R_{eq}



$(6,3) \xrightarrow{P} \frac{6\times3}{6+3} = 2$

$(8,4,2) \xrightarrow{S}$ 8 4 2

$R_{eq} = 8+4+2 = 14\ \Omega$

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(2) $\Delta V = 42$ volt

what is the current in each Resistance

$I = \frac{\Delta V}{R_{eq}}$

$= \frac{42}{14}$

$= 3\ A$

$I_8 = I_4 = I = 3A$

$\Delta V_1 = \Delta V_2$

$6 I_1 = 3 I_2$

$I_2 = 2 I_1$ —(1)

$I_1 + I_2 = 3$ —(2)

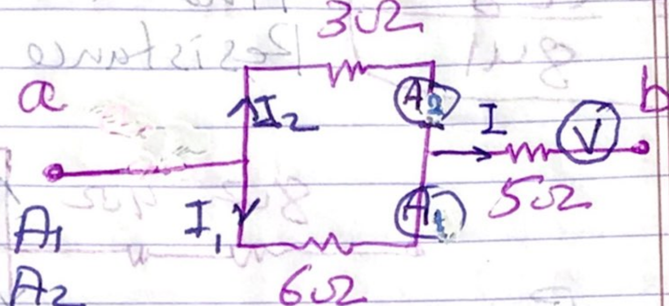
$I_1 = 1\ A$

$I_2 = 2A$

EX

Find

1. R_{eq}
2. Reading of A_1
3. Reading of A_2
4. Reading of (V)



Sol.

① $(3,6) \xrightarrow{P} \frac{3\times6}{3+6} = 2$

$(2,5) \xrightarrow{S} R_{eq} = 2+5 = 7\Omega$

② + ③ $V_1 = V_2$

$I_1 R_1 = I_2 R_2$

$6 I_1 = 3 I_2$

$I_2 = 2 I_1$ —— ①

$I_1 + I_2 = I$

$I_1 + I_2 = 3$ —— ②

subsitute ① in ②

$I_1 + 2I_1 = 3 \longrightarrow I_1 = 1\ A$

$\longrightarrow I_2 = 2\ A$

③ Readings of V

$V = IR$

$= 3\times5 = 15$ volt

General Circuit Equation

معادلة الدائرة العامة

$$I = \frac{\Sigma \mathcal{E}}{\Sigma R}$$

$\Sigma \mathcal{E}$: aljebric sum of Electromotive forces of all batteries

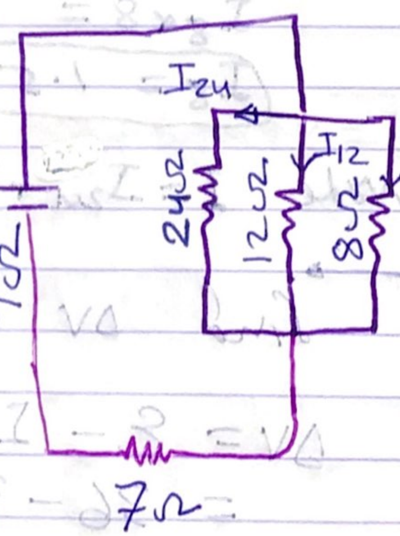
$\Sigma \mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 + \dots$ (aljebric sum)

$\Sigma R = R_{eq} + \Sigma r$

I: current passes through batteries

I: main current.

Ex Find I



(24, 12, 8) → P

$\frac{1}{R} = \frac{1}{24} + \frac{1}{12} + \frac{1}{8}$

$= \frac{1+2+3}{24}$

$\rightarrow R = 4$

$(4, 7) \xrightarrow{S} R_{eq} = 11\ \Omega$

$\Sigma r = 1$

$\rightarrow \Sigma R = 1 + 11 = 12\ \Omega$

$\rightarrow I = \frac{\Sigma \mathcal{E}}{\Sigma R} = \frac{36}{12} = 3$ A

Find current of each Resistance

$I_{24} \times 24 = I \times R_p$ ($V_{24} = V_p$)

