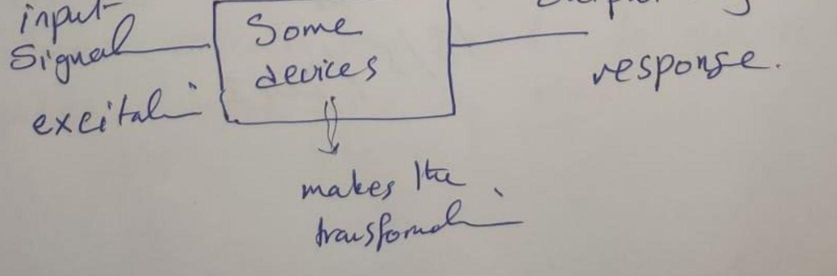
Signals and Systems

Signal: is defined as a physical Phenomenon which Carries Some information or data. It is usually a function of time The value of the Signal at all time may be real. in this Case it is Called a real valued Signal. If the value of the Signal at any instant of time is Complex it is Called a Complex valued Signal.

System: A system Can be viewed as any process that results in transformation of a signal. Thus the system has an input Signal an output Signal related to the input through the System transformation

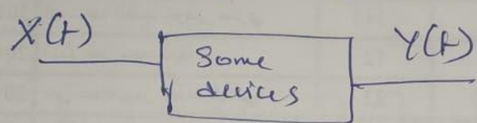


In general we have two types of signals.

1) Continous time Signals (CT) denoted by $X(t)$, $Y(t)$, $h(t)$, $Z(t)$.

is a signal which is specified for every instant of time 't'

A continous time system is a system in which continous time signals is transformed into continous time output signals.

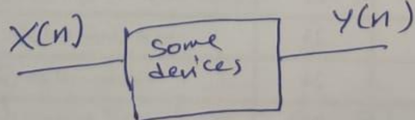


2) Discrete time signals (DT Signals)

Signals that are specified at discrete value of time denoted by $X(n)$, $Y(n)$, $h(n)$ — —

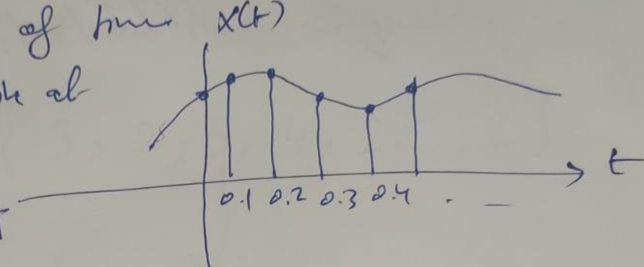
where n is an integer.

Discrete time systems is a system that transforms discrete time inputs to discrete time outputs.



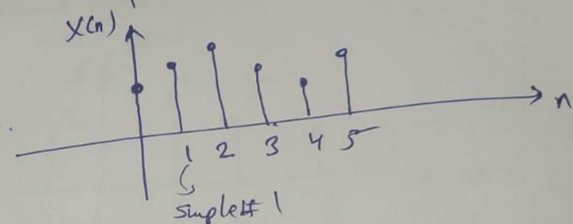
In general a discrete time signal, is a continuous time signal sampled at some instants of time

if we take a sample at every 0.1 sec:



we obtain a DT signal x(n)

here n denotes the number of the sample.



A discrete time signal can be represented in two ways:

1) To write the signal as a function

for example;

$$x(n) = \begin{cases} (1/a)^n & n \geq 0 \\ 0 & n < 0. \end{cases}$$

2) To write the signal as a sequence:

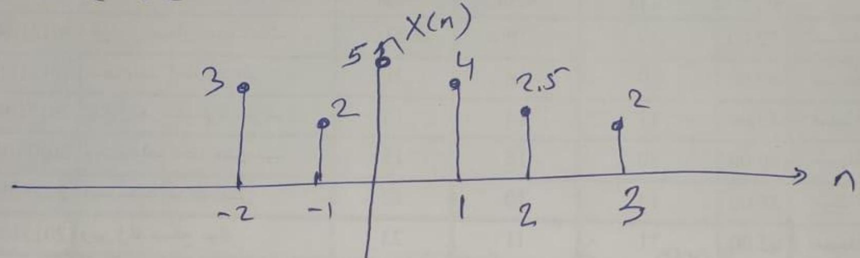
$$x(n) = \{ \underset{\uparrow}{1}, \frac{1}{a}, \frac{1}{a^2}, \frac{1}{a^3} \cdots \}.$$

and an arrow indicates the value of $X(n)$ at $n=0$.

for Example: Draw the Signal.

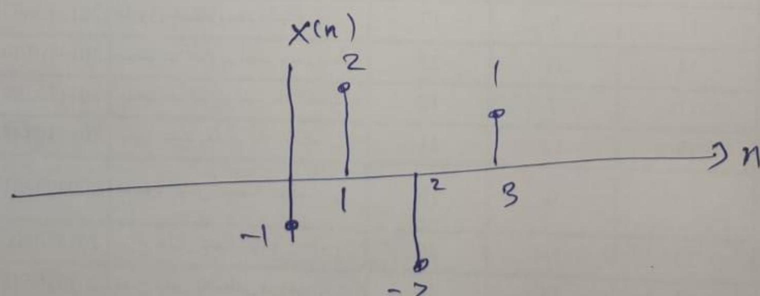
$$X(n) = \{3, 2, \underset{\uparrow}{5}, 4, 2.5, 2\}$$

This can be drawn as:



If no arrow is marked for a sequence this means that the sequence begins at "0"

EX $X(n) = \{\overset{0}{-1}, \overset{1}{2}, \overset{2}{-2}, \overset{3}{1}\}$.



Basic Continuous time Signals.

① Unit impulse Signal (Dirac delta).

denoted by $\delta(t)$

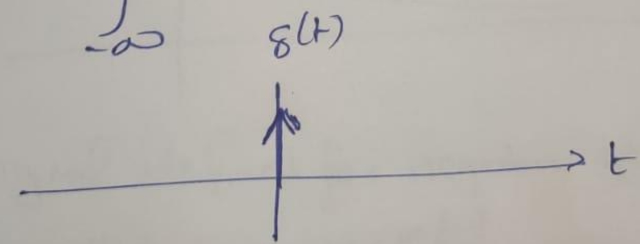
It is defined as having zero amplitude every where except at $t=0$. where it is infinitely large. is such a way it have Unit area under its Curve.

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$

The area Under $\delta(t)$ is one.

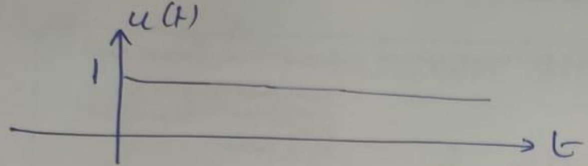
which means

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$



② Unit step signal. denoted by $u(t)$

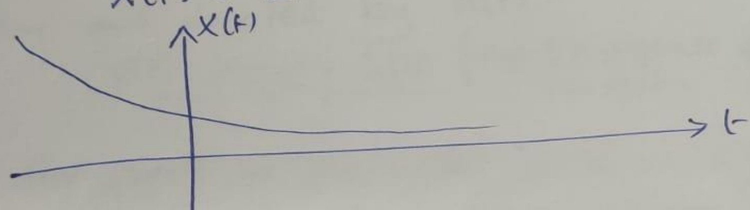
$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



Any signal which is defined for positive values only it is multiplied by u(t).

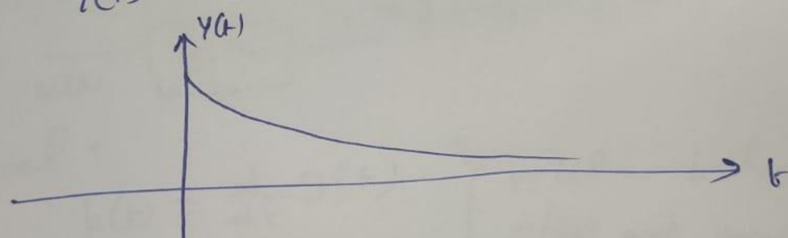
for example:

$x(t) = e^{-at}$ is drawn as below.



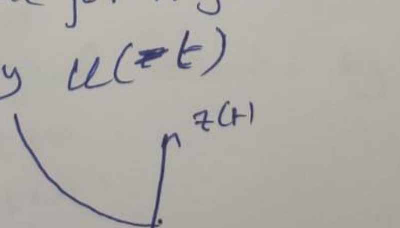
but if we want to draw:

$y(t) = e^{-at} u(t)$ is drawn as:



Any signal defined for negative values only it is multiplied by $u(-t)$

$z(t) = e^{-at} u(-t)$



The relation between $\delta(t)$ and $u(t)$ is defined as:

$$\delta(t) = \frac{d}{dt} u(t)$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau .$$

If the input to a system is unit impulse the output is called impulse response of the system and denoted by $h(t)$.

$$\delta(t) \longrightarrow \boxed{\text{system}} \longrightarrow h(t) \quad \{\text{impulse response of the system}\}.$$

If the input to the system is a unit step then the output is called step response of the system denoted by $s(t)$.

$$u(t) \longrightarrow \boxed{\text{system}} \longrightarrow s(t) \quad \{\text{step response}\}.$$

in general:

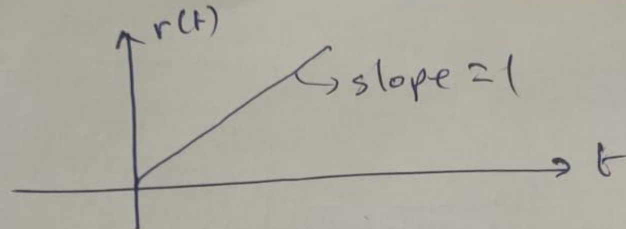
$$\left.\begin{aligned} h(t) &= \frac{d}{dt} s(t) \\ s(t) &= \int_{-\infty}^{t} h(\tau)\, d\tau \end{aligned}\right\} \text{relation between step and impulse response.}$$

(3) Unit ramp signal denoted by $r(t)$.

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

also it can be defined as $r(t) = t\,u(t)$.

