1

Fourier Series analysis of periodic signals.

Let $x(t)$ be a periodic signal with fundamental period T_0 and fundamental radian frequency ω_0. Then signal can be resolved into infinite sum of Sine and Cosine terms

Then $x(t)$ can be written as :—

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t.$$

This form is called Trigonometric Fourier Series representation of the periodic signal $x(t)$

where: $$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\, dt.$$

a_0 represents the average or dc value of $x(t)$.

$\frac{\text{area}}{T_0}$

and:

$$a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos n\omega_0 t\, dt$$

$$b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin n\omega_0 t\, dt.$$

a_n and b_n are Called The Trigonometric Fourier Series coefficients of $x(t)$. a_n and b_n are the amplitudes of the Sine and Cosine terms.

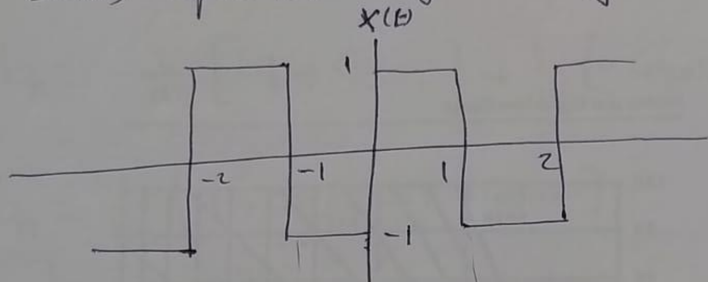
In finding The trigonometric Fourier Series Coefficients of $x(t)$:-

(1) If $x(t)$ is antiseymmetric with respect to time axis Then $a_0 = 0$

(2) If $x(t)$ is even Signal only Cosine terms exist this means $b_n = 0$

(3) If $x(t)$ is odd only Sine terms exist this means $a_n = 0$.

<u>Example</u> Find the Trigonometric Fourier (3)
Series representation of the signal below:



$T_0 = 2 \qquad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2} = \pi.$

$x(t)$ is odd $\left(x(-t) = -x(t) \quad \forall t \right) . \Rightarrow a_n = 0.$

$x(t)$ is antisymmetric around the time axis

$$\Longrightarrow a_0 = 0.$$

Then :

$$x(t) = \sum_{n=1}^{\infty} b_n \sin n\omega_0 t = \sum_{n=1}^{\infty} b_n \sin n\pi t.$$

$$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin n\pi t \, dt.$$ (integration over one period)

$$b_n = \frac{2}{2} \int_{-1}^{0} - \sin n\pi t \, dt + \frac{2}{2} \int_{0}^{1} \sin n\pi t \, dt =$$

(4)

$$b_n = -\frac{1}{n\pi}\left[-\cos n\pi t \Big|_{-1}^{0}\right] + \frac{1}{n\pi}\left[-\cos n\pi t \Big|_{0}^{1}\right]$$

$$b_n = -\frac{1}{n\pi}\left[-1 + \cos n\pi\right] + \frac{1}{n\pi}\left[-\cos n\pi + 1\right].$$

$$b_n = \frac{1}{n\pi}\left[1 - \cos n\pi + 1 - \cos n\pi\right].$$

$$b_n = \frac{1}{n\pi}\left[2 - 2\cos n\pi\right] = \frac{2}{n\pi}\left[1 - \cos n\pi\right].$$

for n odd $\cos n\pi = -1$

for n even $\cos n\pi = 1$

$\Rightarrow b_n = \frac{4}{n\pi}$ n odd

$b_n = 0$ n even.

$$\Rightarrow X(t) = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{4}{n\pi} \sin n\pi t.$$

$X(t)$ can be written as:

$$X(t) = \frac{4}{\pi}\sin \pi t + \frac{4}{3\pi}\sin 3\pi t + \frac{4}{5\pi}\sin 5\pi t + \cdots$$

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Also $x(t)$ Can be written as;

$$x(t) = \sum_{\substack{n=1 \\ n-odd}}^{\infty} \frac{4}{n\pi} \cos(n\pi t - 90^\circ).$$

This is Called the phase.

Discrete spectrum

The representation of the Signal by Fourier Series is equivalent to the resolution of the Signal into its various harmonic Components. We Can Say that the periodic Signal $x(t)$ with Fundamental radian frequency $\underline{\underline{w_0}}$ have components of frequencies.

$$0, \mp w_0, \mp 2w_0, \mp 3w_0 \text{ ---}$$

While $x(t)$ exists in time domain. The frequency domain representation of the Signal Consists of frequencies $0, w_0, 2w_0, 3w_0$ ----

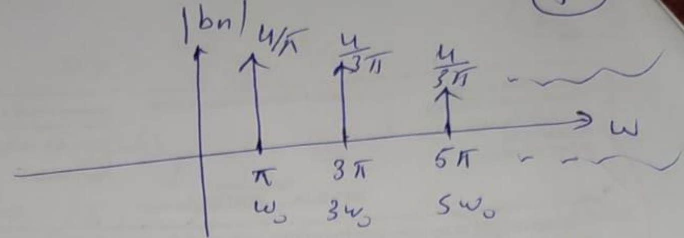
So any Periodic Signal Can be represented in two ways:

1) Time domain representation of the Signal where the signal is defined as a function of time.