$I_{24} \times 24 = 3 \times 4$

$I_{24} = 0.5\ A$

$I_{12} \times 12 = I \times R_p$

$I_{12} \times 12 = 3 \times 4$

$I_{12} = 1\ A$

$I_8 \times 8 = I \times R_p$

$I_8 \times 8 = 3 \times 4$

$I_8 = 1.5\ A$

Note $I = I_{24} + I_{12} + I_8$

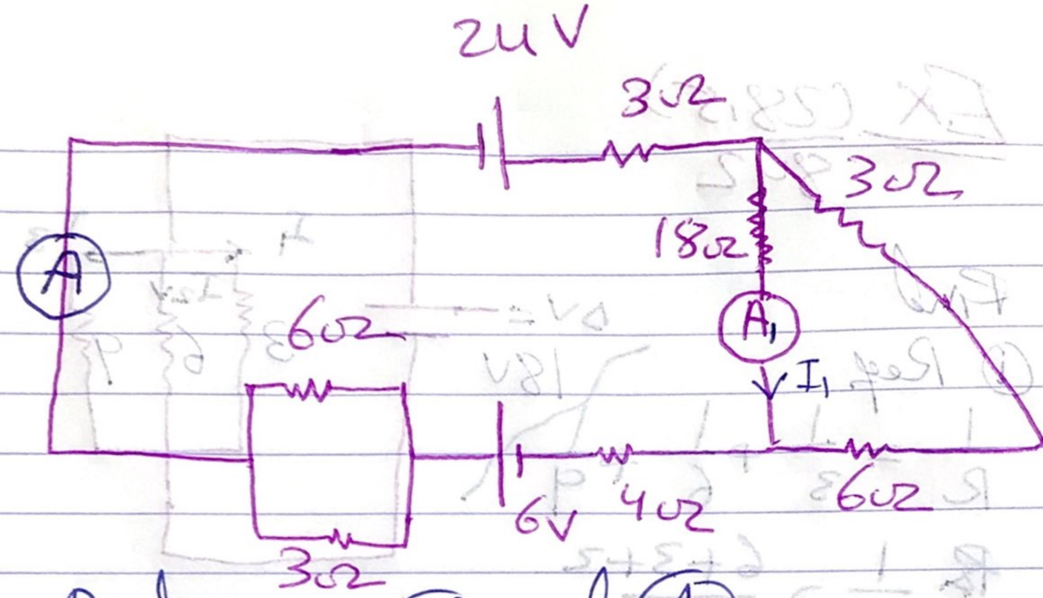
→ find ΔV for the battery

$\Delta V = \mathcal{E} - Ir$

$= 36 - 3 \times 1 = 33$ volt

→ find Power delivered to $12\,\Omega$

$P = I^2 R = (1)^2 (12) = 12$ watt.



Find Reading of (A) and (A₁)

$(3,6) \xrightarrow{S} 3+6=9$

$(9,18) \xrightarrow{P} \frac{9\times18}{9+18} = 6$

$(6,3) \xrightarrow{P} \frac{6\times3}{6+3} = 2$

$(3,6,4,2) \xrightarrow{S} R_{eq} = 3+6+4+2 = 15$

$\Sigma \varepsilon = 6+24 = 30$ volt.

→ $I = \frac{\Sigma \varepsilon}{\Sigma R} = \frac{30}{15} = 2\ A$ (A)

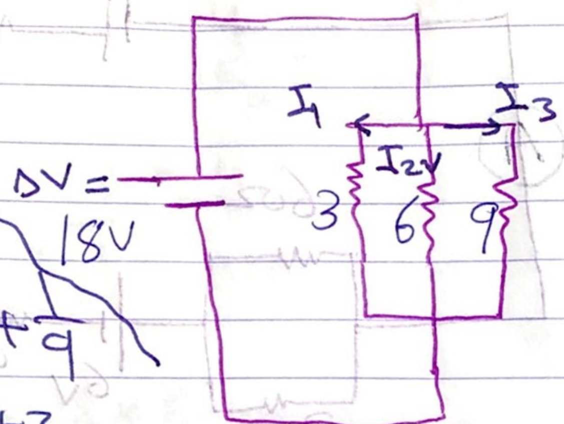
→ $V' = V' = V_p$

$I_1 \times 18 = I \times 6$

$I_1 \times 18 = 2 \times 6$

$I_1 = \frac{2}{3}\ A$

Ex (28.5)
842



Find

① R_{eq}

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{6} + \frac{1}{9}$$

$$\frac{1}{R} = \frac{6+3+2}{18}$$

$$R = \frac{18}{11}\ \Omega$$

② current of each Resistance

$$I_1 = \frac{\Delta V}{R_1} = \frac{18}{3} = 6\ A$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18}{6} = 3\ A$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18}{9} = 2\ A$$

③ power delivered to each Resistance

$$P_3 = I^2 R = (6)^2(3) = 108\ W$$

$$P_6 = (3)^2(6) = 54\ W$$

$$P_9 = (2)^2(9) = 36\ W$$

④ power delivered to the combination

$$P_{comb} = \frac{(\Delta V)^2}{R_{eq}} = \frac{(18)^2}{\frac{18}{11}} = 198\ W$$

or $P_{comb} = P_1 + P_2 + P_3$