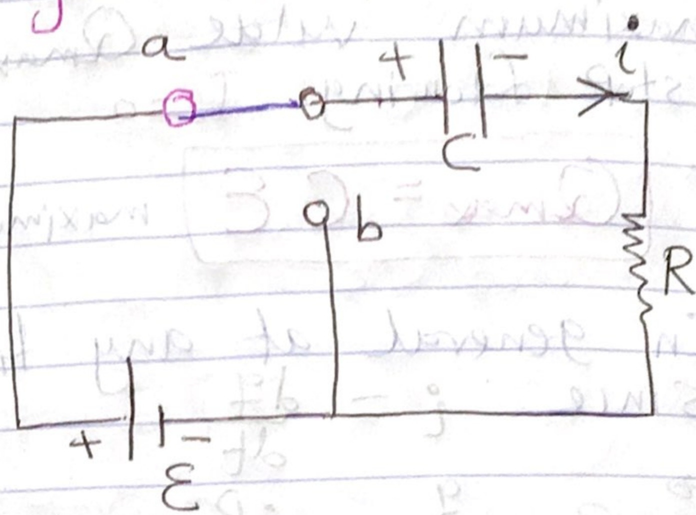
RC - Circuits - part 3

A circuit containing a series combination of a resistor and a capacitor is called an RC circuit.

Charging a Capacitor:

When the switch is thrown to position a the capacitor begins to charge up.



Initially the capacitor is uncharged the switch is thrown at position (a).

Applying Kirchhoff's voltage Loop (Loop Rule)

$$\varepsilon - V_c - iR = 0$$

$$\varepsilon - \frac{q}{C} - iR = 0$$

potential diff across resistor

potential diff across capacitor

at $t=0$ the charge on the capacitor is zero

$Q_i = 0$

Thus $\boxed{I_i = \frac{\mathcal{E}}{R}}$

after some long time the charge on the capacitor reaches its maximum value Q_{max} So I stop flowing $I=0$

$\boxed{Q_{max} = C\mathcal{E}}$ maximum charge

In general at any time (t)

since $i = \frac{dq}{dt}$

$\rightarrow \mathcal{E} - \frac{q}{C} - iR = 0$

$$i = \frac{\mathcal{E}}{R} - \frac{q}{RC}$$

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC}$$

$$\frac{dq}{dt} = \frac{C\mathcal{E}}{RC} - \frac{q}{RC}$$

$$= \frac{-(q - C\mathcal{E})}{RC}$$

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$$\int \frac{dq}{q - C\varepsilon} = -\frac{1}{RC}\int dt$$

Integrate the expression ($q=0$ at $t=0$)

$$\ln_e(q - C\varepsilon)\Big|_0^Q = \frac{-t}{RC}$$

$$Q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)$$

$$Q(t) = Q_{max}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$Q_{max} = C\varepsilon$$

let $\boxed{\tau = RC}$ τ: time constant

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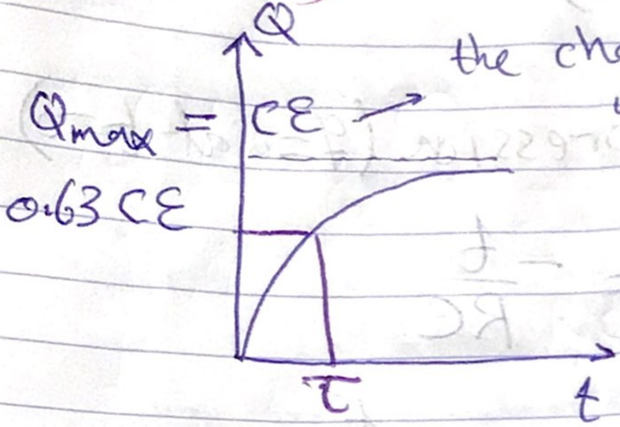
derive $Q(t)$ $\left(i(t) = \frac{dq}{dt}\right)$

$$\rightarrow i(t) = C\varepsilon\left(0 - \left(\frac{-1}{RC}\right) e^{-\frac{t}{RC}}\right)$$

