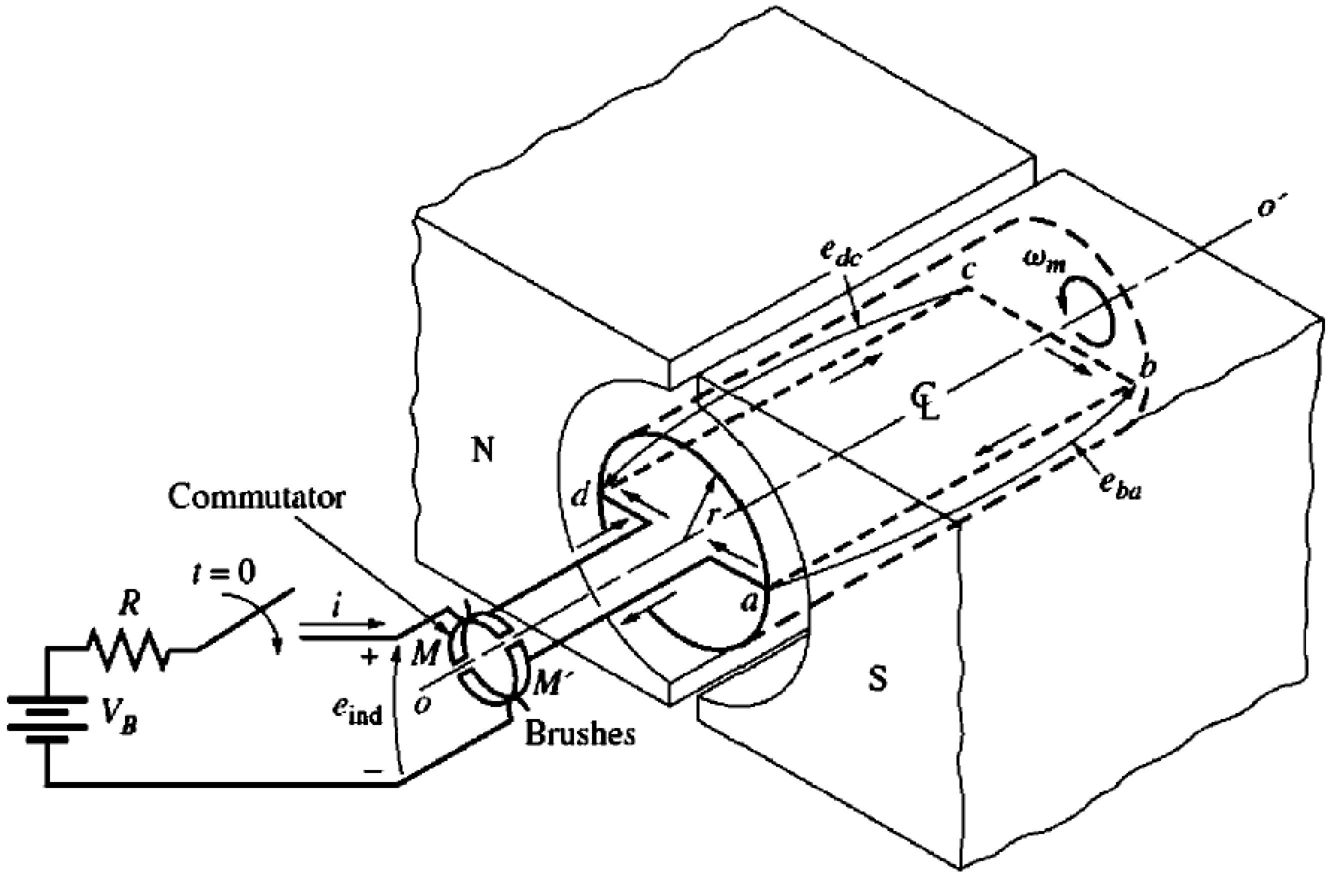

Example 8–1. Figure 8–6 shows a simple rotating loop between curved pole faces connected to a battery and a resistor through a switch. The resistor shown models the total resistance of the battery and the wire in the machine. The physical dimensions and characteristics of this machine are

$$r = 0.5\ \text{m} \qquad l = 1.0\ \text{m}$$
$$R = 0.3\ \Omega \qquad B = 0.25\ \text{T}$$
$$V_B = 120\ \text{V}$$

(*a*) What happens when the switch is closed?

(*b*) What is the machine's maximum starting current? What is its steady-state angular velocity at no load?

(*c*) Suppose a load is attached to the loop, and the resulting load torque is 10 N • m. What would the new steady-state speed be? How much power is supplied to the shaft of the machine? How much power is being supplied by the battery? Is this machine a motor or a generator?

(*d*) Suppose the machine is again unloaded, and a torque of 7.5 N • m is applied to the shaft in the direction of rotation. What is the new steady-state speed? Is this machine now a motor or a generator?

(*e*) Suppose the machine is running unloaded. What would the final steady-state speed of the rotor be if the flux density were reduced to 0.20 T?

