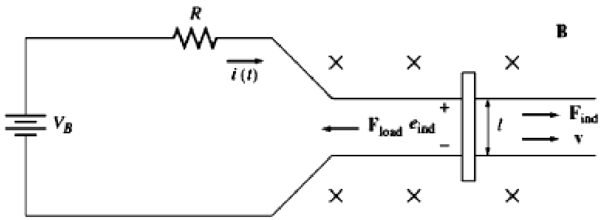

FIGURE 1–22
The linear dc machine as a motor.

3. The bar accelerates to the right, producing an induced voltage e_{ind} as it speeds up.
4. This induced voltage reduces the current flow $i = (V_B - e_{ind}\uparrow)/R$.
5. The induced force is thus decreased ($F = i\downarrow lB$) until eventually $F = 0$. At that point, $e_{ind} = V_B$, $i = 0$, and the bar moves at a constant no-load speed $v_{ss} = V_B/Bl$.

This is precisely the behavior observed in real motors on starting.

The Linear DC Machine as a Motor

Assume that the linear machine is initially running at the no-load steady-state conditions described above. What will happen to this machine if an external load is applied to it? To find out, let's examine Figure 1–22. Here, a force $\mathbf{F}_{load}$ is applied to the bar opposite the direction of motion. Since the bar was initially at steady state, application of the force $\mathbf{F}_{load}$ will result in a net force on the bar in the direction *opposite* the direction of motion ($\mathbf{F}_{net} = \mathbf{F}_{load} - \mathbf{F}_{ind}$). The effect of this force will be to slow the bar. But just as soon as the bar begins to slow down, the induced voltage on the bar drops ($e_{ind} = v\downarrow Bl$). As the induced voltage decreases, the current flow in the bar rises:

$$i\uparrow = \frac{V_B - e_{ind}\downarrow}{R} \tag{1–47}$$

Therefore, the induced force rises too ($F_{ind} = i\uparrow lB$). The overall result of this chain of events is that the induced force rises until it is equal and opposite to the load force, and the bar again travels in steady state, but at a lower speed. When a load is attached to the bar, the velocity v, induced voltage e_{ind}, current i, and induced force F_{ind} are as sketched in Figure 1–23.

There is now an induced force in the direction of motion of the bar, and power is being *converted from electrical form to mechanical form* to keep the bar moving. The power being converted is

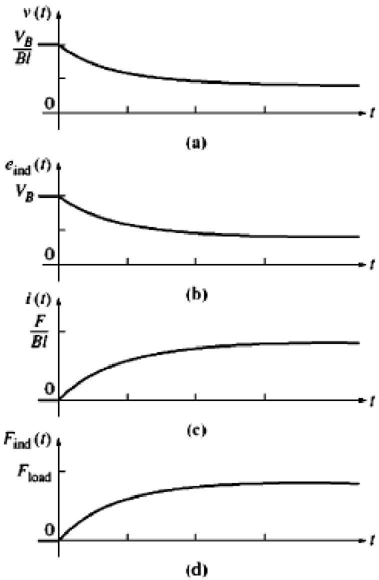

FIGURE 1–23
The linear dc machine operating at no-load conditions and then loaded as a motor. (a) Velocity $v(t)$ as a function of time; (b) induced voltage $e_{ind}(t)$; (c) current $i(t)$; (d) induced force $F_{ind}(t)$.

$$P_{conv} = e_{ind}i = F_{ind}v \tag{1–51}$$

An amount of electric power equal to $e_{ind}i$ is consumed in the bar and is replaced by mechanical power equal to $F_{ind}v$. Since power is converted from electrical to mechanical form, this bar is operating as a *motor*.

To summarize this behavior:

1. A force $\mathbf{F}_{load}$ is applied opposite to the direction of motion, which causes a net force $\mathbf{F}_{net}$ opposite to the direction of motion.
2. The resulting acceleration $a = F_{net}/m$ is negative, so the bar slows down ($v\downarrow$).
3. The voltage $e_{ind} = v\downarrow Bl$ falls, and so $i = (V_B - e_{ind}\downarrow)/R$ increases.
4. The induced force $F_{ind} = i\uparrow lB$ increases until $|\mathbf{F}_{ind}| = |\mathbf{F}_{load}|$ at a lower speed v.
5. An amount of electric power equal to $e_{ind}i$ is now being converted to mechanical power equal to $F_{ind}v$, and the machine is acting as a motor.

