Physics 102

(Session 103)

Ch-27

Circuits

Lecture 24-25

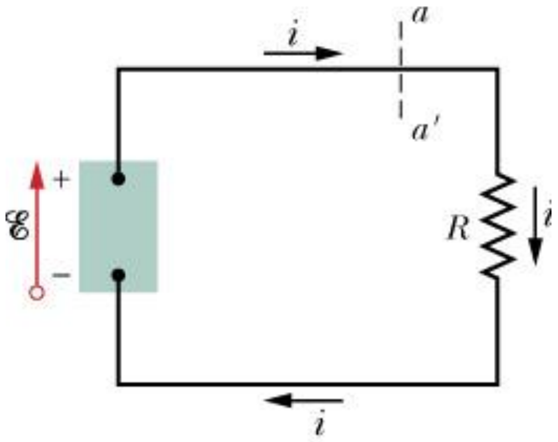
(Week 05)

Chapter Summery

In this chapter we will cover the following topics:

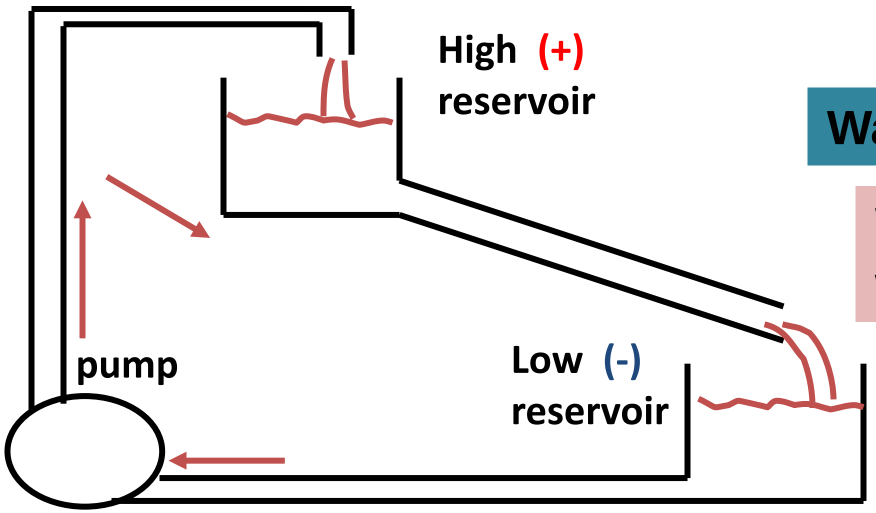
-Electromotive force (emf)
-Ideal and real emf devices
-Kirchhoff's loop rule
-Kirchhoff's junction rule
-Multi-loop circuits
-Resistors in series
-Resistors in parallel
-RC circuits, charging and discharging of a capacitor

Electric Current



Battery driving current in circuit

Charges (+ve) flowing from higher potential (level) to lower potential (level)



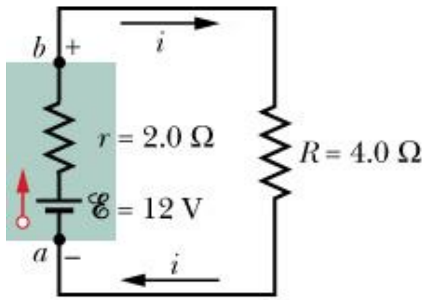
Water flows from high level to low level

Water pump is like "Battery" shifting water from low to high level

Ideal and Real source of emf

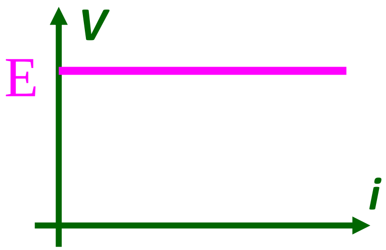
Ideal emf source (no internal resistance)

Terminal potential difference is same as ξ (emf)



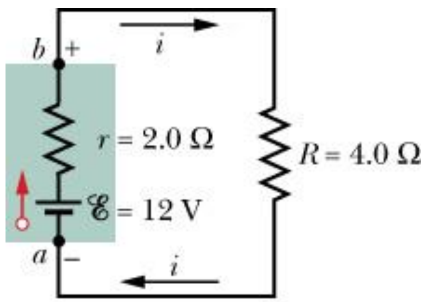
Ideal emf device

$$V = E$$



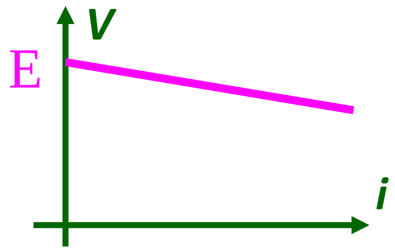
$$V = E - ir$$

Real emf source (internal resistance is not zero)



Real emf device

$$V = E - ir$$



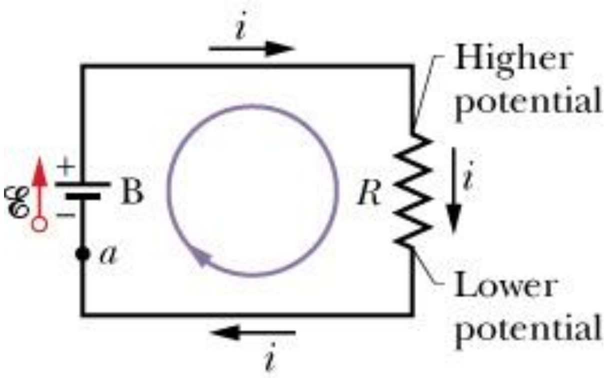
Current in Single loop

Kirchhoff Voltage law: Sum of voltages in closed must be zero

Kirchhoff Loop Rule: KLR

$$\xi - iR = 0$$

$$i = \frac{\xi}{R}$$



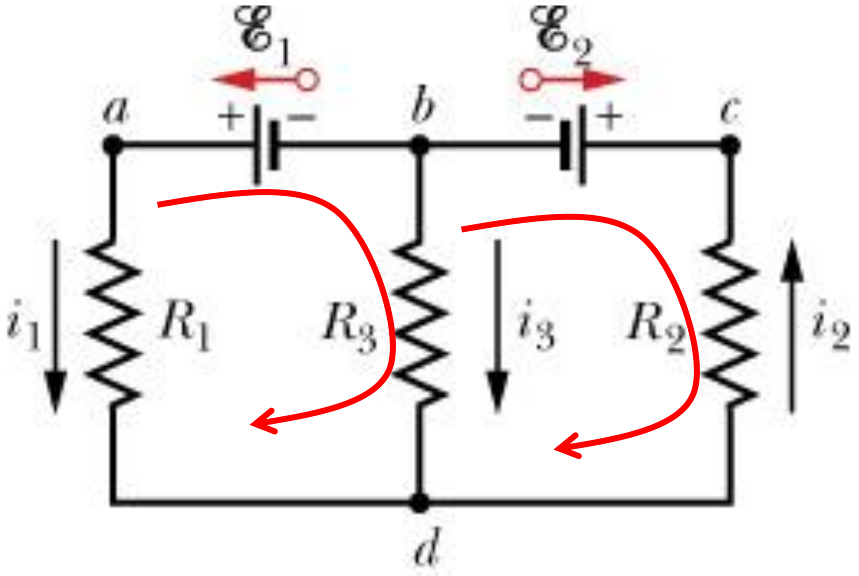
Emf term will be +ve if direction of motion in loop along battery is from –Ve terminal to +Ve terminal of i

