Chapter 30

Sources of the Magnetic Field

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Magnetic Fields

The origin of the magnetic field is moving charges.

The magnetic field due to various current distributions can be calculated.

Ampère's law is useful in calculating the magnetic field of a highly symmetric configuration carrying a steady current.

Magnetic effects in matter can be explained on the basis of atomic magnetic moments.

Biot-Savart Law – Introduction

Biot and Savart conducted experiments on the force exerted by an electric current on a nearby magnet.

They arrived at a mathematical expression that gives the magnetic field at some point in space due to a current.

The magnetic field described by the Biot-Savart Law is the field *due to* a given current carrying conductor.

- Do not confuse this field with any external field applied to the conductor from some other source.

Biot-Savart Law – Observations

The vector $d\vec{\mathbf{B}}$ is perpendicular to both $d\vec{\mathbf{s}}$ and to the unit vector $\hat{\mathbf{r}}$ directed from $d\vec{\mathbf{s}}$ toward P.

The magnitude of $d\vec{\mathbf{B}}$ is inversely proportional to r^2, where r is the distance from $d\vec{\mathbf{s}}$ to P.

The magnitude of $d\vec{\mathbf{B}}$ is proportional to the current and to the magnitude *ds* of the length element $d\vec{\mathbf{s}}$.

The magnitude of $d\vec{\mathbf{B}}$ is proportional to sin θ, where θ is the angle between the vectors $d\vec{\mathbf{s}}$ and $\hat{\mathbf{r}}$.

Biot-Savart Law – Equation

The observations are summarized in the mathematical equation called the **Biot-Savart law**:

$$d\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{I\,d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

The constant μ_o is called the **permeability of free space.**

$\mu_o = 4\pi \times 10^{-7}$ T· m / A

Total Magnetic Field

$d\vec{\mathbf{B}}$ is the field created by the current in the length segment ds.

To find the total field, sum up the contributions from all the current elements I $d\vec{\mathbf{s}}$

$$\vec{\mathbf{B}} = \frac{\mu_o I}{4\pi} \int \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

- The integral is over the entire current distribution.

The law is also valid for a current consisting of charges flowing through space.

- For example, this could apply to the beam in an accelerator.

Magnetic Field Compared to Electric Field

Distance

- The magnitude of the magnetic field varies as the inverse square of the distance from the source.
- The electric field due to a point charge also varies as the inverse square of the distance from the charge.

Direction

- The electric field created by a point charge is radial in direction.
- The magnetic field created by a current element is perpendicular to both the length element $d\vec{\mathbf{s}}$ and the unit vector. $\hat{\mathbf{r}}$

Magnetic Field Compared to Electric Field, cont.

Source

- An electric field is established by an isolated electric charge.
- The current element that produces a magnetic field must be part of an extended current distribution.
  - Therefore you must integrate over the entire current distribution.

Magnetic Field for a Long, Straight Conductor

Find the field contribution from a small element of current and then integrate over the current distribution.

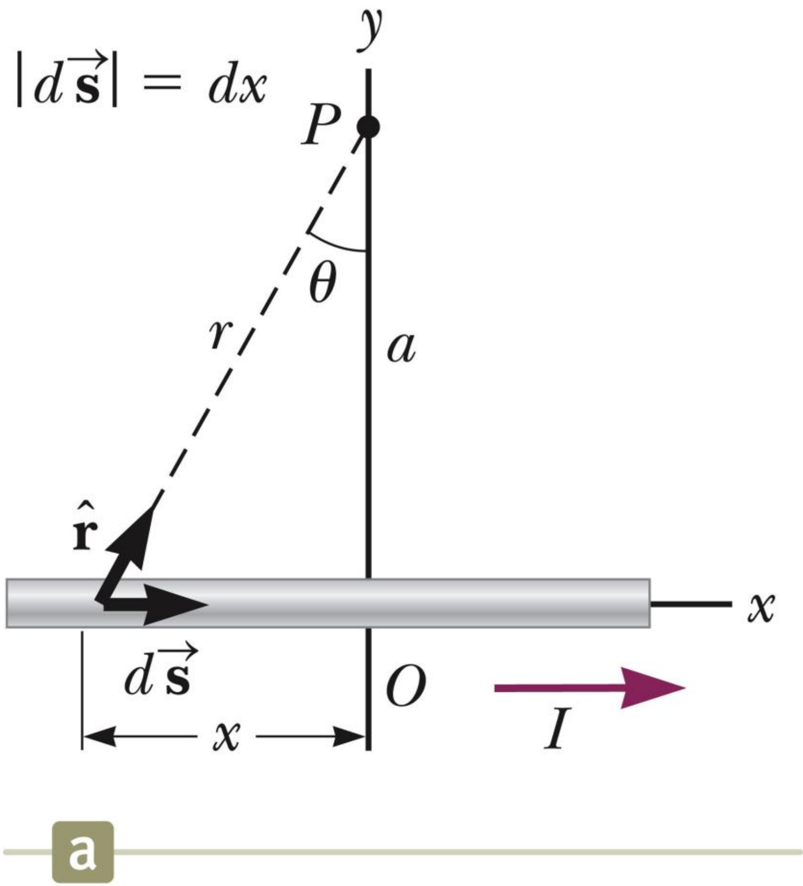
The thin, straight wire is carrying a constant current

$$d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = (dx \sin\theta)\hat{\mathbf{k}}$$

Integrating over all the current elements gives

$$B = -\frac{\mu_o I}{4\pi a}\int_{\theta_1}^{\theta_2} \cos\theta \, d\theta$$

$$= \frac{\mu_o I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

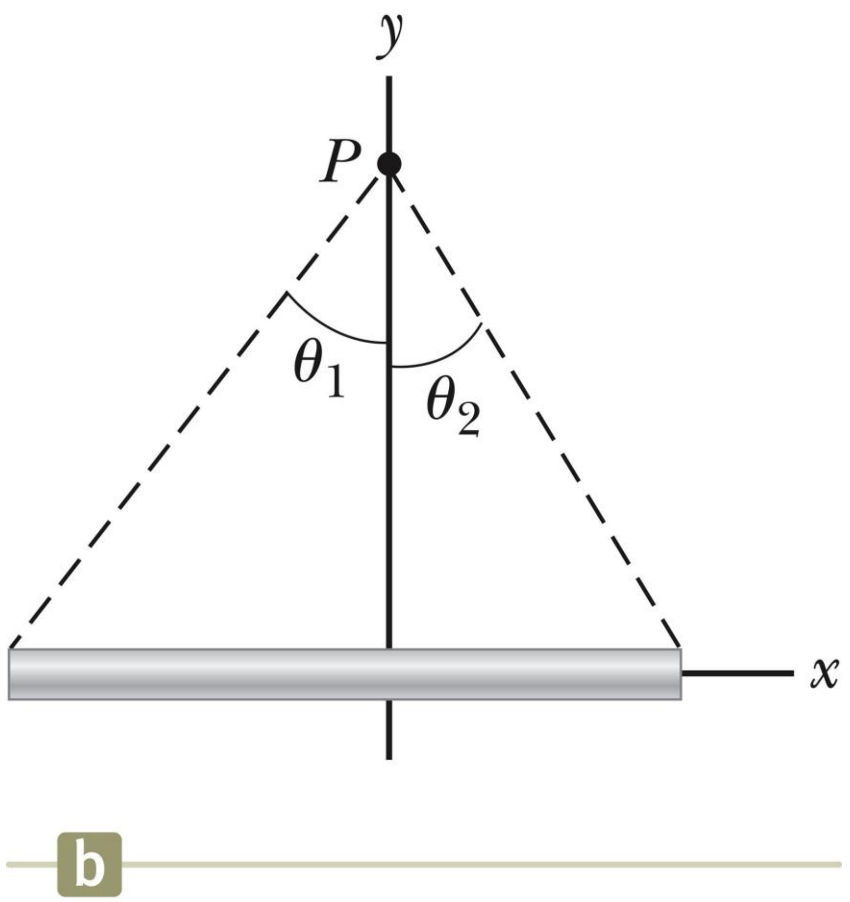


Magnetic Field for a Long, Straight Conductor, Special Case

If the conductor is an infinitely long, straight wire, $\theta_1 = \pi/2$ and $\theta_2 = -\pi/2$

