Chapter 5 - Part 3

AC – Bridges

Comparison Bridges

Inductance

Measurements

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2. Inductance Bridges

The circuit of the inductance comparison bridge is similar to that of the capacitance bridge except that inductors are involved instead of capacitors.

There are two pure resistive arms. So, the phase balance depends on the remaining two inductive arms.

5.6.2 INDUCTANCE COMPARISON BRIDGE

This bridge circuit is used for medium inductances and can be arranged to yield results of *considerable precision*.

In the given bridge, the unknown inductance represented by it's equivalent series inductance L_S and resistance R_S.

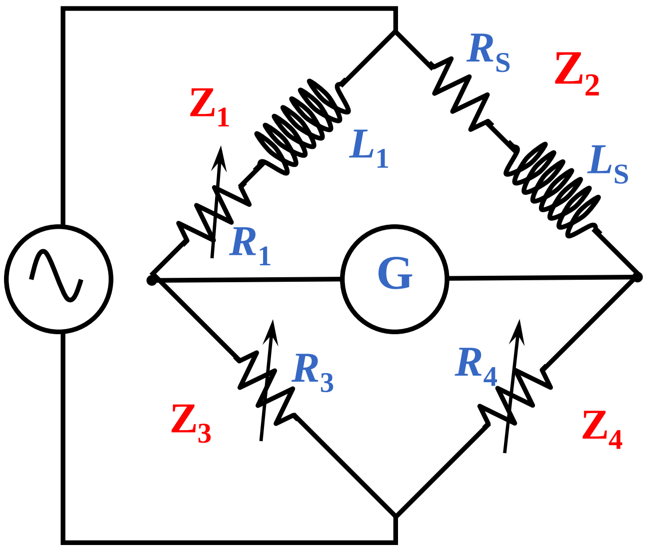
L_1 is the standard inductance and R_1 is a variable resistor to balance R_S.

R_3 or R_4 used to balance the bridge.

The load impedances are:

$$Z_1 = R_1 + jwL_1 \quad ; \quad Z_3 = R_3$$

$$Z_2 = R_S + jwL_S \quad ; \quad Z_4 = R_4$$



When the bridge is balanced:

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3$$

$$(R_1 + jwL_1).R_4 = (R_s + jwL_s).R_3$$

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Then , the equation is simplified as:

$$R_1 \cdot R_4 + jwL_1 \cdot R_4 = R_s.R_3 + jwL_s.R_3$$

After equating the real terms in both sides, we get:

$$R_1 \cdot R_4 = R_s.R_3 \quad \Rightarrow \quad R_s = \frac{R_1 R_4}{R_3}$$

After equating the Imaginary terms in both sides:

$$jwL_1 \cdot R_4 = jwL_s.R_3 \quad \Rightarrow \quad L_s = \frac{L_1 R_4}{R_3}$$

(ii) Maxwell-Wein Bridge

The positive phase angle of an inductive impedance may be compensated by the negative phase angle of a capacitive impedance put in the opposite arm. The unknown inductance then becomes known in terms of this capacitance.

This bridge is found to be most suitable for measuring coils with a low Q-factor (1 < Q < 10) (ωL_S is not much larger than R_S).

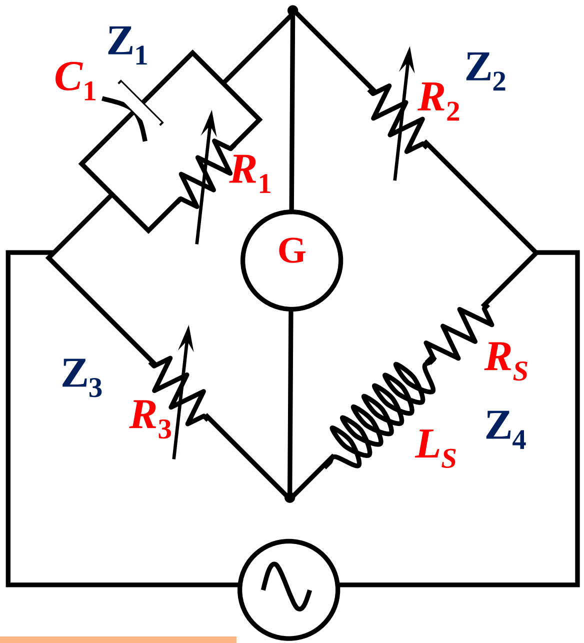
The load impedances are:

