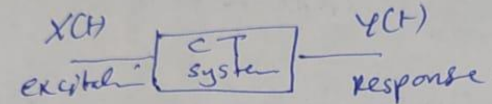
Discrete and Continous time Systems :-

①

A system can be viewed as any process that results in transformation of signals, so the system in general have input, the output which are related by the system transformation

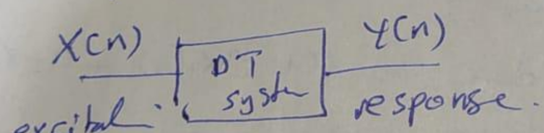
A Continous time system, continous time signals are transformed into continous output signal.



and can be represented as:

$$X(t) \longrightarrow Y(t).$$

Discrete time system, an input discrete time signal is transformed to a discrete time output signal.



$$X(n) \longrightarrow Y(n)$$

for a CT signal in general the input and the output are related by a differential equation

for a DT signal the input and output in general are related by a difference equation

$$y(n) + a_1 Y(n-1) + a_2 Y(n) = b X(n).$$

Properties of CT and DT Signals:

②

① Linear and non-linear systems.

Any system that obeys the superposition theorem is called linear system.

This means if we have two inputs to the system $X_1(t), X_2(t)$ or $X_1(n), X_2(n)$

$X_1(t) \longrightarrow Y_1(t)$ or $X_1(n) \longrightarrow Y_1(n)$

$X_2(t) \longrightarrow Y_2(t)$ $X_2(n) \longrightarrow Y_2(n)$

Then the composite signal:

$X(t) = a_1 X_1(t) + a_2 X_2(t)$

The output is:-

$Y(t) = a_1 Y_1(t) + a_2 Y_2(t)$

and for a DT system;

The output $Y(n)$ due to $X(n) = a_1 X_1(n) + a_2 X_2(n)$

is $Y(n) = a_1 Y_1(n) + a_2 Y_2(n)$.

This means: for a system to be linear the weighted sum of several inputs results in a weighted sum of the outputs.

To check a system for linearity:-

(1) Assume two inputs to the system, $x_1(t)$, $x_2(t)$ or $x_1(n)$, $x_2(n)$ find the output for each input.

$$y_1(t) = f(x_1(t)) \qquad y_2(t) = f(x_2(t))$$

or

$$y_1(n) = f(x_1(n)) \qquad y_2(n) = f(x_2(n)).$$

Then find the weighted sum of the outputs.

$$y_3(t) = a_1 y_1(t) + a_2 y_2(t).$$

(2) Find the output due to the weighted sum of the inputs

$$y_4(t) = f(a_1 x_1(t) + a_2 x_2(t))$$

or

$$y_4(n) = f(a_1 x_1(n) + a_2 x_2(n))$$

(3) If $y_3(t) = y_4(t) \Rightarrow$ the system is linear.

If $y_3(t) \neq y_4(t) \Rightarrow$ the system is nonlinear

the same for DT system.

Ex is the following system linear.

$$x(t) \rightarrow \boxed{\text{system}} \rightarrow y(t) = [2x(t)]^2$$

1) Assume two inputs to the system $x_1(t), x_2(t)$

$$y_1(t) = [2x_1(t)]^2 = 4x_1^2(t)$$

$$y_2(t) = [2x_2(t)]^2 = 4x_2^2(t).$$

Then the weighted sum of the outputs is:

$$y_3(t) = 4a_1 x_1^2(t) + 4a_2 x_2^2(t).$$

2) The weighted sum of the inputs is

$$x(t) = a_1 x_1(t) + a_2 x_2(t)$$

The output due to the weighted sum of the inputs is

$$y_4(t) = \left[2\left(a_1 x_1(t) + a_2 x_2(t)\right)\right]^2.$$

$$y_4(t) = 4\left[a_1^2 x_1^2(t) + 2a_1 a_2 x_1(t) x_2(t) + a_2^2 x_2^2(t)\right].$$

(3) $y_3(t) \neq y_4(t) \Rightarrow$ the system is non-linear system.

Ex $y(t) = x(t^2)$.

(1) Assume two inputs $x_1(t), x_2(t)$

$y_1(t) = x_1(t^2)$ $\quad y_2(t) = x_2(t^2)$.

$$y_3(t) = a_1 x_1(t^2) + a_2 x_2(t^2).$$

$$y_4(t) = a_1 x_1(t^2) + a_2 x_2(t^2)$$

$y_3(t) = y_4(t)$

$\Rightarrow$ the system is linear.

Ex

$Y(n) = 2X(n) + 3$.

Assume two inputs $X_1(n), X_2(n)$

Then;

$Y_1(n) = 2X_1(n) + 3$

$Y_2(n) = 2X_2(n) + 3$

The weighted sum of the two outputs is

$Y_3(n) = 2a_1X_1(n) + 3a_1 + 2a_2X_2(n) + 3a_2$.

The output due to weighted sum of the input

$X(n) = a_1X_1(n) + a_2X_2(n)$ is.

$Y_4(n) = 2a_1X_1(n) + 2a_2X_2(n) + 3$.

$Y_3(n) \neq Y_4(n) \Rightarrow$ the system is nonlinear.

Ex $Y(n) = X(n) + n\,X(n+1)$

$Y_1(n) = X_1(n) + n\,X_1(n+1)$

$Y_2(n) = X_2(n) + n\,X_2(n+1)$

The ~~output due to~~ weighted sum of the outputs;

$Y_3(n) = a_1X_1(n) + a_1 n\,X_1(n+1) + a_2X_2(n) + a_2 n X_2(n+1)$

The weighted sum of the inputs:

$X(t) = a_1X_1(n) + a_2X_2(n)$

$Y_4(n) = a_1X_1(n) + a_2X_2(n) + n a_1 X_1(n+1) + n a_2 X_2(n+1)$

$Y_3(n) = Y_4(n) \Rightarrow$ the system is linear.

Ex

$$y(t) = 10\int_{-\infty}^{t} x(\tau)\,d\tau.$$

Assume two inputs $x_1(t)$, $x_2(t)$.

$$y_1(t) = 10\int_{-\infty}^{t} x_1(\tau)\,d\tau \qquad y_2(t) = 10\int_{-\infty}^{t} x_2(\tau)\,d\tau.$$

$$y_3(t) = 10a_1\int_{-\infty}^{t} x_1(\tau)\,d\tau + 10a_2\int_{-\infty}^{t} x_2(\tau)\,d\tau.$$

The weighted sum of the inputs is:

$$x(t) = a_1x_1(t) + a_2x_2(t)$$

$$y_4(t) = 10\int_{-\infty}^{t} \left(a_1x_1(\tau) + a_2x_2(\tau)\right) d\tau.$$

$$y_4(t) = 10a_1\int_{-\infty}^{t} x_1(\tau)\,d\tau + 10a_2\int_{-\infty}^{t} x_2(\tau)\,d\tau.$$

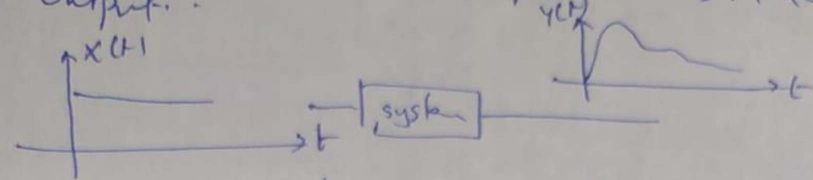
$y_3(t) = y_4(t) \Rightarrow$ the system is linear.

2 Time Invariant and time varying systems.

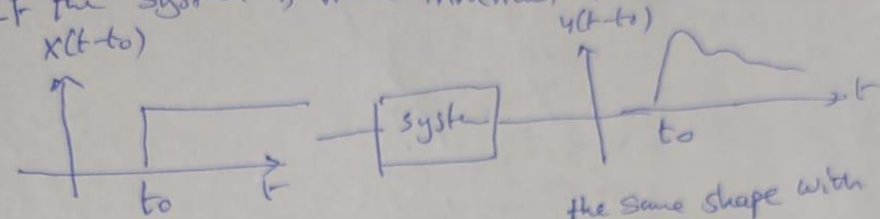
A continuous or discrete time system is said to be time invariant if the parameters of the system does not change with Time.