The field becomes

$$B = \frac{\mu_o I}{2\pi a}$$



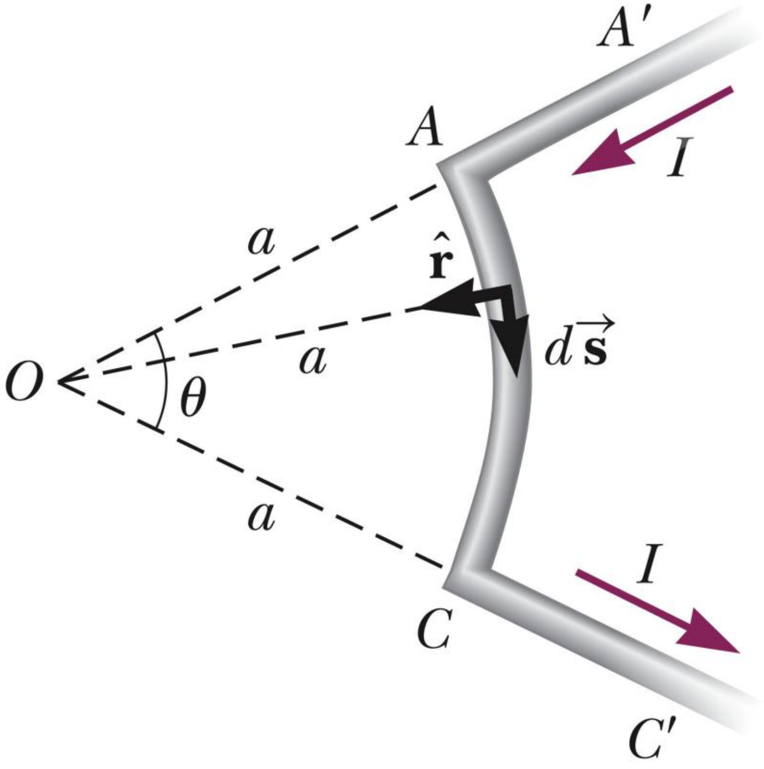
Magnetic Field for a Curved Wire Segment

Find the field at point *O* due to the wire segment.

Integrate, remembering *I* and *R* are constants

$$B = \frac{\mu_o I}{4\pi a}\theta$$

- θ will be in radians



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Magnetic Field for a Circular Loop of Wire

Consider the previous result, with a full circle

- $\theta = 2\pi$

$$B = \frac{\mu_o I}{4\pi a}\theta = \frac{\mu_o I}{4\pi a}2\pi = \frac{\mu_o I}{2a}$$

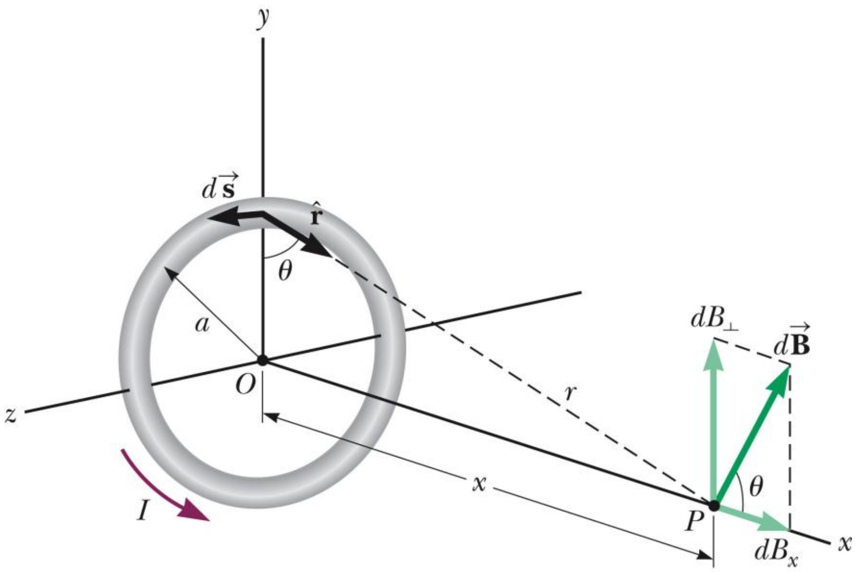
This is the field at the center of the loop.

Magnetic Field for a Circular Current Loop

The loop has a radius of R and carries a steady current of I.

Find the field at point P:

$$B_x = \frac{\mu_o I a^2}{2\left(a^2 + x^2\right)^{3/2}}$$



Comparison of Loops

Consider the field at the center of the current loop.

At this special point, $x = 0$

Then,

$$B_x = \frac{\mu_o I a^2}{2\left(a^2 + x^2\right)^{3/2}} = \frac{\mu_o I}{2a}$$

- This is exactly the same result as from the curved wire.

Magnetic Field Lines for a Loop

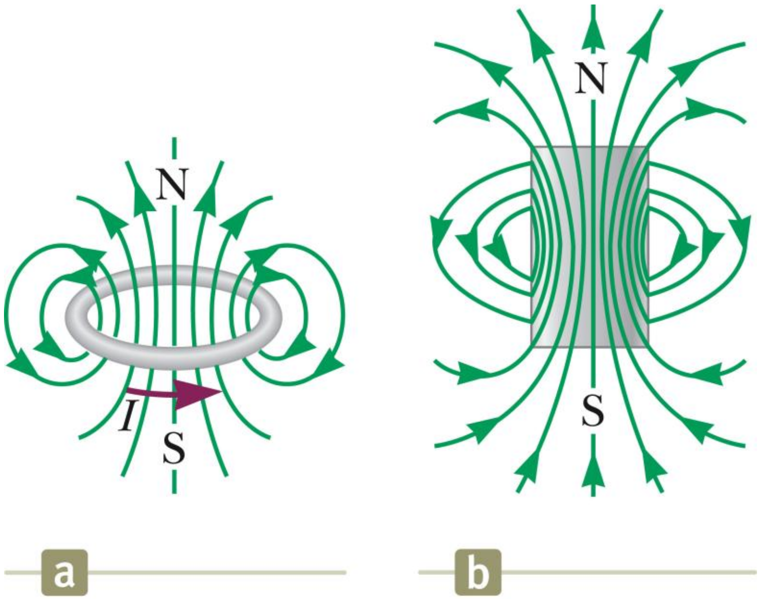


Figure (a) shows the magnetic field lines surrounding a current loop.

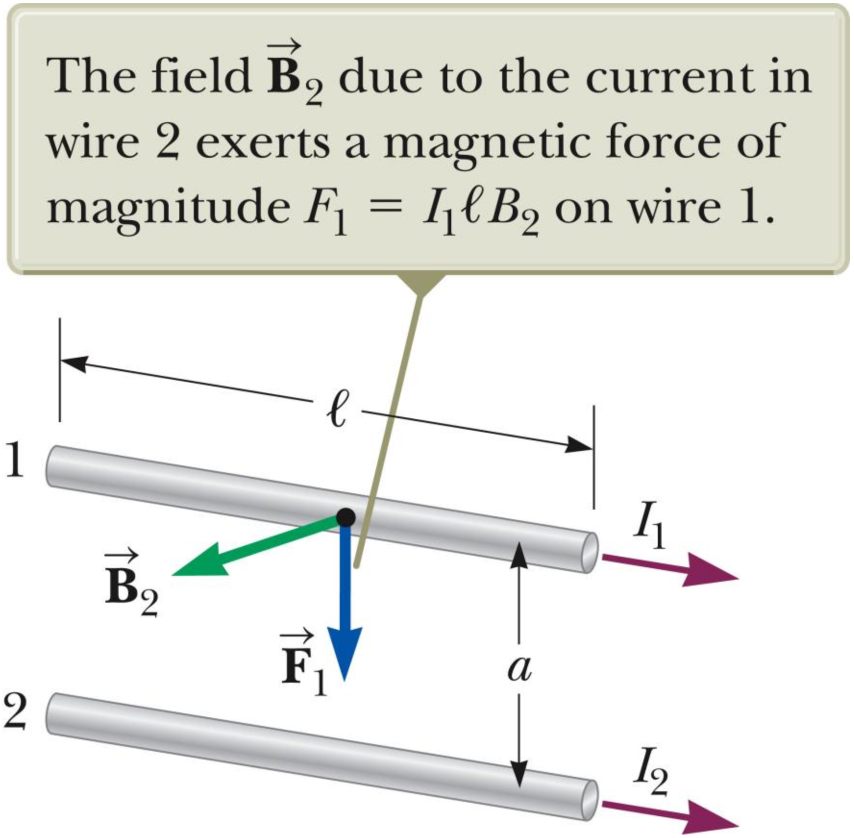
Figure (b) compares the field lines to that of a bar magnet.

Notice the similarities in the patterns.

Magnetic Force Between Two Parallel Conductors

Two parallel wires each carry a steady current.

The field $\vec{\mathbf{B}}_2$ due to the current in wire 2 exerts a force on wire 1 of $F_1 = I_1 \ell B_2$.



Magnetic Force Between Two Parallel Conductors, cont.