A real dc motor behaves in a precisely analogous fashion when it is loaded: As a load is added to its shaft, the motor begins to slow down, which reduces its internal voltage, increasing its current flow. The increased current flow increases its induced torque, and the induced torque will equal the load torque of the motor at a new, slower speed.

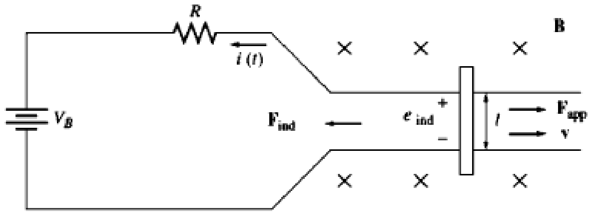

FIGURE 1–24
The linear dc machine as a generator.

Note that the power converted from electrical form to mechanical form by this linear motor was given by the equation $P_{conv} = F_{ind}v$. The power converted from electrical form to mechanical form in a real rotating motor is given by the equation

$$P_{conv} = \tau_{ind}\omega \qquad (1\text{–}52)$$

where the induced torque τ_{ind} is the rotational analog of the induced force F_{ind}, and the angular velocity ω is the rotational analog of the linear velocity v.

The Linear DC Machine as a Generator

Suppose that the linear machine is again operating under no-load steady-state conditions. This time, apply a force *in the direction of motion* and see what happens.

Figure 1–24 shows the linear machine with an applied force $\mathbf{F}_{app}$ in the direction of motion. Now the applied force will cause the bar to accelerate in the direction of motion, and the velocity v of the bar will increase. As the velocity increases, $e_{ind} = v\uparrow Bl$ will increase and will be larger than the battery voltage V_B. With $e_{ind} > V_B$, the current reverses direction and is now given by the equation

$$i = \frac{e_{ind} - V_B}{R} \qquad (1\text{–}53)$$

Since this current now flows *up* through the bar, it induces a force in the bar given by

$$F_{ind} = ilB \qquad \text{to the left} \qquad (1\text{–}54)$$

The direction of the induced force is given by the right-hand rule. This induced force opposes the applied force on the bar.

Finally, the induced force will be equal and opposite to the applied force, and the bar will be moving at a *higher* speed than before. Notice that now *the battery is charging*. The linear machine is now serving as a generator, converting mechanical power $F_{ind}v$ into electric power $e_{ind}i$.

To summarize this behavior:

1. A force $\mathbf{F}_{app}$ is applied in the direction of motion; $\mathbf{F}_{net}$ is in the direction of motion.
2. Acceleration $a = F_{net}/m$ is positive, so the bar speeds up ($v\uparrow$).
3. The voltage $e_{ind} = v\uparrow Bl$ increases, and so $i = (e_{ind}\uparrow - V_B)/R$ increases.
4. The induced force $F_{ind} = i\uparrow lB$ increases until $|\mathbf{F}_{ind}| = |\mathbf{F}_{load}|$ at a higher speed v.
5. An amount of mechanical power equal to $F_{ind}v$ is now being converted to electric power $e_{ind}i$, and the machine is acting as a generator.

Again, a real dc generator behaves in precisely this manner: A torque is applied to the shaft *in the direction of motion*, the speed of the shaft increases, the internal voltage increases, and current flows out of the generator to the loads. The amount of mechanical power converted to electrical form in the real rotating generator is again given by Equation (1–52):

$$P_{conv} = \tau_{ind}\omega \qquad (1\text{–}52)$$

It is interesting that the same machine acts as *both motor and generator*. The only difference between the two is whether the externally applied forces are in the direction of motion (generator) or opposite to the direction of motion (motor). Electrically, when $e_{ind} > V_B$, the machine acts as a generator, and when $e_{ind} < V_B$, the machine acts as a motor. Whether the machine is a motor or a generator, both induced force (motor action) and induced voltage (generator action) are present at all times. This is generally true of all machines—both actions are present, and it is only the relative directions of the external forces with respect to the direction of motion that determine whether the overall machine behaves as a motor or as a generator.

