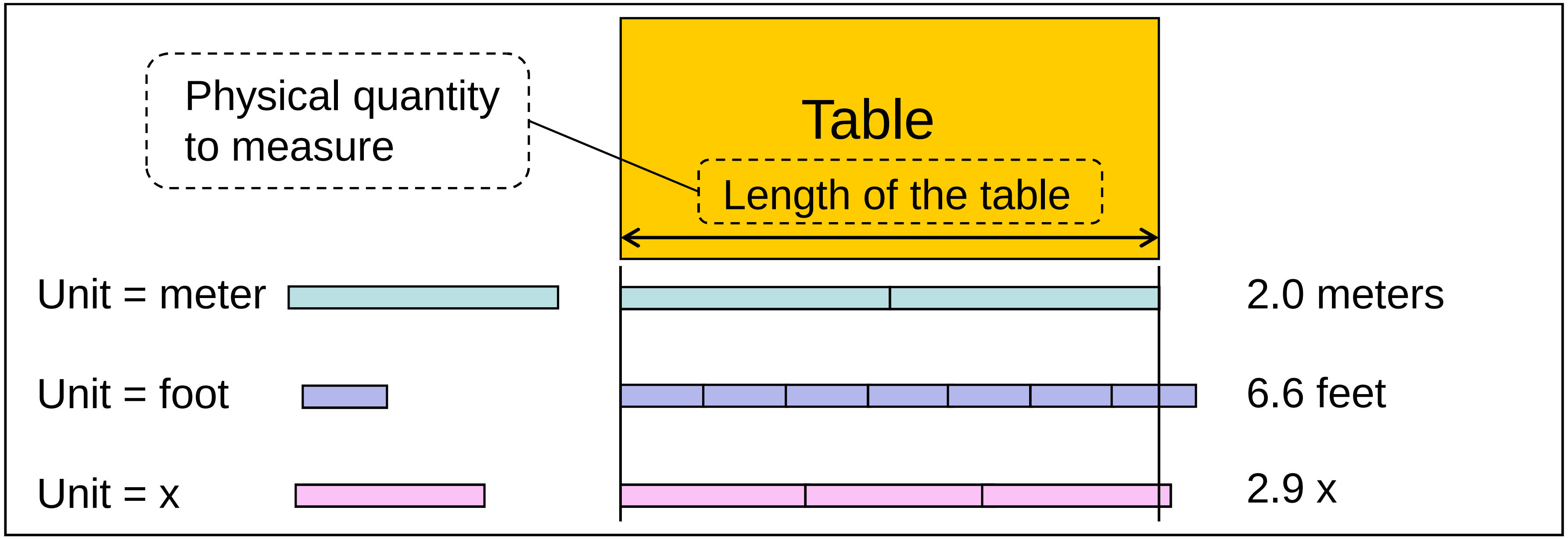
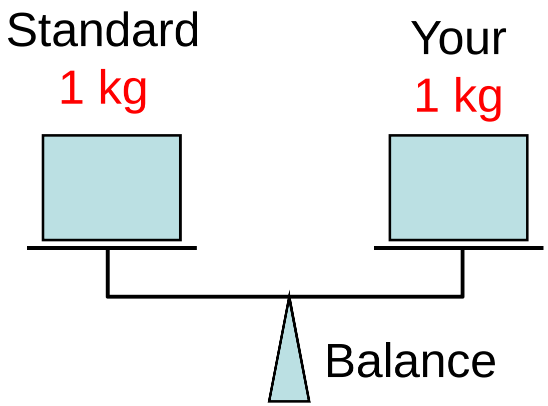
Chapter 1
Measurement

1-1 Measuring Things
Units



For the unit to be useful, people should agree on its definition.
We can use a standard or naturally occurring phenomenon to define a measuring unit.



Standard

A day is the time Earth takes to make one revolution about itself.

naturally occurring phenomenon

1-1 Measuring Things
Base physical quantities

There are many physical quantities, for example,
pressure, mass, force, …

We can derive the units of these quantities from the units of a small number of physical quantities called base physical quantities.

The selection of the base physical quantities is not unique.

A set of base physical quantities has been selected by an agreement.