Voltage across resistance (iR) term will -Ve if if direction of motion in loop is same as direction of current

Multiple loop

Loop -1

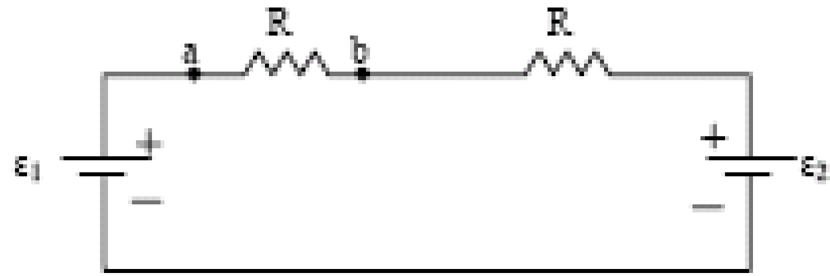
$$-\mathscr{E}_1 + (-i_3 R_3) + i_1 R_1 = 0 -----(1)$$



Loop -2

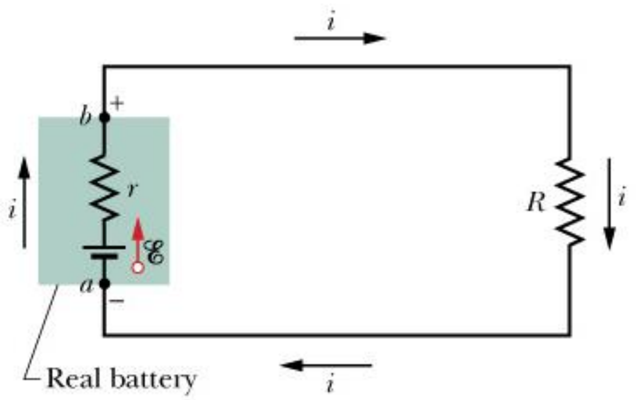
$$\mathscr{E}_2 + i_2 R_2 + i_3 R_3 = 0 ---(2)$$

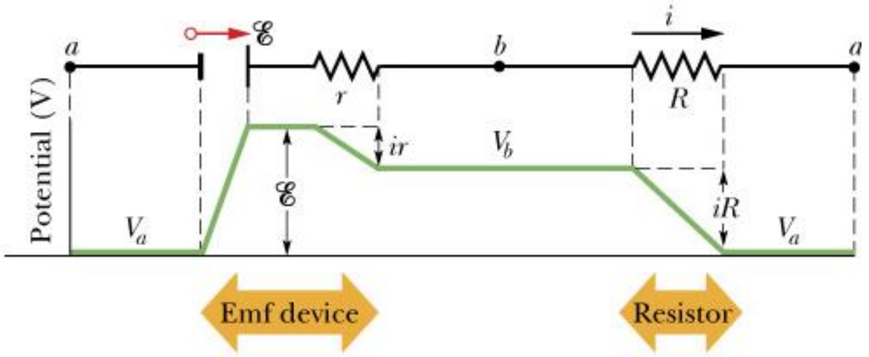
T072Q#13 The Fig. 1 shows two resistors, each of the resistance R, connected to two ideal batteries of emf $\varepsilon1$ and $\varepsilon2$ ($\varepsilon1 > \varepsilon2$). The potential difference Va – Vb is equal to $\varepsilon1/5$. What is the ratio $\varepsilon2/\varepsilon1$?(Ans: 3/5)



Voltage Variation in Circuit "point to point"

Assuming -ve terminal of battery is at Zero





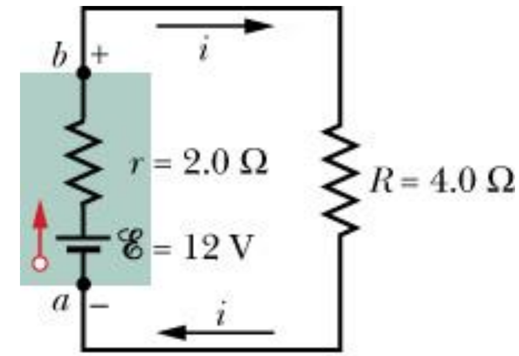
Kirchhoff Loop Rule: KLR

$$\xi - ir - iR = 0$$

$$i = \frac{\xi}{R + r}$$

Potential Difference between Two points

$$i = \frac{\xi}{R+r} = \frac{12}{6} = 2A$$



Voltage equation from point ‘a’ to ‘b’

From the left

$$V_a + \xi - ir = V_b$$

$$V_b - V_a = \xi - ir = 12 - 2\times 2$$

$$V_b - V_a = 8V$$

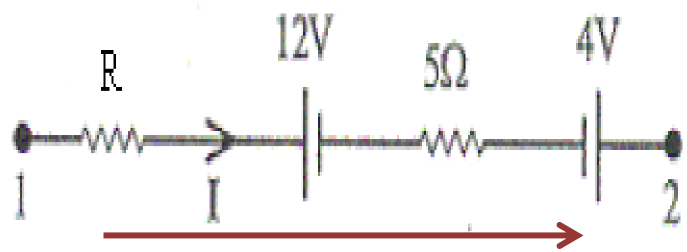
From the right

$$V_b - iR = V_a$$

$$V_b - V_a = iR = 2\times 4$$

$$V_b - V_a = 8V$$

T51-Q#7. In the figure 2 shown, the potential difference between point 1 and 2, (V2-V1), is -40 V, and the current is equal to 4.0 A, then, the value of the resistance R is