That is if the input is delayed t_0 or n_0 the shape of the output will be the same but shifted by an amount of t_0 or n_0.

this means a shift in the input results in a shift in the output.



If the system is time invariant then:



and this is the same for DT systems.

To check for time invariance:

(1) For a delayed input $x(t-t_0)$ or $x(n-n_0)$ obtain the output $y(t,t_0)$ or $y(n,n_0)$.

(2) ~~For a delayed~~ Obtain the expression for the delayed output $y(t-t_0)$, $y(n-n_0)$ by replacing every t by $t-t_0$ in the output, n by $n-n_0$ in the output.

(3) If $y(t,t_0) = y(t-t_0)$ or $y(n,n_0) = y(n-n_0)$ } the system is time invariant.

If $y(t,t_0) \neq y(t-t_0)$ or $y(n,n_0) \neq y(n-n_0)$ the system is time varying system.

REDMI NOTE 8
AI QUAD CAMERA

Ex

$x(t) \rightarrow \boxed{\text{syst}} \rightarrow y(t) = t\,x(t)$.

$y(t) = t\,x(t)$ is the system time invariant

1. for a delayed input $x(t-t_0)$ obtain the output
$y(t,t_0) = t\,x(t-t_0)$.

2. $y(t-t_0) = (t-t_0)\,x(t-t_0)$.

$y(t,t_0) \neq y(t-t_0) \Rightarrow$ the system is time varying system

Ex $y(n) = x(4n-1)$. is the system time varying or time invariant.

1. The output due to a delayed input $x(n-n_0)$ is
$y(n,n_0) = x(4n-n_0-1)$.

2. The delayed output:
$y(n-n_0) = x(4(n-n_0)-1)$
$= x(4n-4n_0-1)$

$y(n,n_0) \neq y(n-n_0) \Rightarrow$ Time varying system

system

$y(t) = t\,x(t)$

$y(t) = t\,x(t)$

is the sys

Ex $y(n) = x^2(n)$.

① for a delayed input $x(n - n_0)$

$$y(n, n_0) = x^2(n - n_0).$$

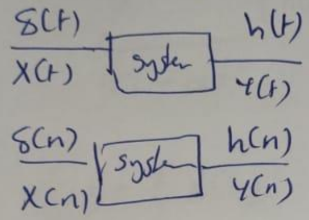
② The delayed output

$$y(n - n_0) = x^2(n - n_0).$$

$$y(n, n_0) = y(n - n_0) \Rightarrow \text{Time invariant system}$$

(3) Stable and non-Stable Systems. (1)

If we have a LTI system. If the input to the system is a unit impulse the output is called impulse response of the system



$h(t)$ and $h(n)$ are called impulse response of the system.

A linear time invariant system is called Bounded input bounded output (BIBO) stable. If for any bounded input results in a Bounded output. that is if $X(t)$ or $X(n)$ is bounded such that:

$$\left.\begin{array}{l} |X(t)| < A < \infty \\ \text{OR} \quad |X(n)| < A < \infty \end{array}\right\} \Rightarrow \begin{array}{l} |Y(t)| < B < \infty \\ |Y(n)| < B < \infty . \end{array}$$

And this implies that for a CT system to be stable the area under the impulse response of the system must be finite. that is:

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty .$$

And for a DT system to be stable ②
the summation of its impulse response must be finite.
that is:

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty .$$

EX $y(t) = t\,x(t)$

$x(t)$ is bounded. However $y(t)$ increases as time increases and tends to ∞.
So the output is unbounded.
$\Rightarrow$ The is unstable system.

EX $h(t) = e^{-2|t|}$ is this system stable

for stability the area under its impulse response must be finite. So,

$$h(t) = \begin{cases} e^{-2t} & t \geq 0 \\ e^{2t} & t < 0 . \end{cases}$$

So the area under $h(t)$ is;

SHOT ON MI 9T
AI TRIPLE CAMERA

(3)

$$\text{area} = \int_{-\infty}^{\infty} h(t)\,dt = \int_{0}^{\infty} e^{-2t}\,dt + \int_{-\infty}^{0} e^{2t}\,dt.$$

$$\text{area} = -\frac{1}{2}\left[e^{-2t}\Big|_{0}^{\infty}\right] + \frac{1}{2}\left[e^{2t}\Big|_{-\infty}^{0}\right].$$

$$\text{area} = \frac{1}{2} + \frac{1}{2} = 1 < \infty$$

$\Rightarrow$ The system is BIBO Stable.

<u>Ex</u> $\quad Y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau \quad$ is this system BIBO Stable.

for this system to be Stable the area under its impulse response must be finite. So find $h(t)$ and find the area under it.

$$h(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau = u(t)$$

$$\Rightarrow \text{The area} = \int_{0}^{\infty} 1\,dt = \infty$$

$\Rightarrow$ the system is Unstable.

SHOT ON MI 9T
AI TRIPLE CAMERA

Ex $y(n) = \sin(x(n))$ stable. 4

Ex $y(n) = e^{x(n)}$

$x(n)$ is bonded so $e^{x(n)}$ will be bonded

$\Rightarrow$ the system is stable.

Ex $h(n) = e^{-|n|}$

for the system to be stable $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$.

$$h(n) = \begin{cases} e^{-n} & n \geq 0 \\ e^{n} & n < 0. \end{cases}$$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0}^{\infty} e^{-n} + \sum_{n=-\infty}^{-1} e^{n}.$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{e}\right)^{n} + \sum_{m=1}^{\infty} \left(\frac{1}{e}\right)^{m}.$$

$$= \frac{1}{1-\frac{1}{e}} + \frac{1}{1-\frac{1}{e}} - 1 < \infty$$

$\Rightarrow$ the system is stable.

Ex $h(n) = 2^n u(-n)$.

5

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{-\infty}^{0} 2^n$$

$$= \sum_{m=0}^{\infty} \left(\tfrac{1}{2}\right)^m = \frac{1}{1-\frac{1}{2}} < \infty$$

$\Rightarrow$ Stable.

(4) Causal and non-Causal systems.

A CT or DT system is said to be Causal if the output at any instant of time depends on the input at that time and past values of the input. That is the output does not depend on future value of the input.

Ex $y(t) = x(t/4)$.

Lets try it:

$y(0) = x(0)$

$y(4) = x(1)$ $\rightarrow$ this is a future value.

$y(-4) = x(-1)$

$\Rightarrow$ the system is nonCausal

SHOT ON MI 9T
AI TRIPLE CAMERA

$\underline{EX}$ $Y(n) = X(n-1) \rightarrow$ Causal. (6)

$Y(n) = \sum_{n=-\infty}^{n} X(k)$ Causal.

$Y(n) = \sum_{n=-\infty}^{n+4} X(n)$ non Causal.

(5) Static and dynamic systems.

The system is said to be static (memoryless) if the output at any instant of time depends only on the input at that time only.

The system is said to be dynamic (with memory) if the output depends on future or past values of the input.

$\underline{EX}$ $Y(n) = X(3n)$ — dynamic

Ex $Y(n) = \sin X(n) \rightarrow$ static

SHOT ON MI 9T
AI TRIPLE CAMERA

7

EX Is the following system dynamic, causal, linear, time invariant

$$Y(n) = 5\,X(3^n).$$

① $Y(1) = 5\,X(3)$ depends on future value.
$\Rightarrow$ dynamic system,

② $Y(1) = 5\,X(3)$ depends on future value
$\Rightarrow$ non causal.

③ For time invariance
for a shifted input.

$$Y(n, n_0) = 5\,X(3^n - n_0).$$

The shifted output:

$$Y(n-n_0) = 5\,X(3^{(n-n_0)})$$

$Y(n, n_0) \neq Y(n-n_0) \Rightarrow$ Time varying system.

④ $Y_1(n) = 5\,X_1(3^n)$

$Y_2(n) = 5\,X_2(3^n)$

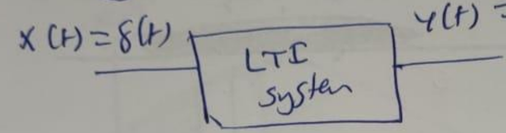
$Y_3(n) = 5a_1 X_1(3^n) + 5a_2 X_2(3^n)$

$Y_4(n) = 5[a_1 X_1(3^n) + a_2 X_2(3^n)]$

$Y_3 = Y_4$
$\Rightarrow$ linear system

Convolution integral. (8)

For a LTI system if the input is a unit impulse the output is called the impulse response of the system denoted by $h(t)$.