In phys101, we will only deal with three base physical quantities: length, mass, and time. The units of all other quantities in phys101 can be derived from the units of these three quantities.

What is the unit of speed?

Derived physical quantity (speed) — Base physical quantities (length, time)

$$\text{speed} = \frac{\text{length}}{\text{time}}$$

$$\text{unit of speed} = \frac{\text{unit of length}}{\text{unit of time}} = \frac{\text{meter}}{\text{second}}$$

1-2 The International System of Units
Seven base quantities

The International System of Units (the SI system of units) was established in 1971.

The SI system of units has seven base quantities.

In phys101, we will only deal with three base physical quantities: length, mass, and time. The units of all other quantities in phys101 can be derived from the units of these three quantities.

Quantity	Unit name	Unit symbol
Length	meter	m
Time	second	s
Mass	kilogram	kg

For example, the SI unit of energy is the joule which can be written in terms of SI base units as follows

SI derived unit (joule) — SI base units ($\frac{kg\ m^2}{s^2}$)

$$\text{joule} = \frac{kg\ m^2}{s^2}$$

One joule is one kilogram-meter squared per second squared.

1-2 The International System of Units
Meter, second, and kilogram

The meter is defined as the length of the path traveled by light in a vacuum during a time interval of 1/299 792 458 of a second.
The time interval was chosen so that the speed of light c is exactly
c = 299 792 458 m/s.

One second is defined as the time taken by 9 192 631 770 oscillations of the light emitted by the cesium atom.

The SI standard of mass is a platinum-iridium cylinder kept a the International Bureau of Weights and Measures near Paris and assigned a mass of 1 kilogram.

The masses of atoms can be compared with one another more precisely than they can be compared with the standard kilogram. For this reason, **the mass of carbon atom is used as a second mass standard**. By agreement,

mass of carbon atom = 12 atomic mass unit (u).

$$1 \text{ u} = 1.66054 \times 10^{-27} \text{ kg}$$

1-2 The International System of Units
Scientific notation

Scientific notation is used to simplify expressing very large or very small quantities.

Quantity	In scientific notation	With prefixes
2 560 000 joule	2.56×10^{6} J	2.56 megajoule = 2.56 MJ
0.000 003 21 second	3.21×10^{-6} s	3.21 microsecond = 3.21 μs
5 460 meter	5.46×10^{3} m	5.46 kilometer = 5.46 km

The number is in scientific notation when it is expressed as some power of ten multiplied by another number between 1 and 10.

Factor	Prefix	Symbol
10^{9}	giga-	G
10^{6}	mega-	M
10^{3}	kilo-	k
10^{-2}	centi-	c
10^{-3}	mille-	m
10^{-6}	micro	μ
10^{-9}	nano-	n
10^{-12}	pico-	p

In some calculators, "exponent to ten" is written as "E"
6.52×10^{-7} written as 6.52 E-7

1-3 Changing Units

Conversion factor

$$3\text{ min} = (3\text{ min})(1) = (3\text{ min})\left(\frac{60\text{ s}}{1\text{ min}}\right) = 180\text{ s}$$

conversion factor

$$180\text{ s} = (180\text{ s})(1) = (180\text{ s})\left(\frac{1\text{ min}}{60\text{ s}}\right) = 3\text{ min}$$

A conversion factor is a ratio of units that is equal to one.

Multiplying any quantity by unity leaves the quantity unchanged.

Appendix D of your textbook gives conversion factors between SI and other system of units.

How many centimeters are there in 5.30 inches?

From Appendix D, 1 inch = 2.540 cm

$$5.30\text{ in} = (5.30\text{ in})\left(\frac{2.540\text{ cm}}{1\text{ in}}\right) = 13.5\text{ cm}$$

conversion factor

1-3 Changing Units
Example 1

A car moves at speed of 1.14 miles per minute. Use the following conversion factors to find its speed in kilometers per hour (km/h)

