Uncertainty management in Rule-based expert systems

Certainty factors theory and evidential reasoning

dr. Mohamed Khalil

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- Introduction, or what is uncertainty?
- Certainty factors theory and evidential reasoning
- Basic probability theory
- Bayesian reasoning
- Bias of the Bayesian method

Introduction, or what is uncertainty?

- Information can be incomplete, inconsistent, uncertain, or all three. In other words, information is often unsuitable for solving a problem.

- **Uncertainty** is defined as the **lack of the exact knowledge that would enable us to reach a perfectly reliable conclusion**.

Introduction, or what is uncertainty?

- Classical logic permits only exact reasoning. It assumes that perfect knowledge always exists and the **law of the excluded middle** can always be applied:

IF *A* **is true**
THEN *A* **is not false**

IF *A* **is false**
THEN *A* **is not true**

Certainty factors theory and evidential reasoning

A certainty factor (*cf*), a number to measure the expert's belief.

The maximum value of the certainty factor is, say, **+1.0 (definitely true)** and the **minimum −1.0 (definitely false).**

For example, if the expert states that some evidence is **almost certainly** true, a *cf* value of **0.8** would be assigned to this evidence.

Uncertain terms and their interpretation in MYCIN

Term	*Certainty Factor*
Definitely not	−1.0
Almost certainly not	−0.8
Probably not	−0.6
Maybe not	−0.4
Unknown	−0.2 to +0.2
Maybe	+0.4
Probably	+0.6
Almost certainly	+0.8
Definitely	+1.0

In expert systems with certainty factors, the knowledge base consists of a set of rules that have the **following syntax:**

IF **<evidence>** *condition*
THEN **<hypothesis>** **{*cf*}** *conclusion*

where *cf* represents belief in hypothesis *H* given that evidence *E* has occurred.

The net certainty factor for single-antecedent rule

$$cf\,(H,E) = cf\,(E) \times cf$$

For example:

IF **sky is clear**
THEN **the forecast is sunny** {*cf* 0.8}

and the current certainty factor of **sky is clear is 0.5**, then **$cf\,(H,E) = 0.5 \times 0.8 = 0.4$**

the net certainty factor for multiple-antecedent rule conjunction (AND)

the certainty of hypothesis *H*, is established as follows:

$$cf\,(H, E_1 \cap E_2 \cap \ldots \cap E_n) = \boldsymbol{min}\ [cf\,(E_1), cf\,(E_2), \ldots, cf\,(E_n)]\ \mathbf{x}\ cf$$

For example:

IF	sky	is clear
AND	the forecast	is sunny
THEN	the action	is 'wear sunglasses' **{*cf* 0.8}**

and the certainty of **sky is clear is 0.9** and the certainty of the forecast of **sunny is 0.7**, then

$$cf\,(H, E_1 \cap E_2) = \boldsymbol{min}\ [0.9, 0.7] \times \mathbf{0.8} = 0.7 \times 0.8 = 0.56$$

the net certainty factor for multiple-antecedent rule disjunctive (OR)

The certainty of hypothesis H, is established as follows:

cf (H, E1 or E2 or... or En) = max [cf (E1), cf (E2),..., cf (En)] x cf

<u>For example:</u>

IF	sky	is overcast
OR	the forecast	is rain
THEN	the action	is 'take an umbrella' {cf 0.9}

and the certainty of sky is overcast is 0.6 and the certainty of the forecast of rain is 0.8, then

$$cf\ (H, E1 \cup E2) = \mathbf{max}\ [0.6, 0.8] \times 0.9 = 0.8 \times 0.9 = 0.72$$

If(smart(W) or can_read(W) and wealthy(W)) then happy(w) (0,5)

OR

AND

smart(W) =0.2.

can_read(W)=0.6.

wealthy(W)=0.3.

Min(max(0.2,0.6),0.3)*0.5=0.15.

When the same consequent is obtained as a result of two or more rules

Suppose the knowledge base consists of the following rules:

Rule 1:	IF	A is X
	THEN	**C is Z** {cf 0.8}
Rule 2:	IF	B is Y
	THEN	**C is Z** {cf 0.6}

When the same consequent is obtained as a result of two or more rules

Common sense suggests that, if we have two pieces of evidence (A is X and B is Y) from different sources (Rule 1 and Rule 2) supporting the same hypothesis (C is Z), then the confidence in this hypothesis should increase and become stronger than if only one piece of evidence had been obtained.