$$V_1 - iR - 12 - i \times 5 + 4 = V_2$$

$$4R - 12 - 4 \times 5 + 4 = V_2 - V_1$$

$$4R - 28 = -40$$

$$R = 3\Omega$$

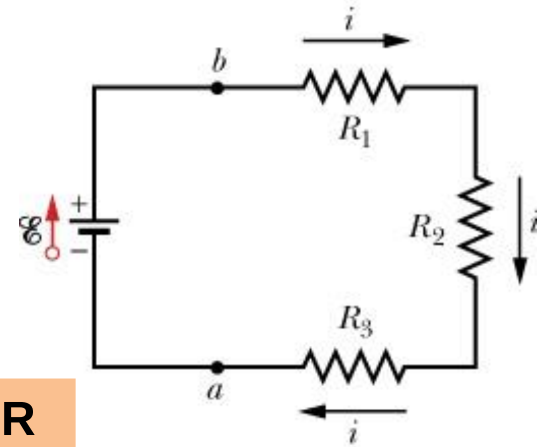
Equivalent Resistance

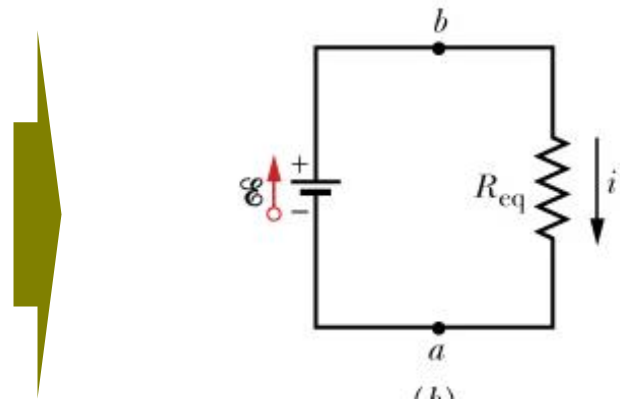
Series combination

Three resistors are connected in series

Resistors connected in series must have same current

Voltage divide





Using KLR

$$\xi - iR_1 - iR_2 - iR_3 = 0$$

$$\xi - iR_{eq} = 0$$

$$i = \frac{\xi}{R_1 + R_2 + R_3}$$

$$i = \frac{\xi}{R_{eq}}$$

$$R_{eq} = R_1 + R_2 + R_3$$

Expression valid for any number of resistors

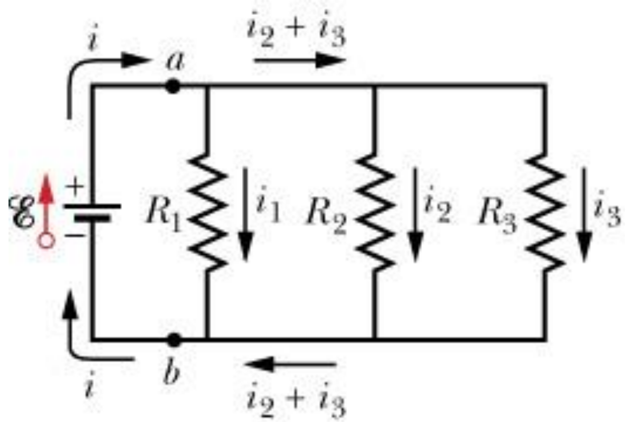
Equivalent Resistance

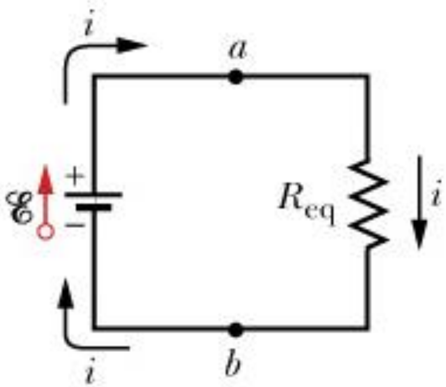
Parallel Combination

Three resistors are connected in parallel

Resistors connected in parallel must have same voltage

Current divide





Using Kirchhoff Junction rule: KJR

$$i = i_1 + i_2 + i_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$i = \frac{V}{R_{eq}}$$

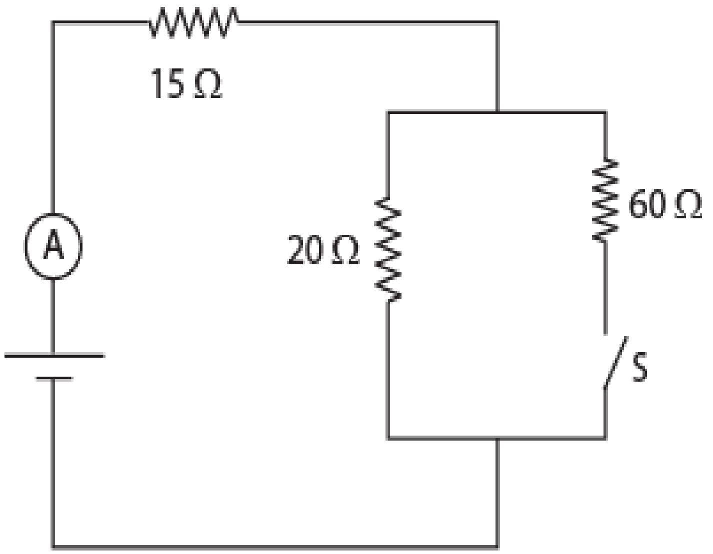
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Expression valid for any number of resistors

T81-Q3.: When switch S is open, the ammeter in the circuit shown in Fig 2 reads 2.0 A. When S is closed, the ammeter reading: (increases/ decreases)

Net resistance in circuit will decrease

increases



T071-Q13. A single loop circuit contains two external resistors and two emf sources as shown in the figure 1. Assume the emf sources are ideal, what is the power dissipation across resistor R1.