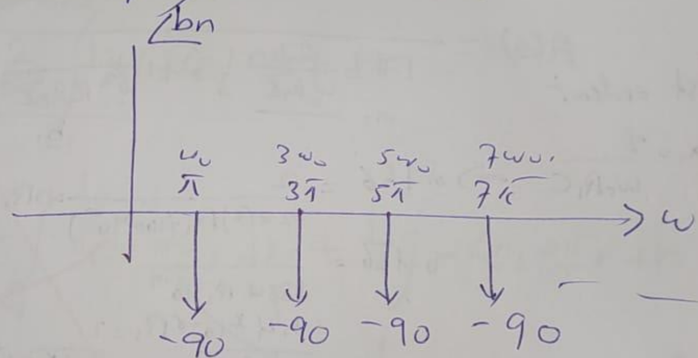
(2) frequency domain representation of X(f) which represents the Signal by its frequency spectrum. Spectrum is a graph that represents all frequency components in the Signal and the effect or the amplitude of each frequency component. And the phase of each frequency component. So we have two spectrums

(1) Amplitude Spectrum which represents all frequency components represented in a signal and the amplitude of each component. for our example.

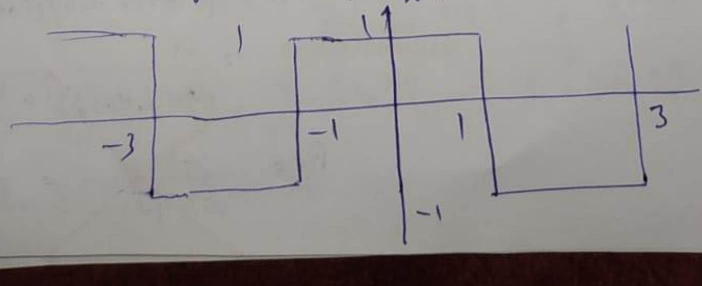
⑦



(2) phase spectrum: which represent, all frequency components of the signal and the phase of each frequency. For our example.



EXAMPLE-2: Find the Trigonometric Fourier Series representation of the signal below. draw the amplitude and phase spectrum



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$a_n = 0$ n even.

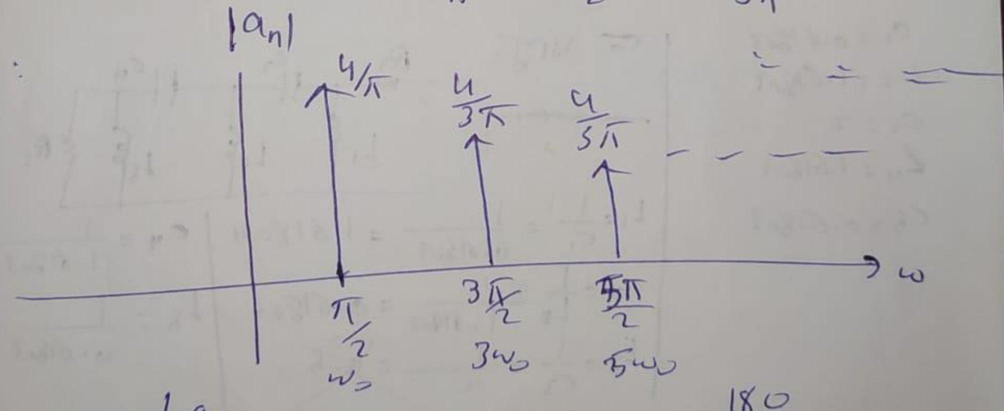
$a_n = \frac{4}{n\pi}$ $n = 1, 5, 9, 13 \ldots$

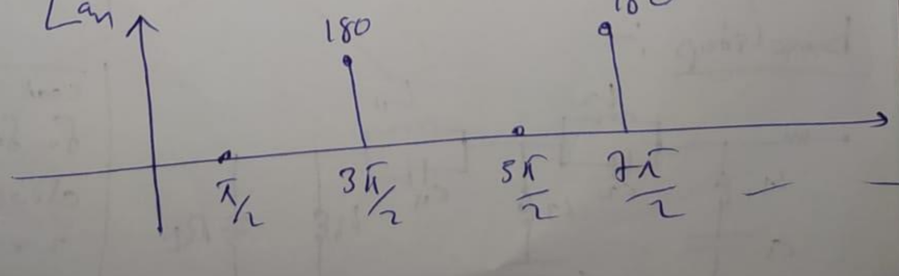
$a_n = \frac{-4}{n\pi}$ $n = 3, 7, 11 \ldots$

$$\Rightarrow X(t) = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{4}{n\pi} \sin \frac{n\pi}{2} \cos \frac{n\pi}{2} t.$$

$$X(t) = \frac{4}{\pi} \cos \frac{\pi}{2} t - \frac{4}{3\pi} \cos \frac{3\pi}{2} t + \frac{4}{5\pi} \cos \frac{5\pi}{2} t + \ldots$$

So:





If the Signal is not even and not odd so b_n and a_n exist (1.0)