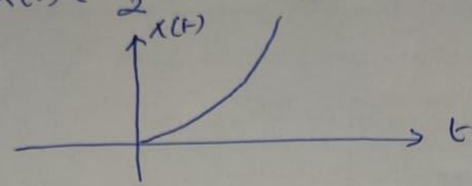
⑧



$$\frac{d}{dt} r(t) = u(t)$$

4 Unit Parabolic Signal

$x(t) = \frac{1}{2}t^2 \quad t \geq 0.$

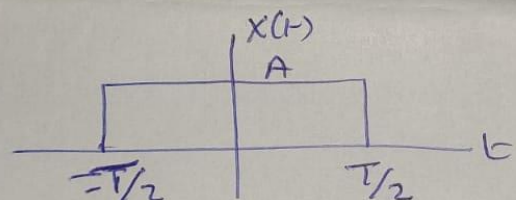


$\frac{dx(t)}{dt} = t \cdot$ for $t \geq 0.$

$\frac{dx(t)}{dt} = ramp(t).$

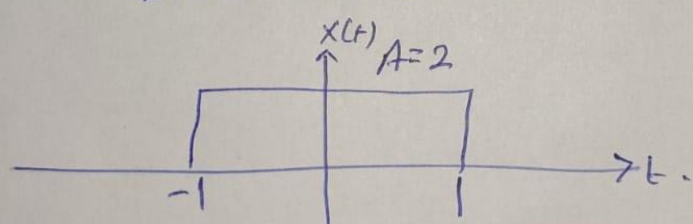
5 Rectangular pulse Signal.

denoted by $x(t) = A\,rect(t/T).$



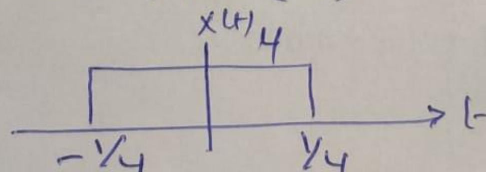
for example:-

Draw $x(t) = 2\,rect(t/2).$



$T = 2$
$A = 2$

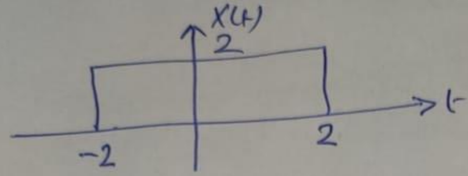
Ex $x(t) = 4\,rect(2t)$



$A = 4$
$T = 1/2$

(2)

$x(t) = 2\, \text{rect}(t/4)$.

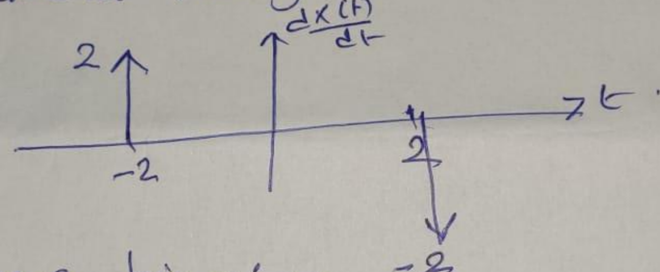


T=4
A=2

What is the derivative of x(t).

$\frac{d}{dt} x(t)$.

when ever we have a jump in the Signal the derivative at that jump is an impulse.



(6) Unit area triangular Signal $x(t) = tri(t)$.

it is defined as:-

$$tri(t) = \begin{cases} 1-|t| & |t| \leq 1 \\ 0 & |t| > 1 \end{cases}$$

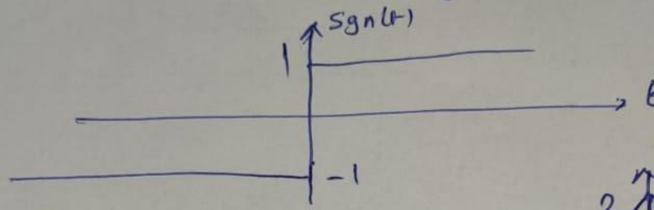
also it Can be written as:

$$tri(t) = \begin{cases} 1-t & 0 \leq t < 1 \\ 1+t & -1 \leq t < 0. \end{cases}$$

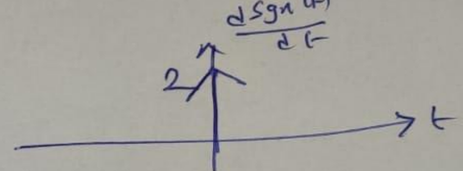
(7) Unit Signum Signal

denoted by $sgn(t)$

$$sgn(t) = \begin{cases} 1 & t>0 \\ 0 & t=0 \\ -1 & t<0. \end{cases}$$



$$\frac{d\,sgn(t)}{dt} = ??$$



(8) Unit Sinc Signal

$x(t) = Sinc(t)$.

It is defined as:

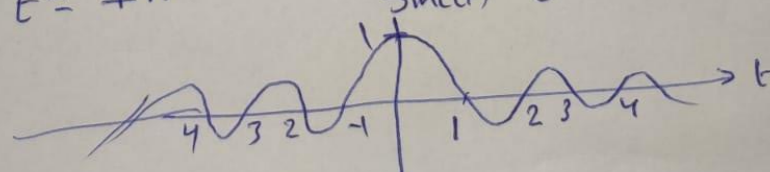
$$Sinc(t) = \frac{\sin \pi t}{\pi t}.$$

This Signal Cross the time axis when:

$$\frac{\sin \pi t}{\pi t} = 0 \Rightarrow \sin \pi t = 0$$

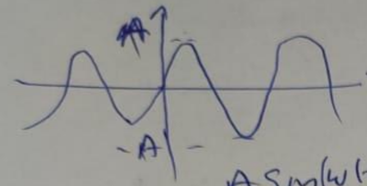
$$\Rightarrow \pi t = \mp n\pi$$

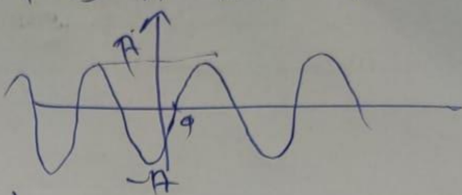
$\Rightarrow t = \mp n$ n is an integer.

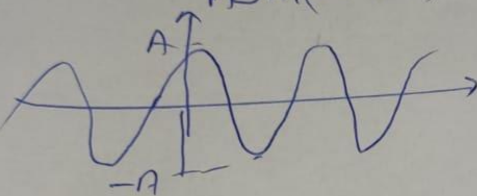


(9) Sinusoidal Signal

$x(t) = A\sin(\omega t \mp \phi) \cdot A\sin(\omega t - \phi)$







(10) Real Exponential Signal.

In general a complex exponential can be written as $x(t) = e^{st}$ $s = \sigma + j\omega$.

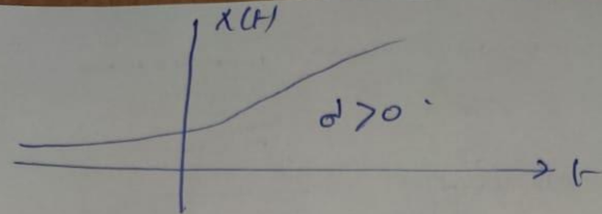
So $x(t) = e^{\sigma t} e^{j\omega t}$.

If $\omega = 0$ it is called a real exp.

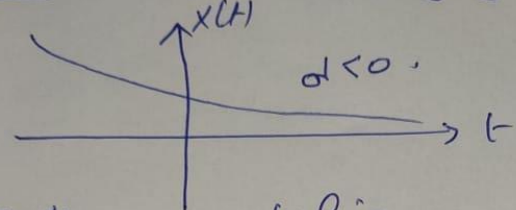
$\Rightarrow x(t) = e^{\sigma t}$.

depending on σ we have two exponentials :-

$\sigma > 0$ the signal is a growing exponential.



for $\sigma<0$ It is a decaying exponential.



(ii) Complex exponential:-

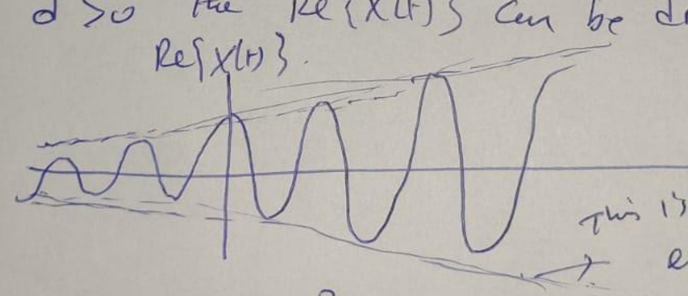
$x(t) = e^{st} = e^{\sigma t} e^{j\omega t}$.

$x(t) = e^{\sigma t}(\cos\omega t + j\sin\omega t)$.

$Re\{x(t)\} = e^{\sigma t}\cos\omega t$

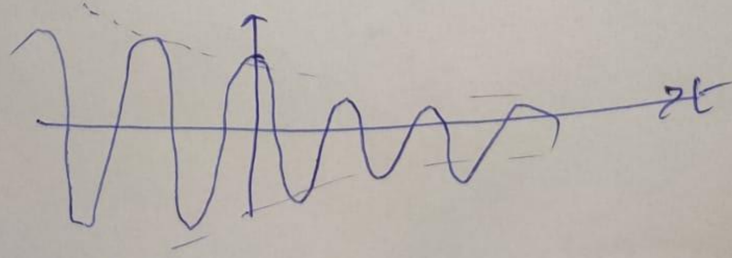
$Im\{x(t)\} = e^{\sigma t}\sin\omega t$.

for $\sigma > 0$ the $Re\{x(t)\}$ can be drawn as.



This is called envelope $= e^{\sigma t}$.

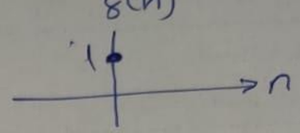
for $\sigma<0$ $Re\{x(t)\}$.



Basic Discrete time Signals :-

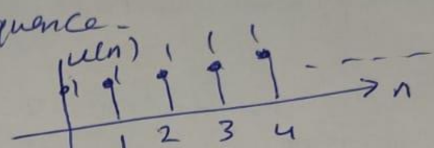
(1) Unit impulse sequence

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



(2) Basic Unit Step sequence.