Substituting the equation for the magnetic field (B_2) gives

$$F_1 = \frac{\mu_o I_1 I_2}{2\pi a} \ell$$

- Parallel conductors carrying currents in the same direction attract each other.
- Parallel conductors carrying current in opposite directions repel each other.

Magnetic Force Between Two Parallel Conductors, final

The result is often expressed as the magnetic force between the two wires, $F_{B.}$

This can also be given as the force per unit length:

$$\frac{F_B}{\ell} = \frac{\mu_o I_1 I_2}{2\pi a}$$

The derivation assumes both wires are long compared with their separation distance.

- Only one wire needs to be long.
- The equations accurately describe the forces exerted on each other by a long wire and a straight, parallel wire of limited length, ℓ.

Definition of the Ampere

The force between two parallel wires can be used to define the ampere.

When the magnitude of the force per unit length between two long, parallel wires that carry identical currents and are separated by 1 m is 2×10^{-7} N/m, the current in each wire is defined to be 1 A.

Definition of the Coulomb

The SI unit of charge, the coulomb, is defined in terms of the ampere.

When a conductor carries a steady current of 1 A, the quantity of charge that flows through a cross section of the conductor in 1 s is 1 C.

Andre-Marie Ampère

1775 – 1836

French physicist

Credited with the discovery of electromagnetism

- The relationship between electric current and magnetic fields

Also worked in mathematics

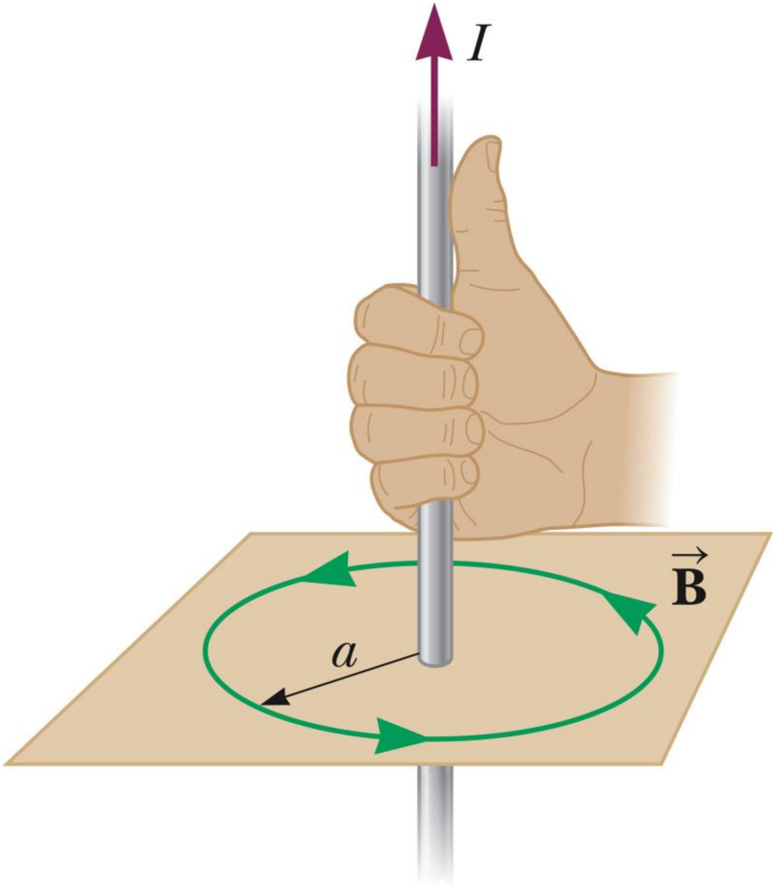
Magnetic Field for a Long, Straight Conductor: Direction

The magnetic field lines are circles concentric with the wire.

The field lines lie in planes perpendicular to the wire.

The magnitude of the field is constant on any circle of radius *a.*

The right-hand rule for determining the direction of the field is shown.



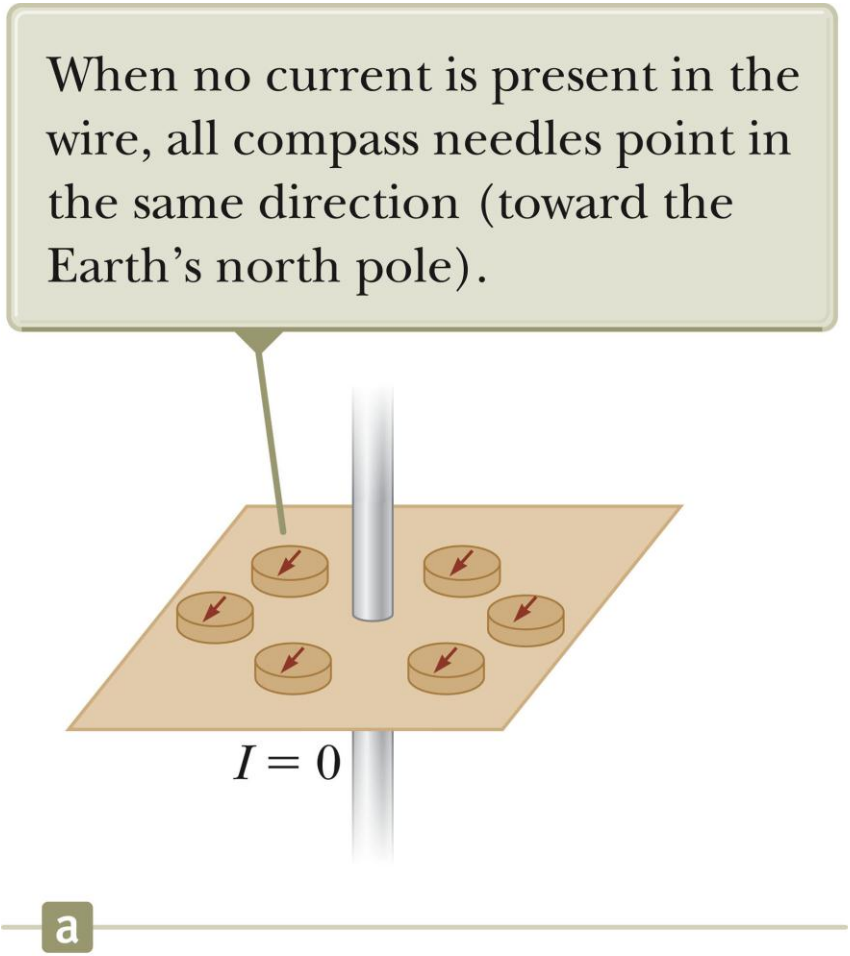
Magnetic Field of a Wire

A compass can be used to detect the magnetic field.

When there is no current in the wire, there is no field due to the current.

The compass needles all point toward the Earth's north pole.

- Due to the Earth's magnetic field



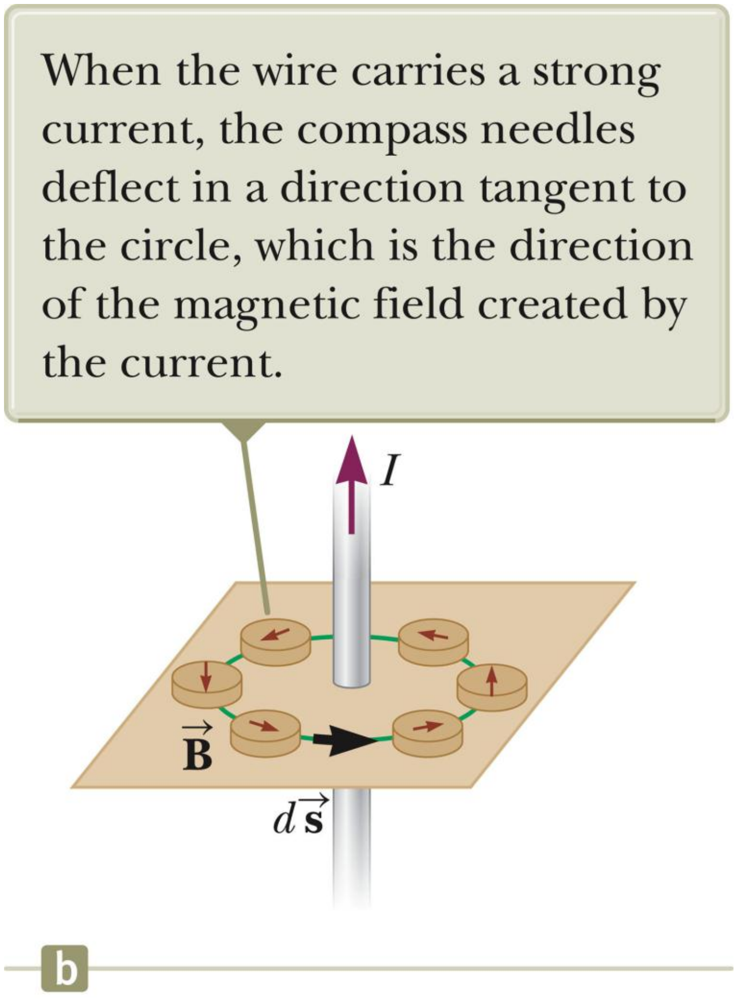
Magnetic Field of a Wire, cont.