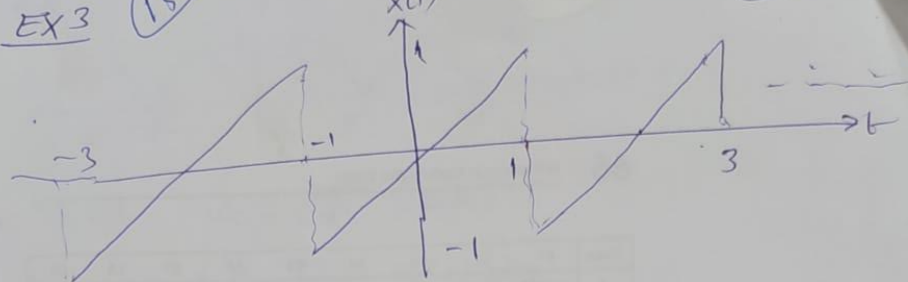
In this Case the amplitude is defined as:

$$|a_n + b_n| = \sqrt{a_n^2 + b_n^2}$$

and the phase is defined as:

$$\text{phase} = \tan^{-1} \frac{b_n}{a_n}$$

EX 3 (15) ②



$a_0 = 0$.

$T_0 = 2 \quad \omega_0 = \frac{2\pi}{2} = \pi$

$x(t)$ is odd $\Rightarrow a_n = 0$.

$$x(t) = \sum_{n=1}^{\infty} b_n \sin n\omega_0 t = \sum_{n=1}^{\infty} b_n \sin n\pi t.$$

$$b_n = \frac{2}{2}\int_{-1}^{1} t \sin n\pi t \, dt = \int_{-1}^{1} t \sin n\pi t \, dt$$

integrate by Parts:

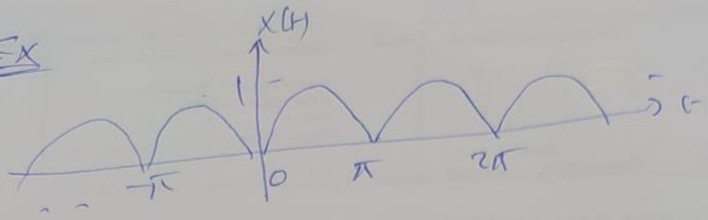
$u = t \qquad du = dt \qquad dv = \sin n\pi t \, dt$

$v = \frac{-1}{n\pi} \cos n\pi t$

$$\Rightarrow b_n = \frac{-2}{n\pi} \cos n\pi$$

$$\Rightarrow x(t) = \sum_{n=1}^{\infty} -\frac{2}{n\pi} \cos n\pi \sin n\pi t.$$

Ex



$T_0 = \pi \qquad \omega_0 = \frac{2\pi}{\pi} = 2$ sec.

Nosy. around the time axis ⇒

$$a_0 = \frac{1}{\pi}\int_0^{\pi} \sin t\, dt = \frac{1}{\pi}\left[-\cos t\Big|_0^{\pi}\right] = \frac{2}{\pi}$$

Symmetric around the vertical axis ⇒ $b_n = 0$.

$$a_n = \frac{2}{\pi}\int_0^{\pi} \sin t \cos n\omega_0 t\, dt$$

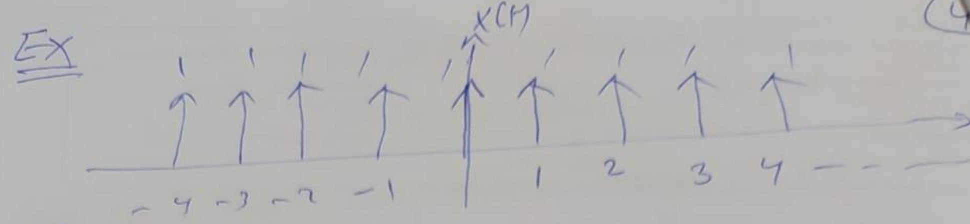
$$= \frac{2}{2\pi}\int_0^{\pi} \sin(2n+1)t\, dt - \frac{2}{2\pi}\int_0^{\pi} \sin(1-2n)t\, dt$$

$$(\sin A \cos B) = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$a_n = \frac{4}{\pi(1-4n^2)}$$

$$x(t) = \frac{2}{\pi} + \sum_{n=1}^{\infty} \frac{4}{\pi}\frac{1}{(1-4n^2)}\cos 2nt$$

Ex



$T_0 = 1 \qquad \omega_0 = \frac{2\pi}{T_0} = 2\pi$

$a_0 = \frac{1}{T_0} \int_{T_0} x(t)\,dt \qquad\qquad x(t) = \delta(t).$

$a_0 = \int_{-0.5}^{0.5} \delta(t)\,dt = 1$

Since $x(t)$ is even:

$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos n\omega_0 t\,dt = 2 \int_{-1/2}^{1/2} \delta(t) \cos 2\pi n t\,dt.$

$= 2.$

$x(t) = 1 + \sum_{n=1}^{\infty} 2\cos(2\pi n t).$

Complex Exponential Fourier Series representation of Signals.

We said that any Periodic signal $x(t)$ can be written as:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t.$$

We Know that:-

$$\cos n\omega_0 t = \frac{1}{2}\left[e^{-jn\omega_0 t} + \frac{1}{2} e^{jn\omega_0 t}\right].$$

$$\sin n\omega_0 t = \frac{1}{2j}\left[e^{jn\omega_0 t} - e^{-jn\omega_0 t}\right].$$

Then $x(t)$ can be written as:

$$x(t) = a_0 + \sum_{n=1}^{\infty} \frac{a_n}{2}\left[e^{jn\omega_0 t} + e^{jn\omega_0 t}\right] + \frac{b_n}{2j}\left[e^{jn\omega_0 t} - e^{jn\omega_0 t}\right].$$

after some mathmatical operations we can say that:

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \quad \text{——} \; (1)$$

where

$$D_n = \frac{1}{T_0}\int_{T_0} x(t) e^{-jn\omega_0 t}\, dt.$$

(6)

(1) is called Complex exponential Fourier Series representation of periodic Signals.