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

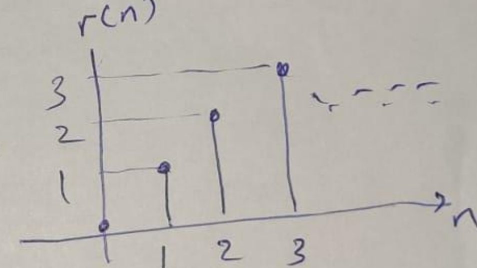


Any discrete time sequence defined for positive values only "$n \geq 0$" it is multiplied by $u(n)$.

And if it is defined for negative values only it is multiplied by $u(-n)$.

(3) Unit ramp sequence.

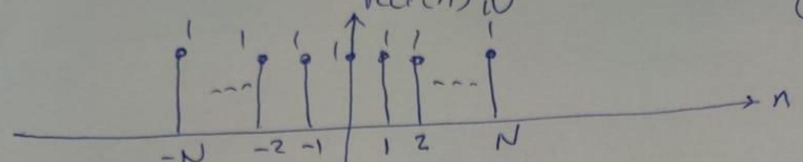
$$r(n) = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$



(4) Unit rectangular sequence.

$$\text{rect}(n)_N = \begin{cases} 1 & |n| \leq N \\ 0 & |n| > N \end{cases}$$

$$\text{rect}(n)_N = 1 \quad \text{for } -N \leq n \leq N$$



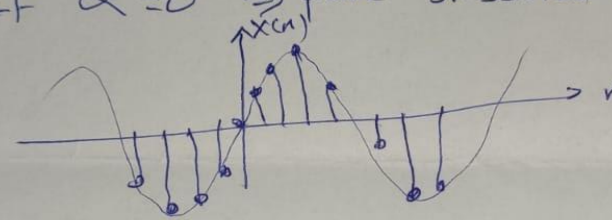
(5) Sinusoidal sequence -

in general it can be written as:-

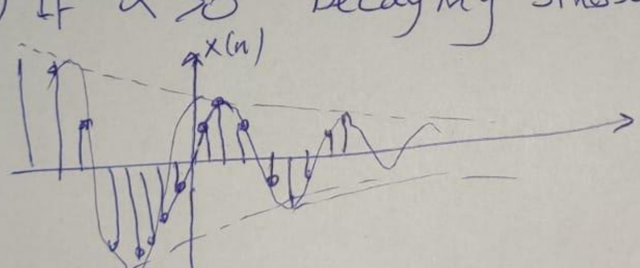
$$X(n) = A e^{-\alpha n} \sin(\omega_0 n + \phi).$$

A and α are real ϕ: phase shift.

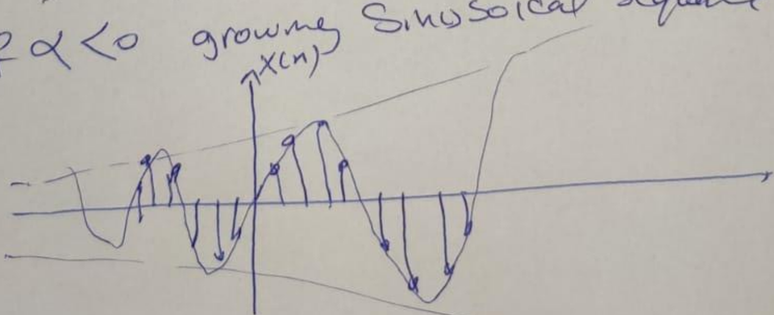
1) If $\alpha = 0 \Rightarrow$ pure Sinusoidal sequence.



2) If $\alpha > 0$ Decaying Sinusoidal sequence



3) If $\alpha < 0$ growing Sinusoidal sequence.

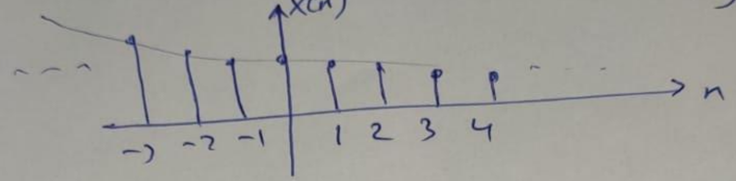


(6) Discrete time real exponential sequence

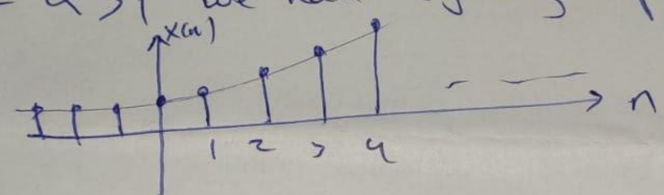
$x(n) = A\alpha^n$. A and α are real.

depending on α:

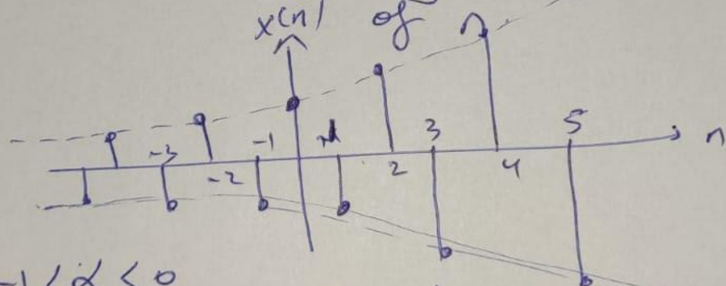
1) If $0 < \alpha < 1$ $x(n)$ is a decaying exp.



2) If $\alpha > 1$ we have a growing exp.

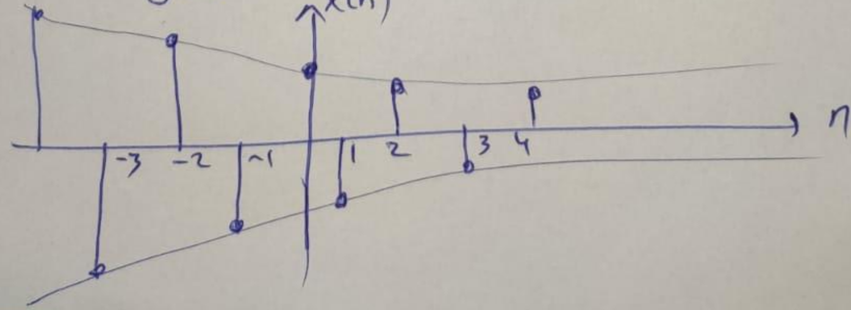


3) $\alpha < -1$ growing for alternate value of n



(4) $-1 < \alpha < 0$

decaying for alternate values of n.



Basic operations on CT Signals: (9)

① Addition: $x_1(t)$ and $x_2(t)$ are two CT Signals. then:

$$x(t) = x_1(t) + x_2(t)$$

Can be obtained by point by point addition.

② Multiplication of two Signals.

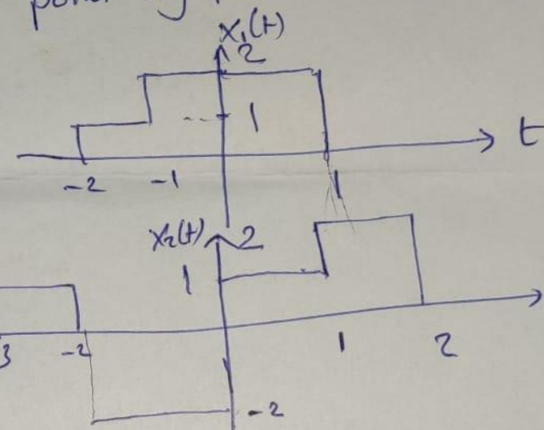
$$x(t) = x_1(t) \times x_2(t)$$

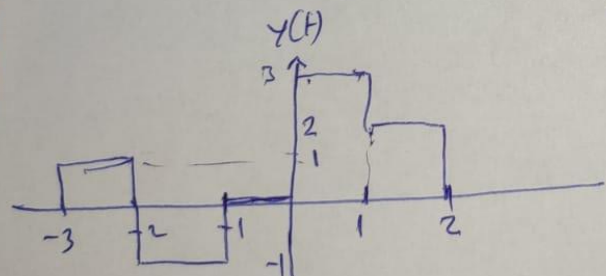
is obtained by point by point multiplication.

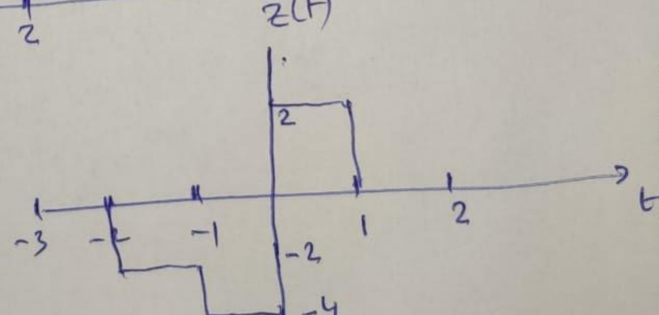
EX

$$y(t) = x_1(t) + x_2(t)$$

$$z(t) = x_1(t) \times x_2(t)$$





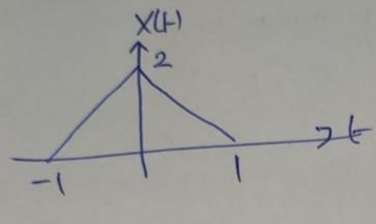


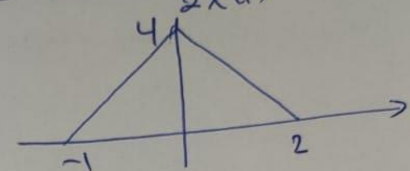
(3) Amplitude Scaling

Multiplication of a signal by factor A

given $x(t)$ find $A\,x(t)$

Ex





(4) Time Scaling of CT Signal.

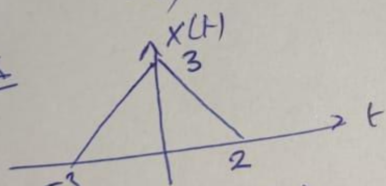
The Compression or expansion of a signal in time.