Here the wire carries a strong current.

The compass needles deflect in a direction tangent to the circle.

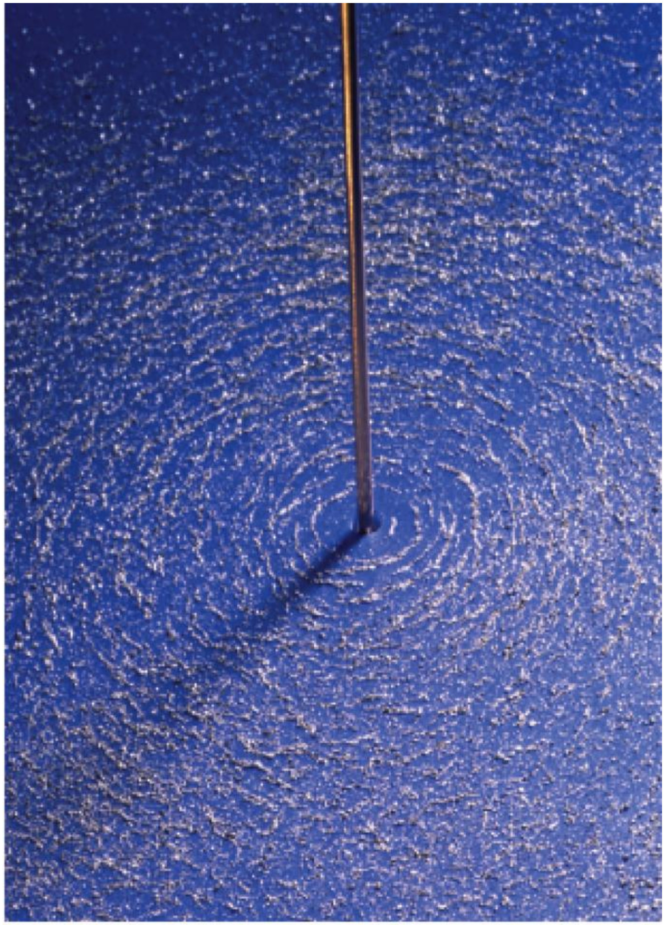
This shows the direction of the magnetic field produced by the wire.

If the current is reversed, the direction of the needles also reverse.



Magnetic Field of a Wire, final

The circular magnetic field around the wire is shown by the iron filings.



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Ampere's Law

The product of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ can be evaluated for small length elements $d\vec{\mathbf{s}}$ on the circular path defined by the compass needles for the long straight wire.

Ampere's law states that the line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around any closed path equals $\mu_o I$ where I is the total steady current passing through any surface bounded by the closed path:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_o I$$

Ampere's law describes the creation of magnetic fields by all continuous current configurations.

- Most useful for this course if the current configuration has a high degree of symmetry.

Put the thumb of your right hand in the direction of the current through the amperian loop and your fingers curl in the direction you should integrate around the loop.

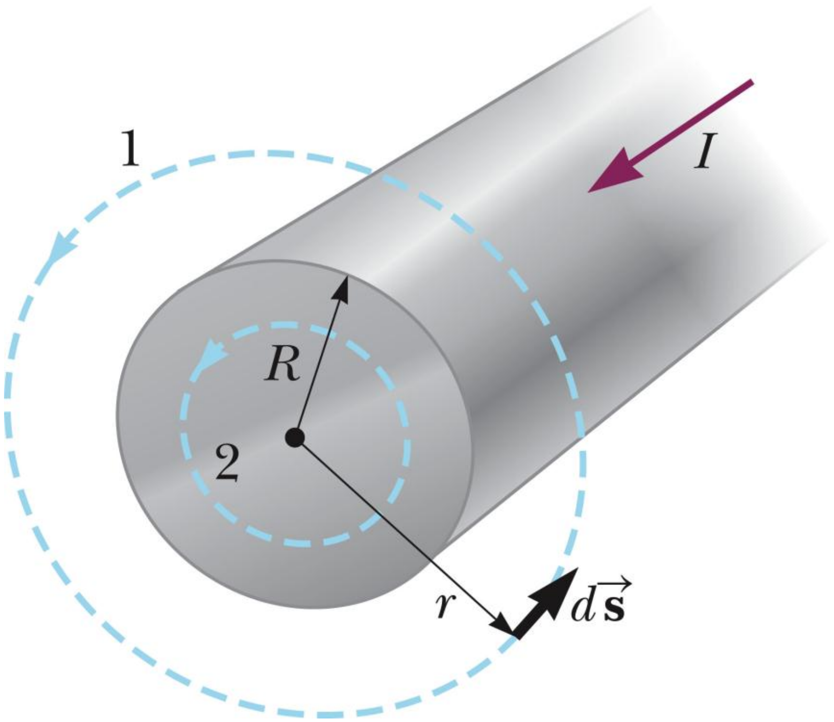
Field Due to a Long Straight Wire – From Ampere's Law

Calculate the magnetic field at a distance *r* from the center of a wire carrying a steady current I.

The current is uniformly distributed through the cross section of the wire.

Since the wire has a high degree of symmetry, the problem can be categorized as a Ampère's Law problem.

- For $r \geq R$, this should be the same result as obtained from the Biot-Savart Law.



Field Due to a Long Straight Wire – Results From Ampere's Law

Outside of the wire, r > R

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_o I \quad \rightarrow \quad B = \frac{\mu_o I}{2\pi r}$$

Inside the wire, we need I', the current inside the amperian circle.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_o I' \quad \rightarrow \quad I' = \frac{r^2}{R^2} I$$

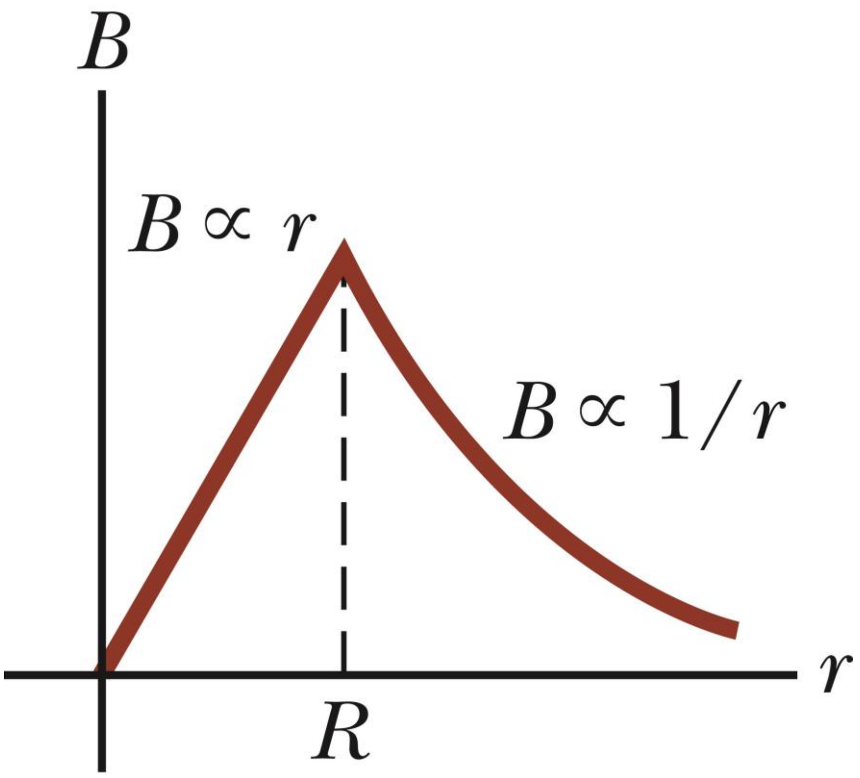
$$B = \left(\frac{\mu_o I}{2\pi R^2} \right) r$$

Field Due to a Long Straight Wire – Results Summary

The field is proportional to r inside the wire.

The field varies as 1/r outside the wire.

Both equations are equal at r = R.