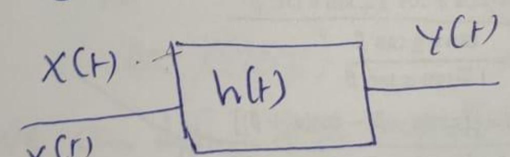


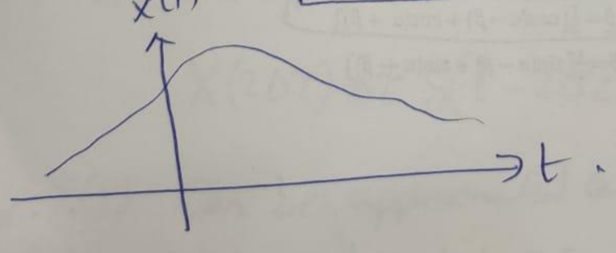
if the input is $2\delta(t)$ the output is $y(t) = 2h(t)$.

if the input is $2\delta(t-t_0)$ the output is $y(t) = 2h(t-t_0)$

shift in the input results in a shift in the output.

Now we have a LTI system with a ~~known~~ impulse response $h(t)$. Can we find the output to the system for an arbitrary input $x(t)$.??

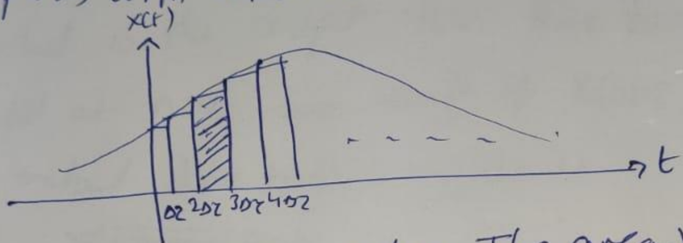




??

To find the output to the system for an arbitrary input $X(t)$.

$X(t)$ is divided into small rectangular pulses with width $\Delta\tau$:



Lets take one of these pulses. The area under this pulse can be found as:

$$\text{area} = \Delta\tau\, X(2\Delta\tau).$$

As $\Delta\tau$ approches to zero, this rectangular pulse can be approxmated as an impulse with strength

$$\Delta\tau\, X(2\Delta\tau)$$

and we can define this impulse as:

$$X(2\Delta\tau)\,\Delta\tau\,\delta(t-2\Delta\tau).$$

So $X(t)$ can be approximated as the sum of shifted impulses denoted as .

$$x(t) = \sum_{n=-\infty}^{\infty} x(n\Delta\tau)\Delta\tau\,\delta(t-n\Delta\tau).$$

10

If the input to the system is $\delta(t)$ the output is $h(t)$

What is the output $y(t)$ due to an impulse located at $n\Delta\tau$ and strength of $x(n\Delta\tau)\Delta\tau$.

the output due to this impulse is

~~$y(t) = x(\ldots$~~

$$y(t) = x(n\Delta\tau)\Delta\tau\,h(t-n\Delta\tau).$$

So the Total output due to $x(t)$ is:

$$y(t) = \sum_{n=-\infty}^{\infty} x(n\Delta\tau)\Delta\tau\,h(t-n\Delta\tau).$$

As $\Delta\tau \longrightarrow 0$.

$$\sum \longrightarrow \int$$

$$n\Delta\tau \longrightarrow \tau$$

$$\Delta\tau \longrightarrow d\tau$$

So $$\boxed{y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau}$$

SHOT ON MI 9T
AI TRIPLE CAMERA

This integral is called Convolution integral denoted by

$$Y(t) = X(t) \circledast h(t)$$

So the Convolution integral gives the output to a LTI system for an arbitrary input $x(t)$ if $h(t)$ is given for a LTI system:

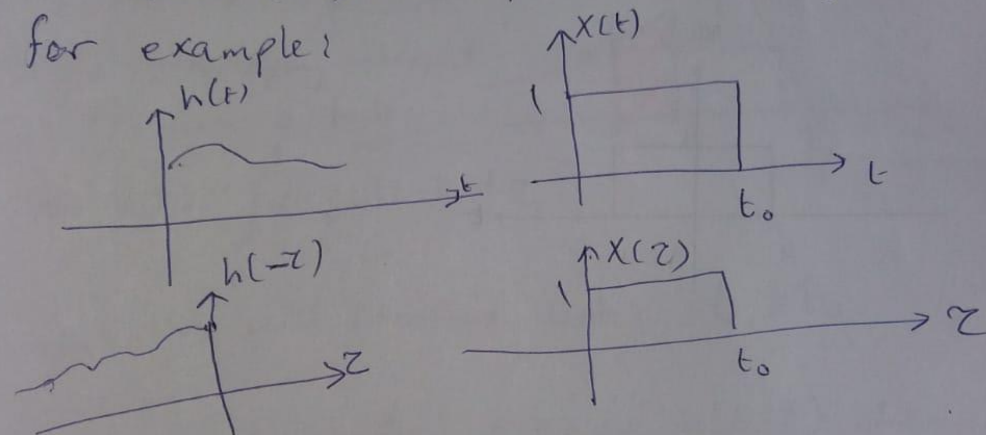
$$Y(t) = X(t) \circledast h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

<u>To solve this integral:</u>

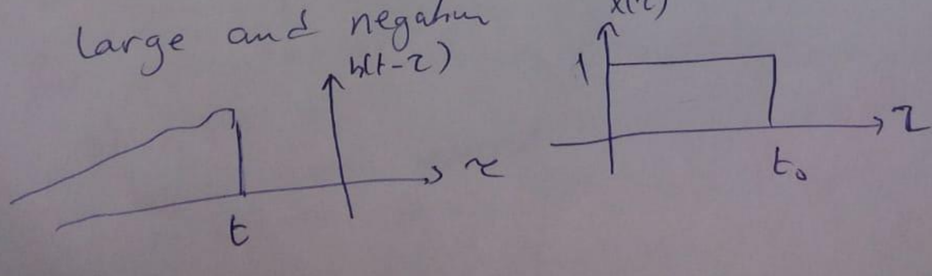
① Let $X(t)$ represent the input, $h(t)$ represent the impulse response of the system. replace each $\underline{\underline{t}}$ in $X(t)$ and $h(t)$ by τ to obtain $X(\tau)$, $h(\tau)$

② Represent $X(\tau)$ in a figure and invert $h(\tau)$ and obtain $h(-\tau)$ and represent it in a figure.

for example:



shift $h(-\tau)$ on the τ axis by an amount "t" to obtain $h(t-\tau)$ at the beginning assume "t" large and negative



3 Multiply the signal $x(\tau)$ by $h(t-\tau)$ and integrate over the overlaping interval.

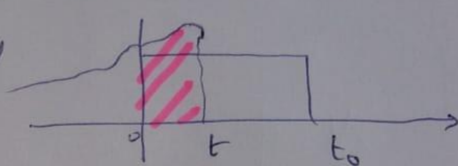
$x(\tau)$ is fixed $h(t-\tau)$ is moved toward the right so that $h(t-\tau)$ and $x(\tau)$ overlaps.

in our example the first overlap occurs when $t>0$. So for $t<0$ no overlap and $y(t)=0$. So we do the following

1) for $t<0$ no over lap $y(t)=0$.

2) for $0<t<t_o$

the overlaping interval is from 0 to t.



$$\Rightarrow y(t) = \int_0^t x(\tau)\, h(t-\tau)\, d\tau$$

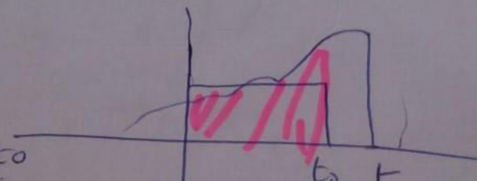
and this will continue until $t>t_o$.

4 When ever either $x(\tau)$ or $h(t-\tau)$ changes a new time shift occurs and $y(t)$ is calculated using step 3

for our example.