Certainty Factors & Evidential Reasoning

To calculate a combined certainty factor we can use the following equation:

$$
(cf_1,\ cf_2) = \begin{cases} cf_1 + cf_2 \times (1 - cf_1) & \text{if } cf_1 > 0 \text{ and } cf_2 > 0 \\[2ex] \dfrac{cf_1 + cf_2}{1 - min\,[\,|cf_1|,\ |cf_2|\,]} & \text{if } cf_1 < 0 \text{ or } cf_2 < 0 \\[2ex] cf_1 + cf_2 \times (1 + cf_1) & \text{if } cf_1 < 0 \text{ and } cf_2 < 0 \end{cases}
$$

where:

cf_1 is the confidence in hypothesis H established by Rule 1;

cf_2 is the confidence in hypothesis H established by Rule 2;

$|cf_1|$ and $|cf_2|$ are absolute magnitudes of cf_1 and cf_2, respectively.

FORECAST: an application of certainty factors

The expert system is required to predict whether it will rain tomorrow, *or in other words* to establish certainty factors for the multivalued object tomorrow.

Rule: 1
if today is rain
then tomorrow is rain {cf 0.5}

Rule: 2
if today is dry
then tomorrow is dry {cf 0.5}

Rule: 3
if today is rain
and rainfall is low
then tomorrow is dry {cf 0.6}

Rule: 4
if today is rain
and rainfall is low
and temperature is cold
then tomorrow is dry {cf 0.7}

Rule: 5
if today is dry
and temperature is warm
then tomorrow is rain {cf 0.65}

Rule: 6
if today is dry
and temperature is warm
and sky is overcast
then tomorrow is rain {cf 0.55}

What is the weather today?
⇒ **rain**

Rule: 1
if today is rain
then tomorrow is rain {cf 0.5}

cf(tomorrow is rain, today is rain) = cf(today is rain) $\times$ cf $= 1.0 \times 0.5 = 0.5$

tomorrow is rain {0.50}

What is the rainfall today?
⇒ **low**

To what degree do you believe the rainfall is low? Enter a numeric certainty between 0 and 1.0 inclusive.
⇒ **0.8**

Rule: 3
if today is rain
and rainfall is low
then tomorrow is dry {cf 0.6}

cf(tomorrow is dry, today is rain $\cap$ rainfall is low)
$= min\,[cf(\text{today is rain}), cf(\text{rainfall is low})] \times cf = min\,[1, 0.8] \times 0.6 = 0.48$

tomorrow is rain	{0.50}
dry	{0.48}

What is the temperature today?
⇒ **cold**

To what degree do you believe the temperature is cold? Enter a numeric certainty between 0 and 1.0 inclusive.
⇒ **0.9**

Rule: 4
if today is rain
and rainfall is low
and temperature is cold
then tomorrow is dry {cf 0.7}

$$cf(\text{tomorrow is dry, today is rain} \cap \text{rainfall is low} \cap \text{temperature is cold})$$
$$= min\,[cf(\text{today is rain}), cf(\text{rainfall is low}), cf(\text{temperature is cold})] \times cf$$
$$= min\,[1, 0.8, 0.9] \times 0.7 = 0.56$$

tomorrow is dry {0.56}
rain {0.50}

$$cf(cf_{\text{Rule:3}}, cf_{\text{Rule:4}}) = cf_{\text{Rule:3}} + cf_{\text{Rule:4}} \times (1 - cf_{\text{Rule:3}})$$
$$= 0.48 + 0.56 \times (1 - 0.48) = 0.77$$

tomorrow is dry {0.77}
rain {0.50}

Now we would conclude that the probability of having a dry day tomorrow is almost certain; however we also may expect some rain!

Basic probability theory
Bayesian reasoning

what is probability?

-Probability provides an exact, mathematically correct, approach to uncertainty management in expert systems.

What is the Bayesian rule?

-The Bayesian rule permits to determine probability of a hypothesis given evidence has been observed.

What requirements must be satisfied before Bayesian reasoning will be effective?

1) the **prior probability** of hypothesis

2) **likelihood of sufficiency ,LS**, to measure belief in the hypothesis if evidence is present

3) **likelihood of necessity, LN**, to measure disbelief in hypothesis if the same evidence is missing.