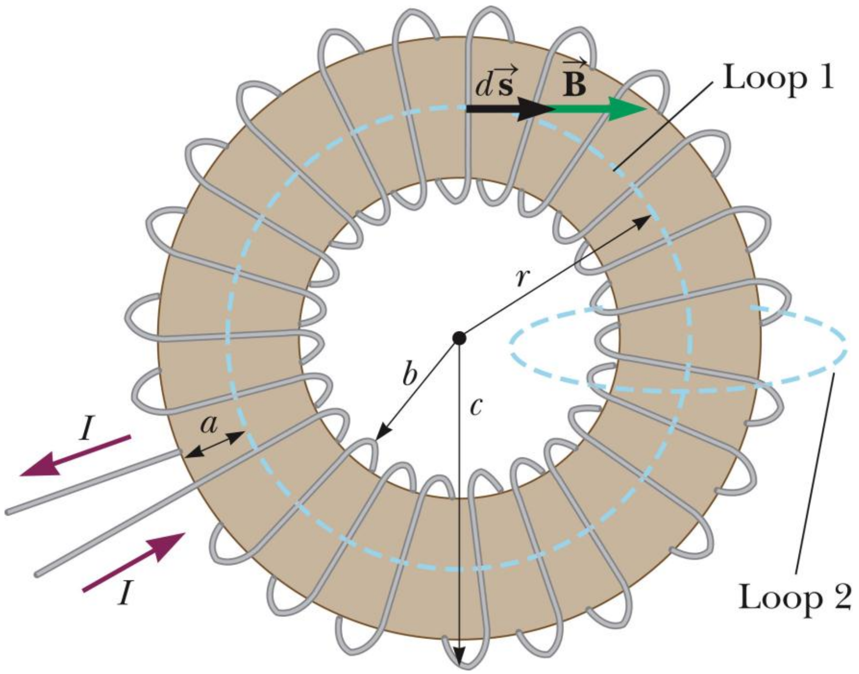
Magnetic Field of a Toroid

Find the field at a point at distance r from the center of the toroid.

The toroid has N turns of wire.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_o N I$$

$$B = \frac{\mu_o N I}{2\pi r}$$

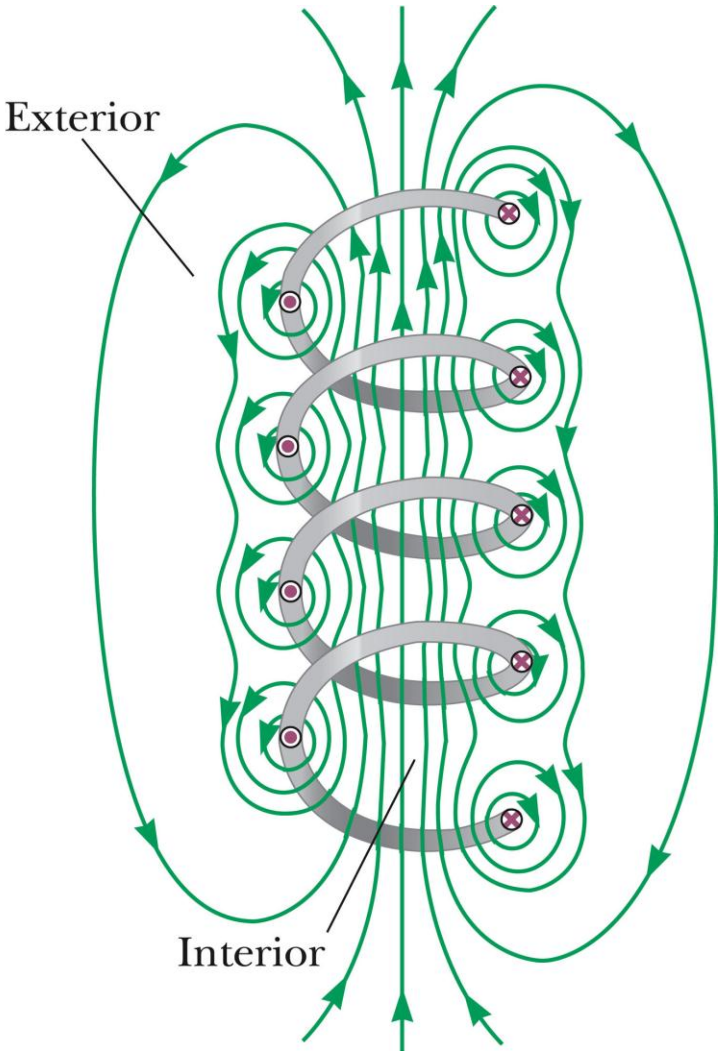


Magnetic Field of a Solenoid

A solenoid is a long wire wound in the form of a helix.

A reasonably uniform magnetic field can be produced in the space surrounded by the turns of the wire.

- The interior of the solenoid



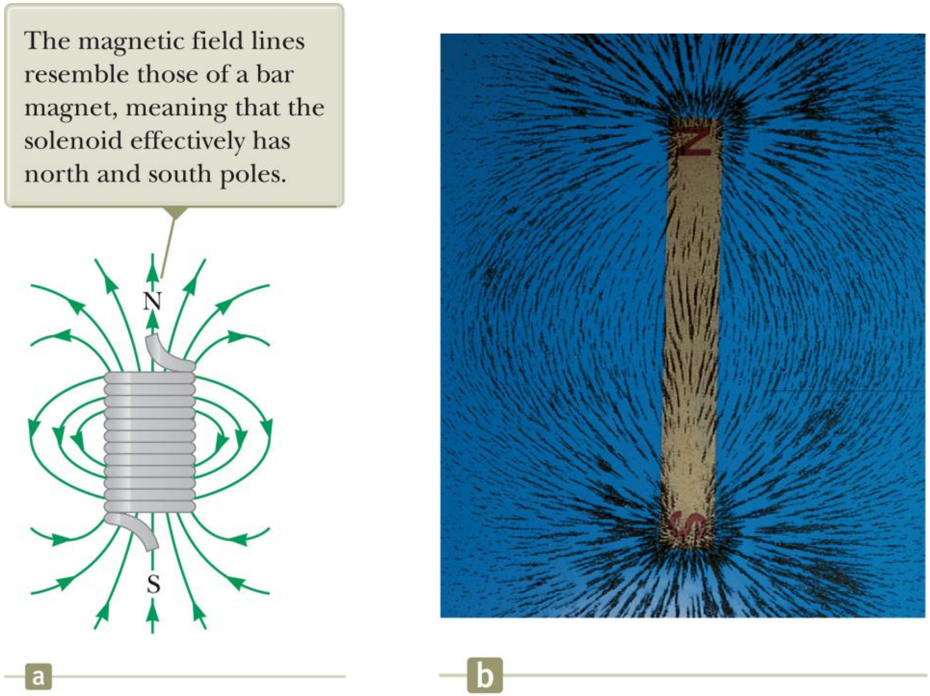
Magnetic Field of a Solenoid, Description

The field lines in the interior are

- Nearly parallel to each other
- Uniformly distributed
- Close together

This indicates the field is strong and almost uniform.

Magnetic Field of a Tightly Wound Solenoid



The field distribution is similar to that of a bar magnet.

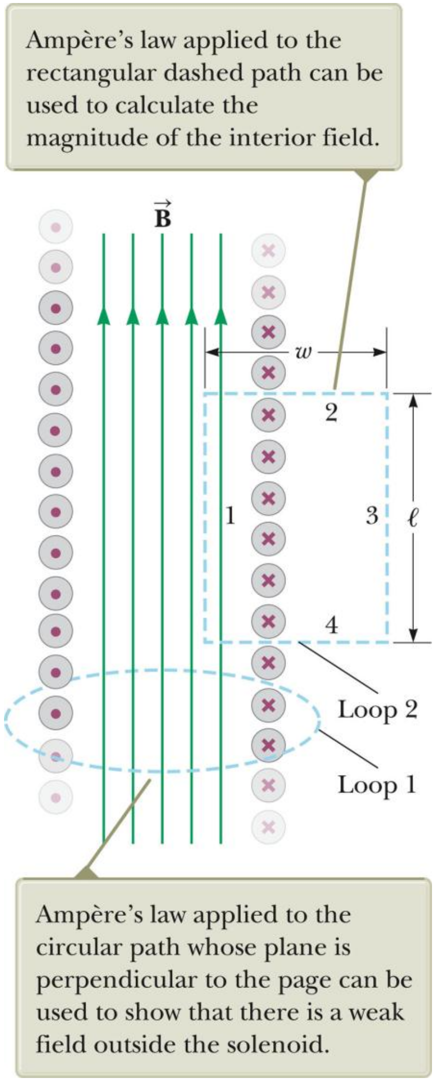
As the length of the solenoid increases,

- The interior field becomes more uniform.
- The exterior field becomes weaker.

Ideal Solenoid – Characteristics

An ideal solenoid is approached when:

- The turns are closely spaced.
- The length is much greater than the radius of the turns.