3) when $t>t_o$

the over laping interval is between zero and t_o

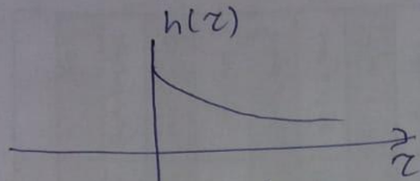


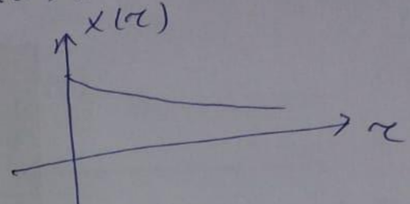
$$y(t) = \int_0^{t_o} x(\tau)\, h(t-\tau)\, d\tau$$

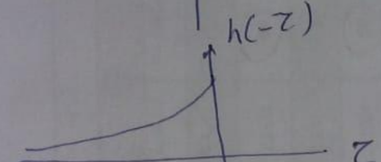
EX $h(t) = e^{-5t}\,u(t)$

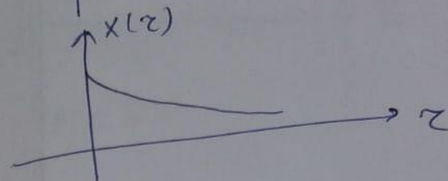
$x(t) = e^{-2t}\,u(t)$

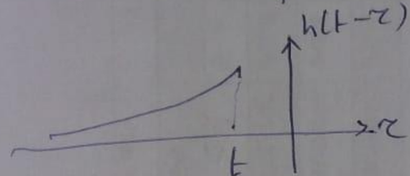
find $y(t) = x(t) * h(t)$.





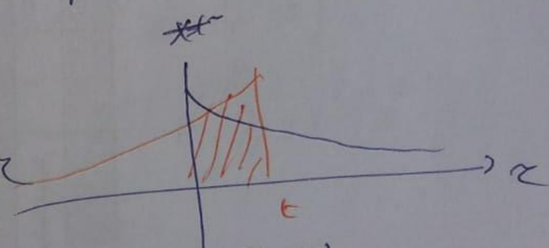






1) for $t<0$ no overlap $\Rightarrow y(t)=0$.

2) for $t>0$



$$y(t) = \int_0^t x(\tau)\,h(t-\tau)\,d\tau$$

$$x(\tau) = e^{-2\tau}u(\tau) \qquad h(t-\tau) = e^{-5(t-\tau)} = e^{-5t}e^{5\tau}$$

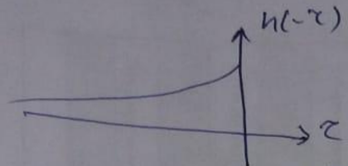
$$\Rightarrow y(t) = \int_0^t e^{-2\tau}e^{5\tau}e^{-5t}\,d\tau = e^{-5t}\int_0^t e^{3\tau}\,d\tau$$

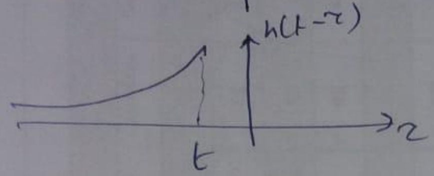
$$y(t) = \frac{1}{3}e^{-5t}\left[e^{3\tau}\Big|_0^t\right] = \frac{1}{3}e^{-5t}\left(e^{3t}-1\right).$$

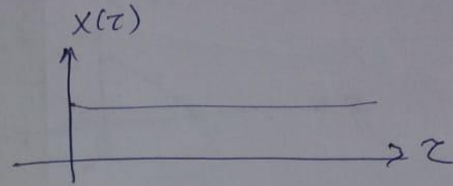
$$= \frac{1}{3}\left(e^{-2t}-e^{-5t}\right)u(t)$$

EX $h(t) = e^{-t} u(t)$

$x(t) = u(t)$



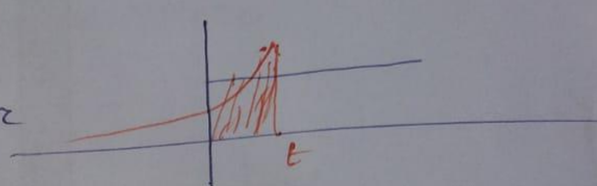




1) for $t<0$ no overlap $y(t) = 0$.

2) for $t>0$

$$y(t) = \int_0^t x(\tau)\, h(t-\tau)\, d\tau$$

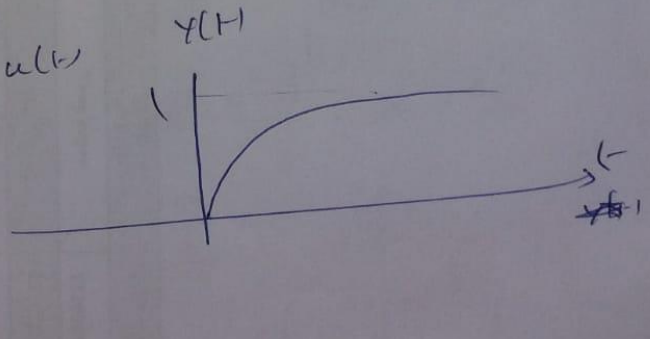


$x(\tau) = u(\tau)$

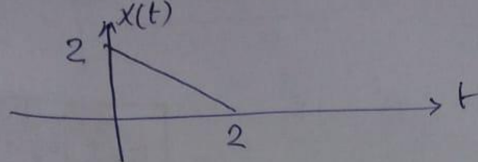
$$h(t-\tau) = e^{-(t-\tau)} u(t-\tau) = e^{-t} e^{\tau} u(t-\tau)$$

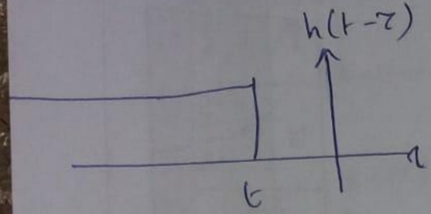
$$y(t) = \int_0^t e^{-t} e^{\tau}\, d\tau = e^{-t}\left[e^{\tau} \Big|_0^t \right] = e^{-t}\left[e^{t} - 1 \right]$$

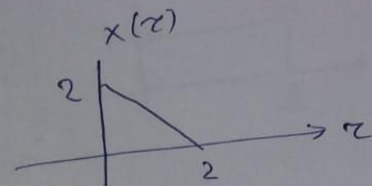
$$y(t) = (1 - e^{-t})\, u(t)$$



Ex $h(t) = u(t)$





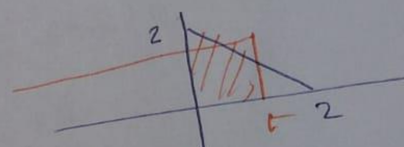


first overlap occurs when $t > 0$.

① for $t < 0$ no overlap $y(t) = 0$

② for $0 < t < 2$

$$y(t) = \int_0^t x(\tau)\, h(t-\tau)\, d\tau$$



$$\cancel{y(t) = \int_0^t (\tau - 2)\, d\tau = \frac{\tau^2}{2} - 2\tau \Big|_0^t}$$

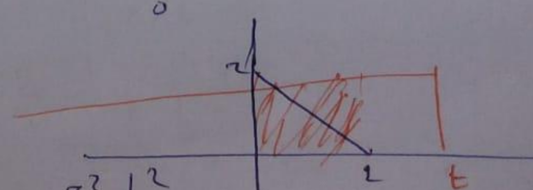
$$\cancel{y(t) = \frac{t^2}{2} - 2t}$$

$\cancel{x(\tau) = -2 + \tau}$

$x(\tau) = 2 - \tau$

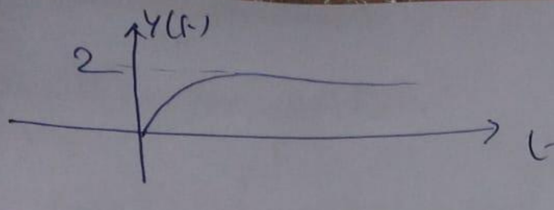
$$y(t) = \int_0^t (2-\tau)\, d\tau = 2\tau - \frac{\tau^2}{2} \Big|_0^t = 2t - t^2/2$$