$$i(t) = \frac{\varepsilon}{R} \cdot e^{-t/RC}$$

Graphical Representation of $(Q-t)$ and $(i-t)$ and V_c

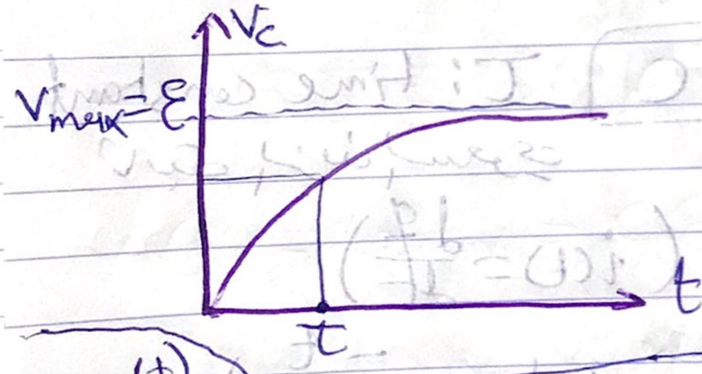


the charge equal $C\mathcal{E}$ when $t \to \infty$

after time $t = \tau \to$
$Q = 0.63\, Q_{max}$
$= 0.63\, C\mathcal{E}$

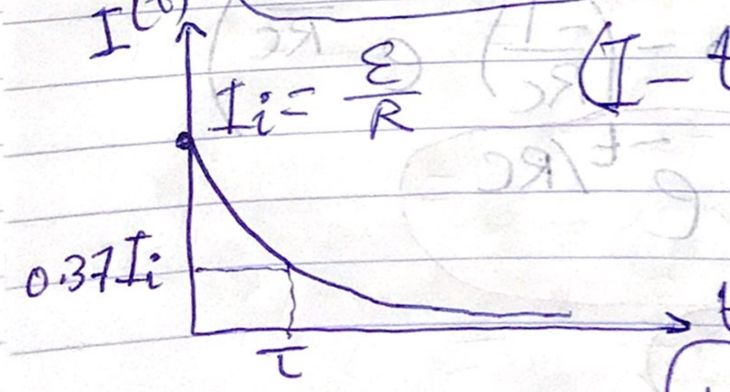
when $t = \tau$
$\to$ $Q = 0.63\, Q_{max}$
$Q = 0.63\, C\mathcal{E}$

$(V_c - t)$



at $t = \tau$
$V_c = 0.63\, V_{max}$
$= 0.63\, \mathcal{E}$

$(I - t)$



at $t = \tau$
$i = 0.37\, I_i$
$= 0.37\, \frac{\mathcal{E}}{R}$

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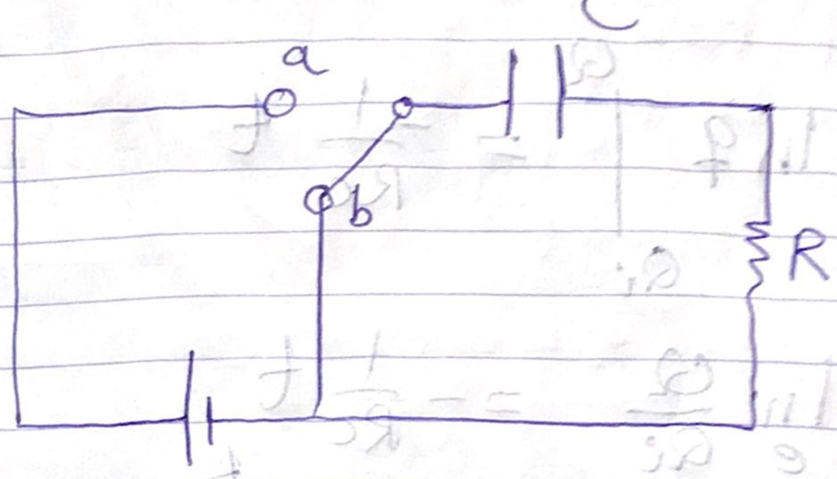
discharging a Capacitor

The capacitor is initially full charged



when the switch is thrown to position b the capacitor discharges

$$-\frac{q}{C} - iR = 0$$

$$i = \frac{dq}{dt}$$

$$-R\frac{dq}{dt} = \frac{q}{C}$$

$$\frac{dq}{q} = -\frac{1}{RC}\,dt$$

Integrating the expression using $q = Q_i = C\mathcal{E}$ $(t=0)$

$$\int_{Q_i} \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln_e\left(\frac{q}{Q_i}\right) = -\frac{t}{RC}$$