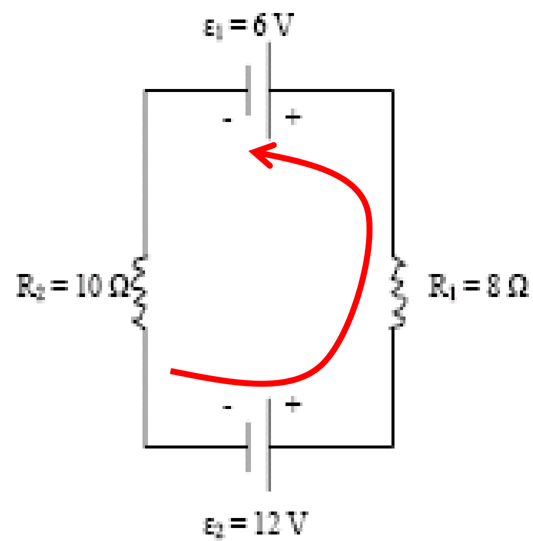
Using KLE

$$12 - iR_1 - 6 - iR_2 = 0$$

$$6 - i \times 8 - i \times 10 = 0$$

$$i = \frac{1}{3} A$$

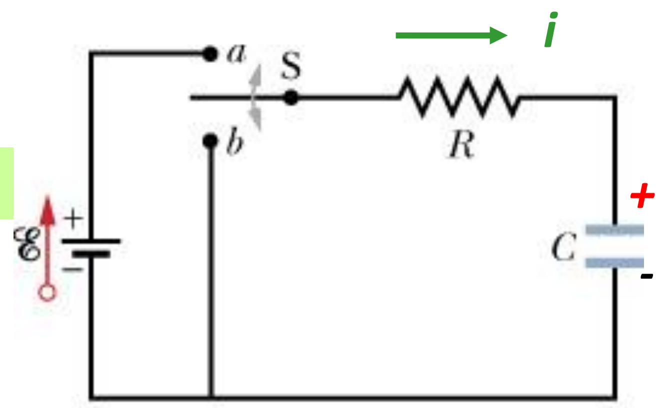
$$P = i^2 R = \frac{1}{9} 8 = 0.9W$$



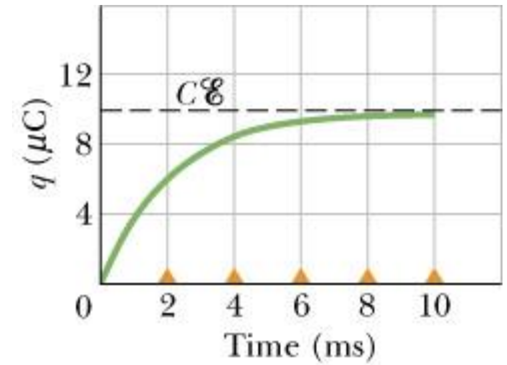
RC Circuit

Switch towards 'a' will charge the capacitor

Switch towards 'b' will discharge the capacitor

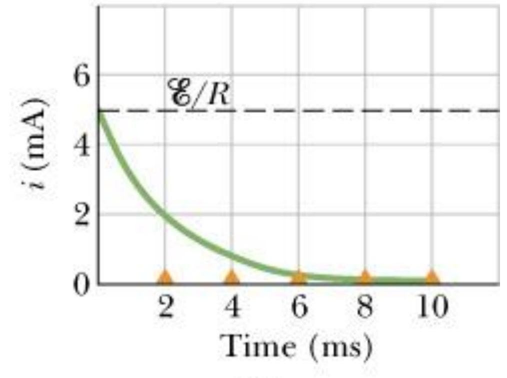


$$Q = Q_o\left(1 - e^{\left(-\frac{t}{RC}\right)}\right)$$



RC is called time constant of a capacitor circuit (RC-circuit)

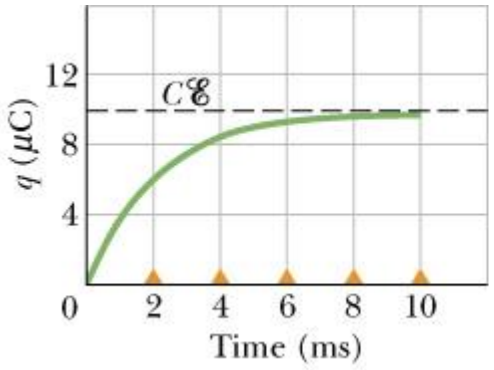
$$I = I_o e^{\left(-\frac{t}{RC}\right)}$$



RC Circuit

Time Constant

$$Q = Q_o\left(1 - e^{\left(-\frac{t}{RC}\right)}\right)$$



RC is called time constant of a capacitor circuit (RC-circuit)

$$I = I_o e^{\left(-\frac{t}{RC}\right)}$$

$$t = RC$$

$$Q = Q_o\left(1 - e^{\left(-\frac{RC}{RC}\right)}\right) = 0.693Q_o$$

Sample Problem

Problem 19 (093)

A resistance thermometer made from platinum has a resistance of 50.0 Ω at 20°C. When immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω. Find the melting point of indium. (For platinum, α = 3.92 × 10^{-3} $(C°)^{-1}$).

$$\rho - \rho_0 = \rho_0 \alpha (T - T_o)$$

$$R - R_0 = R_0 \alpha (T - T_o)$$

$$76.8 - 50 = 50 \times (3.92 \times 10^{-3})(T - 293)$$

$$T = 430K = 157^o C$$

Problem 20 (093)

An unknown resistor is connected between the terminals of a 3.00 V battery. Energy is dissipated in the resistor at the rate of 0.540 W. The same resistor is then connected between the terminals of a 1.50 V battery. At what rate is energy dissipated now?

$$P = \frac{V^2}{R}$$

$$0.54 = \frac{3^2}{R} \Rightarrow R = 16.7\Omega$$

$$P = \frac{V^2}{R}$$

$$P = \frac{1.5^2}{16.7} = 0.135W$$

Problem 19

The resistivity of silver is 1.5 × 10^{-8} Ω.m, and that of aluminium is 2.8 × 10^{-8} Ω.m. Two wires, one made of silver and the other made of aluminium, have the same dimensions and are connected to the same battery. What is the ratio of the current passing in the silver wire to that passing in the aluminium wire?

$$R = \rho \frac{l}{A}$$

Both wires have same dimension, Therefore ratio of resistance of Silver to Aluminum

$$\frac{R_s}{R_a} = \frac{\rho_s}{\rho_a}$$

$$\frac{R_s}{R_a} = \frac{1.5}{2.8}$$

$$R_s = 0.54 R_a$$

$$i = \frac{V}{R}$$

$$i_s = \frac{V}{R_s} \quad and \quad i_a = \frac{V}{R_a}$$

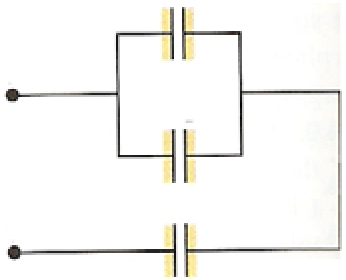
$$\frac{i_s}{i_a} = \frac{V/R_s}{V/R_a} = \frac{R_a}{R_s}$$

$$\frac{i_s}{i_a} = \frac{R_a}{0.54 R_a}$$

$$\frac{i_s}{i_a} = \frac{1}{0.54} = 1.9$$

Problem 20

In Fig, the capacitors are identical, and each has a capacitance C. The equivalent capacitance of the combination is