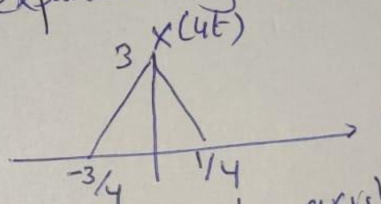
Given a signal $x(t)$ then :-

$x(at) \rightarrow$ is time compressed by a factor a

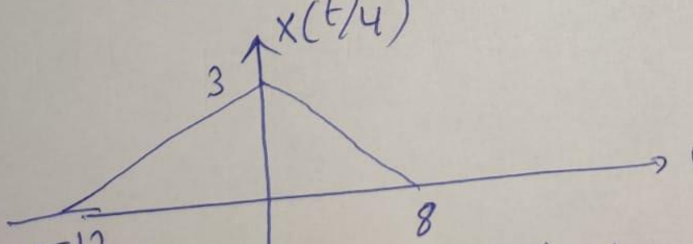
$x(t/a) \rightarrow$ is time expanded by a factor a

Ex





divide the time axis by 4

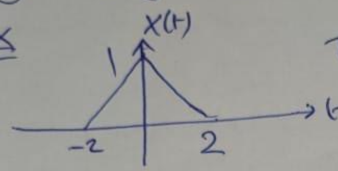


multiply the time axis by 4.

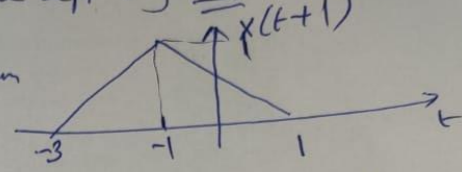
5 Time shifting of CT signals.

1) $x(t)$ is given then $x(t+t_0)$ is obtained by shifting $x(t)$ to the left by $\underline{t_0}$

EX

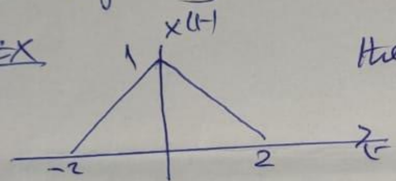


Then

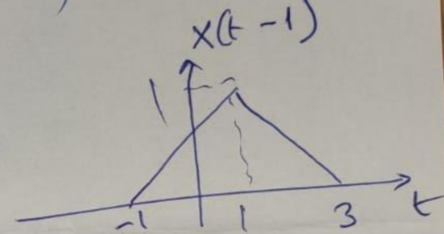


2) $x(t)$ is given then $x(t-t_0)$ is shifted to the right by $\underline{t_0}$

EX

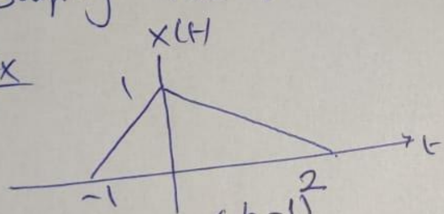


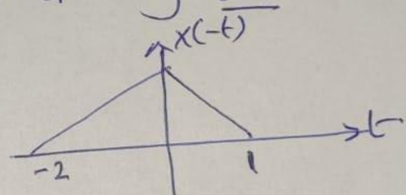
then

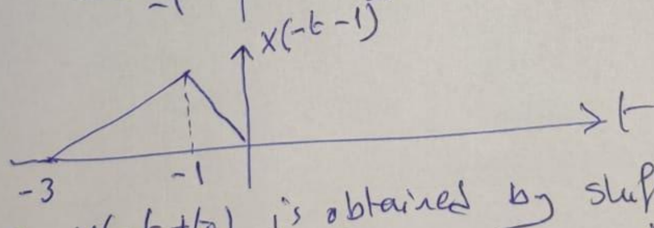


3) $x(t)$ is given then $x(-t-t_0)$ is obtained by shifting $x(-t)$ to the left by $\underline{t_0}$

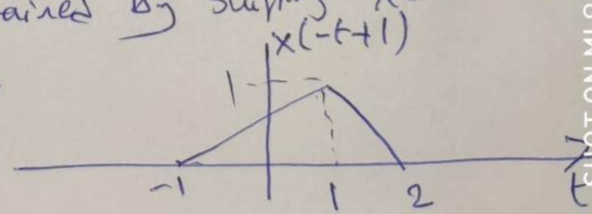
EX







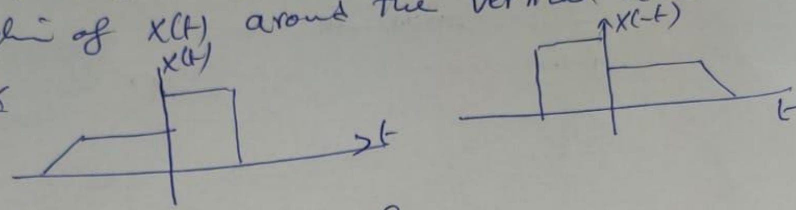
4) $x(-t+t_0)$ is obtained by shifting $x(-t)$ to the right by $\underline{t_0}$



(6) Signal reflection or folding

$x(t)$ is given then $x(-t)$ is obtained by reflection of $x(t)$ around the vertical axis.

EX



(7) Inverted CT signal.

$x(t)$ is given then $-x(t)$ is obtained by reflecting $x(t)$ around the time axis.

(8) Multiple transformation

Given $x(t)$ then how to obtain:

$$y(t) = A\,x\left(\frac{-t-t_0}{a}\right).$$

to plot $y(t)$ the sequence of transformation must be as follows:-

(1) Write $y(t)$ as:

$$y(t) = A\,x\left(\frac{-t}{a} - \frac{t_0}{a}\right).$$

(2) obtain $A\,x(t)$ → Amplitude scaling

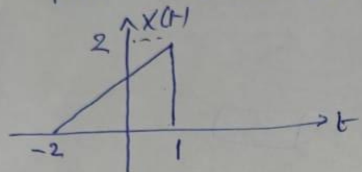
(3) plot $A\,x(-t)$ → reflection.

(4) plot $A\,x\left(-t - \frac{t_0}{a}\right)$ → time shifting

(5) plot $A\,x\left(-t/a - \frac{t_0}{a}\right)$ → Time scaling.

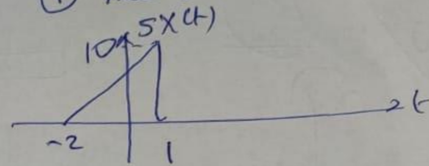
EX Given $x(t)$ as below.

Plot $y(t) = 5x(-3t+1)$.

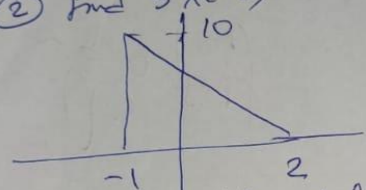


to obtain $y(t)$:

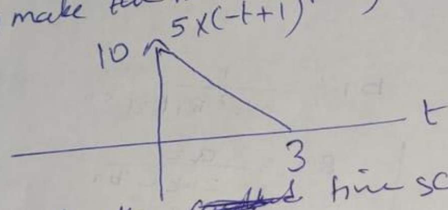
(1) find $5x(t)$



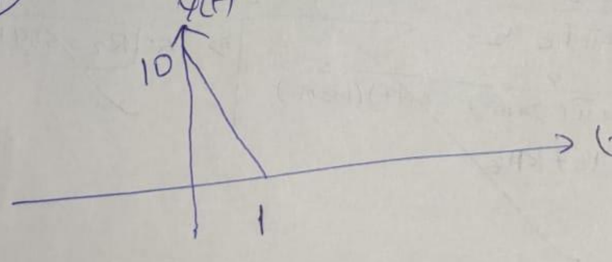
(2) find $5x(-t)$



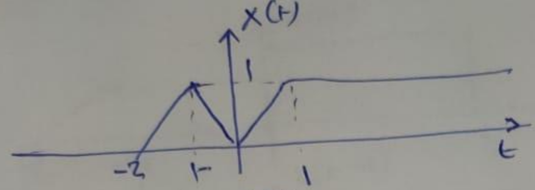
(3) make the time shifting $5x(-t+1)$

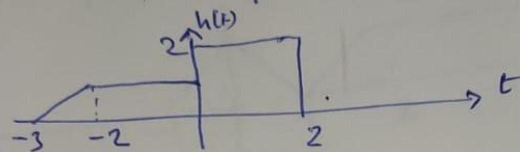


(4) make the ~~[illegible]~~ time scaling

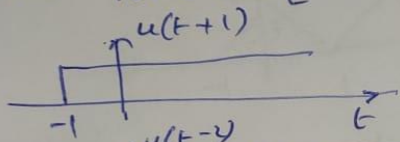


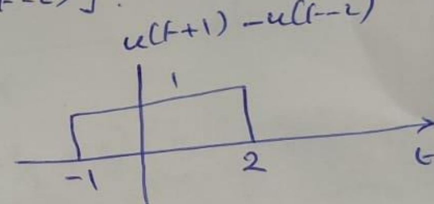
Ex Given $x(t)$ and $h(t)$ as below:

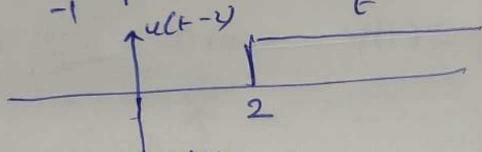


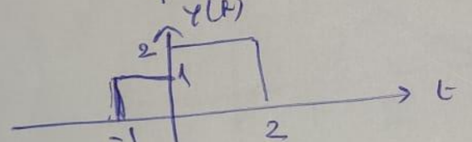


a) Draw $y(t) = h(t)[u(t+1) - u(t-2)]$.

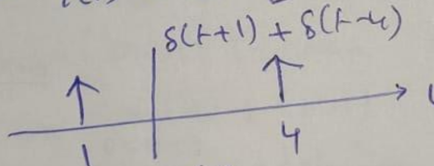


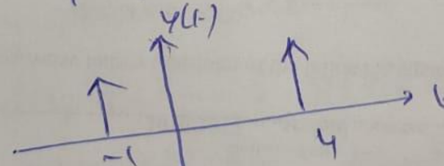






b) Draw $y(t) = x(t)[\delta(t+1) + \delta(t-4)]$.





