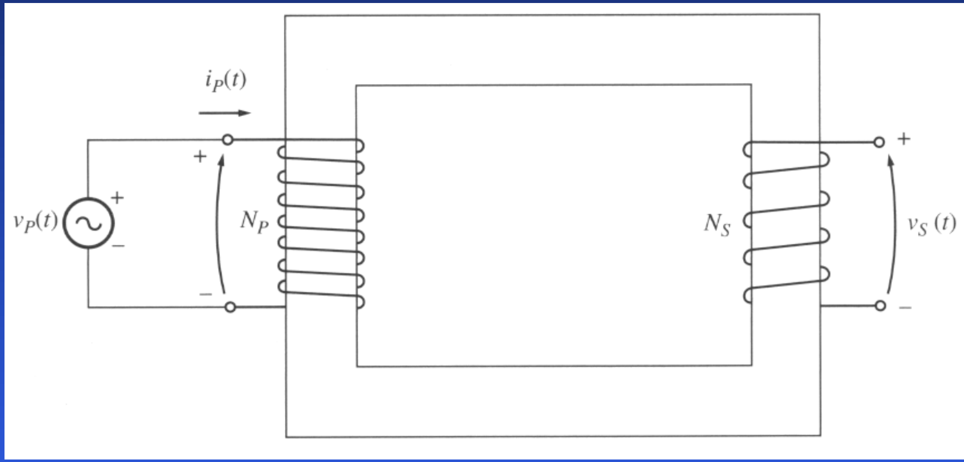
Theory of operation of real transformers

Real transformers approximate ideal ones to some degree.

The basis transformer operation can be derived from Faraday's law:

$$e_{ind} = \frac{d\lambda}{dt} \qquad (4.19.1)$$



Here λ is the flux linkage in the coil across which the voltage is induced:

$$\lambda = \sum_{i=1}^{N} \phi_i \qquad (4.19.2)$$

where ϕ_i is the flux passing through the i^{th} turn in a coil – slightly different for different turns. However, we may use an average flux per turn in the coil having N turns:

$$\bar{\phi} = \lambda / N \qquad (4.19.3)$$

Therefore:

$$e_{ind} = N \frac{d\bar{\phi}}{dt} \qquad (4.19.4)$$

Voltage ratio across a real transformer

If the source voltage $v_p(t)$ is applied to the primary winding, the average flux in the primary winding will be:

$$\bar{\phi} = \frac{1}{N_p} \int v_p(t)dt \qquad (4.20.1)$$

A portion of the flux produced in the primary coil passes through the secondary coil (mutual flux); the rest is lost (leakage flux):

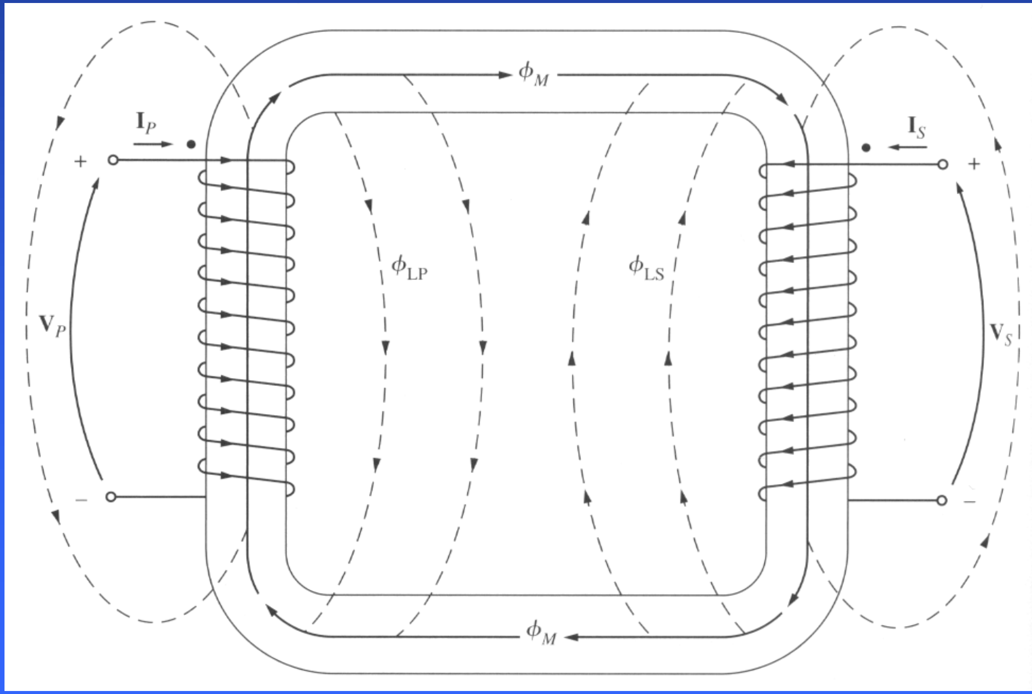
$$\bar{\phi}_p = \phi_m + \phi_{Lp} \qquad (4.20.2)$$