Equivalent Top two capacitors are connected in parallel

C'=2C

C' is connected in series with the capacitor at bottom

Net equivalent capacitance of the circuit is

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} = \frac{3}{2C}$$

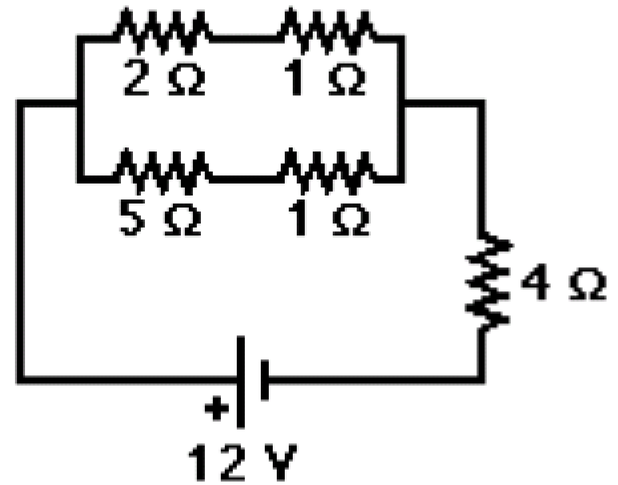
$$C_{eq} = \frac{2}{3}C$$

Problem 14

In the circuit shown in **Figure 8, what power is dissipated in the 4-Ω resistor?**

$$P = i^2 R$$

$$P = i^2 (4)$$



Equivalent of resistors connected in parallel

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{6} = \frac{3}{6}$$

$$R = 2\Omega$$

Total resistance in circuit

$$R_T = 2 + 4 = 6\Omega$$

Total current in circuit

$$i = \frac{12}{6} = 2A$$

$$P = i^2 (4)$$

$$P = (2)^2 (4)$$

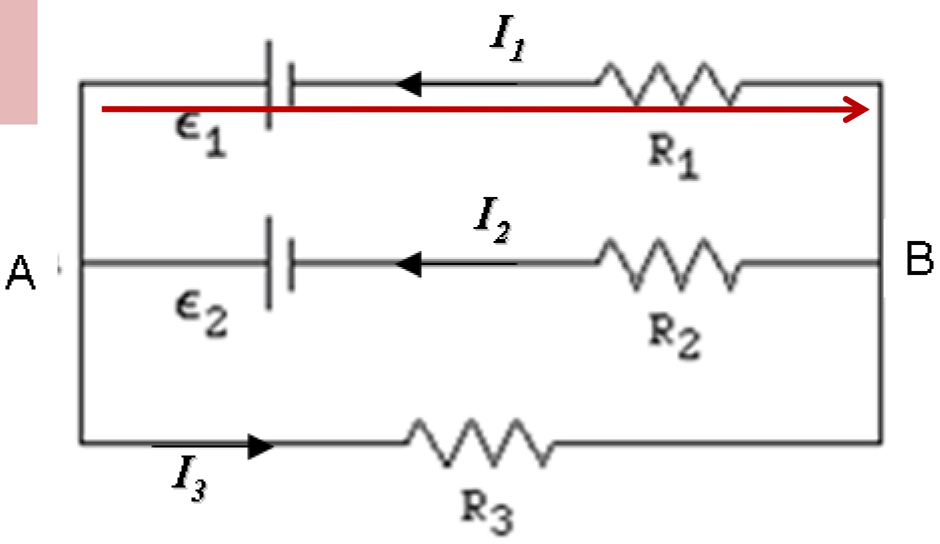
$$P = 16W$$

Problem 15

In the circuit shown in **Fig, ε_1 = 28 V, ε_2 = 42 V, R_1 = 2.0 Ω, R_2 = 5.0 Ω, R_3 = 1.0 Ω,** and *I_1 = 7.5 A. Calculate the potential difference $V_A - V_B$.*

All three horizontal lines are parallel therefore all have potential difference

Writing Kirchhoff's equation along line A-B (blue line)



$$V_A - \varepsilon_1 + I_1 R_1 = V_B$$

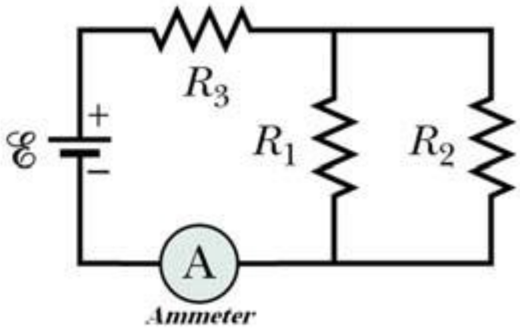
$$V_A - 28 + (7.5)(2) = V_B$$

$$V_A - V_B = 28 - 15$$

$$V_A - V_B = 13V$$

Problem 16

The circuit in Fig shows three identical resistors connected to a battery and an ammeter. The current measured by the ammeter is I_o. *If resistor R_2 is removed, the current measured by* the ammeter will be



Total resistance in original circuit

$$R_{o-T} = R + \frac{R}{2} = \frac{3}{2}R$$

Current in circuit

$$I_0 = \frac{\xi}{\frac{3}{2}R} = \frac{2\xi}{3R}$$

$$\frac{3}{2}I_0 = \frac{\xi}{R}$$

After removing R_2 total resistance in circuit will be

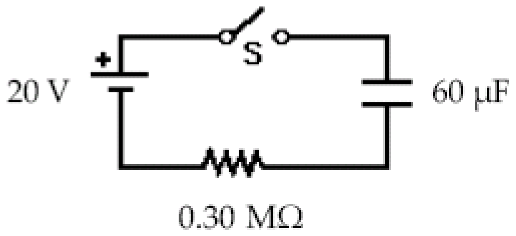
$$R_{o-T} = R + R = 2R$$

$$I_{new} = \frac{\xi}{2R} = \frac{1}{2}\left(\frac{3}{2}I_0\right)$$

$$I_{new} = \frac{3}{4}I_0$$

Problem 17

Initially, for the circuit shown in Fig the switch S is open and the capacitor is uncharged. The switch S is closed at time t = 0. At what time will the current be half its initial value?



Current variation in capacitor circuit is given by

$$I = I_0 e^{-\frac{t}{RC}}$$

We have following data

$$R = 0.3 M\Omega$$

$$C = 60 \mu F$$

$$I = \frac{1}{2} I_0$$

$$I = I_0 e^{-\frac{t}{RC}}$$

$$\frac{I_0}{2} = I_0 e^{-\frac{t}{3\times10^5 \times 6\times10^{-5}}}$$

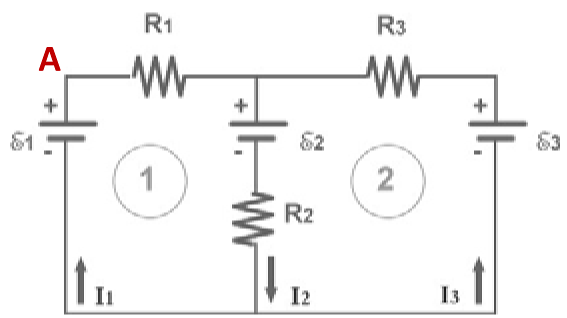
$$\frac{1}{2} = e^{-\frac{t}{18}}$$

$$\ln 2 = \frac{t}{18}$$

$$t = 18 \times 0.693 = 12.4 s$$

Problem 18

For the circuit shown in Figure 4, which equation is correct for loop 2?