$$Q(t) = Q$$

$$\ln q \Big|_{Q_i}^{Q} = -\frac{1}{RC}\, t$$

$$\ln_e \frac{Q}{Q_i} = -\frac{1}{RC}\, t$$

$$\rightarrow \frac{Q}{Q_i} = e^{-\frac{t}{RC}}$$

$$\rightarrow Q(t) = Q_i\, e^{-\frac{t}{RC}}$$

$$Q_i = Q_{max} = C\mathcal{E}$$

$$i(t) = \frac{dQ}{dt} = Q_i\left(-\frac{1}{RC}\right) e^{-\frac{t}{RC}}$$

$$= \cancel{C}\mathcal{E}\left(\frac{-1}{R\cancel{C}}\right) e^{-\frac{t}{RC}}$$

$$i(t) = -\frac{\mathcal{E}}{R}\, e^{-\frac{t}{RC}}$$

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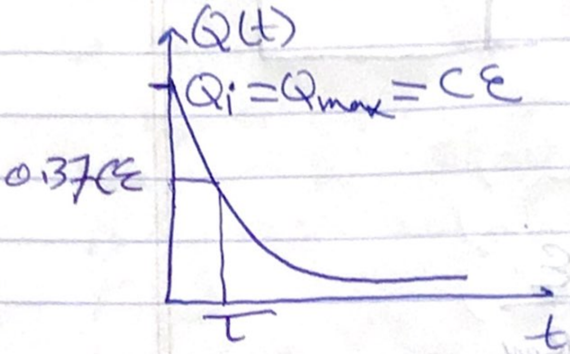
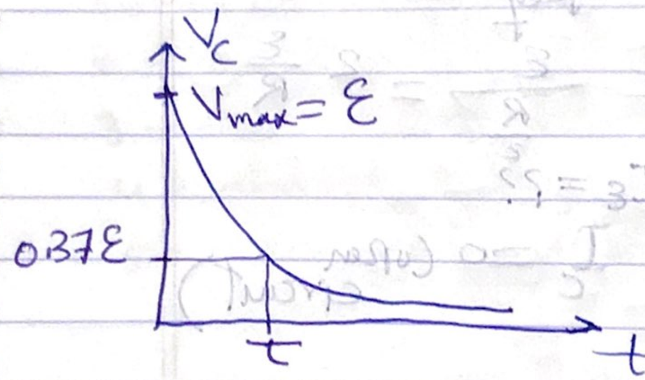
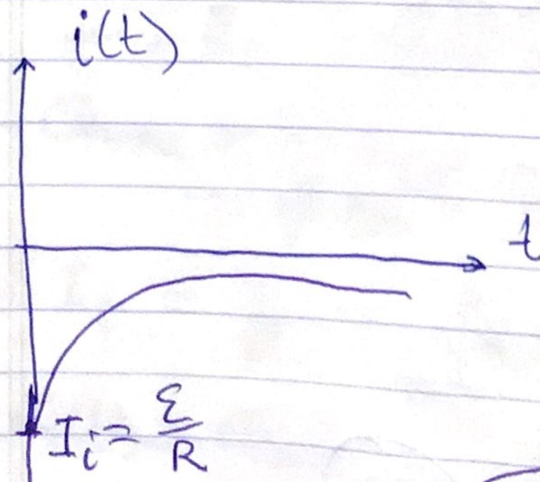
$$V_c(t) = \frac{Q(t)}{C}$$

$$\rightarrow V_c(t) = \mathcal{E} e^{-\frac{t}{RC}}$$

Graphical Representation





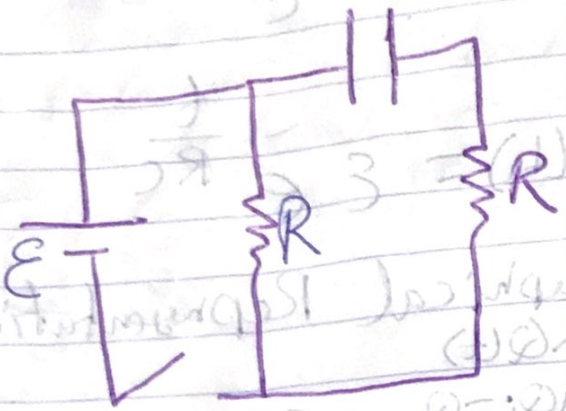


Quiz

~~Ex~~

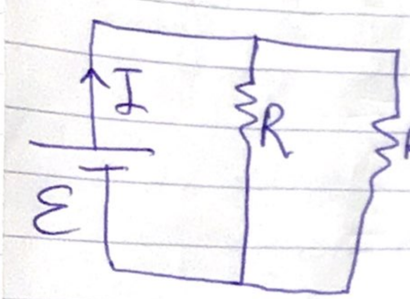
28.5

Consider



(a)

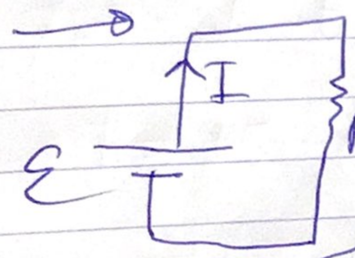
at $t=0$ the begining what is I throught battery



$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{\frac{R}{2}} = 2\frac{\mathcal{E}}{R}$$