1

Note1

$$x(t) \times \delta(t) = x(0)\,\delta(t).$$

$$x(t) \times \delta(t-1) = x(1)\,\delta(t-1)$$

$$x(t) \times \delta(t+1) = x(-1)\,\delta(t+1).$$

Note2

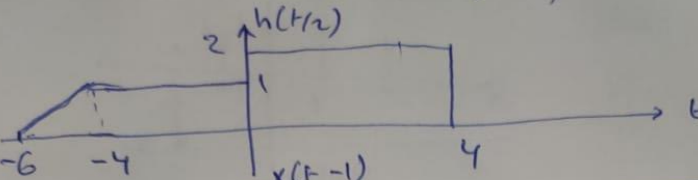
$$\int_{-\infty}^{\infty} \delta(t) = 1$$

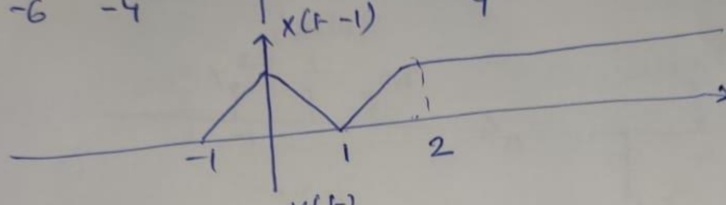
$$\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$$

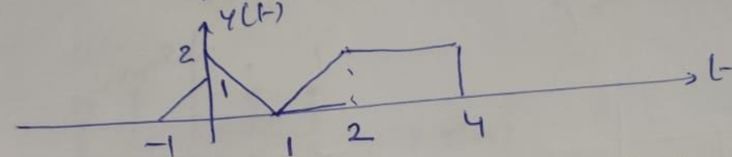
$$\int_{-\infty}^{\infty} x(t)\,\delta(t-1)\,dt = x(1)$$

$$\int_{-\infty}^{\infty} x(t)\,\delta(t+1)\,dt = x(-1)$$

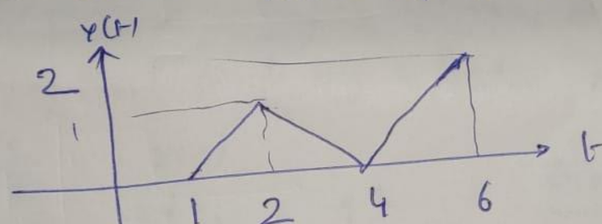
c) Draw $y(t) = x(t-1)\, h(t/2)$.







d) HW Draw $y(t) = x(-t/2 + 2)\,(h(t-4)$.



Basic operations on DT Signals:—

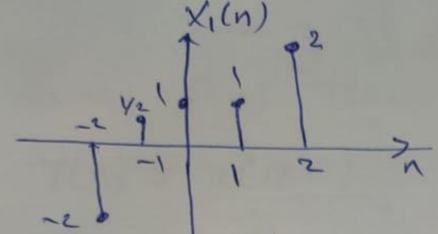
1. Addition: Given $x_1(n), x_2(n)$ then $x(n) = x_1(n) + x_2(n)$ is obtained by point by point addition.
2. Multiplication: $x(n) = x_1(n) \times x_2(n)$. is obtained by point by point multiplication

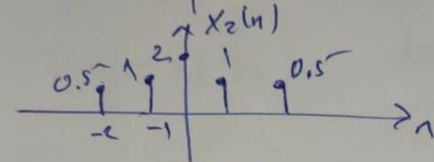
EX

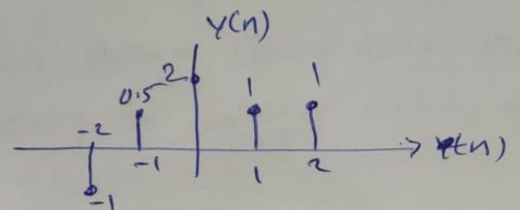
find (3)

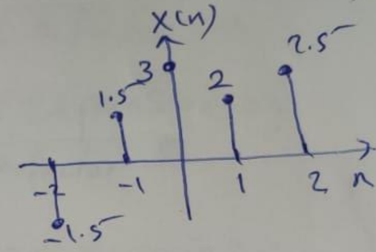
$x(n) = x_1(n) + x_2(n)$

$y(n) = x_1(n) \times x_2(n)$





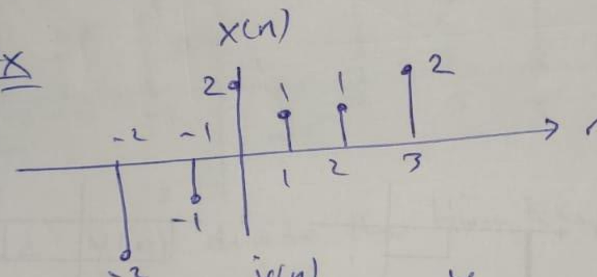


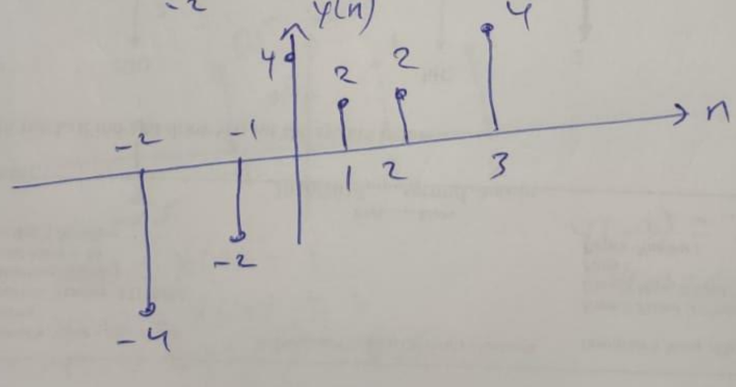


(3) Amplitude scaling: $y(n) = A\,x(n)$.
multiply each point by a factor A.

EX

Draw $y(n) = 2x(n)$





Time scaling of DT signals.

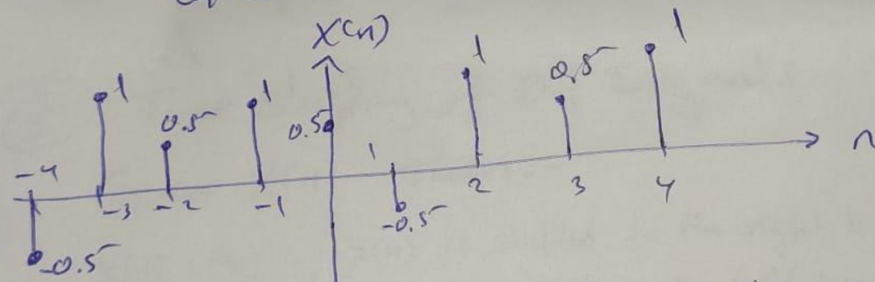
Given $X(n)$ Then:

$Y(n) = X(an)$ is time compressed by a factor a.

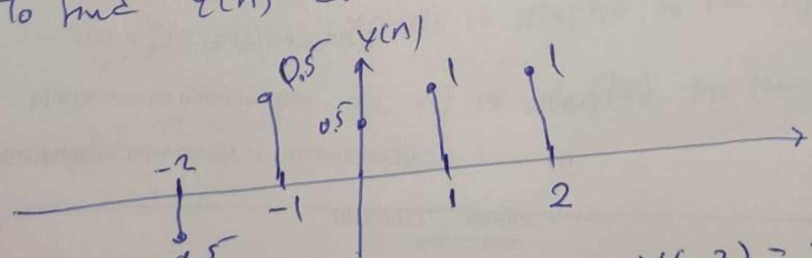
$Y(n) = X(n/a)$ time expansion by a factor a.

In doing compression or expansion only for integer values of $\underline{n}$ a sample exist.

EX Given $X(n)$ find $Y(n) = X(2n)$.



To find $Y(n)$ divide the time axis by $\underline{2}$



$Y(-2) = X(-4) = -0.5$

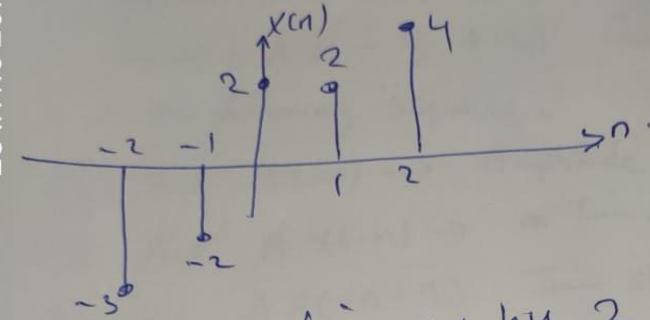
$Y(0) = X(0) = 0.5$

$Y(1) = X(2) = 1$

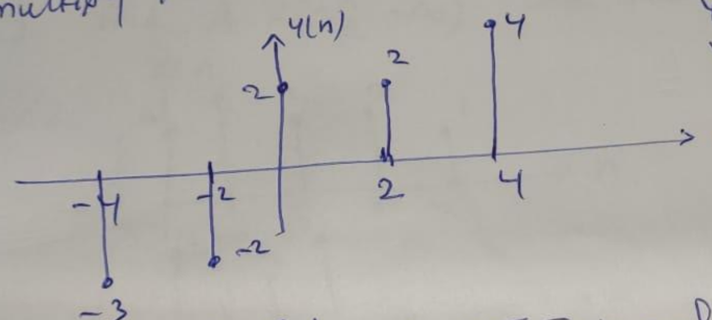
$Y(2) = X(4) = 1$

$Y(-1) = X(-2) = 0.5$

Ex Given $x(n)$ find $y(n) = x(n/2)$.



multiply the time axis by 2



$y(0) = x(0) = 2$
$y(2) = x(1) = 2$
$y(4) = x(2) = 4$
$y(-2) = x(-1) = -2$
$y(-4) = x(-2) = -3$

(5) Time shifting of DT signals.

