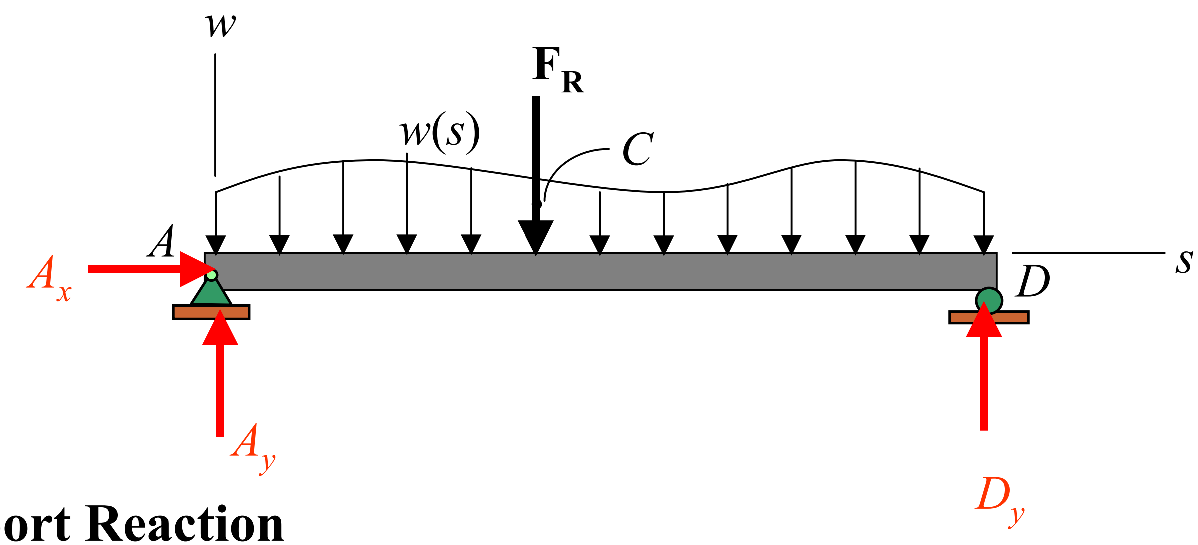
STRESS

- Stress
- Average Normal Stress in an Axially Loaded Bar
- Average Shear Stress
- Allowable Stress
- Design of Simple Connections

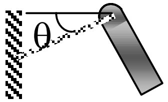
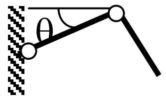
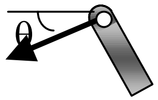
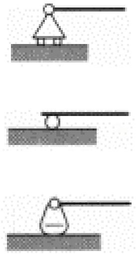
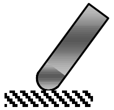
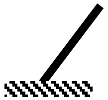
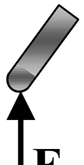
Equilibrium of a Deformable Body

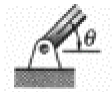
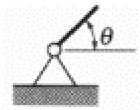
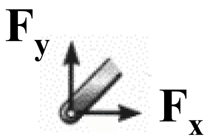
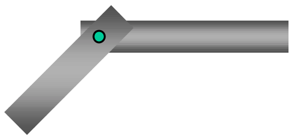
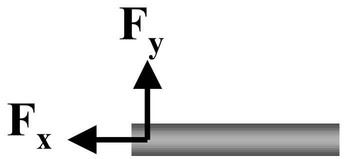
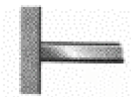
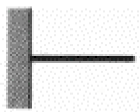
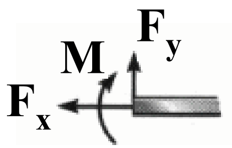
- **Body Force**



- **Support Reaction**

Table 1 Supports for Coplanar Structures

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1)  Light cable			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2)  Rollers and rockers		 **F**	One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3)  Smooth surface		 **F**	One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(4)  Smooth pin or hinge			Two unknowns. The reactions are two force components.
(5)  Internal pin			Two unknowns. The reactions are a force and moment.
(6)  Fixed support			Three unknowns. The reactions are the moment and the two force components.