$$\frac{1}{Z_1} = \frac{1}{R_1} + jwC_1 \Rightarrow Z_1 = \frac{R_1}{1 + jwR_1C_1} \quad ; \quad Z_2 = R_2$$

$$Z_3 = R_3 \quad ; \quad Z_4 = R_S + jwL_S$$

When the bridge is balanced: $Z_1 Z_4 = Z_2 Z_3$

$$\left(\frac{R_1}{1 + jwR_1C_1}\right)(R_S + jwL_S) = R_2R_3$$



After equating the real & Imaginary terms:

$$R_S = \frac{R_2R_3}{R_1} \quad \text{and} \quad L_S = C_1R_2R_3$$

EXAMPLE 10

A Maxwell-Wien bridge as shown in below operates at a supply frequency of 100Hz used to measure inductive impedance. The bridge
balanced at the following values:

$$C_1 = 0.01\mu\text{F},\ R_1 = 470\Omega\ ,\ \ R_2 = 2.2k\Omega\ \text{ and } R_3 = 100\Omega$$

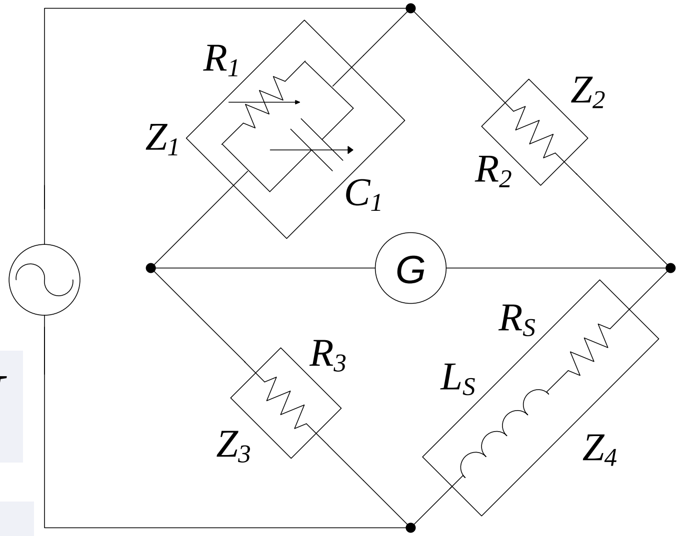
Find the series resistance and inductance and determine its *Q-factor*.

SOLUTION 10

$$R_S = \frac{R_2 R_3}{R_1} = \frac{2.2\text{k}\times 100}{470} = 468.1\Omega$$

$$L_S = C_1 R_2 R_3 = 0.01\mu \times 2.2k \times 100 = 2.2mH$$

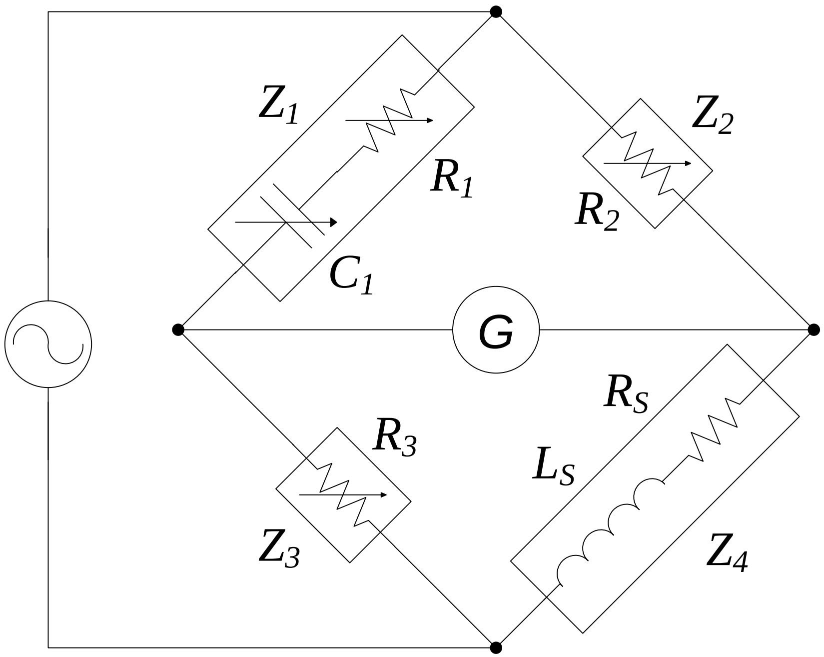
$$Q = \frac{wL_s}{R_s} = \frac{2\pi \times 100 \times 2.2 \times 10^{-3}}{468.1} = 0.00295$$