Kirchhoff's equation for loop-2 from point A in clockwise direction

$$I_3R_3 - \varepsilon_3 + I_2R_2 + \varepsilon_2 = 0$$

Problem 19

The resistivity of silver is 1.5 × 10^{-8} Ω.m, and that of aluminum is 2.8 × 10^{-8} Ω.m. Two wires, one made of silver and the other made of aluminum, have the same dimensions and are connected to the same battery. What is the ratio of the current passing in the silver wire to that passing in the aluminium wire?

$$R = \rho \frac{l}{A}$$

Both wires have same dimension, Therefore ratio of resistance of Silver to Aluminum

$$\frac{R_s}{R_a} = \frac{\rho_s}{\rho_a}$$

$$\frac{R_s}{R_a} = \frac{1.5}{2.8}$$

$$R_s = 0.54 R_a$$

$$i = \frac{V}{R}$$

$$i_s = \frac{V}{R_s} \quad and \quad i_a = \frac{V}{R_a}$$

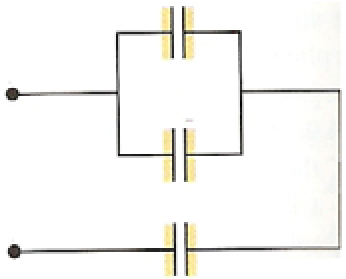
$$\frac{i_s}{i_a} = \frac{V/R_s}{V/R_a} = \frac{R_a}{R_s}$$

$$\frac{i_s}{i_a} = \frac{R_a}{0.54 R_a}$$

$$\frac{i_s}{i_a} = \frac{1}{0.54} = 1.9$$

Problem 20

In Fig, the capacitors are identical, and each has a capacitance C. The equivalent capacitance of the combination is



Equivalent Top two capacitors are connected in parallel

C'=2C

C' is connected in series with the capacitor at bottom

Net equivalent capacitance of the circuit is

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} = \frac{3}{2C}$$

$$C_{eq} = \frac{2}{3}C$$

Suggested Problem Ch 27

2. In Fig. 27-18, the ideal batteries have emfs ε_1=12.0 V and ε_2 = 0.500ε_1, and the resistances are each 4.00 Ω.What is the current in (a) resistance 2 and (b) resistance 3?

Fig. 27-18

Answer: : (a) i_2 = 0.0 A ; (b) i_3 = 1.5 A.

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19. In Fig. 27-30, the current in resistance 6 is i_6 = 2.80 A and the resistances are R_1 = R_2 = R_3 = 2.00 Ω, R_4 = 16.0 Ω, R_5 = 8.00 Ω, and R_6 = 4.00 Ω. What is the emf of the ideal battery?

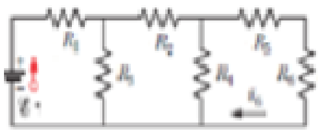

Fig. 27-30 Problem 19

Answer: 96.6 V

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25. In Fig. 27-35, ε = 24.0 V, R_1 = 2000 Ω, R_2 = 3000 Ω, and R_3 = 4000 Ω. What are the potential differences (a) V_A - V_B (b) $V_B - V_C$, (c) $V_C - V_D$, and (d) $V_A - V_C$?

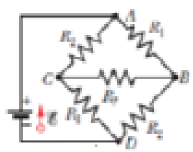

Fig. 27-35 Problem 25

Answer: (a) 10.5 V ; (b) 3.00 V ; (c) 10.5 V ; (d) 13.5 V.

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27. In Fig. 27-36, battery 1 has emf ε_1=12.0 V and internal resistance r_1= 0.025 Ω and battery 2 has emf ε_2=12.0 V and internal resistance r_2= 0.012 Ω. The batteries are connected in series with an external resistance R. (a) What R value makes the terminal- to-terminal potential difference of one of the batteries zero? (b) Which battery is that?

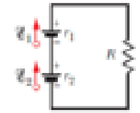


Fig. 27-33

Fig. 27-36 Problem 27

Answer: (a) 0.013 Ω ; (b) Battery 1.

Suggested Problem Ch 27

47. In the circuit of Fig. 27-46, ε =1.2 kV, C = 6.5 µF, R_1 = R_2 = R_3 = 0.73 M Ω. With C completely uncharged, switch S is suddenly closed (at t = 0). At t = 0, what are (a) current i_1 in resistor 1, (b) current i_2 in resistor 2, and (c) current i_3 in resistor 3? At t = ∞ (that is, after many time constants), what are (d) i_1, (e) i_2, and (f) i_3? What is the potential difference V_2 across resistor 2 at (g) t = 0 and (h) t = ∞?

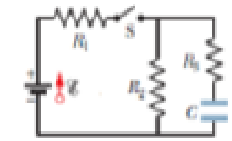


Fig. 27-46 Problem 47

Answer: (a) 1.1 mA ; (b) 0.55 mA ; (c) 0.55 mA ; (d) 0.82 mA ; (e) 0.82 mA ; (f) 0 ; (g) 400 V ; (h) 600 V.

50. When resistors 1 and 2 are connected in series, the equivalent resistance is 20.0 Ω. When they are connected in parallel, the equivalent resistance is 3.75 Ω. What are (a) the smaller resistance and (b) the larger resistance of these two resistors?

Answer: (a) 5.0 Ω ; (b) 15.0 Ω.

60. Figure 27-56 shows five 8.00 Ω resistors. Find the equivalent resistance between points (a) F and H and (b) F and G. (Hint: For each pair of points, imagine that a battery is connected across the pair.)?

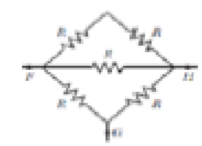


Fig. 27-56

Answer: : (a) 4.00 Ω ; (b) 5.00 Ω

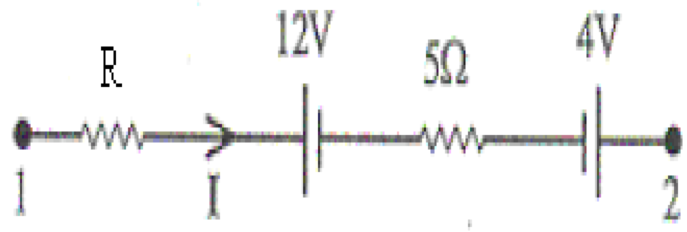
63. The current in a single-loop circuit with one resistance R is 5.0 A. When an additional resistance of 2.0 Ω is inserted in series with R, the current drops to 3.0 A. What is R?

Answer: 3.0 Ω.

65. A total resistance of 5.00 Ω is to be produced by connecting an unknown resistance to a 15.0 Ω resistance. (a) What must be the value of the unknown resistance, and (b) should it be connected in series or in parallel?(c) What is the total resistance if the unknown resistance is connected the other way?