D_n: is called Complex exponential Fourier Series Coefficients.

In this representation. any periodic Signal $x(t)$ Can be written as infinite some of complex exponentials.. With radian frequencies $\mp\omega_0$, $\mp 2\omega_0$ $\mp 3\omega_0$, $\mp 4\omega_0$ - ----

The amplitude of each complex exponential is represented by D_n. D_n: represents the effect of each Complex exponential.

So: We have two representations for any periodic signals:

(1) To represent the Signal as a function of time, This is called Time domain representation of the Signal.

(2) Frequency domain representation of $x(t)$ which represents the signal as a frequency spectrum.

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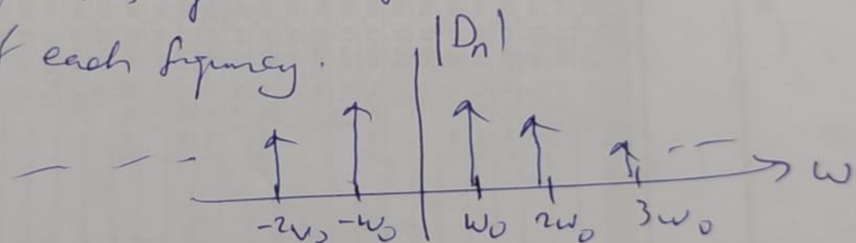
(7)

Frequency spectrum is a graph that shows all frequency components of the signal and the amplitude and phase of each frequency. In general D_n is complex so D_n can be written as:

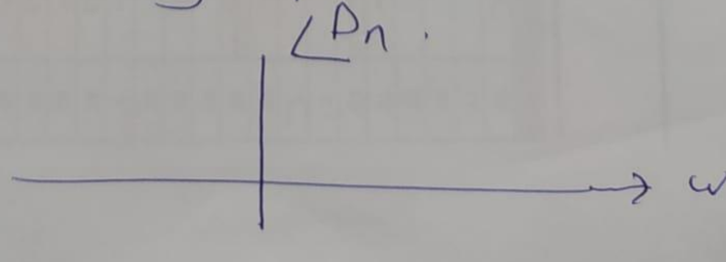
$$D_n = |D_n| e^{j\angle D_n}.$$

So we have two spectrums of the signal.

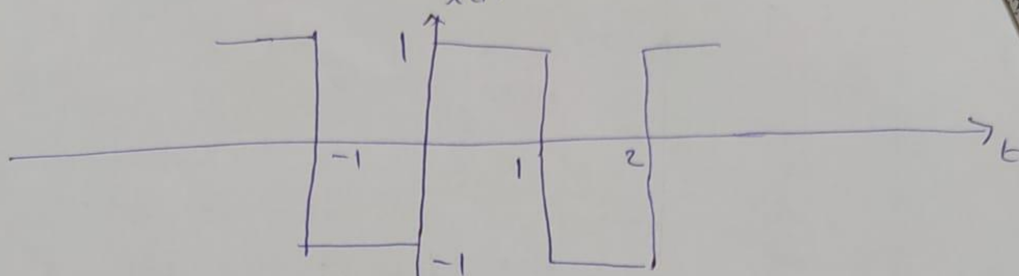
1) Amplitude spectrum: That gives all frequencies of the signal and the amplitude of each frequency.



2) Phase spectrum which gives all frequency components of the signal and the phase of each frequency component.



EX Find the Complex exponential Fourier Series representation of the following Signal.



$T_0 = 2 \quad w_0 = \frac{2\pi}{2} = \pi$

$$D_n = \frac{1}{T_0}\int_{T_0} x(t)e^{-jnw_0 t}\,dt = \frac{1}{2}\int_{-1}^{0} -1\,e^{-jn\pi t}\,dt + \frac{1}{2}\int_{0}^{1} 1\,e^{-jn\pi t}\,dt.$$

$$D_n = \frac{-1}{2}\,\frac{-1}{jn\pi}\left[e^{-jn\pi t}\Big|_{-1}^{0}\right] + \frac{1}{2}\,\frac{-1}{jn\pi}\left[e^{-jn\pi t}\Big|_{0}^{1}\right].$$

$$D_n = \frac{1}{2jn\pi}\left[1 - e^{jn\pi}\right] - \frac{1}{2jn\pi}\left[e^{-jn\pi} - 1\right].$$

$$D_n = \frac{1}{2jn\pi}\left[1 - e^{jn\pi} - e^{-jn\pi} + 1\right].$$

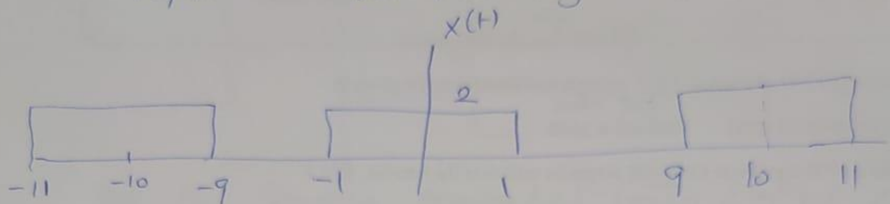
$$D_n = \frac{1}{2jn\pi}\left[2 - 2\cos n\pi\right] = \frac{1}{n\pi}\left[1 - \cos n\pi\right].$$

$D_n = \frac{2}{jn\pi}$ for n-odd

$D_n = 0$ for n-even.