- **Equation of Equilibrium**

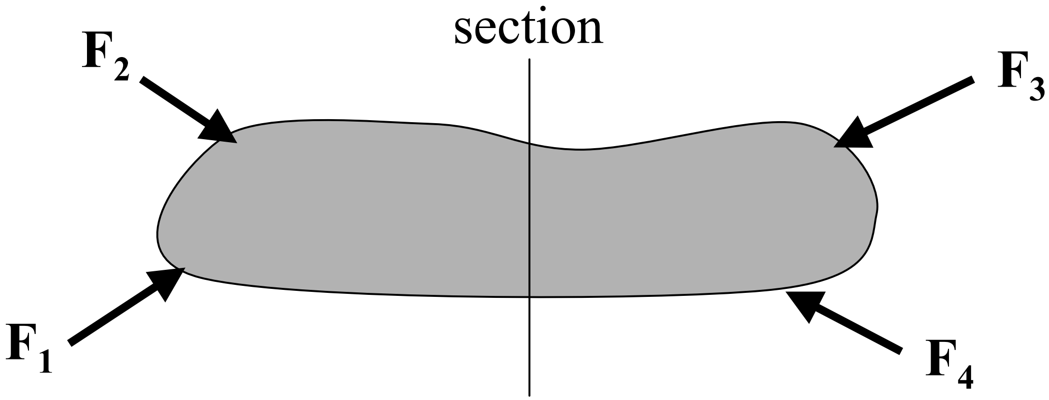
$$\Sigma \mathbf{F} = 0$$

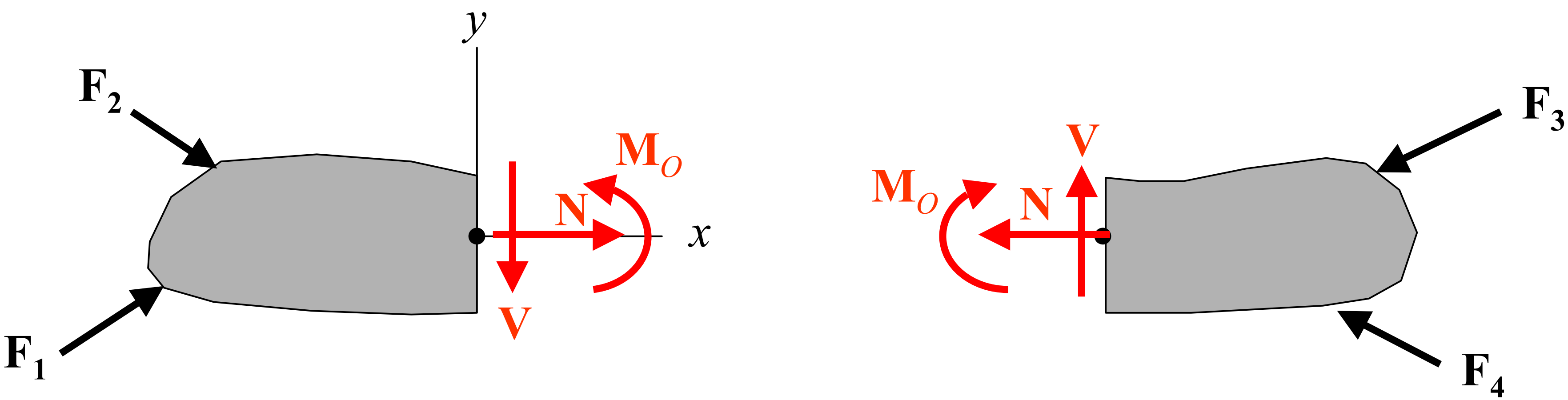
$$\Sigma \mathbf{M}_O = 0$$

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$$

$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0$$