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Ampere's Law Applied to a Solenoid

Consider an amperian loop (loop 1 in the diagram) surrounding the ideal solenoid.

- The loop encloses a small current.
- There is a weak field external to the solenoid.
- A second layer of turns of wire could be used to eliminate the field.

Ampere's law can also be used to find the interior magnetic field of the solenoid.

- Consider a rectangle with side ℓ parallel to the interior field and side w perpendicular to the field.
  - This is loop 2 in the diagram.
- The side of length ℓ inside the solenoid contributes to the field.
  - This is side 1 in the diagram.
  - Sides 2, 3, and 4 give contributions of zero to the field.

Ampere's Law Applied to a Solenoid, cont.

Applying Ampere's Law gives

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \int_{path\,1} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \int_{path\,1} ds = B\ell$$

The total current through the rectangular path equals the current through each turn multiplied by the number of turns.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell = \mu_o NI$$

Solving Ampere's law for the magnetic field is

$$B = \mu_o \frac{N}{\ell} I = \mu_o n I$$

- $n = N / \ell$ is the number of turns per unit length.

This is valid only at points near the center of a very long solenoid.

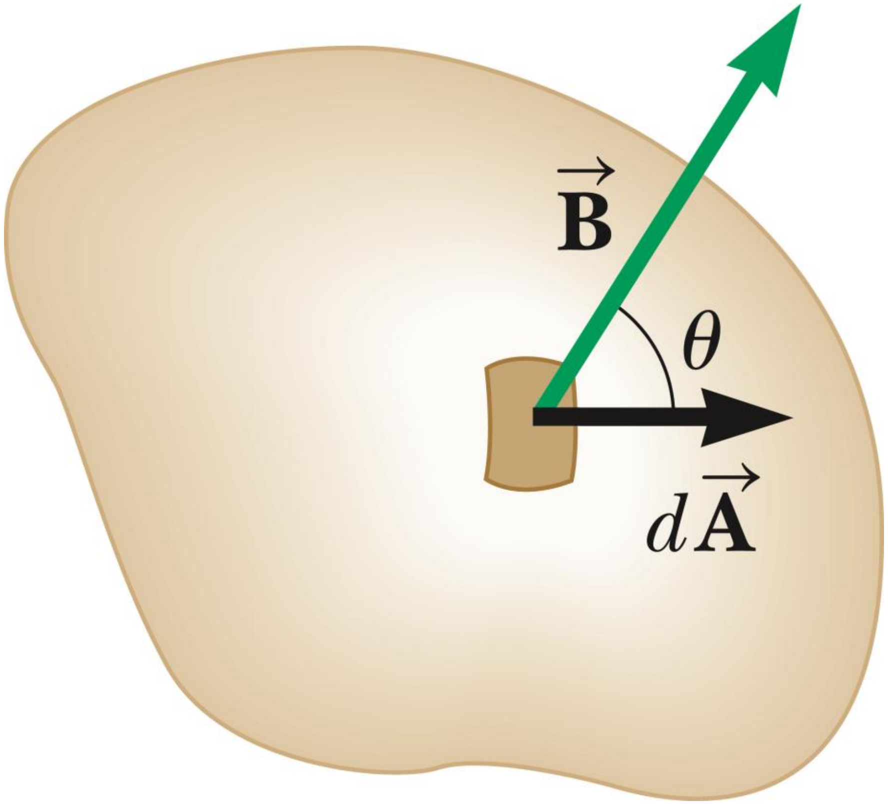
Magnetic Flux

The magnetic flux associated with a magnetic field is defined in a way similar to electric flux.

Consider an area element dA on an arbitrarily shaped surface.

The magnetic field in this element is $\vec{\mathbf{B}}$.

$d\vec{\mathbf{A}}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area *dA.*



Magnetic Flux, cont.

The magnetic flux Φ_B is

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

The unit of magnetic flux is $T \cdot m^2$ = Wb

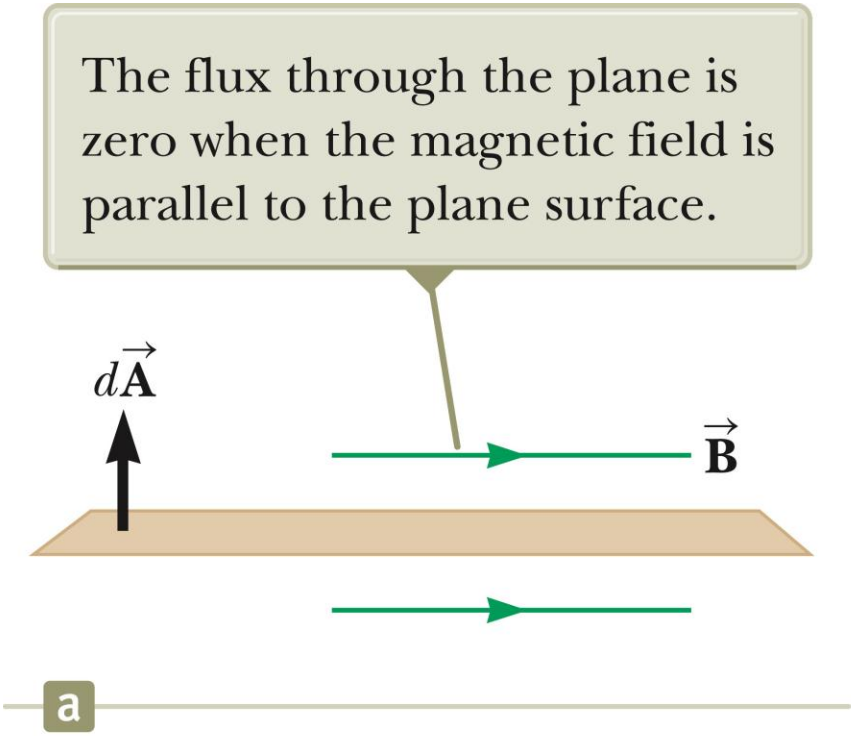
- Wb is a weber

Magnetic Flux Through a Plane, 1

A special case is when a plane of area A makes an angle θ with $d\vec{\mathbf{A}}$.

The magnetic flux is $\Phi_B = BA \cos\theta$.

In this case, the field is parallel to the plane and $\Phi_B = 0$.

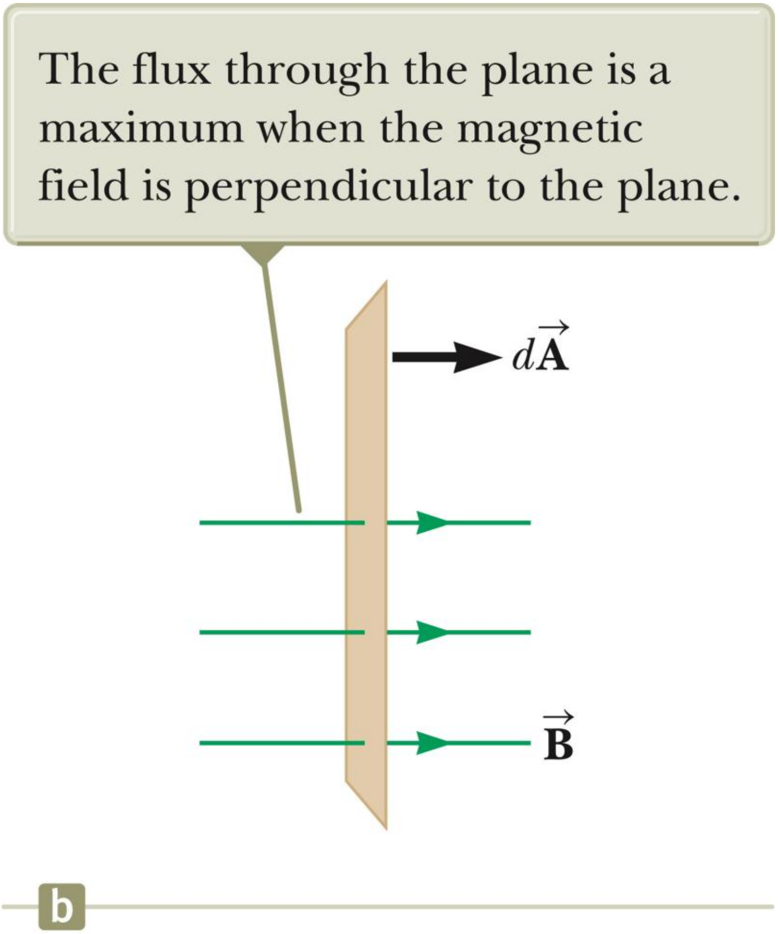


Magnetic Flux Through A Plane, 2

The magnetic flux is $\Phi_B = BA \cos \theta$.

In this case, the field is perpendicular to the plane and $\Phi = BA$.