3) for $t > 2$

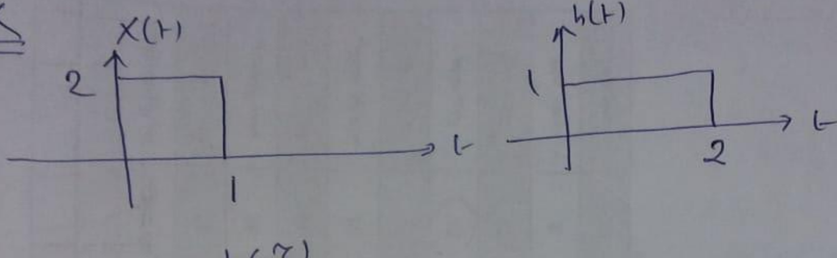


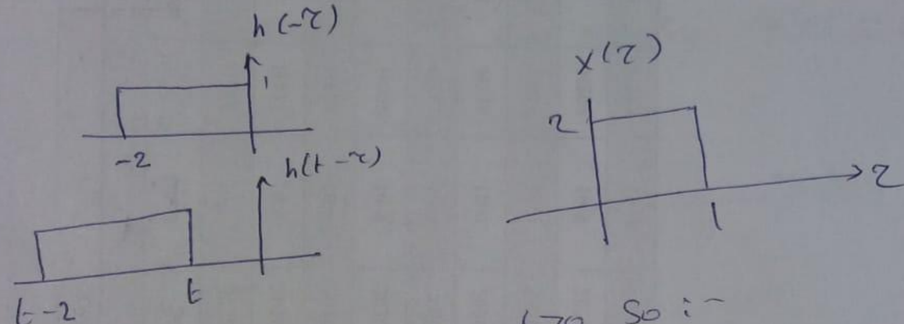
$$y(t) = \int_0^2 (2-\tau)\, d\tau = 2\tau - \tau^2/2 \Big|_0^2$$

$y(t) = 2$



EX



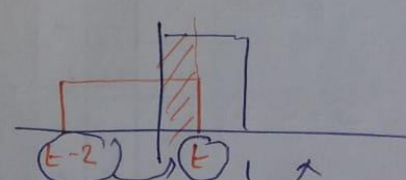


first overlap occurs when $t>0$ So :-

① for $t<0$ no overlap $y(t)=0$

② for $0<t<1$

$$y(t)=\int_0^t 2\times 1\,d\tau = 2\tau\Big|_0^t = 2t.$$

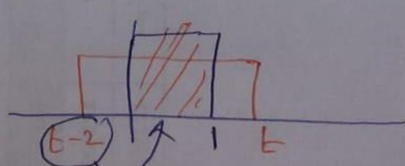


$t-2>0$ or $t>1$

$t>2$

③ for $1<t<2$

$$y(t)=\int_0^1 2\times 1\,d\tau = 2\tau\Big|_0^1 = 2$$

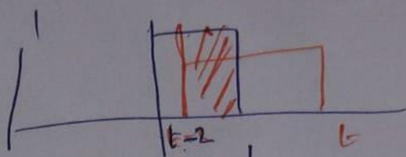


$t-2>0$

$t>2$

(3) for $2<t<3$

$$y(t)=\int_{t-2}^{1} 2\times 1\, d\tau = 2\tau\Big|_{t-2}^{1}$$

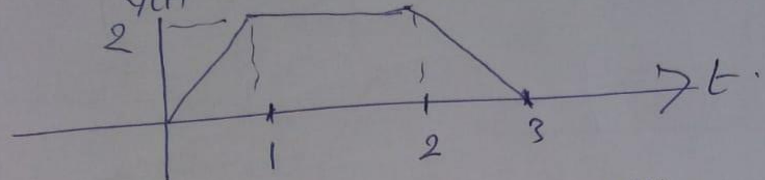


$t-2>1$

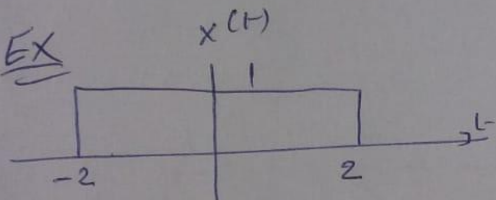
$t>3$.

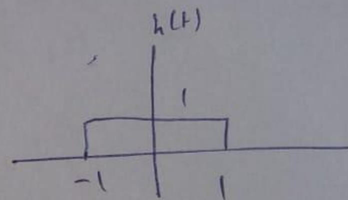
$y(t)=2-2(t-2)=6-2t.$

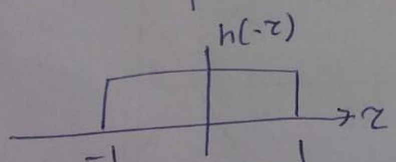
(4) for $t>3$ no overlap $y(t)=0$

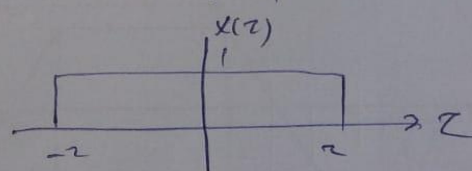


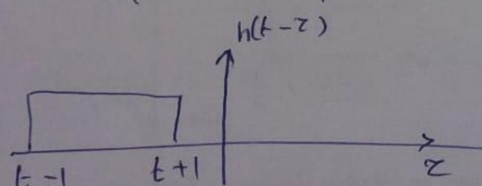
EX

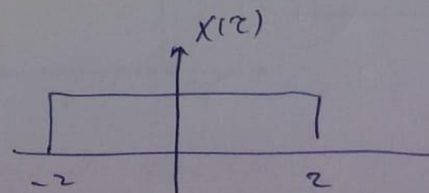












first overlap occurs when $t+1>-2$

$t>-3$

1) for $t<-3$ no overlap.

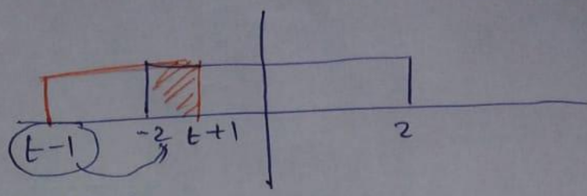
$y(t)=0$

2) for $-3 < t < -1$

$$y(t) = \int_{-2}^{t+1} |x| \, d\tau$$

$$y(t) = \tau \Big|_{-2}^{t+1} = t+1+2$$

$$y(t) = t+3 .$$



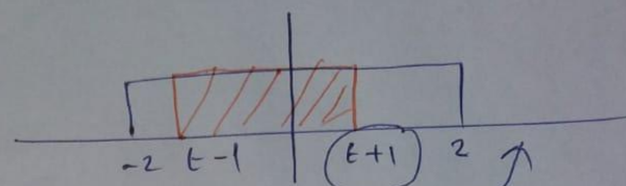
$t-1 > -2$

$t > -1$

3) for $-1 < t < 1$

$$y(t) = \int_{t-1}^{t+1} |x| \, d\tau$$

$$y(t) = \tau \Big|_{t-1}^{t+1} = t+1-(t-1) = 2$$



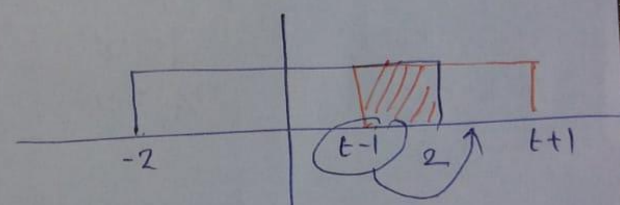
$t+1 > 2$

$t > 1$

4) for $1 < t < 3$

$$y(t) = \int_{t-1}^{2} |x| \, d\tau = \tau \Big|_{t-1}^{2}$$

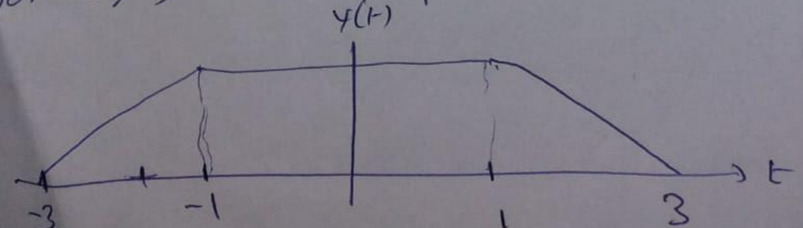
$$y(t) = 2 - t + 1 = 3 - t .$$



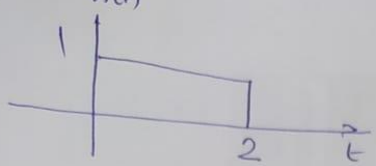
$t-1 > 2$

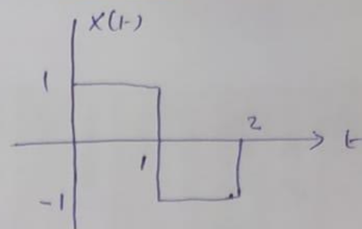
$t > 3$

5) for $t > 3$ no overlap $y(t) = 0$.