Another very interesting fact should be noted: This machine was a generator when it moved rapidly and a motor when it moved more slowly, but whether it was a motor or a generator, it always moved in the same direction. Many beginning machinery students expect a machine to turn one way as a generator and the other way as a motor. *This does not occur.* Instead, there is merely a small change in operating speed and a reversal of current flow.

Starting Problems with the Linear Machine

A linear machine is shown in Figure 1–25. This machine is supplied by a 250-V dc source, and its internal resistance R is given as about 0.10 Ω. (The resistor R models the internal resistance of a real dc machine, and this is a fairly reasonable internal resistance for a medium-size dc motor.)

Providing actual numbers in this figure highlights a major problem with machines (and their simple linear model). At starting conditions, the speed of the bar is zero, so $e_{ind} = 0$. The current flow at starting is

$$i_{start} = \frac{V_B}{R} = \frac{250\ V}{0.1\ \Omega} = 2500\ \text{A}$$

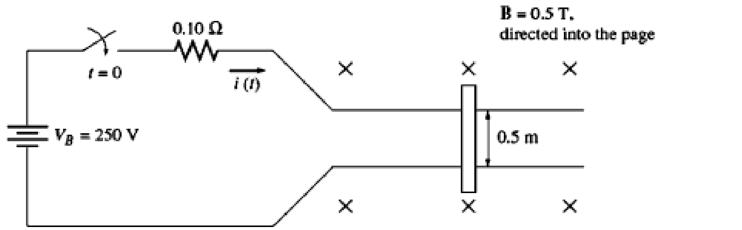

FIGURE 1–25
The linear dc machine with component values illustrating the problem of excessive starting current.

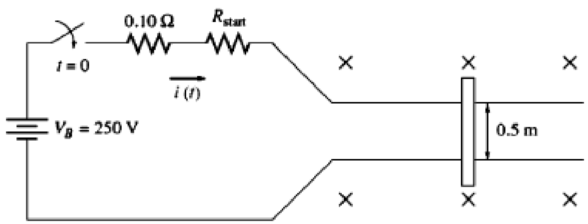

FIGURE 1–26
A linear dc machine with an extra series resistor inserted to control the starting current.

This current is very high, often in excess of 10 times the rated current of the machine. Such currents can cause severe damage to a motor. Both real ac and real dc machines suffer from similar high-current problems on starting.

How can such damage be prevented? The easiest method for this simple linear machine is to insert an extra resistance into the circuit during starting to limit the current flow until e_{ind} builds up enough to limit it. Figure 1–26 shows a starting resistance inserted into the machine circuitry.

The same problem exists in real dc machines, and it is handled in precisely the same fashion—a resistor is inserted into the motor armature circuit during starting. The control of high starting current in real ac machines is handled in a different fashion, which will be described in Chapter 8.

Example 1–10. The linear dc machine shown in Figure 1–27a has a battery voltage of 120 V, an internal resistance of 0.3 Ω, and a magnetic flux density of 0.1 T.

(a) What is this machine's maximum starting current? What is its steady-state velocity at no load?

(b) Suppose that a 30-N force pointing to the right were applied to the bar. What would the steady-state speed be? How much power would the bar be producing or consuming? How much power would the battery be producing or consuming?

