The per-unit system

Another approach to solve circuits containing transformers is the per-unit system. Impedance and voltage-level conversions are avoided. Also, machine and transformer impedances fall within fairly narrow ranges for each type and construction of device while the per-unit system is employed.

The voltages, currents, powers, impedances, and other electrical quantities are measured as fractions of some base level instead of conventional units.

$$Quantity\ per\ unit = \frac{actual\ value}{base\ value\ of\ quantity} \tag{4.40.1}$$

Usually, two base quantities are selected to define a given per-unit system. Often, such quantities are voltage and power (or apparent power). In a 1-phase system:

$$P_{base},\ Q_{base},\ or\ \ S_{base} = V_{base} I_{base} \tag{4.40.2}$$

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{\left(V_{base}\right)^2}{S_{base}} \tag{4.40.3}$$

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$$Y_{base} = \frac{I_{base}}{V_{base}} \tag{4.41.1}$$

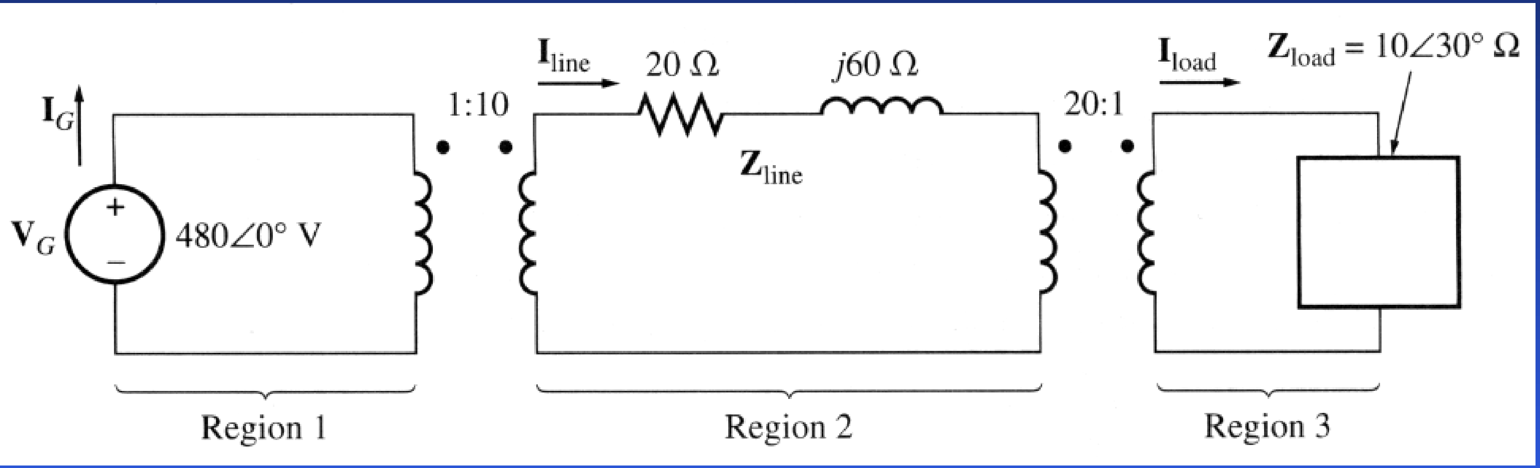
Ones the base values of P (or S) and V are selected, all other base values can be computed form the above equations.

In a power system, a base apparent power and voltage are selected at the specific point in the system. Note that a transformer has no effect on the apparent power of the system, since the apparent power into a transformer equals the apparent power out of a transformer. As a result, the base apparent power remains constant everywhere in the power system.

On the other hand, voltage (and, therefore, a base voltage) changes when it goes through a transformer according to its turn ratio. Therefore, the process of referring quantities to a common voltage level is done automatically in the per-unit system.

The per-unit system: Example

Example 4.3: A simple power system is given by the circuit:



The generator is rated at 480 V and 10 kVA.

a) Find the base voltage, current, impedance, and apparent power at every points in the power system;

b) Convert the system to its per-unit equivalent circuit;

c) Find the power supplied to the load in this system;

e) Find the power lost in the transmission line (Region 2).