average primary flux   mutual flux

Similarly, for the secondary coil:

$$\bar{\phi}_s = \phi_m + \phi_{Ls} \qquad (4.20.3)$$

Average secondary flux



Voltage ratio across a real transformer

From the Faraday's law, the primary coil's voltage is:

$$v_p(t) = N_p \frac{d\bar{\phi}_p}{dt} = N_p \frac{d\phi_m}{dt} + N_p \frac{d\phi_{Lp}}{dt} = e_p(t) + e_{Lp}(t) \qquad (4.21.1)$$

The secondary coil's voltage is:

$$v_s(t) = N_s \frac{d\bar{\phi}_s}{dt} = N_s \frac{d\phi_m}{dt} + N_s \frac{d\phi_{Ls}}{dt} = e_s(t) + e_{Ls}(t) \qquad (4.21.2)$$

The primary and secondary voltages due to the mutual flux are:

$$e_p(t) = N_p \frac{d\phi_m}{dt} \qquad (4.21.3)$$

$$e_s(t) = N_s \frac{d\phi_{Ls}}{dt} \qquad (4.21.4)$$

Combining the last two equations:

$$\frac{e_p(t)}{N_p} = \frac{d\phi_m}{dt} = \frac{e_s(t)}{N_s} \qquad (4.21.5)$$

Voltage ratio across a real transformer

Therefore:

$$\frac{e_p(t)}{e_s(t)} = \frac{N_p}{N_s} = a \quad (4.22.1)$$

That is, the ratio of the primary voltage to the secondary voltage both caused by the mutual flux is equal to the turns ratio of the transformer.

For well-designed transformers:

$$\phi_M >> \phi_{LP} \text{ and } \phi_M >> \phi_{LS} \quad (4.22.2)$$

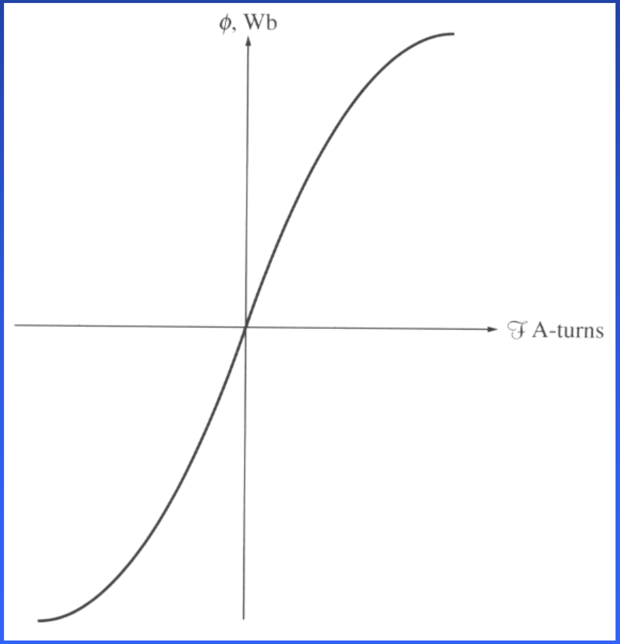
Therefore, the following approximation normally holds:

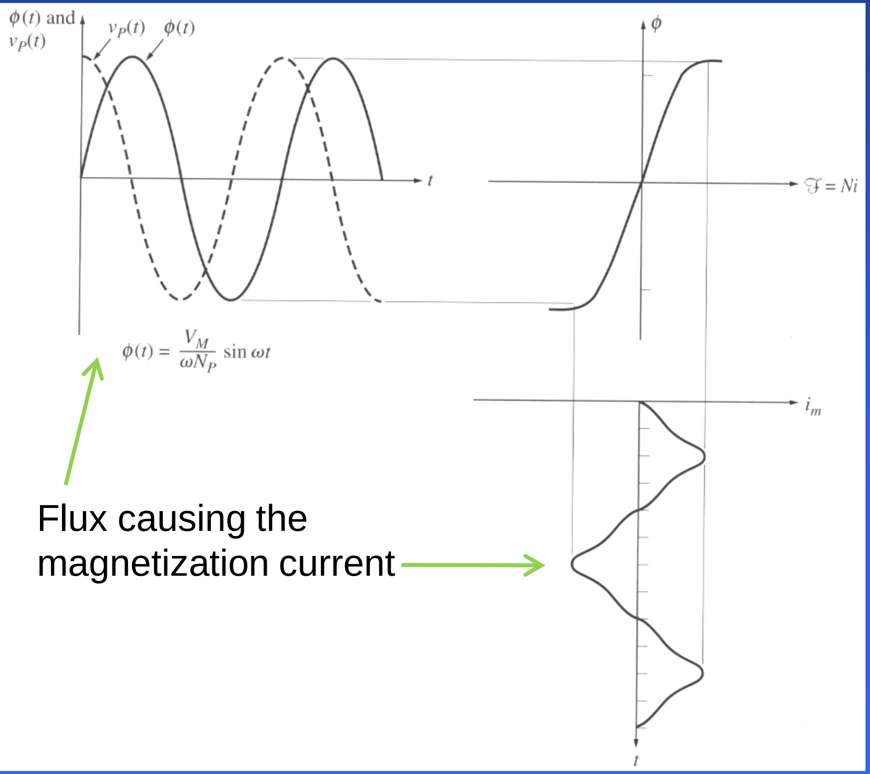
$$\frac{v_p(t)}{v_s(t)} \approx \frac{N_p}{N_s} \approx a \quad (4.22.3)$$