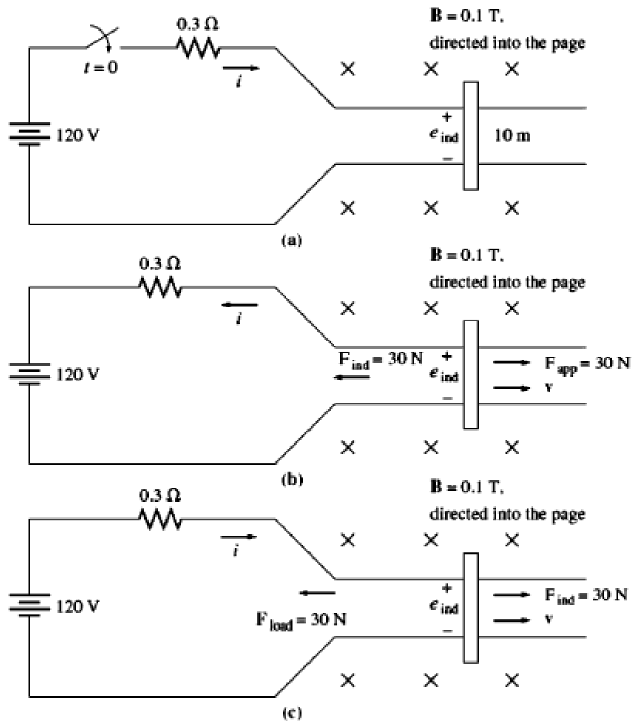

FIGURE 1–27
The linear dc machine of Example 1–10. (a) Starting conditions; (b) operating as a generator; (c) operating as a motor.

Explain the difference between these two figures. Is this machine acting as a motor or as a generator?

(c) Now suppose a 30-N force pointing to the left were applied to the bar. What would the new steady-state speed be? Is this machine a motor or a generator now?

(d) Assume that a force pointing to the left is applied to the bar. Calculate speed of the bar as a function of the force for values from 0 N to 50 N in 10-N steps. Plot the velocity of the bar versus the applied force.

(e) Assume that the bar is unloaded and that it suddenly runs into a region where the magnetic field is weakened to 0.08 T. How fast will the bar go now?

Solution

(a) At starting conditions, the velocity of the bar is 0, so $e_{ind} = 0$. Therefore,

$$i = \frac{V_B - e_{ind}}{R} = \frac{120\text{ V} - 0\text{ V}}{0.3\ \Omega} = 400\text{ A}$$

When the machine reaches steady state, $\mathbf{F}_{ind} = 0$ and $i = 0$. Therefore,

$$VB = e_{ind} = v_{ss}Bl$$

$$v_{ss} = \frac{V_B}{Bl}$$

$$= \frac{120\text{ V}}{(0.1\text{ T})(10\text{ m})} = 120\text{ m/s}$$

(b) Refer to Figure 1–27b. If a 30-N force to the right is applied to the bar, the final steady state will occur when the induced force $\mathbf{F}_{ind}$ is equal and opposite to the applied force $\mathbf{F}_{app}$, so that the net force on the bar is zero:

$$F_{app} = F_{ind} = ilB$$

Therefore,

$$i = \frac{F_{ind}}{lB} = \frac{30\text{ N}}{(10\text{m})(0.1\text{ T})}$$

$$= 30\text{ A} \quad \text{flowing up through the bar}$$

The induced voltage e_{ind} on the bar must be

$$e_{ind} = V_B + iR$$

$$= 120\text{ V} + (30\text{A})(0.3\ \Omega) = 129\text{ V}$$

and the final steady-state speed must be

$$v_{ss} = \frac{e_{ind}}{Bl}$$

$$= \frac{129\text{ V}}{(0.1\text{ T})(10\text{ m})} = 129\text{ m/s}$$

The bar is *producing* $P = (129\text{ V})(30\text{ A}) = 3870$ W of power, and the battery is *consuming* $P = (120\text{ V})(30\text{ A}) = 3600$ W. The difference between these two numbers is the 270 W of losses in the resistor. This machine is acting as a *generator.*

(c) Refer to Figure 1–25c. This time, the force is applied to the left, and the induced force is to the right. At steady state,

$$F_{app} = F_{ind} = ilB$$

$$i = \frac{F_{ind}}{lB} = \frac{30\text{ N}}{(10\text{ m})(0.1\text{ T})}$$

$$= 30\text{ A} \quad \text{flowing down through the bar}$$

The induced voltage e_{ind} on the bar must be

$$e_{ind} = V_B - iR$$

$$= 120\text{ V} - (30\text{ A})(0.3\ \Omega) = 111\text{ V}$$

and the final speed must be

$$v_{ss} = \frac{e_{ind}}{Bl}$$

$$= \frac{111\text{ V}}{(0.1\text{ T})(10\text{ m})} = 111\text{ m/s}$$

This machine is now acting as a *motor,* converting electric energy from the battery into mechanical energy of motion on the bar.