Given $x(n)$ then:-

$x(n - n_0)$	$x(n)$ is shifted to the right by n_0
$x(n + n_0)$	$x(n)$ is shifted to the left by n_0.
$x(-n - n_0)$	$x(-n)$ is shifted to the left by n_0.
$x(-n + n_0)$	$x(-n)$ is shifted to the right by n_0.

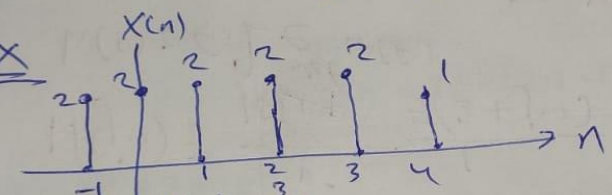
6 Multiple Transformation

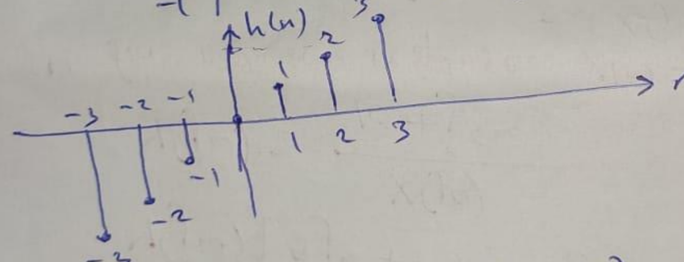
Given $X(n)$ then $Y(n)$

$Y(n) = A\,X\left(-\frac{n}{a} + n_0\right)$ can be drawn in the following sequence.

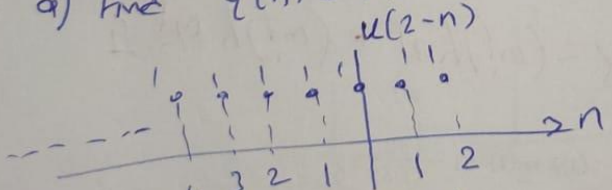
1. Find $A\,X(n) \rightarrow$ Amplitude scaling
2. Find $A\,X(-n) \rightarrow$ Time inverting
3. Find $A\,X(-n-n_0)$ Time shifting.
4. Find $A\,X\left(-\frac{n}{a} - n_0\right)$ Time scaling.

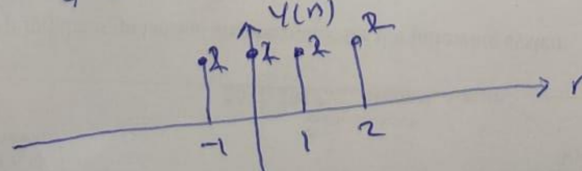
EX





a) Find $Y(n) = X(n)\,u(2-n)$.

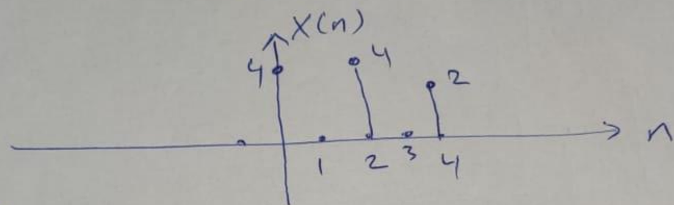




b) find $Y(n) = X(n) + (-1)^n X(n)$.

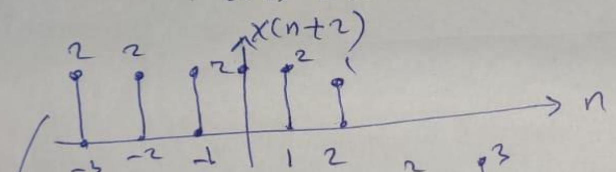
Y(n) can be defined as:.

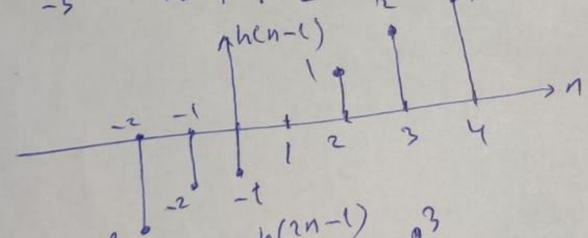
$$Y(n) = \begin{cases} 2X(n) & n \text{ even.} \\ 0 & n \text{ odd.} \end{cases}$$



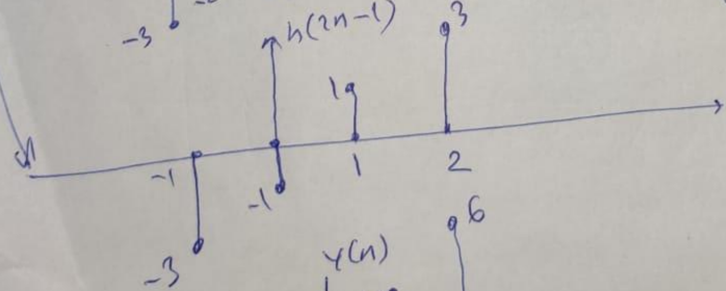
c) Find ~~Y(n) = h(n+1)~~

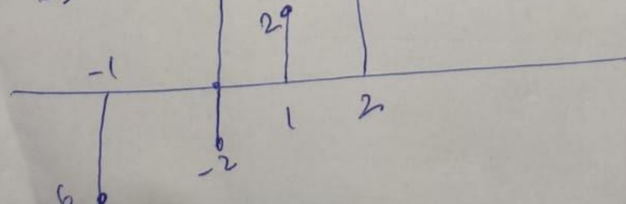
$Y(n) = X(n+2)\, h(2n-1)$





$z(n) = h(2n-1)$
$z(-1) = h(-3)$
$z(0) = h(-1)$
$z(1) = h(1)$
$z(2) = h(3)$





Classification of Signals :-

1. Determinastic or random.
2. Periodic or non-periodic signal,
3. odd or even signals.
4. Power or energy signals.

1. Any signal that can be written as a functi of time is called determinastic. The amplitude of such a signal can be determined at every instant of time. If the signal can not be written as a functi of time it is called random.

2. Periodic and non-periodic signals.

A CT Signal $X(t)$ is said to be periodic ~~with~~ if it repeats itself in a minimum positive interval. The minimum positive interval over which the signal repeats itself is called fundamental period. denoted by T_o.

The signal $X(t)$ is said to be periodic if:-

$$X(t) = X(t + nT_o) \quad \text{for all } t.$$

If a signal is periodic with period T_o it is also periodic with period nT_o.

9

Ex Is the signal $x(t) = A\sin(\omega_0 t + \phi)$ periodic. Find its fundamental period.

the signal $x(t)$ is periodic if:

$$x(t) = x(t + T_0) \ \forall t.$$

$$x(t+T_0) = A\sin(\omega_0(t+T_0) + \phi)$$
$$= A\sin(\omega_0 t + \omega_0 T_0 + \phi).$$

for periodicity:

$$A\sin(\omega_0 t + \phi) \stackrel{?}{=} A\sin(\omega_0 t + \omega_0 T_0 + \phi).$$

This is true if:

$$\omega_0 T_0 = 2\pi \Rightarrow \boxed{T_0 = \frac{2\pi}{\omega_0}}$$

So we can find such $T_0 = \frac{2\pi}{\omega_0}$

So the signal is periodic, with fundamental period $\boxed{T_0 = \frac{2\pi}{\omega_0}}$

Ex $x(t) = e^{j\omega_0 t}$

$$x(t+T_0) = e^{j\omega_0(t+T_0)} = e^{j\omega_0 t} e^{j\omega_0 T_0}.$$

for periodicity: $e^{j\omega_0 t} \stackrel{?}{=} e^{j\omega_0 t} e^{j\omega_0 T_0}.$

$$\Rightarrow 1 = e^{j\omega_0 T_0} \Rightarrow \omega_0 T_0 = 2\pi$$

$$T_0 = \frac{2\pi}{\omega_0}$$

Fundamental period of composite signals:-

If we have n-periodic signals
$x_1(t), x_2(t), x_3(t) \cdots x_n(t)$
with fundamental periods :-
$T_1, T_2, T_3 \cdots, T_n$
Then the composite signal :
$$x(t) = x_1(t) + x_2(t) + \cdots + x_n(t)$$
OR
$$x(t) = x_1(t) \times x_2(t) \times \cdots x_n(t)$$
is periodic iff:- (if and only if)
$$\frac{T_1}{T_2}, \frac{T_1}{T_3}, \frac{T_1}{T_4} \cdots, \frac{T_1}{T_n}$$
are all rational.

and the Fundamental period of the Composite Signal
$$T_0 = LCM(T_1, T_2, T_3, \cdots, T_n)$$
T_0 = Least Common multiple$(T_1, T_2, T_3 \cdots T_n)$.

EX $x(t) = \underbrace{2\cos(2\pi t)}_{x_1} + \underbrace{7\cos 9t}_{x_2}$.
is this signal periodic if periodic find its Fundamental period.

$\omega_1 = 2\pi$ $\qquad$ $\omega_2 = 9$

$T_1 = \frac{2\pi}{2\pi} = 1$ $\qquad$ $T_2 = 2\pi/9$

$\frac{T_1}{T_2} = \frac{1}{2\pi/9} = \frac{9}{2\pi}$ not rational $\Rightarrow x(t)$ not periodic

<u>Ex</u> $\quad x(t) = 5\sin(24\pi t) + 7\sin(6\pi t)$

$\omega_1 = 24\pi$ $\qquad$ $\omega_2 = 6\pi$

$T_1 = \frac{2\pi}{24\pi} = 1/12$ $\qquad$ $T_2 = \frac{2\pi}{6\pi} = 1/3$

$\frac{T_1}{T_2} = \frac{1/12}{1/3} = \frac{3}{12} = 1/4$ rational $\Rightarrow x(t)$ is periodic

To find the Fundamental period T_0:

$T_0 = LCM(T_1, T_2) = LCM(1/12, 1/3)$

multiply T_1 and T_2 by the smallest number z that makes T_1 and T_2 integers.

New (T_1, T_2) is $(1/12, \frac{1}{3}) \times z$.

z : smallest number that makes T_1, T_2 integers

So if we multiply by <u>12</u> Then the new (T_1, T_2) becomes. $(1, 4) \Rightarrow LCM(1, 4) = 4$.