The per-unit system: Example

a. In the generator region: $V_{base\ 1} = 480\ V$ and $S_{base} = 10\ kVA$

$$I_{base\,1} = \frac{S_{base\,1}}{V_{base\,1}} = \frac{10\,000}{480} = 20.83\ A$$

$$Z_{base\,1} = \frac{V_{base\,1}}{I_{base\,1}} = \frac{480}{20.83} = 23.04\,\Omega$$

The turns ratio of the transformer T_1 is $a_1 = 0.1$; therefore, the voltage in the transmission line region is

$$V_{base\,2} = \frac{V_{base\,1}}{a_1} = \frac{480}{0.1} = 4800\,V$$

The other base quantities are

The per-unit system: Example

$$S_{base\,2} = 10\ \ kVA$$

$$I_{base\,2} = \frac{10\,000}{4800} = 2.083\ A$$

$$Z_{base\,2} = \frac{4800}{2.083} = 2304\,\Omega$$

The turns ratio of the transformer T_2 is $a_2 = 20$; therefore, the voltage in the load region is

$$V_{base\,3} = \frac{V_{base\,2}}{a_2} = \frac{4800}{20} = 240\,V$$

The other base quantities are

The per-unit system: Example

$$S_{base\,3} = 10\ \ kVA$$

$$I_{base\,3} = \frac{10\,000}{240} = 41.67\ A$$

$$Z_{base\,3} = \frac{240}{41.67} = 5.76\,\Omega$$

b. To convert a power system to a per-unit system, each component must be divided by its base value in its region. The generator's per-unit voltage is

$$V_{G,pu} = \frac{480\angle 0^\circ}{480} = 1.0\angle 0^\circ\ pu$$

The transmission line's per-unit impedance is

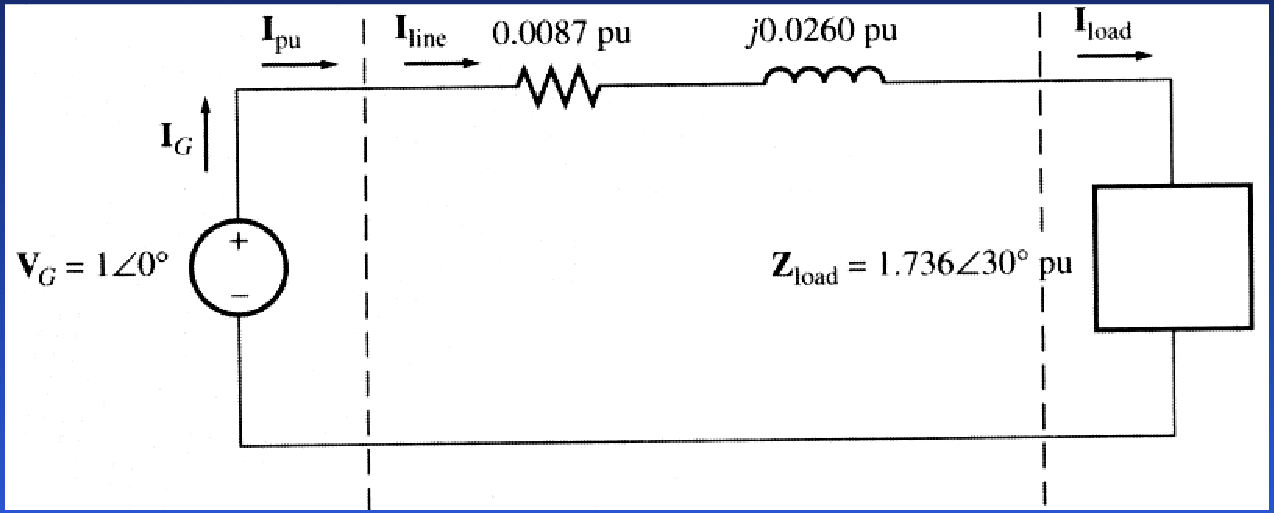
$$Z_{line,\,pu} = \frac{20 + j60}{2304} = 0.0087 + j0.026\ pu$$