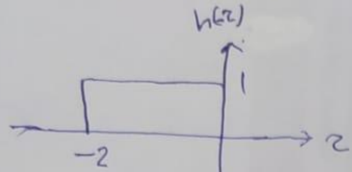


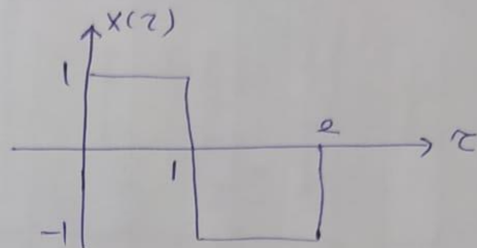
Ex

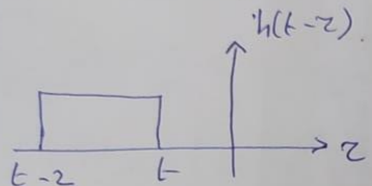




find $y(t)$





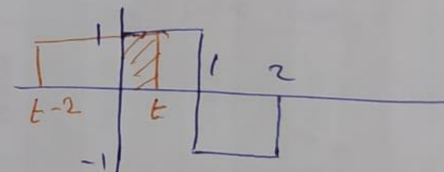


first overlap occurs when $t>0$.

(1) for $t<0$ no overlap $y(t)=0$

(2) for $0<t<1$

$$y(t)=\int_0^t 1\times 1\,d\tau=\tau\Big|_0^t=t.$$



The overlap will change when $t>1$ → occurs first

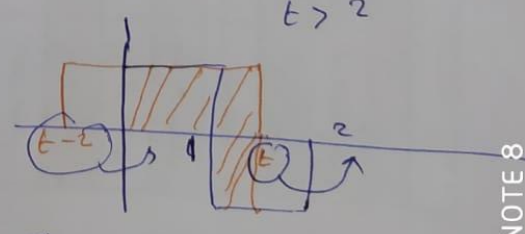
or $t-2>0$

$t>2$

(3) for $1<t<2$

$$y(t)=\int_0^1 1\times 1\,d\tau+\int_1^t 1\times(-1)\,d\tau$$

$$y(t)=\tau\Big|_0^1-\tau\Big|_1^t=2-t$$



The overlap changes when $t>2$

or $t-2>0 \Rightarrow t>2$

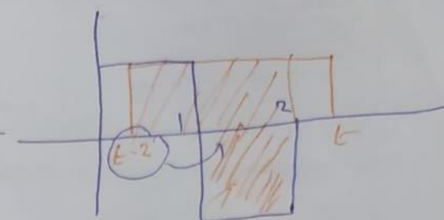
the two occurs at the same time

4 for $2 < t < 3$

$$y(t) = \int_{t-2}^{1} 1 \times 1 \, d\tau + \int_{1}^{2} 1 \times (-1) \, d\tau$$

$$y(t) = \tau \Big|_{t-2}^{1} - \tau \Big|_{1}^{2}$$

$$y(t) = 2 - t$$

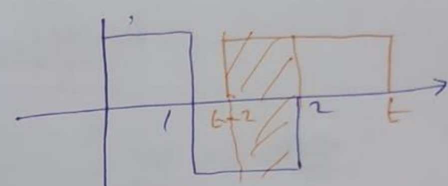


the overlap will change when:
$t-2 > 1$
$t > 3$

5 for $3 < t < 4$

$$y(t) = \int_{t-2}^{2} 1 \times (-1) \, d\tau$$

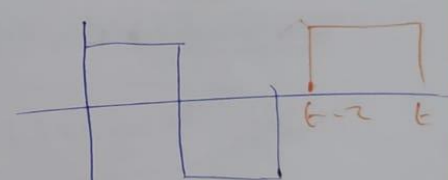
$$y(t) = -\tau \Big|_{t-2}^{2} = -4 + t.$$

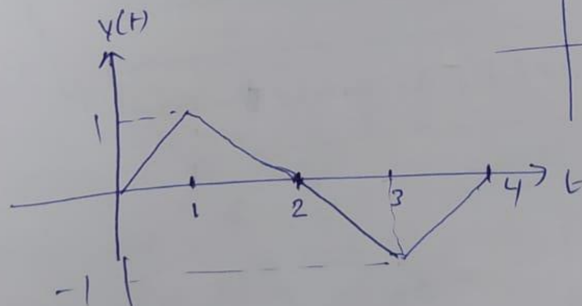


overlap change when
$t-2 > 2$
$t > 4$

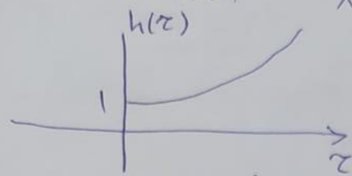
6 for $t > 4$

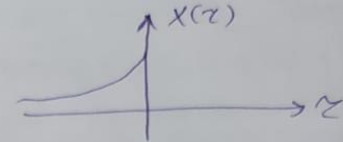
no overlap $y(t) = 0$.

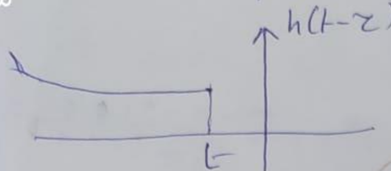


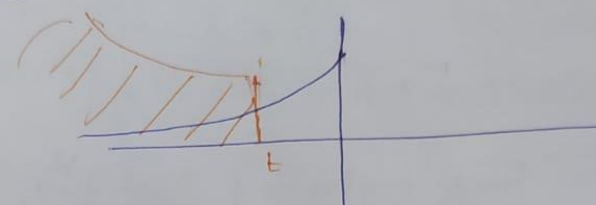


Ex $h(t) = e^{2t}u(t)$ $\quad x(t) = e^{3t}u(-t)$. find $y(t)$









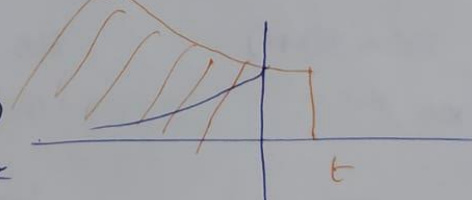
1) for $-\infty < t < 0$

$$y(t) = \int_{-\infty}^{t} x(\tau)\,h(t-\tau)\,d\tau = \int_{-\infty}^{t} e^{3\tau} e^{2(t-\tau)}\,d\tau$$

$$y(t) = e^{2t}\int_{-\infty}^{t} e^{3\tau} e^{-2\tau}\,d\tau = e^{2t}\int_{-\infty}^{t} e^{\tau}\,d\tau$$

$$y(t) = e^{2t} e^{\tau}\Big|_{-\infty}^{t} = e^{2t}e^{t} = e^{3t}$$

2) for $t > 0$



$$y(t) = \int_{-\infty}^{0} e^{3\tau} e^{2(t-\tau)}\,d\tau$$

$$= e^{2t}\left[e^{\tau}\Big|_{-\infty}^{0}\right] = e^{2t}$$

Properties of Convolution:-

(1) The Commutative property.