(16) Ex

Find the Complex exponential Fourier series representation for the signal below.



$T_0 = 10 \quad \omega_0 = \frac{2\pi}{10} = \pi/5$

$$D_n = \frac{1}{T_0}\int_{T_0} x(t) e^{-jn\omega_0 t}\, dt = \frac{1}{10}\int_{-1}^{1} 2 e^{-j\frac{n\pi}{5}t}\, dt$$

$$D_n = \frac{1}{5}\,\frac{-5}{jn\pi}\left[e^{-j\frac{n\pi}{5}t}\Big|_{-1}^{1} \right]$$

$$D_n = \frac{-1}{jn\pi}\left[e^{-j\frac{n\pi}{5}} - e^{j\frac{n\pi}{5}} \right]$$

$$D_n = \frac{1}{jn\pi}\left[e^{j\frac{n\pi}{5}} - e^{-j\frac{n\pi}{5}} \right].$$

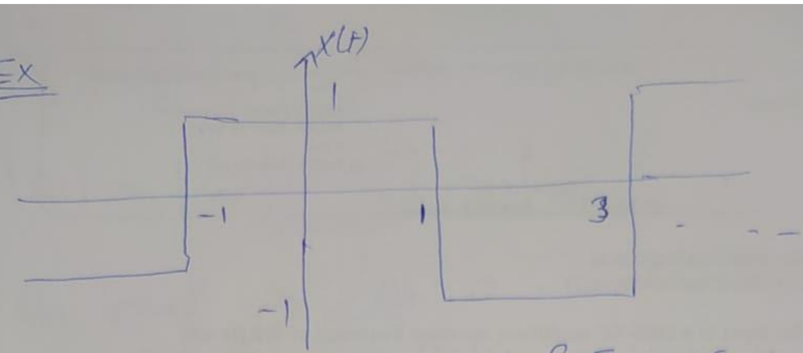
$$D_n = \frac{1}{jn\pi}\, 2j \sin\frac{n\pi}{5} = \frac{2}{n\pi}\sin\frac{n\pi}{5}$$

$D_n = \frac{2}{n\pi}\sin\frac{n\pi}{5}$ for $n \neq 0$.

$$D_0 = \lim_{n\to\infty} \frac{2}{n\pi}\sin\frac{n\pi}{5} = 2/5$$

$$x(t) = \frac{2}{5} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{2}{n\pi}\sin\frac{n\pi}{5}\, e^{jn\frac{\pi}{5}t}$$

EX



Find the complex exponential Fourier Series representation of $x(t)$

$T_0 = 4 \qquad \omega_0 = \frac{2\pi}{4} = \pi/2$

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} D_n e^{-j\frac{n\pi}{2}t}.$$

$$D_n = \frac{1}{4}\int_{-1}^{1} e^{-j\frac{n\pi}{2}t}\,dt - \frac{1}{4}\int_{1}^{3} e^{-j\frac{n\pi}{2}t}\,dt.$$

$$D_n = \frac{1}{4}\,\frac{-2}{jn\pi}\left[e^{-j\frac{n\pi}{2}t}\Big|_{-1}^{1}\right] - \frac{1}{4}\,\frac{-2}{jn\pi}\left[e^{-j\frac{n\pi}{2}t}\Big|_{1}^{3}\right]$$

$$D_n = \frac{-1}{2jn\pi}\left[e^{-j\frac{n\pi}{2}} - e^{j\frac{n\pi}{2}}\right] + \frac{1}{2jn\pi}\left[\underbrace{e^{-j\frac{n\pi 3}{2}}}_{e^{j\frac{n\pi}{2}}} - e^{-j\frac{n\pi}{2}}\right].$$

$$D_n = \frac{1}{n\pi}\sin\frac{n\pi}{2} + \frac{1}{n\pi}\sin\frac{n\pi}{2}$$

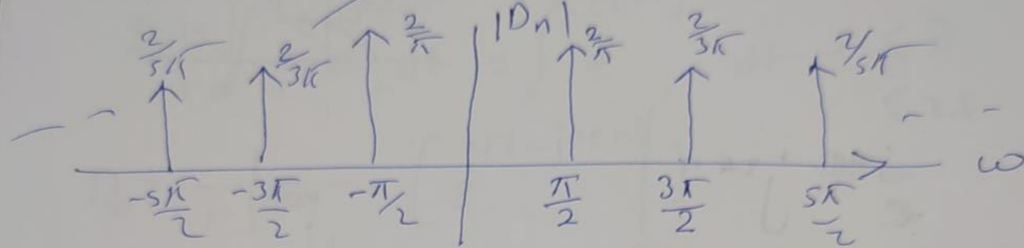
$D_n = \frac{2}{n\pi}\sin n\pi/2.$ for n odd. $D_n = 0$ n even

$$x(t) = \sum_{\substack{n=-\infty \\ n\ \text{odd}}}^{\infty} \frac{2}{n\pi}\sin\frac{n\pi}{2}\, e^{j\frac{n\pi}{2}t}.$$