5.8 HAY BRIDGE (ALSO KNOWN AS OPPOSITE-ANGLE BRIDGE)

This bridge is a modification of the Maxwell-Wien bridge.

It is useful for measuring the resistance and inductance of coils with high Q-factor. (R_1 very low value)



The load impedances are:

$$Z_1 = R_1 - \frac{j}{wC_1} \quad ; \quad Z_2 = R_2$$

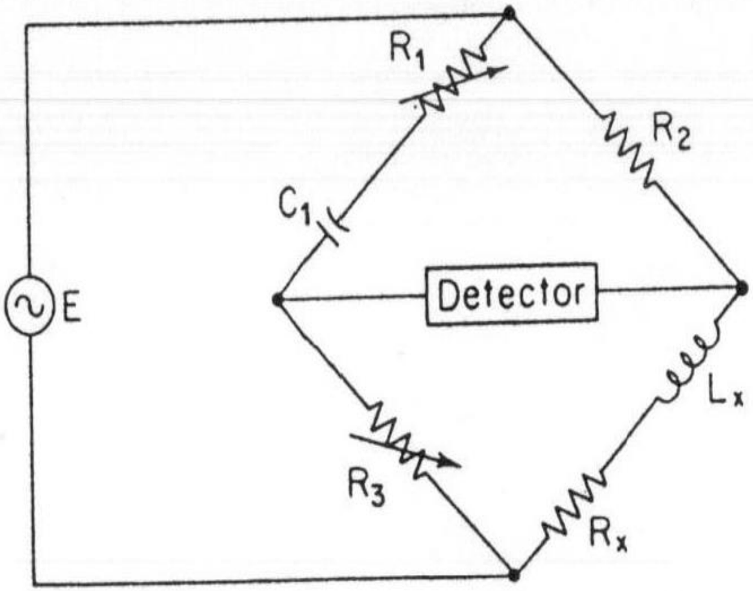
$$Z_3 = R_3 \quad ; \quad Z_4 = R_S + jwL_S$$

When the bridge is balanced:

$$Z_1 Z_4 = Z_2 Z_3 \Rightarrow \left(R_1 - \frac{j}{wC_1} \right)\left(R_S + jwL_S \right) = R_2 R_3$$

After equating the real & Imaginary terms:

$$R_S = \frac{w^2 C_1^2 R_1 R_2 R_3}{1 + w^2 R_1^2 C_1^2} \quad \text{and} \quad L_S = \frac{C_1 R_2 R_3}{1 + w^2 R_1^2 C_1^2}$$



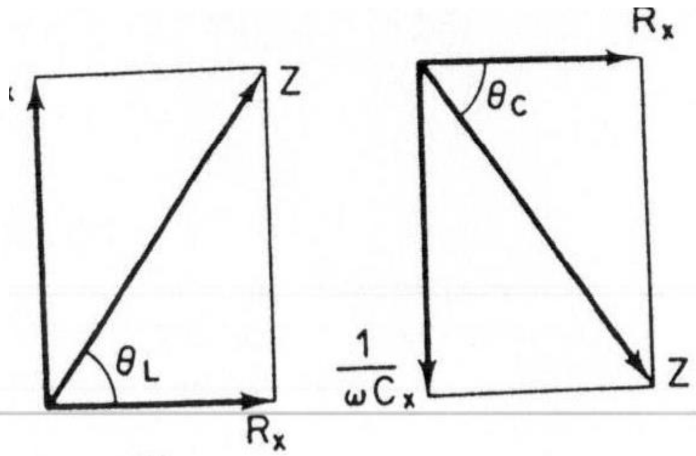


Figure 5-17 Impedance triangles illustrate inductive and capacitive phase angles.

