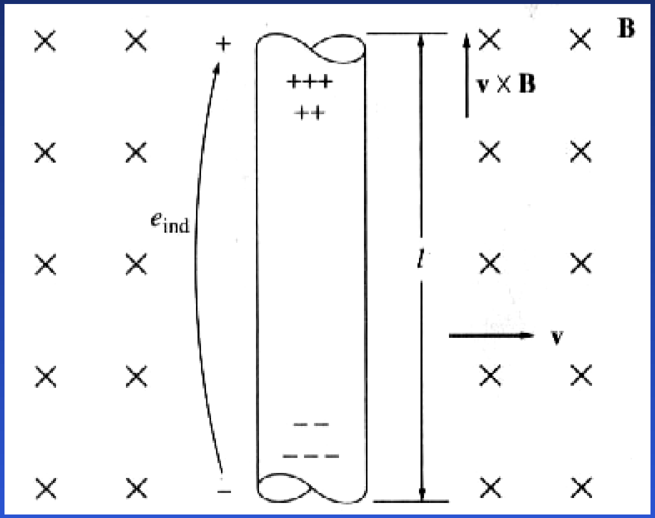
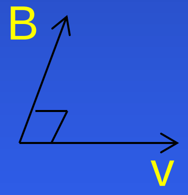
6. Induced voltage on a conductor moving in a magnetic field

The third way in which a magnetic field interacts with its surrounding is by an induction of voltage in the wire with the proper orientation moving through a magnetic field.

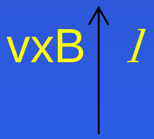


$$e_{ind} = (v \times B) \cdot l$$

Right hand rule



$$e_{ind} = vB\sin\theta\, l \cos\alpha$$

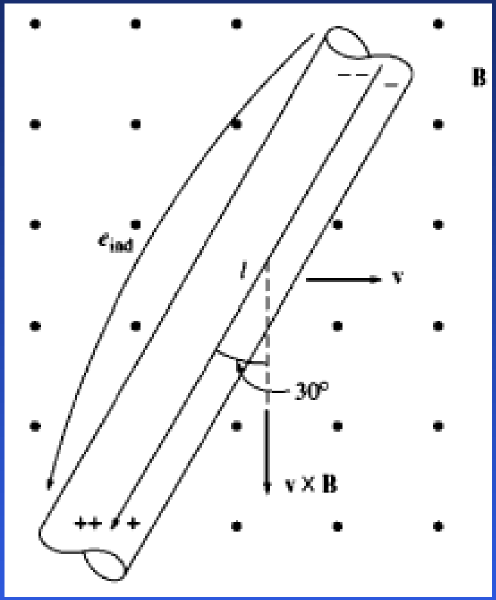


Where *v* is the velocity of the wire, *l* is its length in the magnetic field, *B* – the magnetic flux density

This is a basis for a generator action.

Example: Induced voltage

Figure 1-18 shows a conductor moving with a velocity of 10 m/s to the right in a magnetic field. The flux density is 0.5 T, out of the page, and the wire is 1.0 m in length, oriented as shown. What are the magnitude and polarity of the resulting induced voltage?



$$
\begin{aligned}
e_{ind} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \\
&= (vB \sin 90^\circ)\, l \cos 30^\circ \\
&= (10.0 \text{ m/s})(0.5 \text{ T})(1.0 \text{ m}) \cos 30^\circ \\
&= 4.33 \text{ V}
\end{aligned}
$$

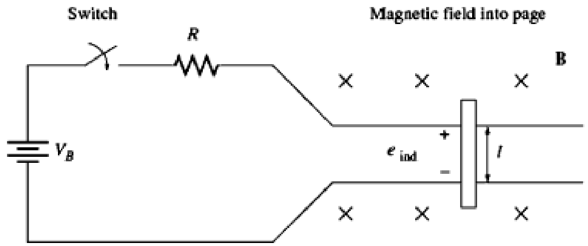


FIGURE 1–19
A linear dc machine. The magnetic field points into the page.

1.8 THE LINEAR DC MACHINE—A SIMPLE EXAMPLE

A *linear dc machine* is about the simplest and easiest-to-understand version of a dc machine, yet it operates according to the same principles and exhibits the same behavior as real generators and motors. It thus serves as a good starting point in the study of machines.

A linear dc machine is shown in Figure 1–19. It consists of a battery and a resistance connected through a switch to a pair of smooth, frictionless rails. Along the bed of this "railroad track" is a constant, uniform-density magnetic field directed into the page. A bar of conducting metal is lying across the tracks.