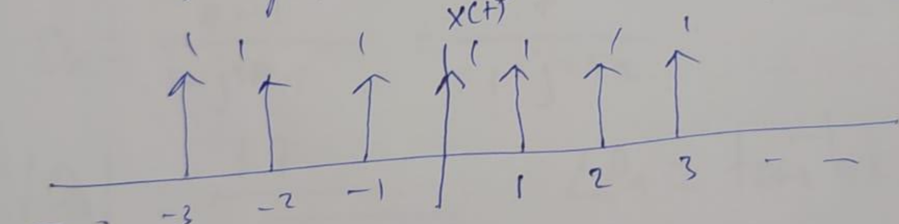
$D_n = \frac{2}{n\pi} \sin\frac{n\pi}{2}$ n odd.

$|D_n| = \left|\frac{2}{n\pi}\right| \left|\sin\frac{n\pi}{2}\right| = \left|\frac{2}{n\pi}\right|$

D_n real $\Rightarrow \angle D_n = 0$.



EX Determine the complex exponential Fourier Series representation of the following signal.

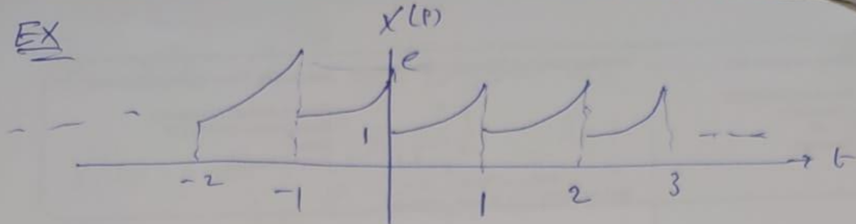


$T_0 = 1 \qquad \omega_0 = \frac{2\pi}{1} = 2\pi$

$$D_n = \frac{1}{T_0}\int_{T_0} x(t) e^{-jn2\pi t}\, dt = \int_{-0.5}^{0.5} \delta(t) e^{-jn2\pi t}\, dt = 1$$

$$x(t) = \sum_{n=-\infty}^{\infty} e^{jn2\pi t}.$$

EX



$T_0 = 1 \qquad \omega_0 = 2\pi$

$$D_n = \int_0^1 e^{t} e^{-j2\pi n t}\, dt = \int_0^1 e^{t(1-j2\pi n)}\, dt$$

$$D_n = \frac{1}{1-j2\pi n}\left[e^{t(1-j2\pi n)} \Big|_0^1 \right]$$

$$D_n = \frac{1}{1-j2\pi n}\left[e^{1-j2\pi n} - 1 \right] = \frac{1}{1-j2\pi n}\left[e^{1} \underbrace{e^{-j2\pi n}}_{1} - 1 \right]$$

$$D_n = \frac{e-1}{1-j2\pi n} = \frac{1.7}{1-j2\pi n}.$$

$$|D_n| = \frac{1.7}{\sqrt{1+(2\pi n)^2}} \qquad \angle D_n = \tan^{-1} 2\pi n.$$

EX Find the complex exponential Fourier series coefficients of:

$$x(t) = 3\cos\left(\tfrac{\pi}{2} t + \tfrac{\pi}{4}\right).$$

Complex exponential fourier series representation of $x(t)$ is to write $x(t)$ as sum of complex exponentials

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So $x(t)$ Can be written as :-

$$x(t) = \frac{3}{2}\left[e^{j\pi/2\, t} + e^{-j\pi/2\, t}\right].$$

$$\omega_0 = \pi/2.$$

$x(t)$ can be written as:

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} D_n e^{jn\pi/2\, t}.$$

$$x(t) = \cdots + D_{-2} e^{j\pi t} + D_{-1} e^{j\pi/2\, t} + D_0 + D_1 e^{-j\pi/2\, t} + D_{+2} e^{-j\pi t} + \cdots$$

compare

$$x(t) = \frac{3}{2} e^{j\pi/2\, t} + \frac{3}{2} e^{-j\pi/2\, t}.$$

$\Rightarrow D_{-1} = 3/2$ $\qquad D_1 = 3/2$.

(16) ... plex exponential Fourier series

(17) (1)

EX Find the complex exponential Fourier Series coefficients of the following signal.

$$X(t) = 2\cos 3t + 3\sin 2t.$$

First you have to show that this signal is periodic and find its fundamental period.