What is a prior probability?

The probability that an event will reflect established beliefs about the event before the arrival of new evidence or information

What is a posterior probability?

The statistical probability that a hypothesis is true calculated in the light of relevant observations.

Bayesian reasoning

statistical approach to uncertainty management in expert systems

Suppose all rules in the knowledge base are represented in the following form:

IF	*E* is true
THEN	*H* is true {**with probability *p***}

This rule implies that if event *E* occurs, then the probability that event *H* will occur is ***p***.

In expert systems, ***H*** usually represents a hypothesis and ***E*** denotes evidence to support this hypothesis.

prior and posterior probability

$$p(H|E) = \frac{p(E|H) \times p(H)}{p(E|H) \times p(H) + p(E|\neg H) \times p(\neg H)}$$

where:

***p*(*H*)** is the prior probability of hypothesis H being true;

***p*(*E*|*H*)** is the probability that hypothesis H being true will result in evidence E;

***p*(¬*H*)** is the prior probability of hypothesis H being false;

***p*(*E*|¬*H*)** is the probability of finding evidence E even when hypothesis H is false.

The Posterior Probability of H

The probability of a hypothesis (H) given an observation (O). This represents your updated degree of belief.

The Likelihood of H

The probability of an observation given a hypothesis. In other words, the probability that the hypothesis confers upon the observation.

The Prior Probability of H

The probability of a hypothesis (H) before the observation. (Not necessarily an *a priori* concept, can sometimes be based on previous empirical evidence)

$$Pr\,(H \mid O) = \frac{Pr\,(O \mid H)\;Pr\,(H)}{Pr\,(O)}$$

The Unconditional Probability of O

The probability of the observation irrespective of any particular hypothesis. Can occasionally be restated as:

$$Pr\,(O|H)\;Pr\,(H) + Pr\,(O|{\sim}H)\;Pr\,({\sim}H)$$

In other words, the unconditional probability of O is represented by the likelihood of H times the prior probability of H, plus the likelihood of not H times the prior probability of not H. This restatement is often useful in the case of a test with a known rate of false positives and false negatives.

•The Bayesian method uses rules of the following form:

IF E is true {LS, LN}
THEN H is true {prior probability}

- **likelihood of sufficiency LS** - represents measure of the expert belief in hypothesis if evidence is present.

- **likelihood of necessity LN**- a measure of discredit to hypothesis if evidence is missing.

The domain expert must provide these values.

Rule 1:

IF today is rain {LS 2.5 LN .6}
THEN tomorrow is rain {prior .5}

Rule 2:

IF today is dry {LS 1.6 LN .4}
THEN tomorrow is dry {prior .5}

High values of LS (LS>>1) indicates that rule strong supports the hypothesis (H) if the evidence is observed.

Low values of LN (0 < LN < 1) suggest that the rule also strong opposes the hypothesis (~H) if the evidence is missing.

prior odds is calculated from the prior probability of the consequent

$$O(H) = \frac{p(H)}{1 - p(H)}$$

The posterior odds is update by LS(if E is true) or LN(if E is false

$$O(H \mid E) = \begin{cases} LS \times O(H), & \text{E is true} \\ LN \times O(H), & \text{E is false} \end{cases}$$

The posterior probability is recovered from the posterior odds

$$p(H \mid E) = \frac{O(H \mid E)}{1 + O(H \mid E)}$$

$$p(H \mid \sim E) = \frac{O(H \mid \sim E)}{1 + O(H \mid \sim E)}$$

Odds--A certain number of points given beforehand to a weaker side in a contest to equalize the chances of all participants.

Knowledge Base

Rule: 1
if today is rain {LS 2.5 LN .6}
then tomorrow is rain {prior .5}

Rule: 2
if today is dry {LS 1.6 LN .4}
then tomorrow is dry {prior .5}

Rule: 3
if today is rain
and rainfall is low {LS 10 LN 1}
then tomorrow is dry {prior .5}

Rule: 4
if today is rain
and rainfall is low
and temperature is cold {LS 1.5 LN 1}
then tomorrow is dry {prior .5}

Rule: 5
if today is dry
and temperature is warm {LS 2 LN .9}
then tomorrow is rain {prior .5}

Rule: 6
if today is dry
and temperature is warm
and sky is overcast {LS 5 LN 1}
then tomorrow is rain {prior .5}

What is the weather today?