- This is the maximum value of the flux.



Gauss' Law in Magnetism

Magnetic fields do not begin or end at any point.

- Magnetic field lines are continuous and form closed loops.
- The number of lines entering a surface equals the number of lines leaving the surface.

Gauss' law in magnetism says the magnetic flux through any closed surface is always zero:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

This indicates that isolated magnetic poles (monopoles) have never been detected.

- Perhaps they do not exist
- Certain theories do suggest the possible existence of magnetic monopoles.

Magnetic Moments

In general, any current loop has a magnetic field and thus has a magnetic dipole moment.

This includes atomic-level current loops described in some models of the atom.

This will help explain why some materials exhibit strong magnetic properties.

Magnetic Moments – Classical Atom

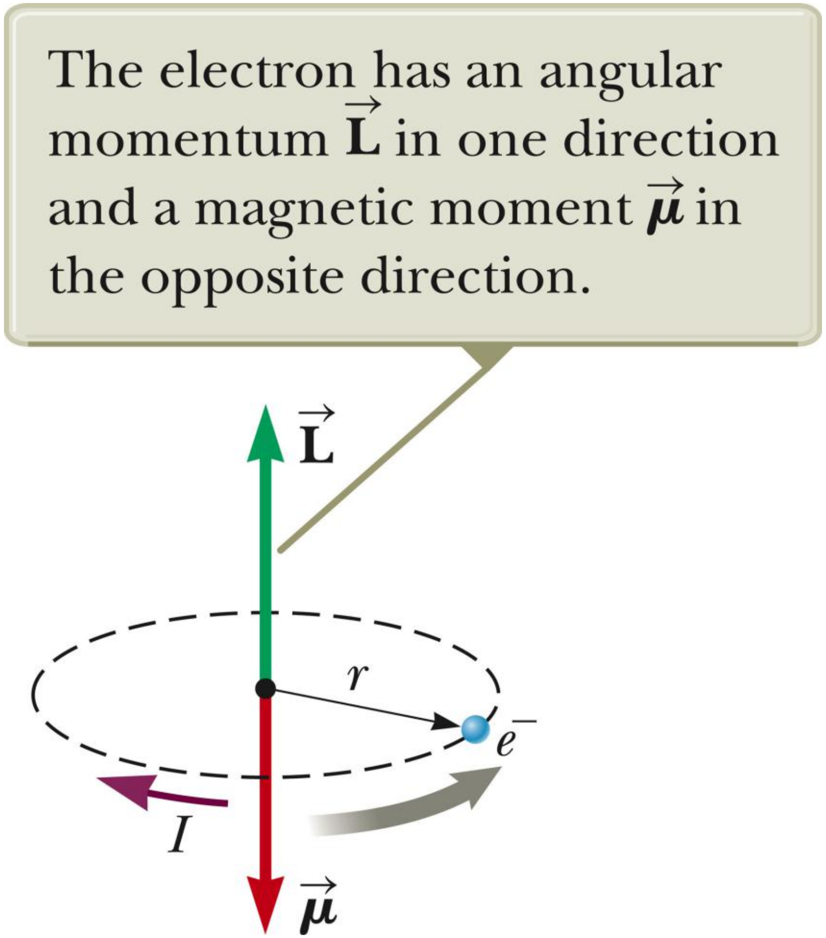
The electrons move in circular orbits.

The orbiting electron constitutes a tiny current loop.

The magnetic moment of the electron is associated with this orbital motion.

$\vec{\mathbf{L}}$ is the angular momentum.

$\vec{\mu}$ is magnetic moment.



Magnetic Moments – Classical Atom, cont.

This model assumes the electron moves:

- with constant speed *v*
- in a circular orbit of radius *r*
- travels a distance $2\pi r$ in a time interval *T*

The current associated with this orbiting electron is

$$I = \frac{e}{T} = \frac{ev}{2\pi r}$$

The magnetic moment is $\mu = I\,A = \frac{1}{2}evr$

The magnetic moment can also be expressed in terms of the angular momentum.

$$\mu = \left(\frac{e}{2m_e}\right)L$$

Magnetic Moments – Classical Atom, final

The magnetic moment of the electron is proportional to its orbital angular momentum.

- The vectors $\vec{\mathbf{L}}$ and $\vec{\mu}$ point in *opposite* directions.
- Because the electron is negatively charged

Quantum physics indicates that angular momentum is quantized.

Magnetic Moments of Multiple Electrons

In most substances, the magnetic moment of one electron is canceled by that of another electron orbiting in the same direction.

The net result is that the magnetic effect produced by the orbital motion of the electrons is either zero or very small.

Electron Spin

Electrons (and other particles) have an intrinsic property called **spin** that also contributes to their magnetic moment.

- The electron is not physically spinning.
- It has an intrinsic angular momentum as if it were spinning.
- Spin angular momentum is actually a relativistic effect

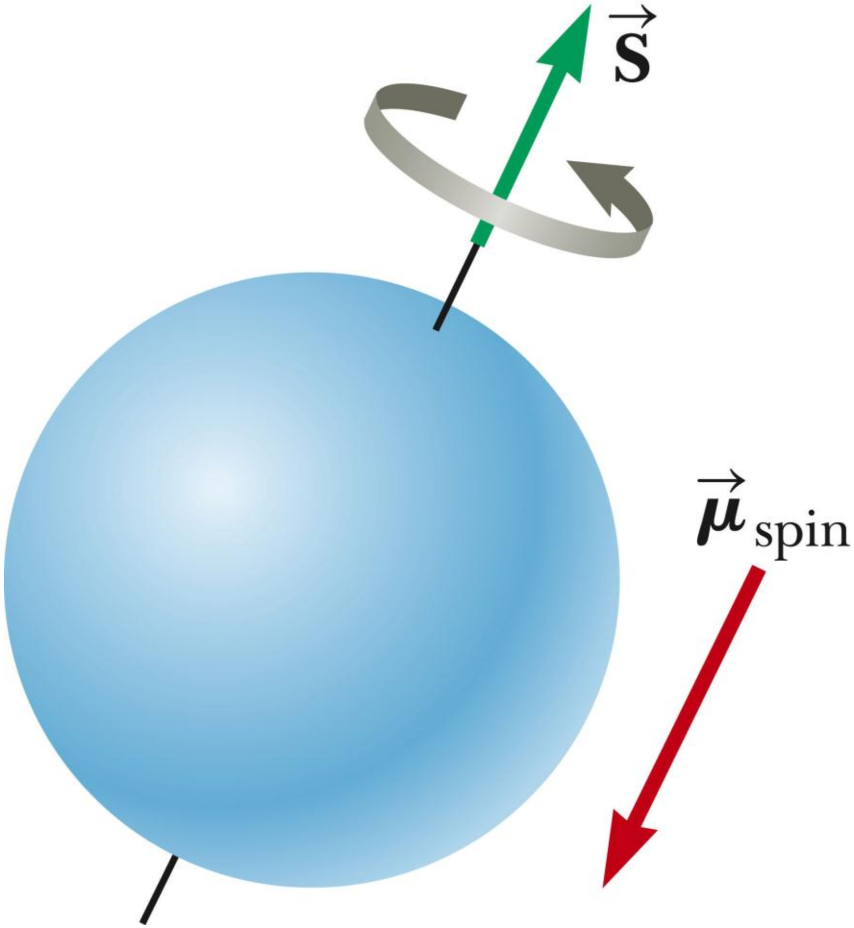
Electron Spin, cont.

The classical model of electron spin is the electron spinning on its axis.

The magnitude of the spin angular momentum is

$$S = \frac{\sqrt{3}}{2}\hbar$$

- $\hbar$ is Planck's constant.



Electron Spin and Magnetic Moment

The magnetic moment characteristically associated with the spin of an electron has the value

$$\mu_{\text{spin}} = \frac{e\hbar}{2m_e}$$

This combination of constants is called the **Bohr magneton** $\mu_{\mathbf{B}}$ = 9.27 x 10^{-24} J/T.

Electron Magnetic Moment, final

The total magnetic moment of an atom is the vector sum of the orbital and spin magnetic moments.

Some examples are given in the table at right.

The magnetic moment of a proton or neutron is much smaller than that of an electron and can usually be neglected.

TABLE 30.1

Magnetic Moments of Some Atoms and Ions

Atom or Ion	Magnetic Moment (10^{-24} J/T)
H	9.27
He	0
Ne	0
Ce^{3+}	19.8
Yb^{3+}	37.1

Ferromagnetism

Some substances exhibit strong magnetic effects called ferromagnetism.

Some examples of ferromagnetic materials are:

- iron
- cobalt
- nickel
- gadolinium
- dysprosium

They contain permanent atomic magnetic moments that tend to align parallel to each other even in a weak external magnetic field.