1 mile = 5280 feet

1 foot = 0.3048 meter

Solution

$$1.14\ \frac{\text{miles}}{\text{min}} = \left(1.14\ \frac{\cancel{\text{miles}}}{\cancel{\text{min}}}\right)\left(\frac{5280\ \cancel{\text{feet}}}{1\ \cancel{\text{mile}}}\right)\left(\frac{0.3048\ \cancel{\text{m}}}{1\ \cancel{\text{foot}}}\right)\left(\frac{1\ \text{km}}{1000\ \cancel{\text{m}}}\right)\left(\frac{60\ \cancel{\text{min}}}{1\ \text{h}}\right)$$

$$= 110\ \text{km/h}$$

1-3 Changing Units
Example 2

How many liters are there in one US fluid gallon, if

1 US fluid gallon = 231 in^3

1 in = 2.540 cm

1 L = 1000 cm^3?

Solution

$$1 \text{ gallon} = (1 \text{ gallon})\left(\frac{231 \text{ in}^3}{1 \text{ gallon}}\right)\left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)^3\left(\frac{1 \text{ L}}{1000 \text{ cm}^3}\right)$$

$$= 3.79 \text{ L}.$$

1-4 Dimensional Analysis
Dimensions

The dimension of a quantity is its property that we measure.
For distances, we measure length. → Dimension of distance = length
For periods, we measure time. → Dimension of period = time

Although any quantity might be measured in different units, it has one unique dimension. For example, a distance can be measured in meters or in feet. The dimension of distance is unique = length.

All quantities in phys101 can be expressed in terms of three dimensions:

Length (L)
Time (T)
Mass (M)

The brackets [ ] is used to denote the dimension of a quantity.

[acceleration] stands for the dimension of acceleration

[distance] = Length = L

$$[\text{speed}] = \frac{\text{Length}}{\text{Time}} = \frac{L}{T}$$

[pure number] = 1
[angle] = 1
[argument of a trigonometric function] = 1

Quantities with dimension 1 are called dimensionless quantities

1-4 Dimensional Analysis
Adding quantities

Quantities can be added or subtracted only if they have the same dimensions.

Acceptable

$$x + v\,t$$

$[x] = L$

$[v\,t] = \frac{L}{T}\,T = L$

The terms have the same dimensions.

Not acceptable

$$\cancel{x + a\,t}$$

$[x] = L$

$[a\,t] = \frac{L}{T^2}\,T = \frac{L}{T}$

The terms have different dimensions.

Given

x = distance	$[x] = L$
t = time	$[t] = T$
v = velocity	$[v] = \frac{L}{T}$
a = acceleration	$[a] = \frac{L}{T^2}$

1-4 Dimensional Analysis
Equating quantities

The terms on both sides of an equation must have the same dimensions.

Acceptable

$$v = a\,t$$

$$[v] = \frac{L}{T}$$

$$[a\,t] = \frac{L}{T^2}\,T = \frac{L}{T}$$

Both sides have the same dimensions.

Not acceptable

~~$v = x\,t$~~

$$[v] = \frac{L}{T}$$

$$[x\,t] = LT$$

The two sides have different dimensions.

Given

x = distance	$[x] = L$
t = time	$[t] = T$
v = velocity	$[v] = \frac{L}{T}$
a = acceleration	$[a] = \frac{L}{T^2}$

1-4 Dimensional Analysis

Equating quantities

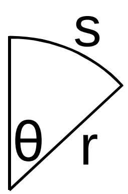
The terms on both sides of an equation must have the same dimensions.

Not Acceptable

~~$\text{area} = \theta\, r$~~

$[\text{area}] = L^2$

$[\theta\, r] = 1\, L = L$



The two sides have different dimensions.

Not acceptable

~~$\text{volume} = \pi\, r^2$~~

$[\text{volume}] = L^3$

$[\pi\, r^2] = 1\, L^2 = L^2$

The two sides have different dimensions.

Given

r = radius $\quad [r] = L$

$[\text{area}] = [\text{distance}^2] = L^2$

$$\theta = \frac{s}{r}$$

$$[\theta] = \frac{[\text{distance}]}{[\text{distance}]} = 1$$

$[\text{volume}] = [\text{distance}^3] = L^3$

1-4 Dimensional Analysis

Example 3

Suppose the distance x is given in terms of acceleration a and time t as in the following expression

$$x = k\, a^n\, t^m,$$

where k is a dimensionless constant. Find m and n.