$\omega_1 = 3 \qquad \omega_2 = 2$

$T_1 = \frac{2\pi}{3} \qquad T_2 = \frac{2\pi}{2} = \pi$

$$\frac{T_1}{T_2} = \frac{\frac{2\pi}{3}}{\pi} = 2/3 \text{ rational} \Rightarrow \text{Periodic}$$

$$T_0 = 2T_2 = 3T_1 = 2\pi \qquad \omega_0 = \frac{2\pi}{2\pi} = 1$$

Then write the signal as exponentials.

$$X(t) = 2\left[\tfrac{1}{2}e^{j3t} + \tfrac{1}{2}e^{-j3t}\right] + \frac{3}{2j}\left[e^{j2t} - e^{-j2t}\right].$$

$$X(t) = e^{j3t} + e^{-j3t} + \frac{3}{2j}e^{j2t} - \frac{3}{2j}e^{-j2t}.$$

$$X(t) = \underbrace{1}_{D_{-3}}e^{j3\omega_0 t} + \underbrace{1}_{D_3}e^{-j3\omega_0 t} + \underbrace{\frac{3}{2j}}_{D_{-2}}e^{j2\omega_0 t} - \underbrace{\frac{3}{2j}}_{D_2}e^{-j2\omega_0 t}.$$

$\Rightarrow$

$D_{-3} = 1 \Rightarrow |D_{-3}| = 1 \quad \angle D_{-3} = 0$

$D_3 = 1 \Rightarrow |D_3| = 1 \quad \angle D_3 = 0$

$D_{-2} = \frac{3}{2j} \Rightarrow |D_{-2}| = 3/2 \quad \angle D_2 = -90^\circ.$

$D_2 \;\; \frac{-3}{2j} \qquad |D_2| = 3/2 \quad \angle D_{-2} = 90^0$

Properties Of Fourier Series :-

(2)

(1) Linearity: $x_1(t), x_2(t)$ are two periodic Signals with the Same period.

$$x_1(t) \longrightarrow D_{n1}$$

$$x_2(t) \longrightarrow D_{n2}$$

Let $x(t) = A x_1(t) + B x_2(t)$

Then $x(t)$ have Fourier series coefficient, D_n

where: $D_n = A D_{1n} + B D_{2n}$

Proof The Fourier series coefficient for $x_1(t)$

$$D_{n1} = \frac{1}{T_0}\int_{T_0} x_1(t)\, e^{-jn\omega_0 t}\, dt$$

and for $x_2(t)$: $D_{n2} = \frac{1}{T_0}\int_{T_0} x_2(t)\, e^{-jn\omega_0 t}\, dt.$

The Fourier series. Coefficients for $x(t)$ is:

$$D_n = \frac{1}{T_0}\int_{T_0} (A x_1(t) + B x_2(t))\, e^{-jn\omega_0 t}\, dt$$

$$D_n = A \underbrace{\frac{1}{T_0}\int_{T_0} A x_1(t)\, e^{-jn\omega_0 t}\, dt}_{D_{n1}} + B \underbrace{\frac{1}{T_0}\int_{T_0} B x_2(t)\, e^{-jn\omega_0 t}\, dt}_{D_{n2}}.$$

$$D_n = A D_{1n} + B D_{2n}$$

(2) Time shifting Property.

$x(t)$ is periodic with period T_0 if $x(t)$ is time shifted. periodicity does not change.

if $x(t) \xrightarrow{F.C} D_n$.

Then $x(t-t_0)$ have Fourier coefficients D_{n0} where:

$$D_{n0} = D_n e^{-jn\omega_0 t_0}.$$

$\Rightarrow$ shift in time multiplied by exponential in frequency..

<u>Proof</u> $x(t)$ is periodic $\Rightarrow D_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jn\omega_0 t} \, dt$

$x(t-t_0)$ is periodic $\Rightarrow$

$$D_{n0} = \frac{1}{T_0} \int_{T_0} x(t-t_0) e^{-jn\omega_0 t} \, dt.$$

$t - t_0 = \tau \Rightarrow dt = d\tau.$

$t = \tau + t_0.$

$$\Rightarrow D_{n0} = \frac{1}{T_0} \int_{T_0} x(\tau) e^{-jn\omega_0 (t_0+\tau)} \, d\tau.$$

$$D_{n0} = \frac{1}{T_0} \int_{T_0} x(\tau) e^{-jn\omega_0 \tau} e^{-jn\omega_0 t_0} \, d\tau.$$

$$D_{n0} = \frac{1}{T_0} e^{-jn\omega_0 t_0} \int_{T_0} x(\tau) e^{-jn\omega_0 \tau} \, d\tau = e^{-jn\omega_0 t_0} D_n.$$

(3) Differentiation property.

$x(t)$ is periodic with period T_0 and have Fourier Coefficients D_n Then $\frac{d}{dt}x(t)$ have Fourier Series Coefficients D_n':

$$D_n' = jn\omega_0 D_n .$$

Proof: $x(t)$ Can be written as:

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}.$$

اشتقاق الطرفان بالنسبة ل t نجد:

$$\frac{dx(t)}{dt} = \sum_{n=-\infty}^{\infty} D_n \frac{d}{dt} e^{jn\omega_0 t}.$$

$$\frac{dx(t)}{dt} = \sum_{n=-\infty}^{\infty} jn\omega_0 D_n e^{jn\omega_0 t}$$

$$\boxed{\Rightarrow \frac{dx(t)}{dt} \longrightarrow jn\omega_0 D_n}$$

(4) Integration property.

$$x(t) \xrightarrow{F.S.C} D_n \quad \text{then}$$

$$\int_{-\infty}^{t} x(\tau)\, d\tau \overset{FSC}{\longleftrightarrow} \frac{1}{jn\omega_0} D_n .$$

(5) Time reversal property (5)

$x(t)$ is periodic with Fundamental period T_0 and have FSC D_n Then:

$$x(-t) \xleftrightarrow{FSC} D_{-n}.$$

(6) Parsevals Theorem:

In time domain we found that the average Power of a periodic Signal $x(t)$ is in one period.