$$R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1 + \omega^2 C_1^2 R_1^2} \qquad (5\text{-}42)$$

$$L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 C_1^2 R_1^2} \qquad (5\text{-}43)$$

phase angle equals

$$\tan\theta_L = \frac{X_L}{R} = \frac{\omega L_x}{R_x} = Q \quad (5\text{-}44)$$

and that of the capacitive phase angle is

$$\tan\theta_C = \frac{X_C}{R} = \frac{1}{\omega C_1 R_1} \quad (5\text{-}45)$$

When the two phase angles are equal, their tangents are also equal and we can write

$$\tan\theta_L = \tan\theta_C \quad \text{or} \quad Q = \frac{1}{\omega C_1 R_1} \quad (5\text{-}46)$$

$$L_x = \frac{R_2 R_3 C_1}{1 + (1/Q)^2}$$

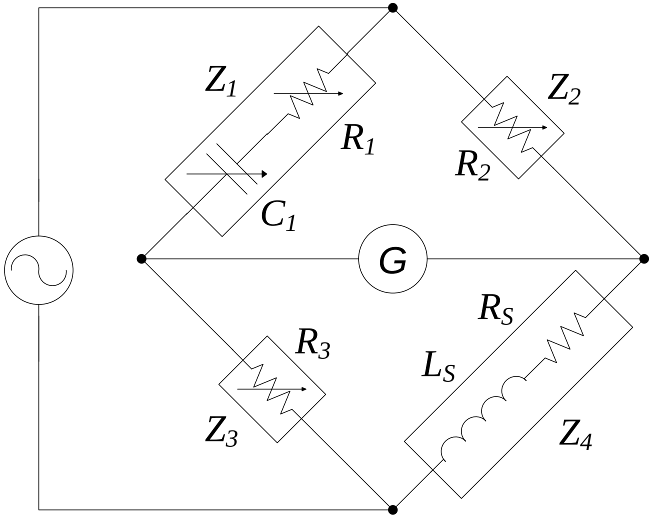
For a value of Q greater than ten, the term $(1/Q)^2$ will be smaller than $\frac{1}{100}$ and

can be neglected. Equation (5-43) therefore reduces to the expression derived for the Maxwell bridge,

$$L_x = R_2 R_3 C_1$$

EXAMPLE 11

Calculate the inductance and resistance of the network that causes a **Hay bridge** as shown in figure below to null with the following component values: **w=3000rad/s, C_1=0.1nF, R_1=20kΩ, R_2=10kΩ** and **R_3=1kΩ.**



SOLUTION 11

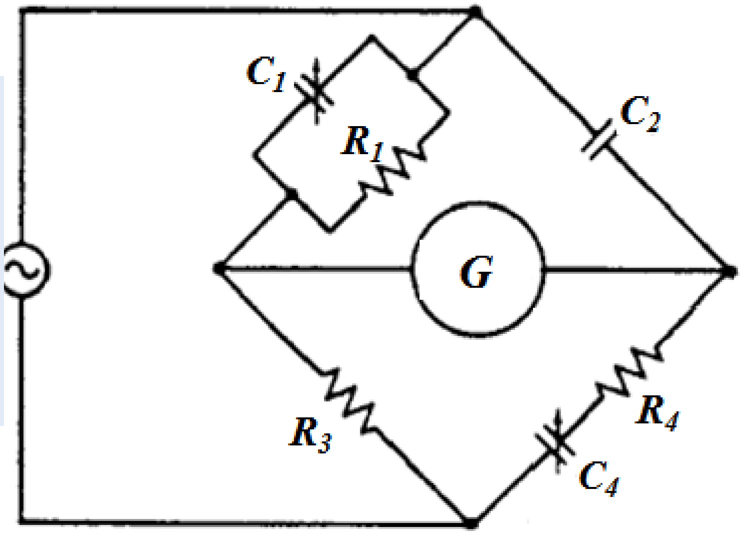
To find the series resistance and inductance, we use the above equations as:

$$L_S = \frac{C_1 R_2 R_3}{1 + w^2 R_1^2 C_1^2} = \frac{0.1n \times 10k \times 1k}{1 + (3000)^2 (20k)^2 (0.1n)^2} = 1mH$$

$$R_S = \frac{w^2 C_1^2 R_1 R_2 R_3}{1 + w^2 R_1^2 C_1^2} = \frac{(3000)^2 (0.1n)^2 \times 20k \times 10k \times 1k}{1 + (3000)^2 (20k)^2 (0.1n)^2} = 0.018\Omega$$