Solution

Both sides of the equation should have the same dimensions.

$$[x] = L$$

$$[k\, a^n\, t^m] = (1)\left(\frac{L}{T^2}\right)^n T^m = L^n\, T^{m-2n}$$

$$L = L^n\, T^{m-2n}$$

$$L = L^1\, T^0$$

$$n = 1$$

$$m - 2n = 0 \rightarrow m = 2n = 2$$

$$x = k\, a\, t^2$$

1-4 Dimensional Analysis

Example 4

Suppose the acceleration a of a particle moving with uniform speed v in a circle of radius r is given by

$$a = k\, v^n\, r^m,$$

where k is a dimensionless constant. Find m and n.

Solution

Both sides of the equation should have the same dimensions.

$$[a] = \frac{L}{T^2}$$

$$[k\, v^n\, r^m] = (1)\left(\frac{L}{T}\right)^n L^m = \frac{L^{n+m}}{T^n}$$

$$\frac{L}{T^2} = \frac{L^{n+m}}{T^n}$$

$$n = 2$$

$$n + m = 1 \rightarrow m = 1 - n = -1$$

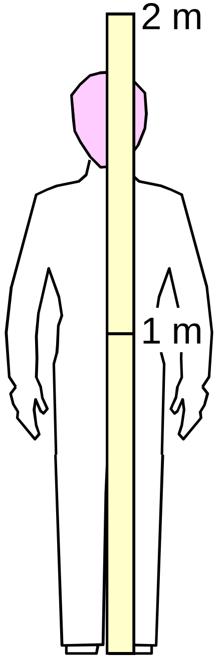
$$a = k\,\frac{v^2}{r}$$

1-5 Significant Figures

Measurements

Numbers

Exact	Inexact
Five fingers in your hand	Any measurement

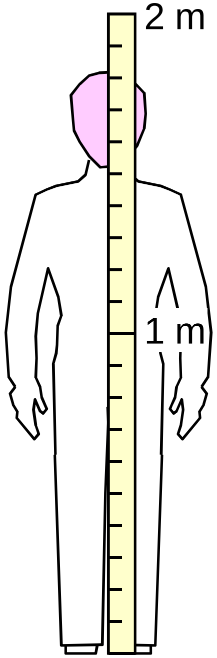


Height = 1.8 m

Correct digit: 1

doubtful digit: 8

2 significant figures

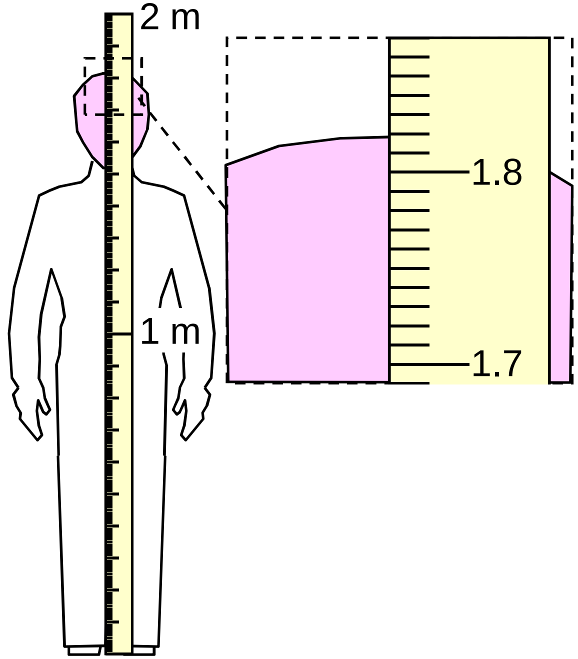


Height = 1.82 m

Correct digits: 1.8

doubtful digit: 2

3 significant figures



Height = 1.819 m

Correct digits: 1.81

doubtful digit: 9

4 significant figures

Number of significant figures depends on the instrument used in the measurement.

1-5 Significant Figures
Number of significant figures

A significant figure is a digit in a number.

↓
15.07 — This number has four significant figures.

The least significant figure is the significant figure farthest to the right.

↓
10.68 — The 8 is the least significant figure.

All leading zeros are <u>not</u> significant figures.

↓
0.00064 — This number has two significant figures.

All trailing zeros to the right of the decimal point are significant figures

↓
12.000 — This number has five significant figures.