(b) after long time $I_{\mathcal{E}} = ??$

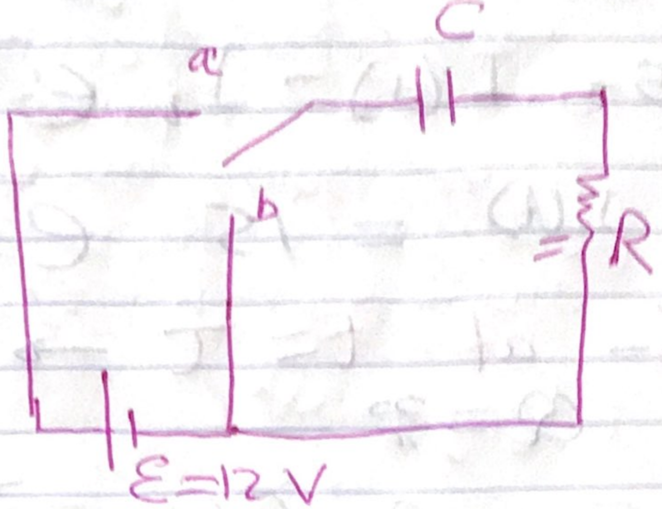
after long time $I_C = 0$ (open circuit)



$$\boxed{I = \frac{\mathcal{E}}{R}}$$

EX (28.9)

An uncharged capacitor and resistor are connected in series to a battery as shown in fig. When the switch is thrown at (a) Find



$\mathcal{E}=12$ V

$R=8\times10^5\ \Omega$

$C=5\ \mu F$

1- time constant
2- maximum charge
3- the initial (maximum) current
4- charge as function of time
5- current as function of time

1- $\tau = RC = 8\times10^5 \times 5\times10^{-6} = 4$ sec

2- $Q_{max} = C\mathcal{E}$

$= 5\times10^{-6}\times 12 = 60\ \mu C$

3- $I_i = \dfrac{\mathcal{E}}{R} = \dfrac{12}{8\times10^5} = 15\ \mu A$

4- $Q(t) = Q_{max}\left(1-e^{-\frac{t}{\tau}}\right)$

$Q(t) = 60(1 - e^{-\frac{t}{4}})$

5- $I(t) = I_i \, e^{-\frac{t}{\tau}}$

$I(t) = 15 \, e^{-\frac{t}{4}}$

6- at $t = \tau \rightarrow$
$Q = ??$

$\because$ $Q = 60(1 - e^{-1})$

$= 60(1 - \frac{1}{e})$

$= 60(1 - \frac{1}{2.72})$

$= 0.63 \times 60$

$Q = 37.8 \ \mu c$

7- at $t = 1$ sec $\rightarrow I = ?$

$I = 15 \, e^{-\frac{1}{4}}$

$= 15 \, e^{-0.25}$

$= (15)(0.778)$

$= 11.68 \ \mu A$

EX 28.10 Discharging the last example when the switch is at (b)

(A) After How many time constants $(t=?)$ is the charge on the capacitor equal one-fourth its initial (max) value

$$Q(t) = Q_i \, e^{-\frac{t}{\tau}} \quad \left(Q_i = Q_{max} = C\mathcal{E}\right.$$

when $Q = \frac{Q_i}{4}$

$$\rightarrow \frac{\cancel{Q_i}}{4} = \cancel{Q_i} \, e^{-\frac{t}{\tau}}$$

$$\ln\left(\frac{1}{4}\right) = \ln\left(e^{-\frac{t}{\tau}}\right)$$

$$-\ln 4 = -\frac{t}{\tau}$$

$$t = (\ln 4)\tau$$

$$t = 1.39\,\tau$$

$$= 1.39 \times 4$$

$$t = 5.56 \text{ sec}$$

(B) The energy stored in the capacitor decreases with time as the capacitor discharges. After How many time constants ($t=?$) is this stored energy one-fourth its initial value?

$$U(t) = \frac{Q^2}{2C}$$

$$= \frac{1}{2C}\left(Q_i e^{-\frac{t}{\tau}}\right)^2$$

$$U(t) = \frac{Q_i^2}{2C} e^{-\frac{2t}{\tau}} \qquad U_i = \frac{Q_i^2}{2C}$$

when $U = \frac{1}{4} U_i \rightarrow$

$$\frac{1}{4}\cancel{U_i} = \cancel{U_i} e^{-\frac{2t}{\tau}}$$

$$-\ln 4 = -\frac{2t}{\tau}$$

$$\rightarrow t = \frac{\tau \ln 4}{2}$$

$$t = 0.69\,\tau$$