How does such a strange device behave? Its behavior can be determined from an application of four basic equations to the machine. These equations are

1. The equation for the force on a wire in the presence of a magnetic field:

$$\boxed{\mathbf{F} = i(\mathbf{l} \times \mathbf{B})} \qquad (1\text{–}43)$$

where $\mathbf{F}$ = force on wire
i = magnitude of current in wire
$\mathbf{l}$ = length of wire, with direction of $\mathbf{l}$ defined to be in the direction of current flow
$\mathbf{B}$ = magnetic flux density vector

2. The equation for the voltage induced on a wire moving in a magnetic field:

$$\boxed{e_{ind} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}} \qquad (1\text{–}45)$$

where e_{ind} = voltage induced in wire
$\mathbf{v}$ = velocity of the wire
$\mathbf{B}$ = magnetic flux density vector
$\mathbf{l}$ = length of conductor in the magnetic field

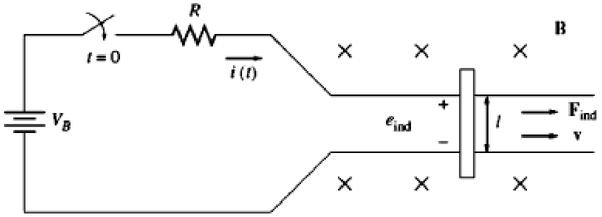


FIGURE 1–20
Starting a linear dc machine.

3. Kirchhoff's voltage law for this machine. From Figure 1–19 this law gives

$$V_B - iR - e_{ind} = 0$$

$$\boxed{V_B = e_{ind} + iR = 0} \qquad (1\text{–}46)$$

4. Newton's law for the bar across the tracks:

$$\boxed{F_{net} = ma} \qquad (1\text{–}7)$$

We will now explore the fundamental behavior of this simple dc machine using these four equations as tools.

Starting the Linear DC Machine

Figure 1–20 shows the linear dc machine under starting conditions. To start this machine, simply close the switch. Now a current flows in the bar, which is given by Kirchhoff's voltage law:

$$i = \frac{V_B - e_{ind}}{R} \qquad (1\text{–}47)$$

Since the bar is initially at rest, $e_{ind} = 0$, so $i = V_B/R$. The current flows down through the bar across the tracks. But from Equation (1–43), a current flowing through a wire in the presence of a magnetic field induces a force on the wire. Because of the geometry of the machine, this force is

$$F_{ind} = ilB \qquad \text{to the right} \qquad (1\text{–}48)$$

Therefore, the bar will accelerate to the right (by Newton's law). However, when the velocity of the bar begins to increase, a voltage appears across the bar. The voltage is given by Equation (1–45), which reduces for this geometry to

$$e_{ind} = vBl \qquad \text{positive upward} \qquad (1\text{–}49)$$

The voltage now reduces the current flowing in the bar, since by Kirchhoff's voltage law

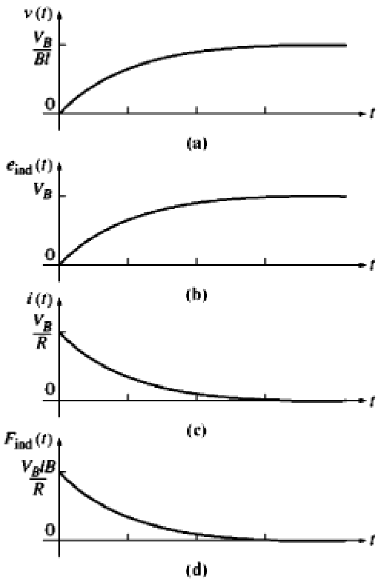


FIGURE 1–21
The linear dc machine on starting.
(a) Velocity $v(t)$ as a function of time;
(b) induced voltage $e_{ind}(t)$; (c) current $i(t)$;
(d) induced force $F_{ind}(t)$.

$$i\downarrow = \frac{V_B - e_{ind}\uparrow}{R} \tag{1–47}$$

As e_{ind} increases, the current i decreases.

The result of this action is that eventually the bar will reach a constant steady-state speed where the net force on the bar is zero. This will occur when e_{ind} has risen all the way up to equal the voltage V_B. At that time, the bar will be moving at a speed given by

$$V_B = e_{ind} = v_{ss}Bl$$

$$v_{ss} = \frac{V_B}{Bl} \tag{1–50}$$

The bar will continue to coast along at this no-load speed forever unless some external force disturbs it. When the motor is started, the velocity v, induced voltage e_{ind}, current i, and induced force F_{ind} are as sketched in Figure 1–21.

To summarize, at starting, the linear dc machine behaves as follows:

1. Closing the switch produces a current flow $i = V_B/R$.
2. The current flow produces a force on the bar given by $F = ilB$.