We find that $y(t) = x(t) * h(t)$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

According to commutative property $y(t)$ can also be expressed as:

$$y(t) = h(t) \circledast x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau.$$

Proof $y(t) = x(t) \circledast h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

Let $t-\tau = u \Rightarrow \tau = t-u$

$-d\tau = du \Rightarrow d\tau = -du.$

as $\tau \to \infty \Rightarrow u \to -\infty$

$\tau \to -\infty \Rightarrow u \to \infty$

$$\Rightarrow y(t) = \int_{\infty}^{-\infty} x(t-u)\, h(u)\, (-du)$$

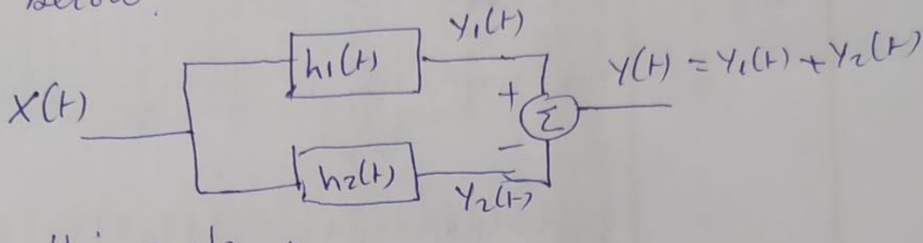
$$= -\int_{\infty}^{-\infty} h(u)\, x(t-u)\, du$$

$$= \int_{-\infty}^{\infty} h(\tau) x(t-\tau)\, d\tau$$

$$= h(t) \circledast x(t)$$

(2) The distributive property (5)

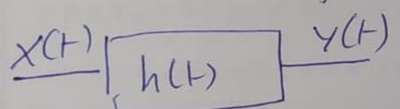
We have two systems connected in parallel with two impulse responses $h_1(t), h_2(t)$ as below:



for this system:

$$\left.\begin{aligned} Y_1(t) &= X(t) \circledast h_1(t) \\ Y_2(t) &= X(t) \circledast h_2(t) \end{aligned}\right\} \Rightarrow Y(t) = X(t) \circledast h_1(t) + X(t) \circledast h_2(t)$$

according to this property this system is equivalent to the following system:



$$Y(t) = X(t) \circledast h(t)$$

where: $h(t) = h_1(t) + h_2(t)$.

this means: $Y(t) = X(t) \circledast h(t)$

$$Y(t) = X(t) \circledast \big(h_1(t) + h_2(t)\big).$$

Proof: $Y_1(t) = \int_{-\infty}^{\infty} X(\tau) h_1(t-\tau) d\tau$

$$Y_2(t) = \int_{-\infty}^{\infty} X(\tau) h_2(t-\tau) d\tau$$

Then:

$$Y(t) = Y_1(t) + Y_2(t)$$

$$= X(t) \circledast h_1(t) + X(t) \circledast h_2(t).$$

This is from the first configuration.

for the second configuration:

$$Y(t) = X(t) \circledast (h_1(t) + h_2(t))$$

$$Y(t) = \int_{-\infty}^{\infty} X(\tau)\left(h_1(t-\tau) + h_2(t-\tau)\right) d\tau$$

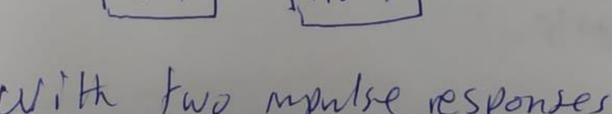
$$Y(t) = \int_{-\infty}^{\infty} X(\tau) h_1(t-\tau) d\tau + \int_{-\infty}^{\infty} X(\tau) h_2(t-\tau) d\tau$$

$$= X(t) \circledast h_1(t) + X(t) \circledast h_2(t).$$

$$\Rightarrow X(t) \circledast (h_1(t) + h_2(t)) = X(t) \circledast h_1(t) + X(t) \circledast h_2(t)$$

(3) Associative property

Consider two systems connected in cascade in the following way:

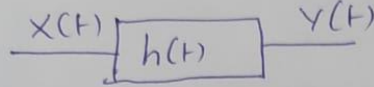


With two impulse responses $h_1(t)$ and $h_2(t)$

in this system:- $y_1(t) = x(t) \circledast h_1(t)$

$$y(t) = y_1(t) \circledast h_2(t) = [x(t) \circledast h_1(t)] \circledast h_2(t).$$

According to this property this system can be replaced by the following system.



where $h(t) = h_1(t) \circledast h_2(t)$

from this:

$$y(t) = x(t) \circledast h(t) = x(t) \circledast [h_1(t) \circledast h_2(t)].$$

$$\Longrightarrow [x(t) \circledast h_1(t)] \circledast h_2(t) = x(t) \circledast [h_1(t) \circledast h_2(t)].$$

(4) The shift property of convolution.

$$x(t) \circledast h(t) = y(t)$$

according to this property:

$$x(t) \circledast h(t-T) = y(t-T).$$

this means shift in the impulse response results in a shift in the output.

also $x(t-P) \circledast h(t-T) = y(t-P-T)$

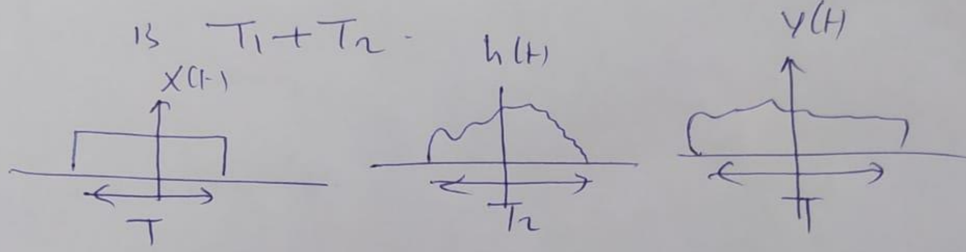
(5) The width property of convolution

Let $x(t)$ is of finite duration T_1

$h(t)$ is of finite duration T_2.

Then the width of $y(t) = x(t) \circledast h(t)$

is $T_1 + T_2$.



$T = T_1 + T_2$.

(6) Convolution of a signal with an impulse.

$$x(t) \circledast \delta(t) = \delta(t) \circledast x(t) = x(t).$$

$$x(t) \circledast \delta(t-t_0) = \delta(t-t_0) \circledast x(t) = x(t-t_0)$$

⑥ Convolution with Unit impulse.

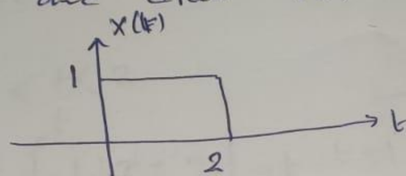
$$x(t) \circledast \delta(t) = \delta(t) \circledast x(t) = x(t) \qquad ①$$

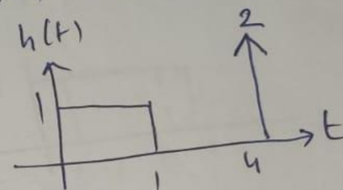
Convolution unit impulse with any signal $x(t)$ is the signal itself.

also:

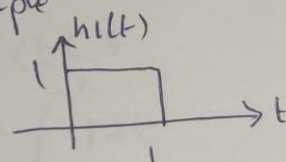
$$x(t) \circledast \delta(t-t_0) = \delta(t-t_0) * x(t) = x(t-t_0)$$

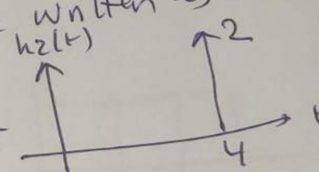
<u>Example</u> Given $x(t)$ and $h(t)$ as below find and draw $y(t)$.





in this example $h(t)$ can be written as two signals.

$h(t) =$


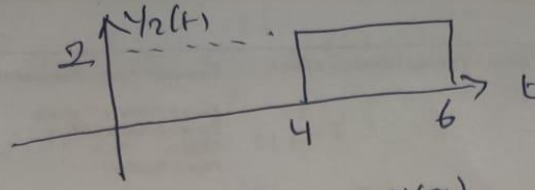
$+$


Then $y(t) = x(t) * [h_1(t) + h_2(t)]$

$$= \underbrace{x(t) \circledast h_1(t)}_{y_1(t)} + \underbrace{x(t) \circledast h_2(t)}_{y_2(t)}$$

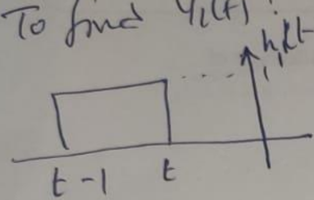
$$y(t) = y_1(t) + y_2(t).$$

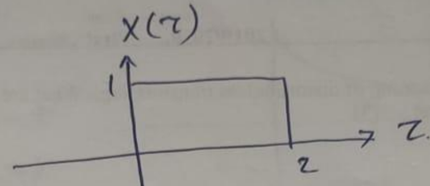
$$y_2(t) = x(t) \circledast 2\delta(t-4) = 2x(t-4).$$



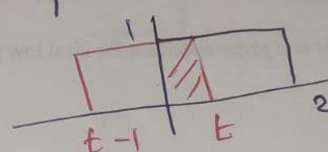
2

To find $y_1(t)$:



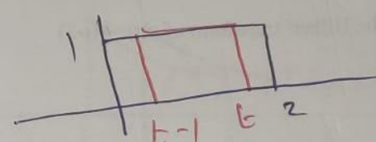


1) for $t<0$ no over lap. $y_1(t)=0$.