$\Rightarrow$ **rain**

Rule: 1

if   today is rain {LS 2.5 LN .6}

then   tomorrow is rain {prior .5}

$$O(\textit{tomorrow is rain}) = \frac{0.5}{1 - 0.5} = 1.0$$

$$O(\textit{tomorrow is rain} \mid \textit{today is rain}) = 2.5 \times 1.0 = 2.5$$

$$p(\textit{tomorrow is rain} \mid \textit{today is rain}) = \frac{2.5}{1 + 2.5} = 0.71$$

tomorrow is rain   {0.71}

Rule: 2
if $\quad$ today is dry {LS 1.6 LN .4}
then $\quad$ tomorrow is dry {prior .5}

$$O(tomorrow\ is\ dry) = \frac{0.5}{1 - 0.5} = 1.0$$

$$O(tomorrow\ is\ dry \mid today\ is\ rain) = 0.4 \times 1.0 = 0.4$$

$$p(tomorrow\ is\ dry \mid today\ is\ rain) = \frac{0.4}{1 + 0.4} = 0.29$$

tomorrow is rain	{0.71}
dry	{0.29}

What is the rainfall today?
⇒ **low**

Rule: 3
if today is rain
and rainfall is low {LS 10 LN 1}
then tomorrow is dry {prior .5}

$$O(tomorrow\ is\ dry) = \frac{0.29}{1 - 0.29} = 0.41$$

$$O(tomorrow\ is\ dry \mid today\ is\ rain \cap rainfall\ is\ low) = 10 \times 0.41 = 4.1$$

$$p(tomorrow\ is\ dry \mid today\ is\ rain \cap rainfall\ is\ low) = \frac{4.1}{1 + 4.1} = 0.80$$

tomorrow is dry {0.80}
rain {0.71}

What is the temperature today?
⇒ **cold**

Rule: 4
if today is rain
and rainfall is low
and temperature is cold {LS 1.5 LN 1}
then tomorrow is dry {prior .5}

$$O(\textit{tomorrow is dry}) = \frac{0.80}{1 - 0.80} = 4$$

$$O(\textit{tomorrow is dry} \mid \textit{today is rain} \cap \textit{rainfall is low} \cap \textit{temperature is cold})$$
$$= 1.50 \times 4 = 6$$

$$p(\textit{tomorrow is dry} \mid \textit{today is rain} \cap \textit{rainfall is low} \cap \textit{temperature is cold})$$
$$= \frac{6}{1 + 6} = 0.86$$

tomorrow is dry {0.86}
rain {0.71}

Rule: 5

if today is dry

and temperature is warm {LS 2 LN .9}

then tomorrow is rain {prior .5}

$$O(tomorrow\ is\ rain) = \frac{0.71}{1 - 0.71} = 2.45$$

$$O(tomorrow\ is\ rain \mid today\ is\ not\ dry \cap temperature\ is\ not\ warm) = 0.9 \times 2.45 = 2.21$$

$$p(tomorrow\ is\ rain \mid today\ is\ not\ dry \cap temperature\ is\ not\ warm) = \frac{2.21}{1 + 2.21} = 0.69$$

tomorrow is dry {0.86}

rain {0.69}

What is the cloud cover today?
⇒ **overcast**

Rule: 6
if today is dry
and temperature is warm
and sky is overcast {LS 5 LN 1}
then tomorrow is rain {prior .5}

$$O(\textit{tomorrow is rain}) = \frac{0.69}{1 - 0.69} = 2.23$$

$$O(\textit{tomorrow is rain} \mid \textit{today is not dry} \cap \textit{temperature is not warm} \cap \textit{sky is overcast})$$
$$= 1.0 \times 2.23 = 2.23$$

$$p(\textit{tomorrow is rain} \mid \textit{today is not dry} \cap \textit{temperature is not warm} \cap \textit{sky is overcast})$$
$$= \frac{2.23}{1 + 2.23} = 0.69$$

tomorrow is dry {0.86}
rain {0.69}

this means we have two potentially true hypothesis *tomorrow is dry* and *tomorrow is rain*, but the likelihood of first one is higher