5.9 SCHERING BRIDGE

- C_1 and R_1 are made adjustable while C_4 and R_4 are the unknown impedance, then we have a **Schering** bridge.
- Schering bridge is used for measuring **unknown capacitance and dissipation factor.**



The load impedances are:

$$\frac{1}{Z_1} = \frac{1}{R_1} + jwC_1 \Rightarrow Z_1 = \frac{R_1}{1 + jwR_1C_1} \;;$$

$$Z_2 = -\frac{j}{wC_2} \;; \qquad Z_3 = R_3 \;; \qquad Z_4 = R_4 - \frac{j}{wC_4}$$

When the bridge is balanced:

$$Z_1Z_4 = Z_2Z_3 \Rightarrow \left(\frac{R_1}{1 + jwR_1C_1}\right)\left(R_4 - \frac{j}{wC_4}\right) = -\frac{jR_3}{wC_2}$$

After equating the real & Imaginary terms:

$$R_4 = \frac{R_3C_1}{C_2} \qquad \text{and} \qquad C_4 = \frac{R_1C_2}{R_3}$$

5.9 Schering bridge

The *power factor* (PF) of a series RC combination is defined as the cosine of the phase angle of the circuit. Therefore the PF of the unknown equals PF $= R_x/Z_x$. For phase angles very close to 90°, the reactance is almost equal to the impedance and we can approximate the power factor to

$$X_X = \frac{1}{wC_X}$$

$$\text{PF} \simeq \frac{R_x}{X_x} = \omega C_x R_x \tag{5-51}$$

The *dissipation factor* of a series RC circuit is defined as the cotangent of the phase angle and therefore, by definition, the dissipation factor

$$D = \frac{R_x}{X_x} = \omega C_x R_x \tag{5-52}$$

Subs. Rx and Cx

$$R_X = \frac{R_3 C_1}{C_2} \quad \text{and} \quad C_X = \frac{R_1 C_2}{R_3}$$

Giving ➔ $D = \omega R_1 C_1$ (5-53)

5.11 WIEN BRIDGE

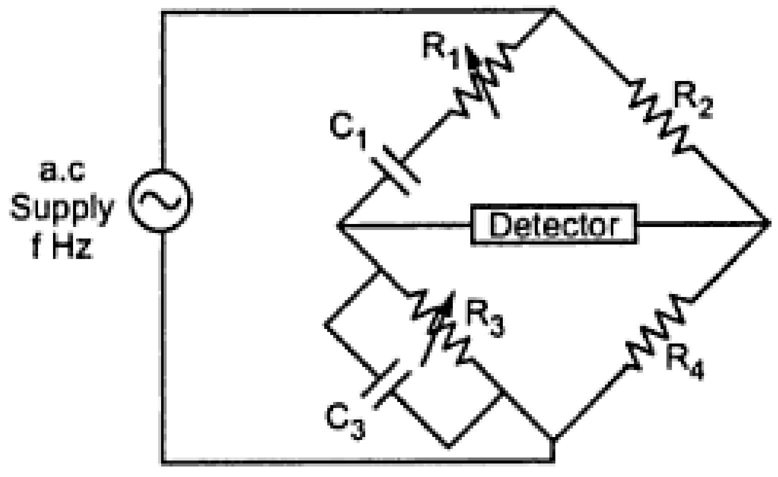


Fig. 2.31 Wien bridge

Basically the bridge is used for the frequency measurement but it is also used for the measurement of the unknown capacitor with great accuracy.

Its one ratio arm consists of a series RC circuit i.e. R_1 and C_1. The second ratio arm consists of a resistance R_2. The third arm consists of the parallel combination of resistance and capacitor i.e. R_3 and C_3. The circuit of the Wien bridge is shown in the Fig. 2.31.