$\Rightarrow T_0 = \frac{LCM}{z} = \frac{4}{12} = 1/3$ sec.

If we have <u>two signals only</u> T_0 can be found as

$\frac{T_1}{T_2} = \frac{1}{4} \Rightarrow T_0 = 4T_1 = T_2 = \frac{4}{12} = 1/3$ sec

Ex $x(t) = 3\sin(8\pi t) + 5\sin(\frac{\pi}{3} t) + \cos 7\pi t.$

$\omega_1 = 8\pi$ $\qquad \omega_2 = \pi/3$ $\qquad \omega_3 = 7\pi$

$T_1 = \frac{2\pi}{8\pi} = 1/4$ $\qquad T_2 = \frac{2\pi}{\pi/3} = 6$ $\qquad T_3 = \frac{2\pi}{7\pi} = 2/7$

$\frac{T_1}{T_2} = \frac{1/4}{6} = 1/24$ $\qquad \frac{T_1}{T_3} = \frac{1/4}{2/7} = 7/8$

all rational $\Rightarrow x(t)$ is periodic.

$T_0 = \mathrm{LCM}(1/4, 6, 2/7) \times 28$

$\Rightarrow$ new $(T_1, T_2, T_3) = (7, 168, 8).$

$\mathrm{LCM}(7, 168, 8)$

$\mathrm{LCM}(7,168,8) = 7 \times 2 \times 2 \times 2 \times 3$
$= 168$

$\Rightarrow T_0 = \frac{168}{28} = 6 \text{ sec}$

7	7	168	8
2	1	24	8
2	1	12	4
2	1	6	2
3	1	3	1
	1	1	1

x(t)

Even and odd Signals:-

a CT signal $x(t)$ is said to be even if:

$x(t) = x(-t) \quad \forall t.$

A CT signal $x(t)$ is said to be odd if

$x(-t) = -x(t) \quad \forall t.$

Any signal $x(t)$ can be expressed as two components. Odd and even components. such that

$x(t) = x_e(t) + x_o(t)$

where: $x_e(t)$ is the even component

$x_o(t)$ is the odd component.

+

$x(-t) = x_e(-t) + x_o(-t).$

$x(-t) = x_e(t) - x_o(t).$

Then; $x(t) + x(-t) = 2x_e(t)$

$\Rightarrow \boxed{x_e(t) = \frac{x(t) + x(-t)}{2}}$

$x(t) - x(-t) = 2x_o(t)$

$\boxed{x_o(t) = \frac{x(t) - x(-t)}{2}}$

EX Show that the product of two even signals is an even signal.

$x_1(t)$, $x_2(t)$ is two even signals

$\Rightarrow x_1(-t) = x_1(t) \qquad x_2(-t) = x_2(t)$

$x(t) = x_1(t)\, x_2(t)$

$x(-t) = x_1(-t)\, x_2(-t)$.

$x(-t) = x_1(t)\, x_2(t)$

$x(t) = x(-t) \Rightarrow x(t)$ is even.

EX Find if the following signals odd or even and find the even and odd components

a) $x(t) = t^2 - 5t + 10$.

$x(-t) = t^2 + 5t + 10$

$x(t) \neq x(-t) \Rightarrow$ not even

$x(-t) \neq -x(t) \Rightarrow$ not odd.

So $x(t)$ have two components,

$$x_e(t) = \frac{x(t) + x(-t)}{2} = t^2 + 10$$

$$x_o(t) = \frac{x(t) - x(-t)}{2} = -5t.$$

Ex

$$x(t) = 10\sin(10\pi t + \pi/4)$$

$$x(-t) = 10\sin(-10\pi t + \pi/4).$$

$x(t) \neq x(-t) \Rightarrow x(t)$ not even.

$x(-t) \neq -x(t) \Rightarrow x(t)$ not odd.

$x(t)$ can be written as:-

$$x(t) = 10[\sin 10\pi t \cos \pi/4 + \cos 10\pi t \sin \pi/4].$$

$$x(t) = \frac{10}{\sqrt{2}}[\sin 10\pi t + \cos 10\pi t].$$

$$x(-t) = 10[-\sin 10\pi t \cos \pi/4 - \cos 10\pi t \sin \pi/4].$$

$$x(-t) = \frac{-10}{\sqrt{2}}[\sin 10\pi t + \cos 10\pi t].$$

$$x_e(t) = \frac{x(t) + x(-t)}{2} = \frac{10}{\sqrt{2}}\cos 10\pi t$$

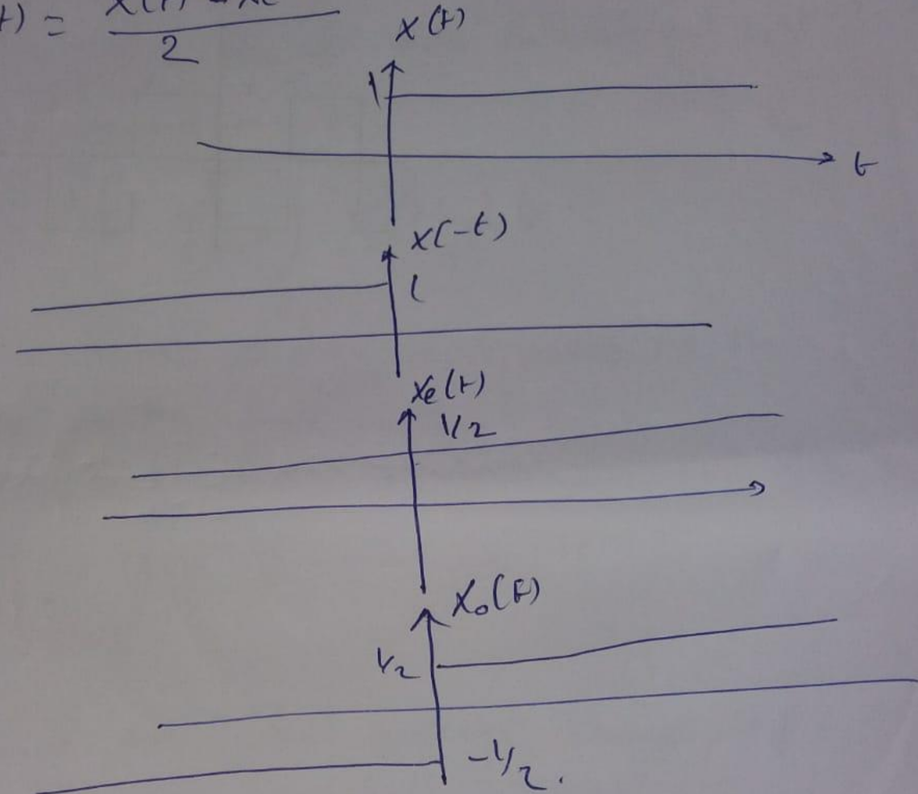
$$x_o(t) = \frac{x(t) - x(-t)}{2} = \frac{10}{\sqrt{2}}\sin 10\pi t.$$

$$\sin(A+B) = \sin(A)\cos B + \cos A \sin(B).$$

Ex find and draw the odd and even compount of $x(t) = u(t)$.

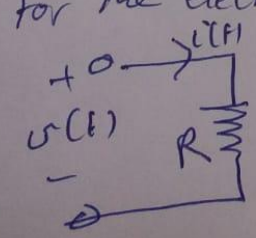
$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$



Energy and Power of CT Signals

for the electric Circuit



The instant power for the resistor

$$P = \frac{v^2(t)}{R} = R\, i^2(t)$$

If $R=1$ it is called normalized power.

⇒ The instant power:

$$P = v^2(t) = i^2(t) = x^2(t)$$

$x(t)$ may be current or voltage.

The average power over the interval $t_1 < t < t_2$ is defined as:-

$$P_{av} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} x^2(t)\, dt.$$

and the total energy dissipated in the resistor in the same interval is

$$E = \int_{t_1}^{t_2} x^2(t)\, dt.$$

In general the Total average Power of a signal $x(t)$ is defined as:

$$P = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2\, dt.$$

and the Total energy is defined as:

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt.$$

Remarks

1. If the total energy of a signal is finite such that $0 < E < \infty$ the signal is called an energy signal. For an energy signal the total average power is zero

2. If the total average power of a signal is finite. Such that $0 < P < \infty$ the signal is called power signal. The total energy of a power signal is ∞

3. Periodic signals are power signals.

4. Non-periodic signals with finite duration are energy signals.

EX: Find the Total energy and power of the following signal.

$$x(t) = A\,u(t)$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_0^T A^2\, dt = \lim_{T \to \infty} \frac{A^2}{2T} \left[t \Big|_0^T \right]$$

$$= \lim_{T \to \infty} \frac{A^2}{2T} T = A^2/2 \qquad E = \infty$$

EX $x(t) = e^{-3t} u(t)$

$$E = \int_0^{\infty} (e^{-3t})^2 dt = \int_0^{\infty} e^{-6t} dt = -\frac{1}{6}\left[e^{-6t}\right]_0^{\infty}$$

$E = 1/6$ joules.

Total energy is finite $\Rightarrow P = 0$.

EX $x(t) = 5\cos(10t + \phi)$

$x(t) = 5\cos(10t + \phi)$.

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 25\cos^2(10t + \phi)\, dt.$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 25\left(\frac{1}{2} + \frac{1}{2}\cos(20t + 2\phi)\right) dt.$$

$$P = \lim_{T\to\infty} \frac{25}{2T} \int_{-T}^{T} \frac{1}{2}\, dt + \lim_{T\to\infty} \frac{25}{2T} \cancel{\int_{-T}^{T} \frac{1}{2}\cos(20t + 2\phi)\, dt}$$
0

$$P = \lim_{T\to\infty} \frac{25}{4T}\left[t\Big|_{-T}^{T}\right] = \lim_{T\to\infty} \frac{25}{4T} 2T = \frac{25}{2}$$

$P = 25/2$ Watt.

P - finite $\quad E = \infty$.

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EX $x(t) = 10\, e^{j2\pi t}\, u(t)$.