Solution

(*a*) When the switch in Figure 8–6 is closed, a current will flow in the loop. Since the loop is initially stationary, $e_{ind} = 0$. Therefore, the current will be given by

$$i = \frac{V_B - e_{\text{ind}}}{R} = \frac{V_B}{R}$$

This current flows through the rotor loop, producing a torque

$$\tau_{\text{ind}} = \frac{2}{\pi}\phi i \qquad \text{CCW}$$

This induced torque produces an angular acceleration in a counterclockwise direction, so the rotor of the machine begins to turn. But as the rotor begins to turn, an induced voltage is produced in the motor, given by

$$e_{\text{ind}} = \frac{2}{\pi}\phi\omega$$

so the current i falls. As the current falls, $\tau_{\text{ind}} = (2/\pi)\phi i\downarrow$ decreases, and the machine winds up in steady state with $\tau_{\text{ind}} = 0$, and the battery voltage $V_B = e_{\text{ind}}$.

This is the same sort of starting behavior seen earlier in the linear dc machine.

(*b*) At *starting conditions*, the machine's current is

$$i = \frac{V_B}{R} = \frac{120\text{ V}}{0.3\ \Omega} = 400\text{ A}$$

At *no-load steady-state conditions*, the induced torque τ_{ind} must be zero. But $\tau_{\text{ind}} = 0$ implies that current i must equal zero, since $\tau_{\text{ind}} = (2/\pi)\phi i$, and the flux is nonzero. The fact that $i = 0$ A means that the battery voltage $V_B = e_{\text{ind}}$. Therefore, the speed of the rotor is

$$V_B = e_{\text{ind}} = \frac{2}{\pi}\phi\omega$$

$$\omega = \frac{V_B}{(2/\pi)\phi} = \frac{V_B}{2rlB}$$

$$= \frac{120\text{ V}}{2(0.5\text{ m})(1.0\text{ m})(0.25\text{ T})} = 480\text{ rad/s}$$

(*c*) If a load torque of 10 N • m is applied to the shaft of the machine, it will begin to slow down. But as ω decreases, $e_{\text{ind}} = (2/\pi)\phi\omega\downarrow$ decreases and the rotor current increases $[i = (V_B - e_{\text{ind}}\downarrow)/R]$. As the rotor current increases, $|\tau_{\text{ind}}|$ increases too, until $|\tau_{\text{ind}}| = |\tau_{\text{load}}|$ at a lower speed ω.

At steady state, $|\tau_{\text{load}}| = |\tau_{\text{ind}}| = (2/\pi)\phi i$. Therefore,

$$i = \frac{\tau_{\text{ind}}}{(2/\pi)\phi} = \frac{\tau_{\text{ind}}}{2rlB}$$

$$= \frac{10\text{ N}\cdot\text{m}}{(2)(0.5\text{ m})(1.0\text{ m})(0.25\text{ T})} = 40\text{ A}$$

By Kirchhoff's voltage law, $e_{ind} = V_B - iR$, so

$$e_{ind} = 120 \text{ V} - (40 \text{ A})(0.3 \ \Omega) = 108 \text{ V}$$

Finally, the speed of the shaft is

$$\omega = \frac{e_{ind}}{(2/\pi)\phi} = \frac{e_{ind}}{2rlB}$$

$$= \frac{108 \text{ V}}{(2)(0.5 \text{ m})(1.0 \text{ m})(0.25 \text{ T})} = 432 \text{ rad/s}$$

The power supplied to the shaft is

$$P = \tau\omega$$

$$= (10 \text{ N} \cdot \text{m})(432 \text{ rad/s}) = 4320 \text{ W}$$

The power out of the battery is

$$P = V_B i = (120 \text{ V})(40 \text{ A}) = 4800 \text{ W}$$

This machine is operating as a *motor,* converting electric power to mechanical power.

(d) If a torque is applied in the direction of motion, the rotor accelerates. As the speed increases, the internal voltage e_{ind} increases and exceeds V_B, so the current flows out of the top of the bar and into the battery. This machine is now a *generator*. This current causes an induced torque opposite to the direction of motion. The induced torque opposes the external applied torque, and eventually $|\tau_{load}| = |\tau_{ind}|$ at a higher speed ω.

The current in the rotor will be

$$i = \frac{\tau_{ind}}{(2/\pi)\phi} = \frac{\tau_{ind}}{2rlB}$$

$$= \frac{7.5 \text{ N} \cdot \text{m}}{(2)(0.5 \text{ m})(1.0 \text{ m})(0.25 \text{ T})} = 30 \text{ A}$$

The induced voltage e_{ind} is

$$e_{ind} = V_B + iR$$

$$= 120 \text{ V} + (30 \text{ A})(0.3 \ \Omega)$$

$$= 129 \text{ V}$$

Finally, the speed of the shaft is

$$\omega = \frac{e_{ind}}{(2/\pi)\phi} = \frac{e_{ind}}{2rlB}$$

$$= \frac{129 \text{ V}}{(2)(0.5 \text{ m})(1.0 \text{ m})(0.25 \text{ T})} = 516 \text{ rad/s}$$

(e) Since the machine is initially unloaded at the original conditions, the speed $\omega =$ 480 rad/s. If the flux decreases, there is a transient. However, after the transient is over, the machine must again have zero torque, since there is still no load on its shaft. If $\tau_{ind} = 0$, then the current in the rotor must be zero, and $V_B = e_{ind}$. The shaft speed is thus

$$\omega = \frac{e_{ind}}{(2/\pi)\phi} = \frac{e_{ind}}{2rlB}$$

$$= \frac{120\text{ V}}{(2)(0.5\text{ m})(1.0\text{ m})(0.20\text{ T})} = 600\text{ rad/s}$$

Notice that when the flux in the machine is decreased, its speed increases. This is the same behavior seen in the linear machine and the same way that real dc motors behave.