↓
3000 — The trailing zeros to the left of the decimal point might or might not be significant figures.
The zeros might not be significant and they are just being used to locate the decimal point.
However, in this course, we will take them as significant figures.

1-5 Significant Figures
Rounding off

When the left-most of the digits to be discarded is 5 or more, the last remaining digit is rounded up; otherwise it is retained as is.

Round up

$$2.36502 = 2.37$$

5 or more

Do not change

$$80.76493 = 80.76$$

Less than 5

1-5 Significant Figures
Multiplication or division

When multiplying or dividing quantities, the result should have the same number of significant figures as the quantity with the lowest number of significant figures.

$$2.31563 \times 0.25 = 0.58$$

6 significant figures (2.31563) · 2 significant figures (0.25, 0.58)

Your calculator gives 0.578908.

You should round off your answer to two significant figures.

$$2.751 \div 3.22 = 0.854$$

4 significant figures (2.751) · 3 significant figures (3.22, 0.854)

Your calculator gives 0.854348.

You should round off your answer to three significant figures.

1-5 Significant Figures
Addition or subtraction

When adding or subtracting quantities, the least significant figure in the result has the same position relative to the decimal point as that of the quantity whose least significant figure is farthest to the left.

Least significant figure

9.1 ← Least significant figure

+ 7.265

16.4

Least significant figure

Your calculator gives 16.365.

Since 9.1 is the quantity with its least significant figure farthest to the left relative to the decimal point, your answer should be rounded off so that the position of its least significant figure match that of 9.1.

Least significant figure

1.02 ← Least significant figure

- 0.9743

0.05

Least significant figure

Your calculator gives 0.0457.

Since 1.02 is the quantity with its least significant figure farthest to the left relative to the decimal point, your answer should be rounded off so that the position of its least significant figure match that of 1.02.

1-6 Order-of-Magnitude Calculations

Order-of-magnitude

An order-of magnitude calculation is a rough estimate that is accurate to within a factor of about 10.

It is useful if you want to get a quick rough answer.

You may use this estimate to check your detailed calculation.

The order of magnitude of a quantity is the power of ten when the quantity is expressed in scientific notation.

$A = 7\,600 = 7.6 \times 10^3$	The order of magnitude of A is 3
$B = 3\,700 = 3.7 \times 10^3$	The order of magnitude of B is 3
$A = 7\,600 \approx 10\,000 = 10^4$	The nearest order of magnitude of A is 4
$B = 3\,600 \approx 1\,000 = 10^3$	The nearest order of magnitude of B is 3

1-6 Order-of-Magnitude Calculations
Example 5

Estimate the number of heart beats during an average human lifetime.

Solution

Average human lifetime ≈ 70 years

Average heart beat per minute ≈ 70 beats

Number of days per year = 365 days ≈ 400 days .

Number of hours per day = 24 hours ≈ 20 hours .

Number of minutes per hour = 60 minutes

Number of heart beats during human lifetime ≈

$$\left(70\ \frac{\text{beats}}{\cancel{\text{min}}}\right)\left(60\ \frac{\cancel{\text{min}}}{\cancel{\text{h}}}\right)\left(20\ \frac{\cancel{\text{h}}}{\cancel{\text{day}}}\right)\left(400\ \frac{\cancel{\text{day}}}{\cancel{\text{yr}}}\right)\left(70\ \frac{\cancel{\text{yr}}}{\text{liftime}}\right) = 2\times10^{9}\ \frac{\text{beats}}{\text{lifetime}}\ .$$

Reasonable!

Compare this estimate with the detailed calculation

$$\left(70\ \frac{\text{beats}}{\text{min}}\right)\left(60\ \frac{\text{min}}{\text{h}}\right)\left(24\ \frac{\text{h}}{\text{day}}\right)\left(365\ \frac{\text{day}}{\text{yr}}\right)\left(70\ \frac{\text{yr}}{\text{liftime}}\right) = 2.5\times10^{9}\ \frac{\text{beats}}{\text{lifetime}}\ .$$

1-7 Density
Definition

The density of a substance ρ is the amount of mass contained in a unit volume

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\rho = \frac{m}{V}$$

ρ is the Greek letter rho