The magnetization current

Even when no load is connected to the secondary coil of the transformer, a current will flow in the primary coil. This current consists of:

1. The magnetization current i_m needed to produce the flux in the core;
2. The core-loss current i_{h+e} hysteresis and eddy current losses.





Typical magnetization curve

The magnetization current

Ignoring flux leakage and assuming time-harmonic primary voltage, the average flux is:

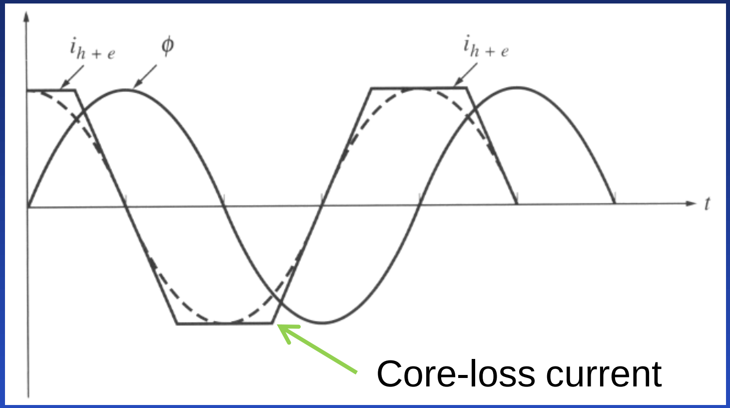
$$\bar{\phi} = \frac{1}{N_p}\int v_p(t)dt = \frac{1}{N_p}\int V_m \cos\omega t dt = \frac{V_m}{\omega N_p}\sin\omega t \quad [Wb] \tag{4.24.1}$$

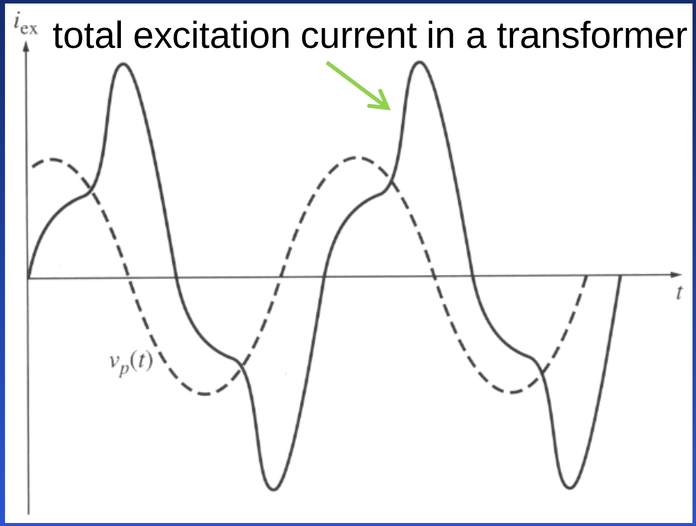
If the values of current are comparable to the flux they produce in the core, it is possible to sketch a magnetization current. We observe:

1. Magnetization current is not sinusoidal: there are high frequency components;
2. Once saturation is reached, a small increase in flux requires a large increase in magnetization current;
3. Magnetization current (its fundamental component) lags the voltage by 90^o;
4. High-frequency components of the current may be large in saturation.

Assuming a sinusoidal flux in the core, the eddy currents will be largest when flux passes zero.

The magnetization current





Core-loss current is:

1. Nonlinear due to nonlinear effects of hysteresis;
2. In phase with the voltage.

The total no-load current in the core is called the excitation current of the transformer:

$$i_{ex} = i_m + i_{h+e} \tag{4.25.1}$$

Laminated core