$$P = \frac{1}{T_0}\int_{T_0} |x(t)|^2 dt.$$

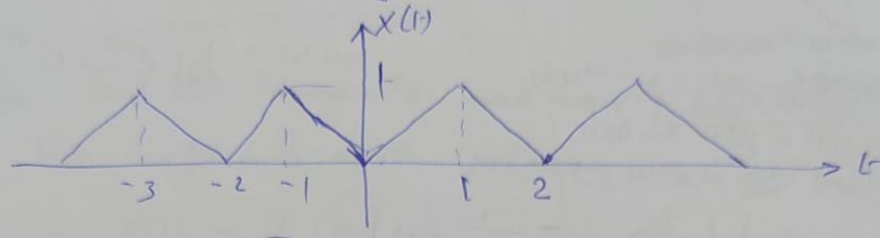
according to thi Theorem the average Power can be found as:

$$P = \sum_{n=-\infty}^{\infty} |D_n|^2.$$

6

Ex $x(t) = \begin{cases} t & 0 \le t \le 1 \\ 2-t & 1 \le t \le 2 \end{cases}$

is a periodic signal with $T_0 = 2$.

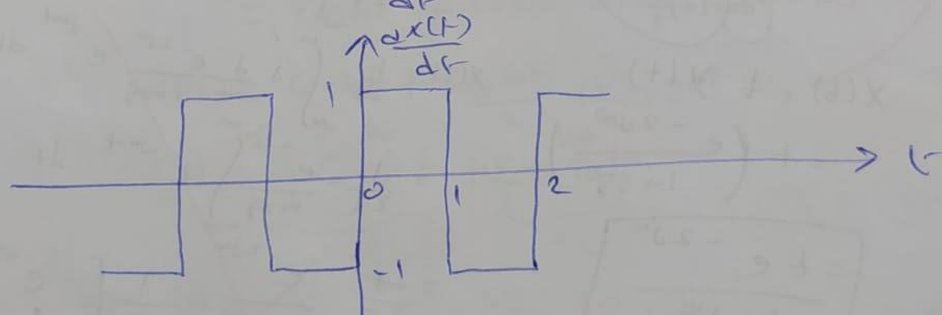


$T_0 = 2 \quad \omega_0 = \frac{2\pi}{2} = \pi$

a) Find a_0.

$a_0 = 1/2$

b) Find the complex exponential Fourier series coefficients for $\frac{d}{dt}x(t)$



we find that $D_n' = \frac{2}{jn\pi}$ for n odd.

So:

$$\frac{dx(t)}{dt} = \sum_{\substack{n=-\infty \\ n \text{ odd}}}^{\infty} \frac{2}{jn\pi} e^{jn\pi t}$$

$$D_n = \frac{1}{jn\pi}\,\frac{2}{jn\pi} = \frac{2}{-n^2\pi^2}.$$

$$D_0 = a_0 = 1/2$$

EX Find the trigonometric and complex exponential FS coefficients of the following signal

$$x(t) = 2\cos^3 10t$$

(1) for trigonometric FS coefficients. $x(t)$ can be written as:

$$x(t) = 2\cos 10t\,\cos^2 10t.$$

$$= 2\cos 10t\left[\tfrac{1}{2} + \tfrac{1}{2}\cos 20t\right].$$

$$= \cos 10t + \cos 10t\cos 20t.$$

$$x(t) = \underset{w_0}{\cos 10t} + \frac{\cos 10t}{2} + \underset{3w_0}{\frac{1}{2}\cos 30t}.$$

$$x(t) = \underbrace{\frac{3}{2}\cos 10t}_{x_1} + \underbrace{\frac{1}{2}\cos 30t}_{x_2}.$$

$w_1 = 10$ $\qquad$ $w_2 = 30$

$T_1 = \frac{2\pi}{10} = \pi/5$ $\qquad$ $T_2 = \frac{2\pi}{30} = \frac{\pi}{15}$

$$\frac{T_1}{T_2} = \frac{\pi/5}{\pi/15} = 3.$$

$T_0 = T_1 = 3T_2 = \pi/5$ $\qquad$ $w_0 = \frac{2\pi}{\pi/5} = 10$

$\Rightarrow a_1 = 3/2 \quad a_3 = 1/2.$

2) To find complex exponential FSC:- (8)

$$x(t) = \frac{3}{2}\cos 10t + \frac{1}{2}\cos 30t$$

write x(t) as complex exponentials

$$X(t) = \frac{3}{4}\left[e^{j10t} + e^{-j10t}\right] + \frac{1}{4}\left(e^{j30t} - e^{-j30t}\right)$$

$$X(t) = \underbrace{\frac{3}{4}}_{D_{-1}} e^{j\overset{\omega_0}{10}t} + \underbrace{\frac{3}{4}}_{D_1} e^{-j\overset{\omega_0}{10}t} + \underbrace{\frac{1}{4}}_{D_{-3}} e^{j\overset{3\omega_0}{30}t} - \underbrace{\frac{1}{4}}_{D_3} e^{-j\overset{3\omega_0}{30}t}.$$

$\Rightarrow D_{-1} = 3/4 \quad D_1 = 3/4 \quad D_3 = 1/4 \quad D_{-3} = -1/4$

$A + B = 1$

$3A + 3B = 2$