Domains

All ferromagnetic materials are made up of microscopic regions called **domains.**

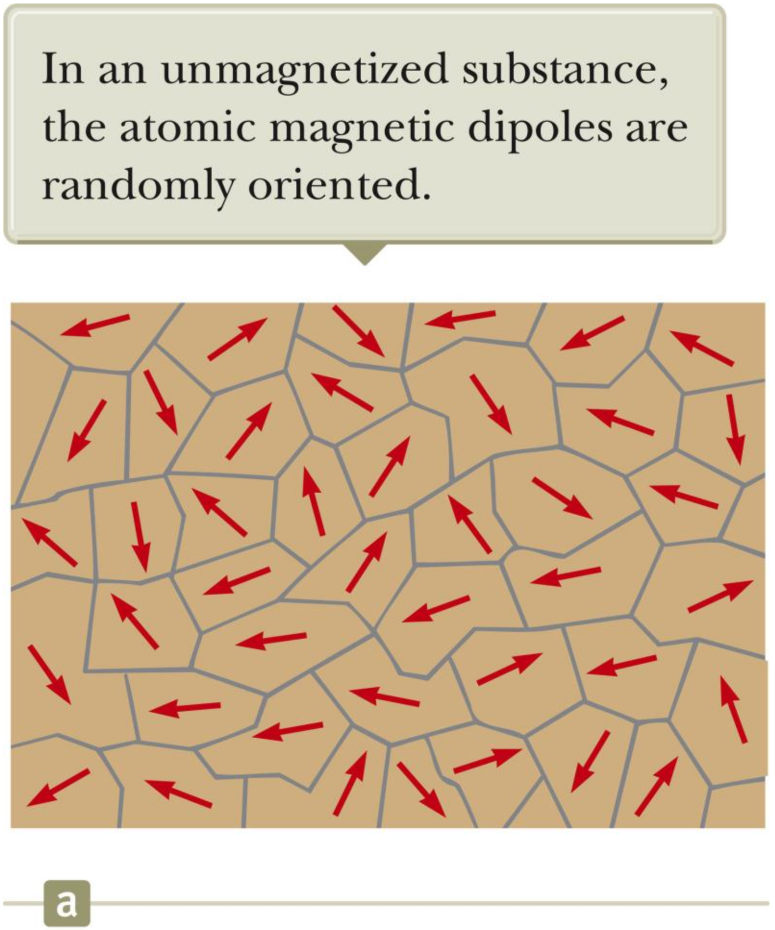
- The domain is an area within which all magnetic moments are aligned.

The boundaries between various domains having different orientations are called **domain walls.**

Domains, Unmagnetized Material

The magnetic moments in the domains are randomly aligned.

The net magnetic moment is zero.

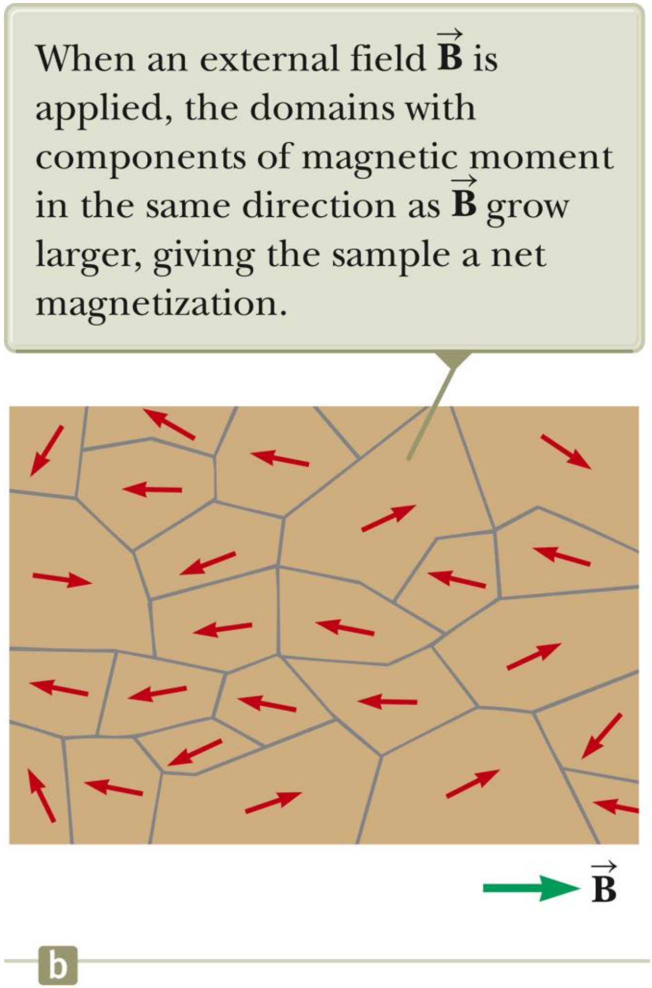


Domains, External Field Applied

A sample is placed in an external magnetic field.

The size of the domains with magnetic moments aligned with the field grows.

The sample is magnetized.

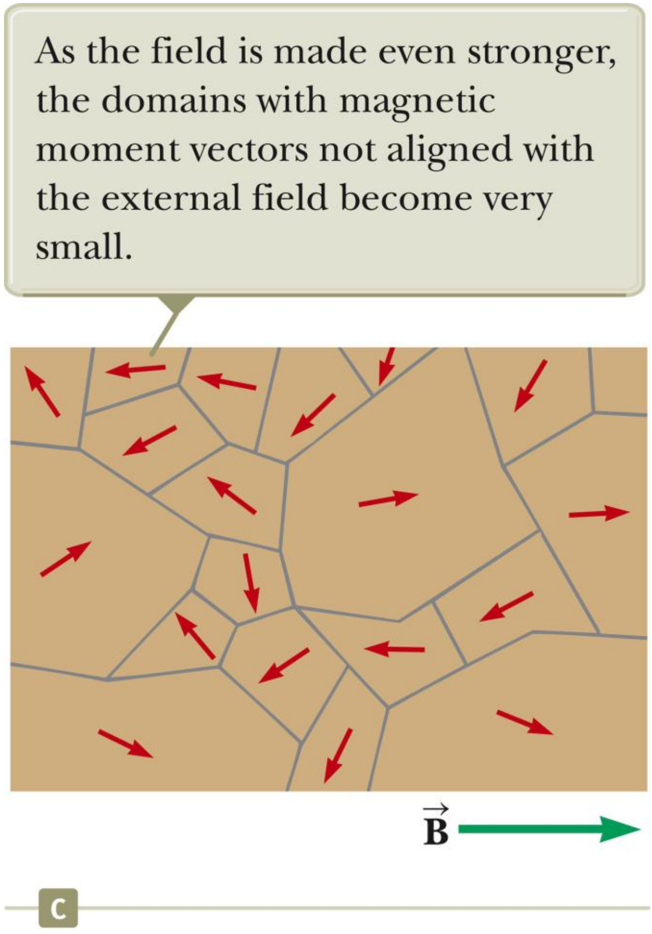


Domains, External Field Applied, cont.

The material is placed in a stronger field.

The domains not aligned with the field become very small.

When the external field is removed, the material may retain a net magnetization in the direction of the original field.



Curie Temperature

The **Curie temperature** is the critical temperature above which a ferromagnetic material loses its residual magnetism.

- The material will become paramagnetic.

Above the Curie temperature, the thermal agitation is great enough to cause a random orientation of the moments.

Table of Some Curie Temperatures

TABLE 30.2 *Curie Temperatures for Several Ferromagnetic Substances*

Substance	T_{Curie} (K)
Iron	1 043
Cobalt	1 394
Nickel	631
Gadolinium	317
Fe_2O_3	893

Paramagnetism

Paramagnetic substances have small but positive magnetism.

It results from the presence of atoms that have permanent magnetic moments.

- These moments interact weakly with each other.

When placed in an external magnetic field, its atomic moments tend to line up with the field.

- The alignment process competes with thermal motion which randomizes the moment orientations.

Diamagnetism

When an external magnetic field is applied to a diamagnetic substance, a weak magnetic moment is induced in the direction opposite the applied field.

Diamagnetic substances are weakly repelled by a magnet.

- Weak, so only present when ferromagnetism or paramagnetism do not exist

Meissner Effect

Certain types of superconductors also exhibit perfect diamagnetism in the superconducting state.

- This is called the **Meissner effect.**

If a permanent magnet is brought near a superconductor, the two objects repel each other.

In the Meissner effect, the small magnet at the top induces currents in the superconducting disk below, which is cooled to 321°F (77 K). The currents create a repulsive magnetic force on the magnet causing it to levitate above the superconducting disk.