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_0^T \left|10\, e^{j2\pi t}\right|^2 dt.$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_0^T 100 \underbrace{\left|e^{j2\pi t}\right|^2}_{1} dt.$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_0^T 100\, dt = \lim_{T\to\infty} \frac{1}{2T} T = 1/2$$

Classification of DT Signals :-

1. Periodic and Non-periodic DT Signals.

a DT signal $x(n)$ is said to be periodic if it repeats its self after each N-Sample. The smallest N that the signal repeats itself is called Fundamental Period denoted by N_0. So: for periodicity:

$$x(n+N) = x(n) \quad \forall n.$$

If a signal is periodic with period N then it is also periodic with period mN. This means

$$x(n+mN) = x(n) \quad \forall n.$$

<u>EX</u> $x(n) = e^{j\omega_0 n}$. is this signal periodic if periodic find $\underline{N_0}$

for periodicity: $x(n+N) \overset{?}{=} x(n) \quad \forall n.$

$$x(n+N) = e^{j\omega_0(n+N)} = e^{j\omega_0 n} e^{j\omega_0 N}.$$

So for periodicity:

$$\cancel{e^{j\omega_0 n}} \overset{?}{=} e^{j\omega_0 n} e^{j\omega_0 N}$$

$\Rightarrow$ for periodicity $e^{j\omega_0 N} = 1$

This means: $\omega_0 N = 2\pi m$ $\quad$ m integer.

So ~~at~~ $N = \frac{2\pi}{\omega_0} m$ $\quad$ (N must be integer)

$\Rightarrow$ for periodicity $\frac{N}{m} = \frac{2\pi}{\omega_0}$ must be rational.

So for periodicity $\frac{2\pi}{\omega_0}$ must be rational

So $e^{j\omega_0 n}$ is periodic if $\frac{2\pi}{\omega_0}$ is rational. ②

and $N_0 = \frac{2\pi}{\omega_0} m$.

m is the smallest integer that makes N_0 integer.

EX $x(n) = \sin(\omega_0 n + \phi)$.

$$x(n+N) = \sin(\omega_0 n + \omega_0 N + \phi)$$

for periodicity:

$$\sin(\omega_0 n + \phi) \overset{?}{=} \sin(\omega_0 n + \omega_0 N + \phi)$$

and this is true if:

$$\omega_0 N = 2\pi m.$$

$$\Rightarrow \quad \frac{N}{m} = \frac{2\pi}{\omega_0}$$

So for periodicity $\frac{2\pi}{\omega_0}$ must be rational.

and $N_0 = \frac{2\pi}{\omega_0} m$ m - Smallest integer that makes N_0 integer.

Fundamental period of Composite signals

$x_1(n)$, $x_2(n)$ .... $x_m(n)$ are all periodic signals with Fundamental periods:

$N_1, N_2, N_3 --- N_m$.

Then the composite signal:

$$x(n) = x_1(n) + x_2(n) + ----$$

OR $x(n) = x_1(n) \times x_2(n) \times ---$

is periodic with Fundamental period

$$N_0 = LCM(N_1, N_2, N_3 ---).$$

Ex $x(n) = e^{j\pi n}$ is this signal periodic if periodic find N_0.

$\omega_0 = \pi$

$\frac{2\pi}{\omega_0} = \frac{2\pi}{\pi} = 2$ rational $\Rightarrow x(n)$ is periodic

$N_0 = \frac{2\pi}{\omega_0} m = \frac{2\pi}{\pi} m = 2m$.

$m = 1$

$\Rightarrow N_0 = 2$

Ex $x(n) = \cos(\frac{n}{8} - \pi)$.

$\omega_0 = 1/8$

$\frac{2\pi}{\omega_0} = 2\pi / 1/8 = 16\pi$

$\frac{2\pi}{\omega_0}$ not rational.
$\Rightarrow x(n)$ is not periodic

Ex $x(n) = \sin^2 \pi/4 \, n$.

$x(n) = \underbrace{\frac{1}{2}}_{x_1(n)} - \underbrace{1/2 \cos \frac{2\pi}{4} n}_{x_2(n)}$

$x_1(n)$ is periodic with fundamental period $N_1 = 1$

$\omega_2 = \pi/2 \quad \frac{2\pi}{\omega_2} = \frac{2\pi}{\pi/2} = 4$ rational.
$\Rightarrow x_2(n)$ is periodic

$N_2 = \frac{2\pi}{\omega_2} m = \frac{2\pi}{\pi/2} m = 4m$.

$\Rightarrow N_2 = 4$ for $m = 1$

$N_0 = LCM(N_1, N_2) = LCM(4, 1) = 4$ Sample

EX $x(n) = \cos\frac{\pi}{2}n + \sin\frac{\pi}{8}n + \cos\pi/4\, n$.

$\omega_1 = \pi/2$ $\qquad \omega_2 = \pi/8$ $\qquad \omega_3 = \pi/4$

$\frac{2\pi}{\omega_1} = \frac{2\pi}{\pi/2} = 4$ $\qquad \frac{2\pi}{\omega_2} = \frac{2\pi}{\pi/8} = 16$ $\qquad \frac{2\pi}{\omega_3} = \frac{2\pi}{\pi/4} = 8$

All rational $\Rightarrow$ $x(n)$ is periodic.

$N_1 = \frac{2\pi}{\omega_1} m = 4m$ $\qquad N_2 = \frac{2\pi}{\omega_2} m$ $\qquad N_3 = \frac{2\pi}{\omega_3} m$

$\Rightarrow N_1 = 4$ for $m=1$ $\qquad N_2 = 16m$ $\qquad N_3 = 8m$

$\Rightarrow N_2 = 16$ for $m=1$ $\qquad N_3 = 8$ for $m=1$

$N_0 = LCM(4, 16, 8) = 16$ Sample.

EX $x(n) = e^{j\frac{2\pi}{3}n} + e^{j\frac{3\pi}{4}n}$

$\omega_1 = \frac{2\pi}{3}$ $\qquad \omega_2 = 3\pi/4$

$\frac{2\pi}{\omega_1} = \frac{2\pi}{\frac{2\pi}{3}} = 3$ $\qquad \frac{2\pi}{\omega_2} = \frac{2\pi}{3\pi/4} = 8/3$

rational $\qquad$ rational.

$\Rightarrow x(n)$ is periodic

$N_1 = 3m \Rightarrow N_1 = 3$ for $m=1$

$N_2 = \frac{8}{3} m \Rightarrow N_2 = 8$ for $m=3$.

$N_0 = LCM(3, 8) = 24$ Samples.

Odd and even DT signals:

$x(n)$ is said to be even if:

$$x(n) = x(-n) \quad \forall n.$$

$x(n)$ is said to be odd if:

$$x(-n) = -x(n) \quad \forall n.$$

Any signal $x(n)$ can be written as two components

$$x(n) = x_e(n) + x_o(n)$$

$$x_e(n) = \frac{x(n) + x(-n)}{2} \qquad x_o(n) = \frac{x(n) - x(-n)}{2}.$$

Ex

$$x(n) = \{-2, 1, \underset{\uparrow}{3}, -5, 4, 2\}$$

$$x(-n) = \{2, 4, -5, \underset{\uparrow}{3}, 1, -2\}$$

$$x_e(n) = \{1, 1, -2, \underset{\uparrow}{3}, -2, 1, 1\}.$$

$$x_o(n) = \{-1, -3, 3, \underset{\uparrow}{0}, -3, 3, 1\}$$

Energy and power of DT signals:

The Total energy of a DT Signal $x(n)$ is defined as:

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

and Total average Power is defined as:

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 .$$

Remarks

1) $x(n)$ is energy signal if $0 < E < \infty$
   If E is finite the total average power is zero.
2) $x(n)$ is a power signal if $0 < P < \infty$
   if the Power is finite $\Rightarrow E = \infty$.
3) All periodic signals are power signal,
4) All non-periodic signals with finite dural: are energy signal.

~~Reme~~

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a} \quad a \neq 1,$$
$$= N \quad \text{for } N = 1$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad a < 1$$

$$\sum_{n=m}^{\infty} a^n = \frac{a^m}{1-a} \quad a < 1$$

$$\sum_{n=0}^{\infty} n a^n = \frac{a}{(1-a)^2} \quad a < 1$$

Ex Find the Total energy and Power of $x(n) = A\,\delta(n)$.

$x(n) = A$ for $n = 0$

$x(n) = 0$ for $n \neq 0$.

$$E = \sum_{n=0}^{0} A^2 = A^2$$

$P = 0$.

Ex $x(n) = u(n)$.

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1 \qquad \sum_{n=0}^{N} 1 = N+1$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1}\, N+1 = \frac{1}{2}$$

$E = \infty$

Ex $x(n) = \text{ramp}(n)$.

$\text{ramp}(n) = n$ for $n \geq 0$.

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} n^2$$

$$\sum_{n=0}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \frac{N(N+1)(2N+1)}{6} = \infty$$

$$E = \sum_{n=0}^{\infty} n^2 = \infty$$

the signal is not power not energy.

(8)

EX $x(n) = 2e^{j(n\pi+\theta)}$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left|2e^{j(n\pi+\theta)}\right|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 4 \qquad \left|e^{j(n\pi+\theta)}\right| = 1$$

$$P = \lim_{N\to\infty} \frac{4}{2N+1} \sum_{n=-N}^{N} 1 = \lim_{N\to\infty} \frac{4}{\cancel{2N+1}} \cancel{2N+1}$$

$$\Rightarrow E = \infty \qquad = 4.$$

EX

$$x(n) = \begin{cases} (1/3)^n & n \geq 0 \\ 3^n & n < 0 \end{cases}$$

$$E = \sum_{n=0}^{\infty} (1/3)^{2n} + \sum_{-\infty}^{-1} 3^{2n}$$

$$E = \sum_{n=0}^{\infty} (1/9)^n + \sum_{-\infty}^{-1} 9^n.$$

Assume $n = -m$

$$E = \sum_{n=0}^{\infty} (1/9)^n + \sum_{m=1}^{\infty} 9^{-m}.$$

$$E = \frac{1}{1-1/9} + \sum_{m=0}^{\infty} (1/9)^m - 1$$

$$E = \frac{1}{1-1/9} + \frac{1}{1-1/9} - 1$$