From the Fig. 2.31 we can write,

$$Z_1 = R_1 - j\left(\frac{1}{\omega C_1}\right)$$

$$Z_2 = R_2$$

$$Z_3 = R_3 \,||\, C_3$$

$$\therefore \quad Y_3 = \frac{1}{R_3} + j\omega C_3$$

and

$$Z_4 = R_4$$

The balance condition is,

$$\overline{Z_1 Z_4} = \overline{Z_2 Z_3}$$

$$\therefore \quad \overline{Z_2} = \frac{\overline{Z_1 Z_4}}{\overline{Z_3}} = Z_1 \overline{Z_4 Y_3}$$

$$\therefore \quad R_2 = \left[R_1 - j\left(\frac{1}{\omega C_1}\right)\right] R_4 \left[\frac{1}{R_3} + j\omega C_3\right]$$

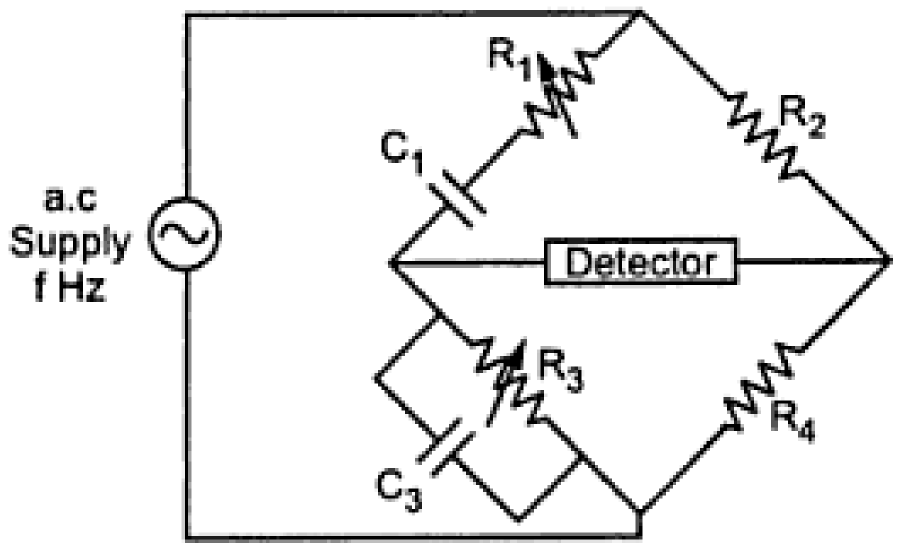
$$\therefore \quad R_2 = R_4 \left[\frac{R_1}{R_3} + j\omega R_1 C_3 - j\frac{1}{\omega C_1 R_3} + \frac{C_3}{C_1}\right]$$

$$\therefore \quad R_2 = R_4 \left[\frac{R_1}{R_3} + \frac{C_3}{C_1}\right] + jR_4 \left[\omega R_1 C_3 - \frac{1}{\omega C_1 R_3}\right]$$

Equating real parts of both sides,

$$R_2 = \frac{R_4 R_1}{R_3} + \frac{C_3 R_4}{C_1}$$

$$\therefore \quad \boxed{\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}}$$



Equating imaginary parts of both sides,

$$\omega R_1 C_3 - \frac{1}{\omega C_1 R_3} = 0$$

$$\therefore \qquad \omega^2 = \frac{1}{R_1 R_3 C_1 C_3}$$

$$\therefore \qquad \omega = \frac{1}{\sqrt{R_1 C_1 R_3 C_3}} \qquad \dots (2)$$

$$\therefore \qquad \boxed{f = \frac{1}{2\pi\sqrt{R_1 C_1 R_3 C_3}}} \qquad \dots (3)$$

The equation (1) gives the resistance ratio while the equation (3) gives the frequency of applied voltage.

Generally in Wien bridge, the selection of the components is such that

$$R_1 = R_3 = R$$

and $$C_1 = C_3 = C$$

$$\therefore \qquad \frac{R_2}{R_4} = 2 \qquad \dots (4)$$

and $$\boxed{f = \frac{1}{2\pi RC}} \qquad \dots (5)$$

The equation (5) is the general equation for the frequency of the bridge circuit.

2.13.1 Applications

The bridge is used to measure the frequency in audio range. The audio range is 20-200-2k-20kHz. The resistances are used for the range changing while the capacitors are used for fine frequency control.

The bridge can be used for capacitance measurement if the operating frequency is known.

The accuracy of 0.5% - 1% can be readily obtained using this bridge.

The generally accepted standard range of audible frequencies is 20 to 20,000 Hz