(2) for $0<t<1$

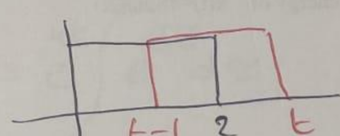
$$y_1(t)=\int_0^t 1\times 1\, d\tau = t.$$



(3) for $1<t<2$

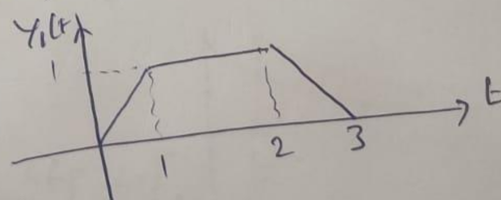
$$y_1(t)=\int_{t-1}^{t} 1\, d\tau = \tau\Big|_{t-1}^{t} = t-t+1 = 1$$

4 for $2<t<3$



$$y(t) = \int_{t-1}^{2} 1\, d\tau$$

$$= \tau\Big|_{t-1}^{2} = 2-(t-1) = 3-t$$



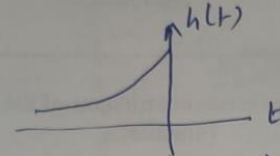
$$y(t) = y_1(t) + y_2(t)$$

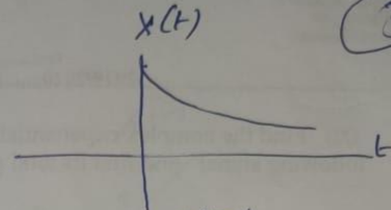
SHOT ON MI 9T
AI TRIPLE CAMERA

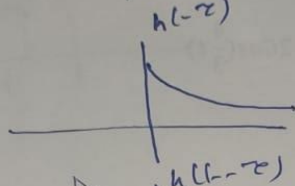
Ex $\quad x(t) = e^{-2t}\, u(t)$

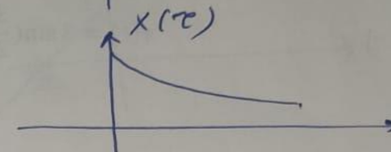
$h(t) = e^{4t}\, u(-t).$ find $y(t)$.

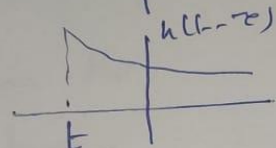
3



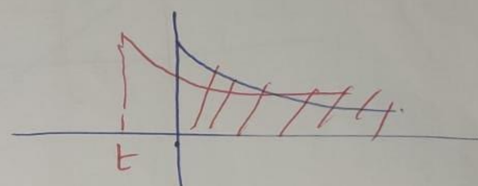








(1) for $t<0$

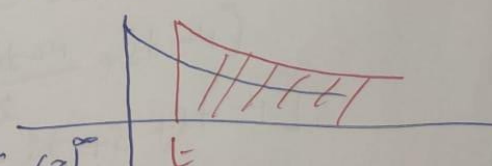


$$y(t) = \int_0^\infty e^{-2\tau}\, e^{4(t-\tau)}\, d\tau$$

$$= \int_0^\infty e^{-2\tau}\, e^{4t}\, e^{-4\tau}\, d\tau = e^{4t}\int_0^\infty e^{-6\tau}\, d\tau$$

$$= -\frac{1}{6} e^{4t}\left[e^{-6\tau}\Big|_0^\infty \right] = \frac{1}{6} e^{4t}.$$

(2) for $t>0$



$$y(t) = \int_t^\infty e^{4t}\, e^{-6\tau}\, d\tau = -\frac{1}{6} e^{4t}\left[e^{-6\tau}\Big|_t^\infty \right]$$

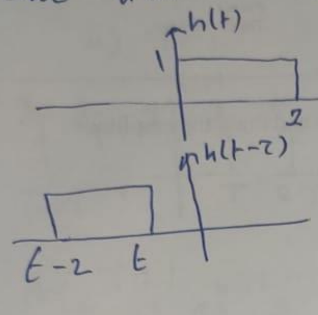
$$= -\frac{1}{6} e^{4t}\left[-e^{-6t} \right]$$

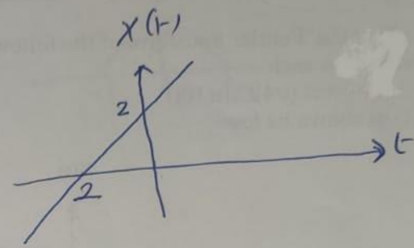
$$= \frac{1}{6} e^{-2t}.$$

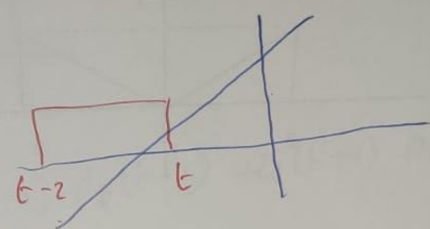
4

<u>Ex</u> If $x(t) = t + 2$

and $h(t)$ as below: find $y(t)$.







for any time the overlap between $x(\tau)$ and $h(t-\tau)$ is between $t-2$ and t.

So: $$y(t) = \int_{t-2}^{t} x(\tau)\, h(t-\tau)\, d\tau.$$

$$= \int_{t-2}^{t} (\tau + 2)\, d\tau = \left. \frac{\tau^2}{2} + 2\tau \right|_{t-2}^{t}$$

$$= \frac{t^2}{2} + 2t - \left[\frac{(t-2)^2}{2} + 2(t-2)\right].$$

$$= \frac{t^2}{2} + 2t - \left[\frac{t^2 - 4t + 4}{2} + 2t - 4\right].$$

$$= t^2/2 + 2t - \left[\frac{t^2 - 4t + 4}{2} + 2t - 4\right].$$

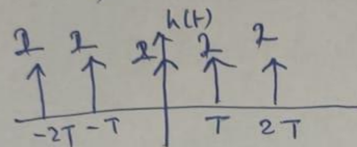
$$= t^2/2 + 2t - \left[t^2/2 - \cancel{2t} + 2 + \cancel{2t} - 4\right].$$

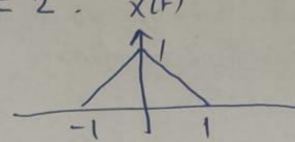
$$= t^2/2 + 2t - t^2/2 + 2 = 2t + 2$$

EX Given $x(t)$ and $h(t)$ as below. find and draw $y(t)$ if:

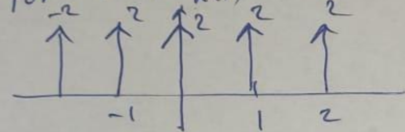
a) $T=1$ b) $T=2$.

5





a) for $T=1$



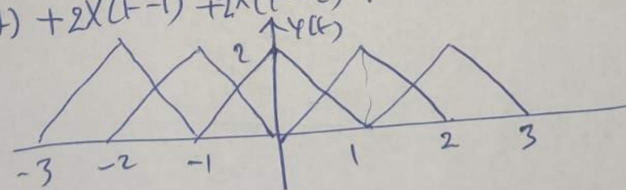
$\Rightarrow y(t) = x(t) \circledast h(t)$

$h(t) = 2\delta(t) + 2\delta(t-1) + 2\delta(t-2) + 2\delta(t+1) + 2\delta(t+2)$.

$y(t) = h(t) \circledast [x(t)] = x(t) \circledast [2\delta(t) + 2\delta(t-1) + 2\delta(t-2) + 2\delta(t+1) + 2\delta(t+2)]$

$y(t) = 2x(t) \circledast \delta(t) + 2x(t) \circledast \delta(t-1) + 2x(t) \circledast \delta(t+1) + 2x(t) \circledast \delta(t+2) + 2x(t) \circledast \delta(t-2) + 2x(t) \circledast \delta(t+1)$

$= 2x(t) + 2x(t-1) + 2x(t-2) + 2x(t+1) + 2x(t+2)$



b) for $T=2$