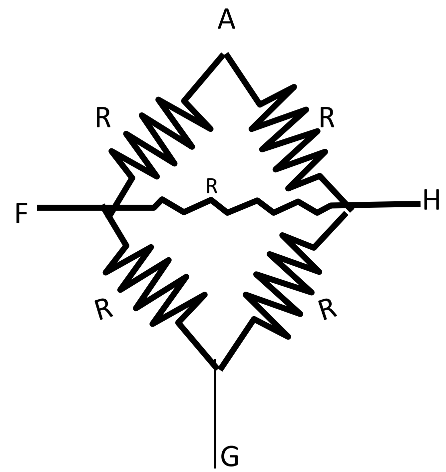
Answer: (a) 7.0 Ω ; (b) In parallel. (c) 22.5 Ω

T51-Q#7. In the figure 2 shown, the potential difference between point 1 and 2, (V2-V1), is -40 V, and the current is equal to 4.0 A, then, the value of the resistance R is [Ans: 3Ω]



Solve this in your note bo
and bring to recitation

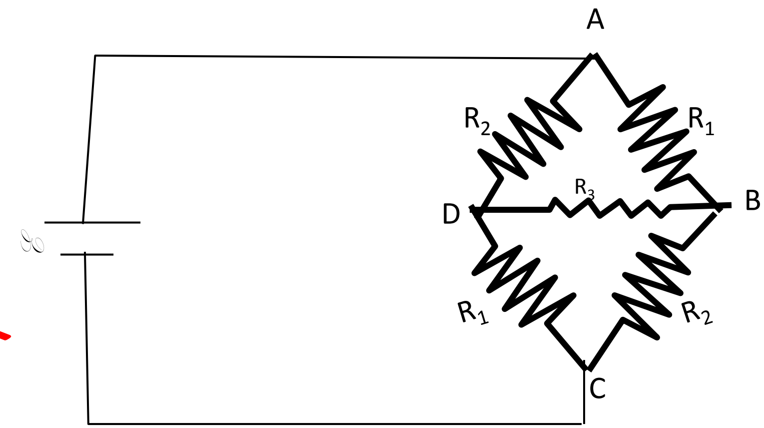
Ch27-26: Fig 27-39 shows five 5 Ω resistors. Find the equivalent resistance between points (a) F and H and (b) F and G (Hint: for each pair of points, imagine that a battery is connected across the pair)



Solve this in your note book and bring to recitation

Answers: (a) 2.5 Ω (b)3.13 Ω

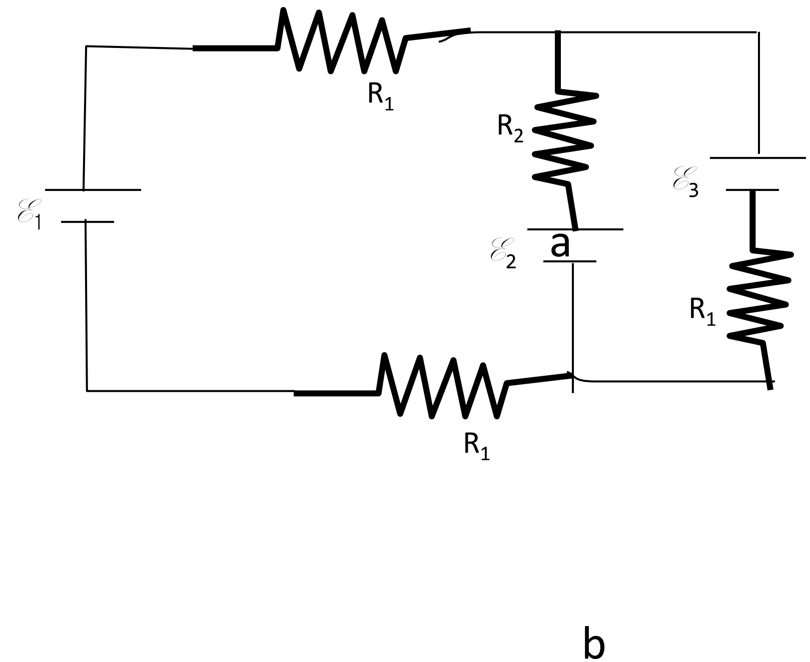
Ch27-39: In the circuit of Fig 27-50, $\mathscr{E}$ = 12V, R_1 =2000 Ω, R_2 =3000 Ω, R_3 = 4000Ω. what are potential differences (a) $V_A - V_B$, (b) $V_B - V_C$ (c) $V_C – V_D$ and(d) $V_A - V_C$?



Solve this in your note book and bring to recitation

Answers: (a) 5.25 V (b)1.5V (c) 5.25V (d) 6.75 V

Ch27-37: In Fig. 27-48, the resistances R1 = 1 ohm and R2 = 2 Ohm. And the ideal battries have emfs $\mathscr{E}_1$ = 2V an $\mathscr{E}_2$ = $\mathscr{E}_3$ = 4V. What are (a) size and (b) direction (up or down) of the current in battery 1, the (c) size and (d) direction (up or down) of the current in battery 2, the (e) size and (f) direction (up or down) of the current in battery 3? What is the potential difference $V_a - V_b$?



Solve this in your note book and bring to recitation

Answers: (a) 0.67 A, (b) downward (c)0.33 A (d) upward (e) 0.33(f) upward(g) 3.3 V

T001-Q#3: The circuit in Figure 3 has been connected for a long time. Find the potential difference Vb - Va. (Ans: 8 V)

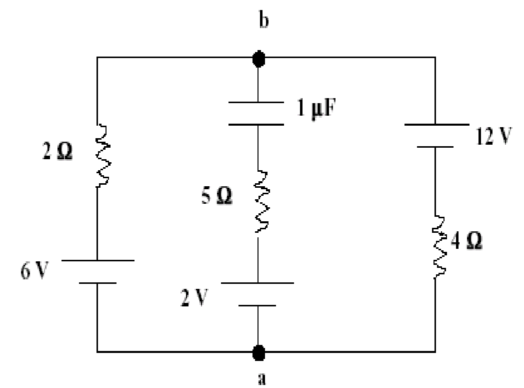
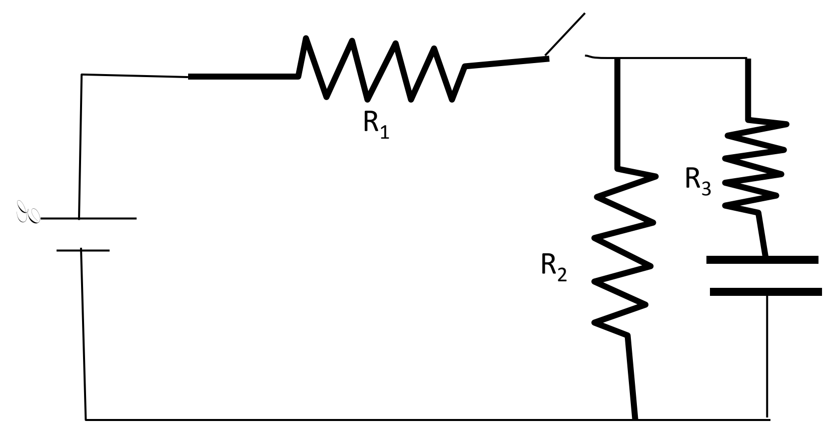