The per-unit system: Example

The load's per-unit impedance is

$$Z_{load,\,pu} = \frac{10\angle 30^\circ}{5.76}$$
$$= 1.736\angle 30^\circ \ pu$$

The per-unit equivalent circuit



c. The current flowing in this per-unit power system is

$$I_{pu} = \frac{V_{pu}}{Z_{tot,\,pu}} = \frac{1\angle 0^\circ}{0.0087 + j0.026 + 1.736\angle 30^\circ} = 0.569\angle -30.6^\circ \ pu$$

The per-unit system: Example

Therefore, the per-unit power on the load is

$$P_{load,\,pu} = I_{pu}^{2} R_{pu} = 0.569^{2} \cdot 1.503 = 0.487$$

The actual power on the load is

$$P_{load} = P_{load,\,pu} S_{base} = 0.487 \cdot 10\,000 = 487\,W$$

d. The per-unit power lost in the transmission line is

$$P_{line,\,pu} = I_{pu}^{2} R_{line,\,pu} = 0.569^{2} \cdot 0.0087 = 0.00282$$

The actual power lost in the transmission line

$$P_{line} = P_{line,\,pu} S_{base} = 0.00282 \cdot 10\,000 = 28.2\,W$$

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When only one device (transformer or motor) is analyzed, its own ratings are used as the basis for per-unit system. When considering a transformer in a per-unit system, transformer’s characteristics will not vary much over a wide range of voltages and powers. For example, the series resistance is usually from 0.02 to 0.1 pu; the magnetizing reactance is usually from 10 to 40 pu; the core-loss resistance is usually from 50 to 200 pu. Also, the per-unit impedances of synchronous and induction machines fall within relatively narrow ranges over quite large size ranges.

If more than one transformer is present in a system, the system base voltage and power can be chosen arbitrary. However, the entire system must have the same base power, and the base voltages at various points in the system must be related by the voltage ratios of the transformers.

System base quantities are commonly chosen to the base of the largest component in the system.

The per-unit system

Per-unit values given to another base can be converted to the new base either through an intermediate step (converting them to the actual values) or directly as follows:

$$\left(P, Q, S\right)_{pu,\,base\,2} = \left(P, Q, S\right)_{pu,\,base\,1} \frac{S_{base\,1}}{S_{base\,2}} \qquad (4.49.1)$$

$$V_{pu,\,base\,2} = V_{pu,\,base\,1} \frac{V_{base\,1}}{V_{base\,2}} \qquad (4.49.2)$$

$$\left(R, X, Z\right)_{pu,\,base\,2} = \left(R, X, Z\right)_{pu,\,base\,1} \frac{V_{base\,1}^2 S_{base\,1}}{V_{base\,2}^2 S_{base\,2}} \qquad (4.49.3)$$

The per-unit system: Example

Example 4.4: Sketch the appropriate per-unit equivalent circuit for the 8000/240 V, 60 Hz, 20 kVA transformer with $R_c = 159$ kΩ, $X_M = 38.4$ kΩ, $R_{eq} = 38.3$ Ω, $X_{eq} = 192$ Ω.

To convert the transformer to per-unit system, the primary circuit base impedance needs to be found.

$$V_{base\,1} = 8\,000\,V; \quad S_{base\,1} = 20\,000\,VA$$

$$Z_{base\,1} = \frac{V_{base\,1}^2}{S_{base\,1}} = \frac{8\,000^2}{20\,000} = 3\,200\,\Omega$$

$$Z_{SE,pu} = \frac{38.4 + j192}{3\,200} = 0.012 + j0.06\ pu$$

$$R_{C,pu} = \frac{159\,000}{3\,200} = 49.7\ pu$$

$$X_{M,pu} = \frac{38\,400}{3\,200} = 12\ pu$$

The per-unit system: Example

Therefore, the per-unit equivalent circuit is shown below: