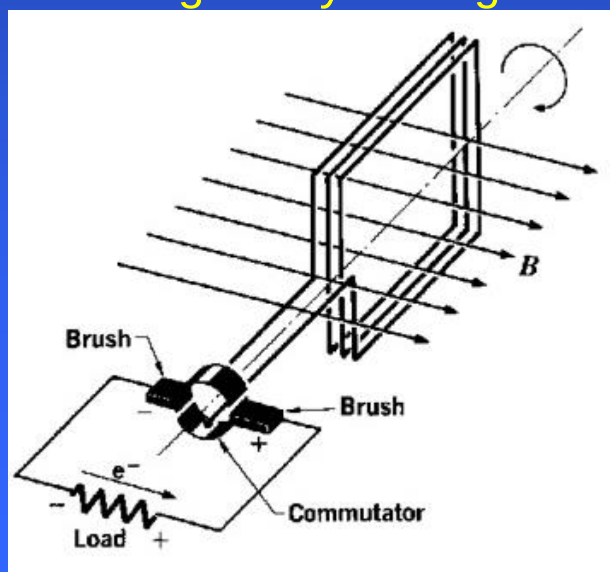
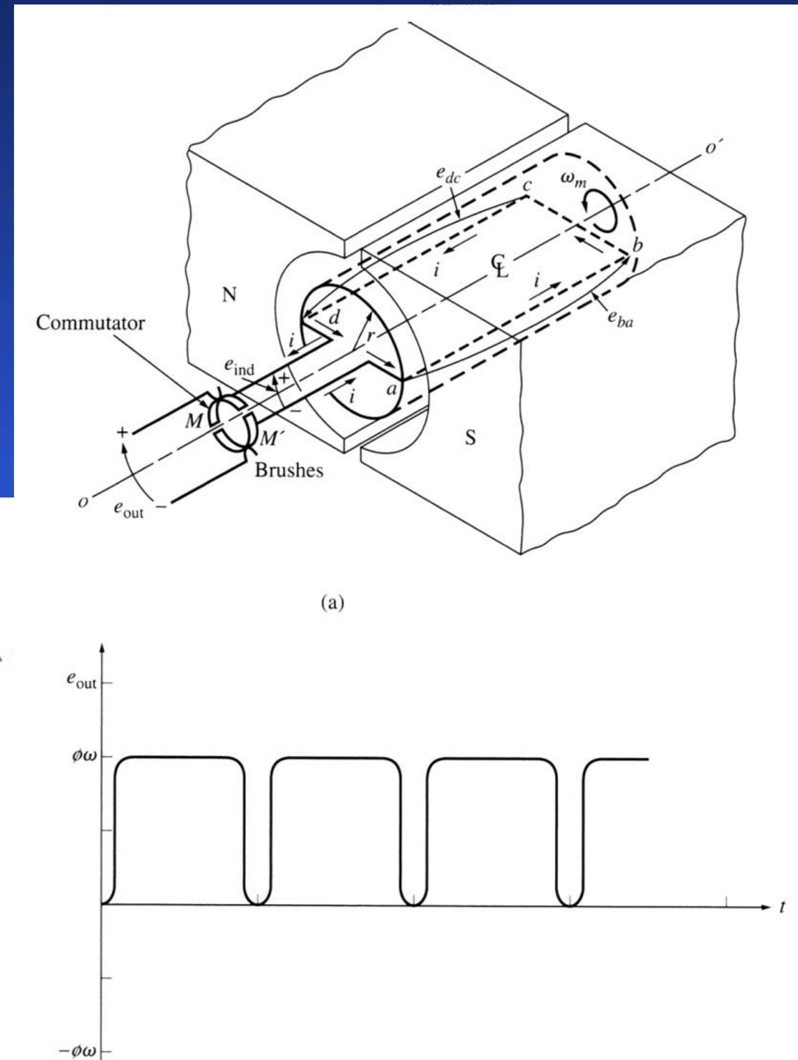
The simplest DC machine

2. Getting DC voltage out of a rotating loop

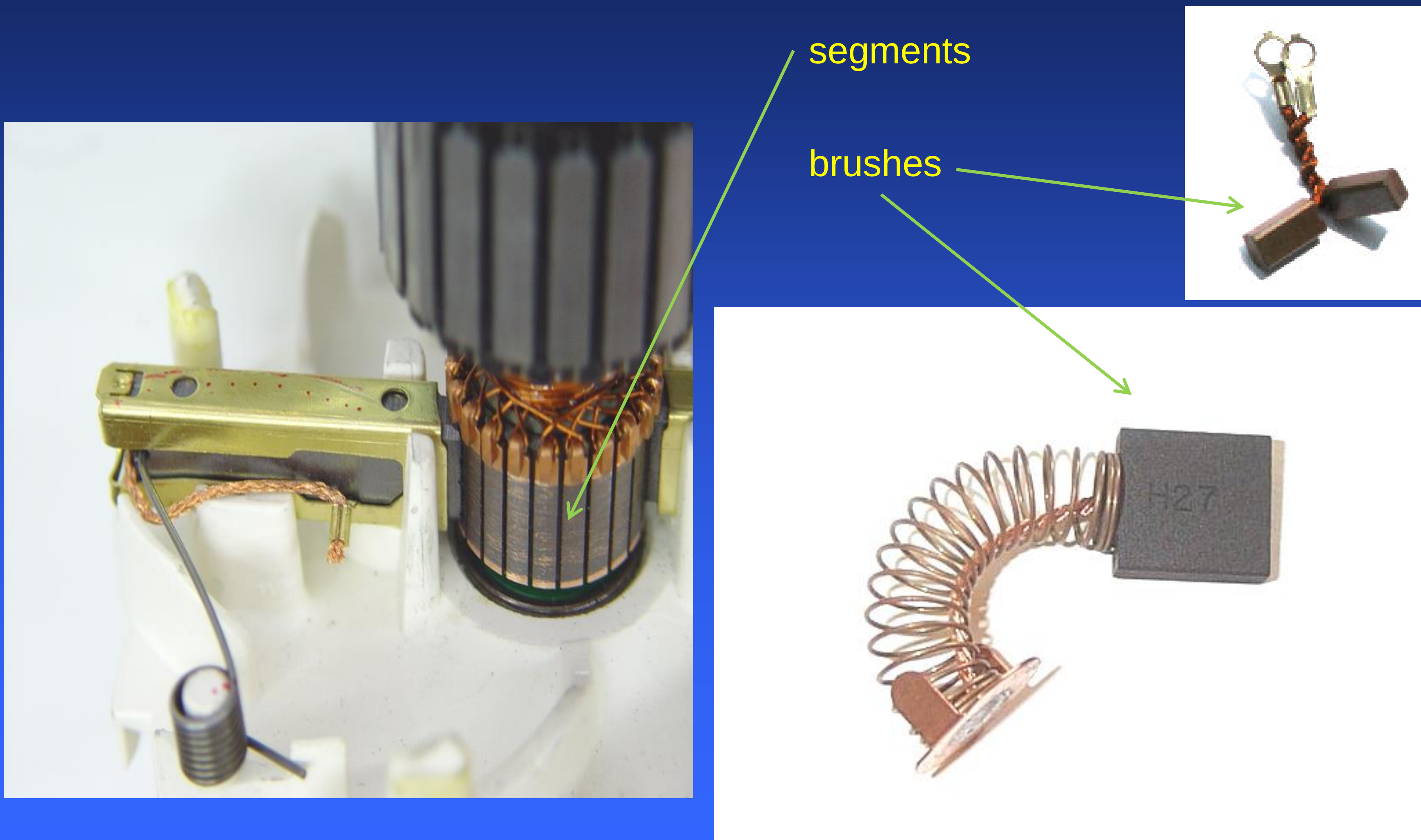
A voltage out of the loop is alternatively a constant positive value and a constant negative value.

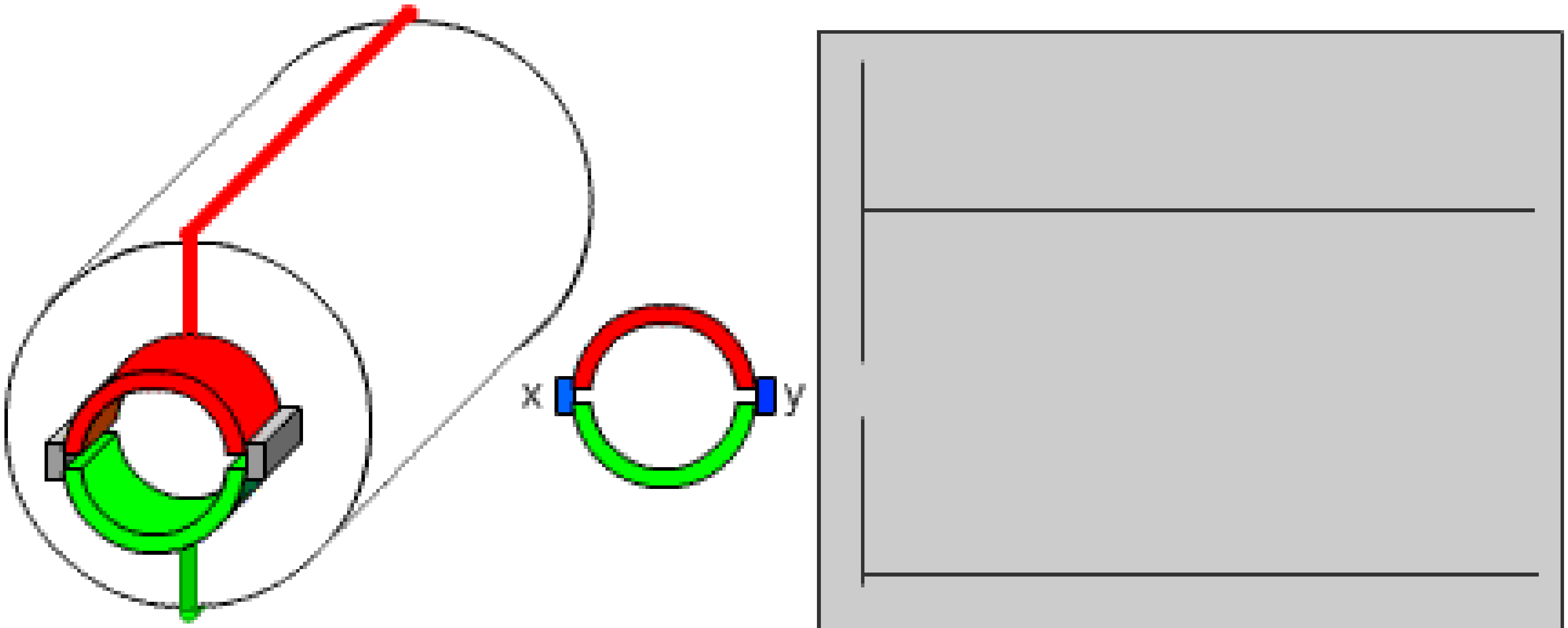
One possible way to convert an alternating voltage to a constant voltage is by adding a commutator segment/brush circuitry to the end of the loop. Every time the voltage of the loop switches direction, contacts switch connection.



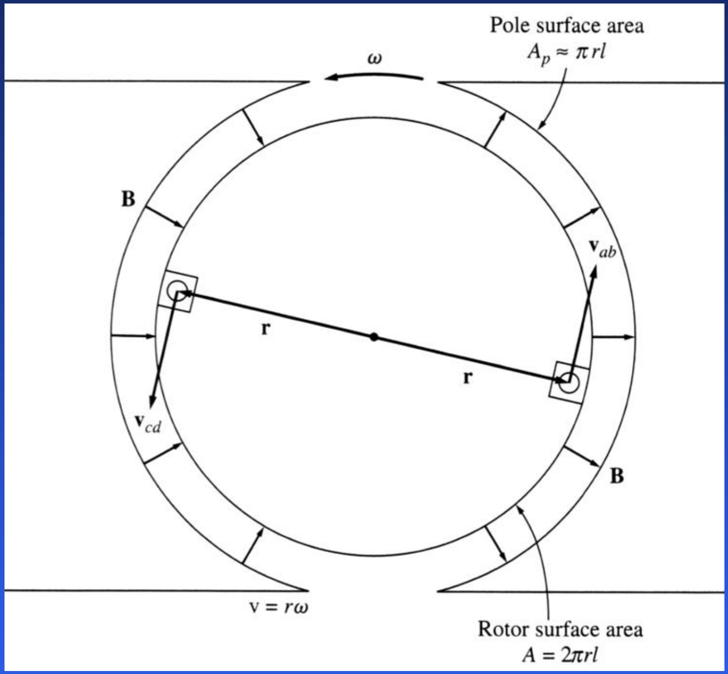


The simplest DC machine




x
y

The simplest DC machine



The tangential velocity of the loop's edges is

$$v = r\omega \qquad (5.8.1)$$

where r is the radius from the axis of rotation to the edge of the loop. The total induced voltage:

$$e_{tot} = \begin{cases} 2r\omega Bl & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases} \qquad (5.8.2)$$

The rotor is a cylinder with surface area $2\pi rl$. Since there are two poles, the area of the rotor under each pole is $A_p = \pi rl$. Therefore:

$$e_{tot} = \begin{cases} \frac{2}{\pi} A_p B\omega & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases} \qquad (5.8.3)$$

The simplest DC machine

Assuming that the flux density B is constant everywhere in the air gap under the pole faces, the total flux under each pole is

$$\phi = A_p B \tag{5.9.1}$$

The total voltage is

$$e_{tot} = \begin{cases} \frac{2}{\pi}\phi\omega & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases} \tag{5.9.2}$$

The voltage generated in any real machine depends on the following factors:

1. The flux inside the machine;
2. The rotation speed of the machine;
3. A constant representing the construction of the machine.