FIGURE 3

Solve this in your note book and bring to recitation

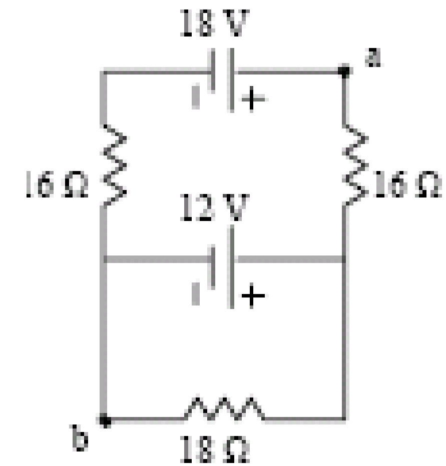
Ch27-65: In the circuit of Fig 27-64, $\mathscr{E}$ = 1.2kV, C = 6.5 µF, $R_1 = R_2 = R_3$ = 0.73 MΩ. With C completely uncharged, switch S is suddenly closed (at t = 0). At t =0, what are (a) current i_1 in resistor 1, (b) current i_2 in resistor 2 (c) current i_3 in the resistor 3? At t = ∞, what are (d) i_1, (e) i_2 (f) i_3? What is the potential difference V_2 across resistor 2 at (g) t = 0 and (h) t = ∞

Solve this in your note book and bring to recitation

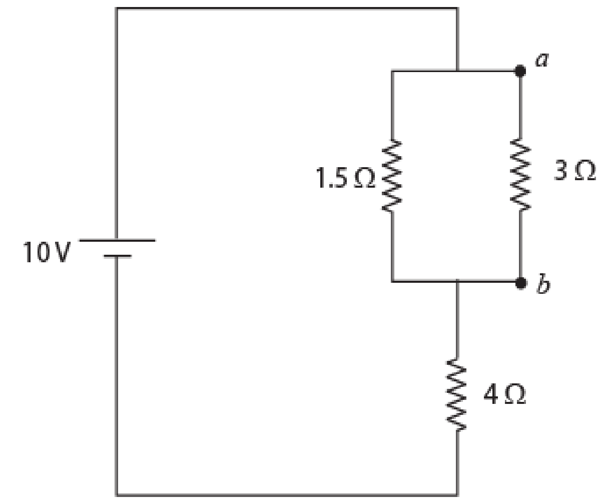


Answers: (a) 1.1 mA, (b) 0.55mA (c)0.55mA (d) 0.82mA (e)0.82mA (f)0 mA (g) 400V (h) 600V

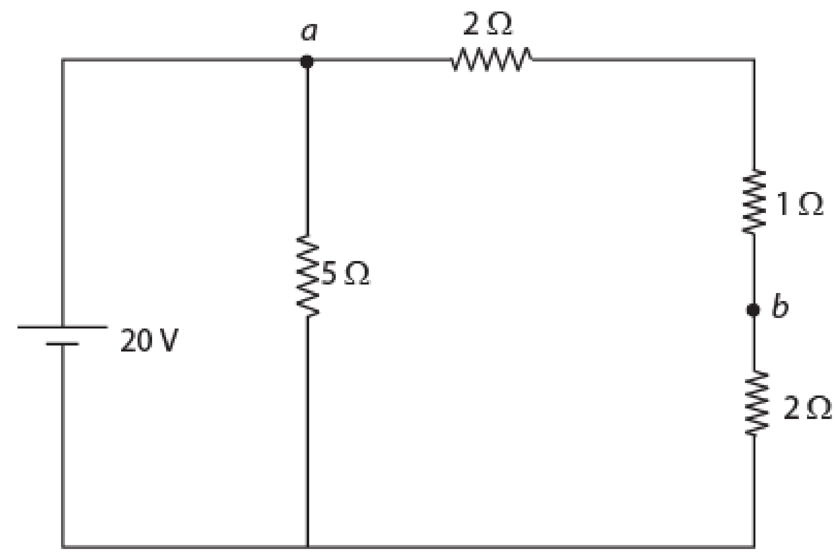
T72-Q14. Three resistors and two batteries are connected as shown in Fig. 2. What is the potential difference Va – Vb? (Ans: 15 V)



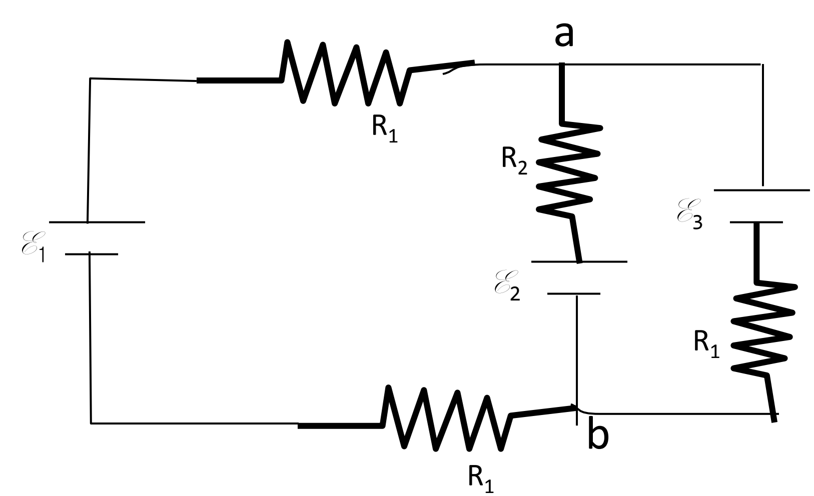
T81-Q1. Fig 1 shows two resistors 3.0 Ω and 1.5 Ω connected in parallel and the combination is connected in series to a 4.0 Ω resistor and a 10 V emf device. The potential difference V_a - V_b is: (Ans: 2.0 V)



T81-Q4. In Fig 3, what is the potential difference V_a-V_b? (Ans: 12 V)



Ch27-37: In Fig. 27-48, the resistances R1 = 1 ohm and R2 = 2 Ohm. And the ideal battries have emfs $\mathscr{E}_1$ = 2V an $\mathscr{E}_2$ = $\mathscr{E}_3$ = 4V. What are (a) size and (b) direction (up or down) of the current in battery 1, the (c) size and (d) direction (up or down) of the current in battery 2, the (e) size and (f) direction (up or down) of the current in battery 3? What is the potential difference $V_a - V_b$?



Answers: (a) 0.67 A, (b) downward (c)0.33 A (d) upward (e) 0.33(f) upward(g) 3.3 V