The simplest DC machine

3. The induced torque in the rotating loop

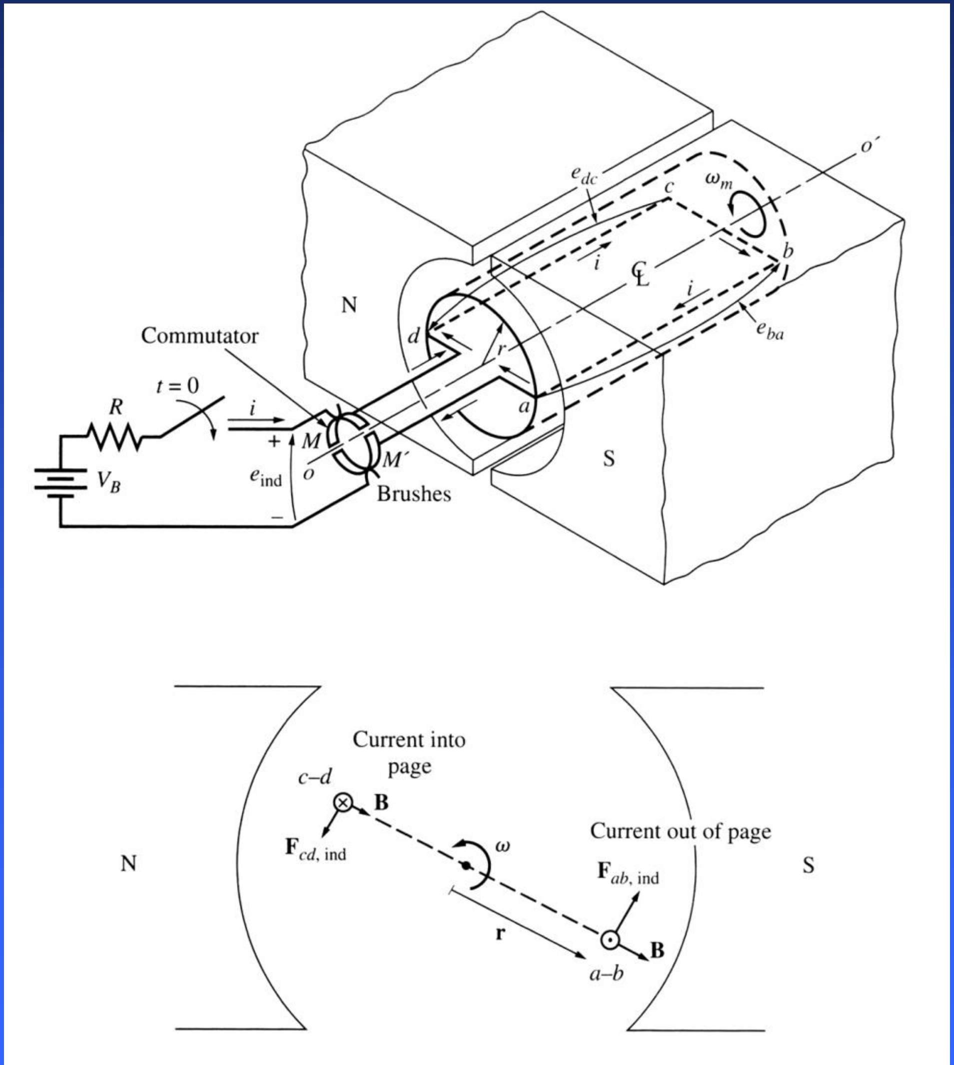
Assuming that a battery is connected to the DC machine, the force on a segment of a loop is:

$$F = i\left(\mathbf{l}\times\mathbf{B}\right) \quad (5.12.1)$$

And the torque on the segment is

$$\tau = rF\sin\theta \quad (5.12.2)$$

Where θ is the angle between r and F. Therefore, the torque is zero when the loop is beyond the pole edges.



The simplest DC machine

When the loop is under the pole faces:

1. Segment *ab*: $$F_{ab} = i\left(\mathbf{l} \times \mathbf{B}\right) = ilB \qquad (5.13.1)$$
$$\tau_{ab} = rF\sin\theta = r\left(ilB\right)\sin 90° = rilB \quad ccw \qquad (5.13.2)$$

2. Segment *bc*: $$F_{ab} = i\left(\mathbf{l} \times \mathbf{B}\right) = 0 \qquad (5.13.3)$$
$$\tau_{ab} = rF\sin\theta = 0 \qquad (5.13.4)$$

3. Segment *cd*: $$F_{ab} = i\left(\mathbf{l} \times \mathbf{B}\right) = ilB \qquad (5.13.5)$$
$$\tau_{ab} = rF\sin\theta = r\left(ilB\right)\sin 90° = rilB \quad ccw \qquad (5.13.6)$$

4. Segment *da*: $$F_{ab} = i\left(\mathbf{l} \times \mathbf{B}\right) = 0 \qquad (5.13.7)$$
$$\tau_{ab} = rF\sin\theta = 0 \qquad (5.13.8)$$

The simplest DC machine

The resulting total induced torque is

$$\tau_{ind} = \tau_{ab} + \tau_{bc} + \tau_{cd} + \tau_{da} \qquad (5.14.1)$$

$$\tau_{ind} = \begin{cases} 2rilB & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases} \qquad (5.14.2)$$

Since $A_p \cong \pi r l$ and $\phi = A_p B$ (5.14.3)

$$\tau_{ind} = \begin{cases} \dfrac{2}{\pi}\phi i & \text{under the pole faces} \\ 0 & \text{beyond the pole edges} \end{cases} \qquad (5.14.4)$$

The torque in any real machine depends on the following factors:

1. The flux inside the machine;
2. The current in the machine;
3. A constant representing the construction of the machine.