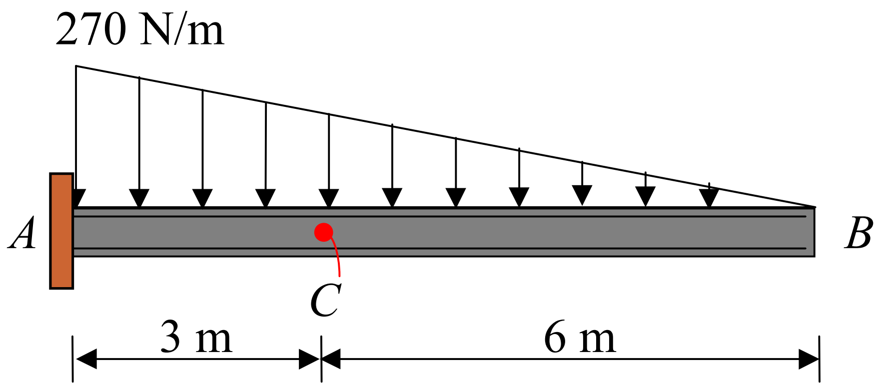
• **Internal Resultant Loadings**



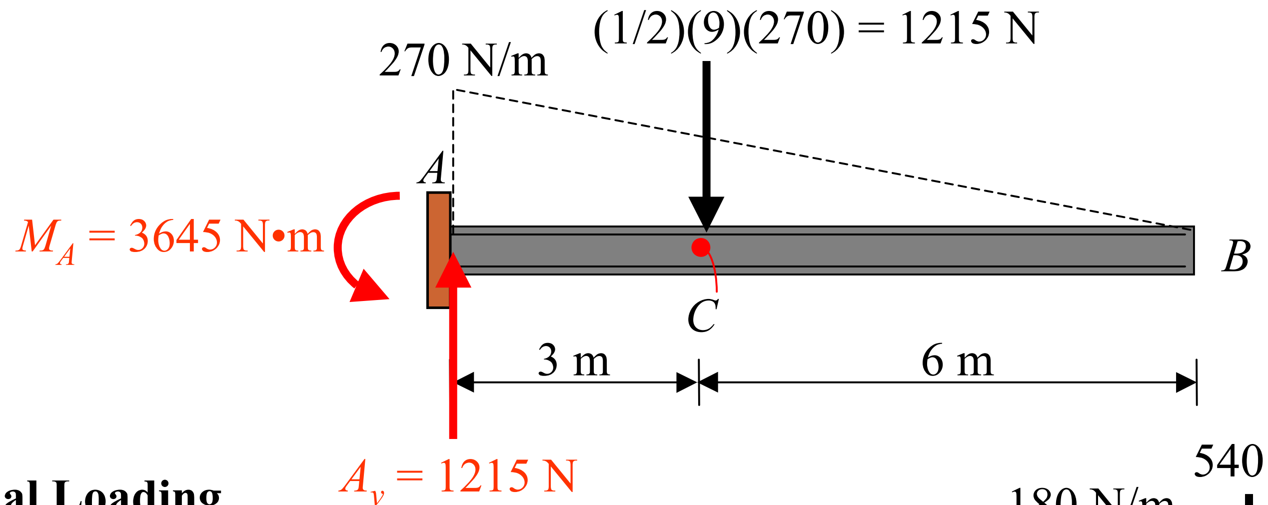


Example 1

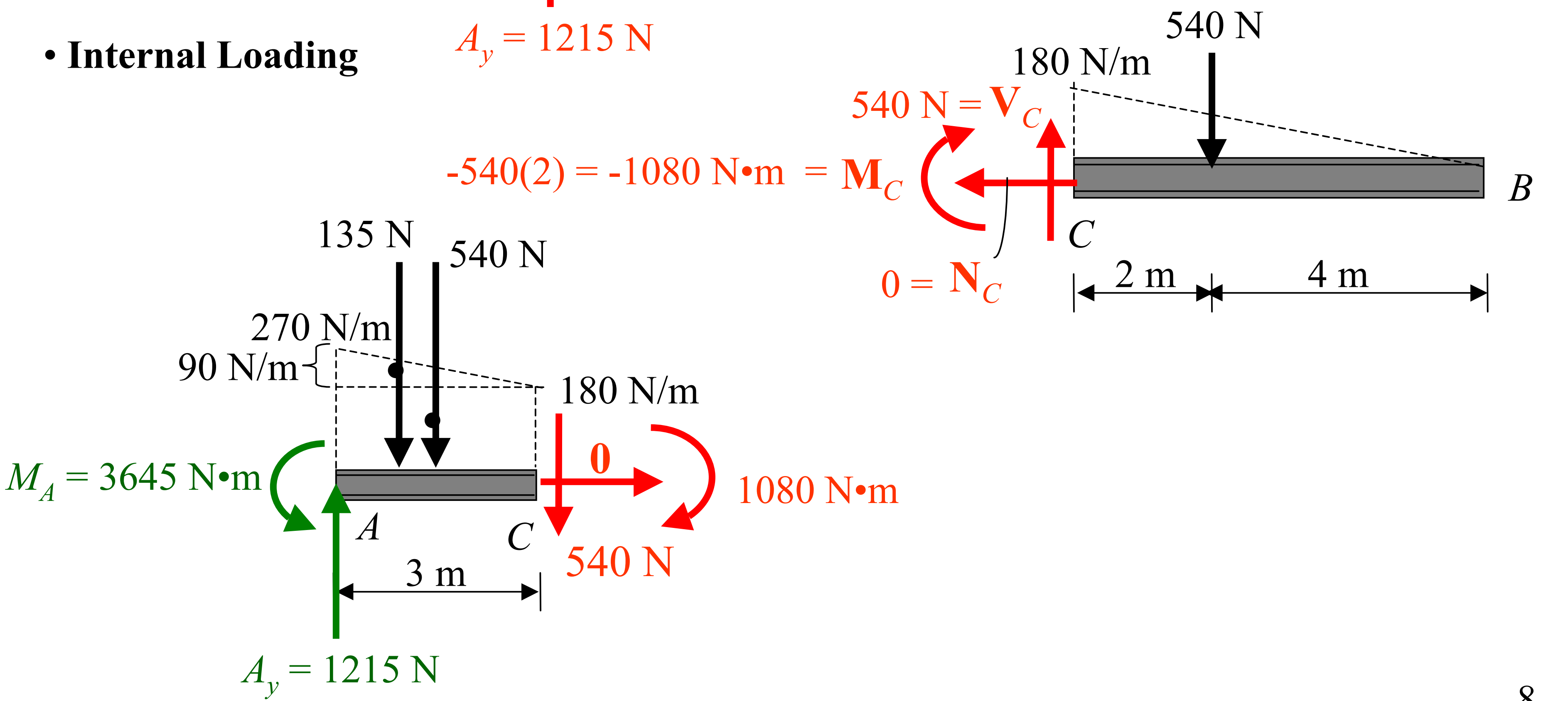
Determine the resultant internal loadings acting on the cross section at *C* of the beam shown.



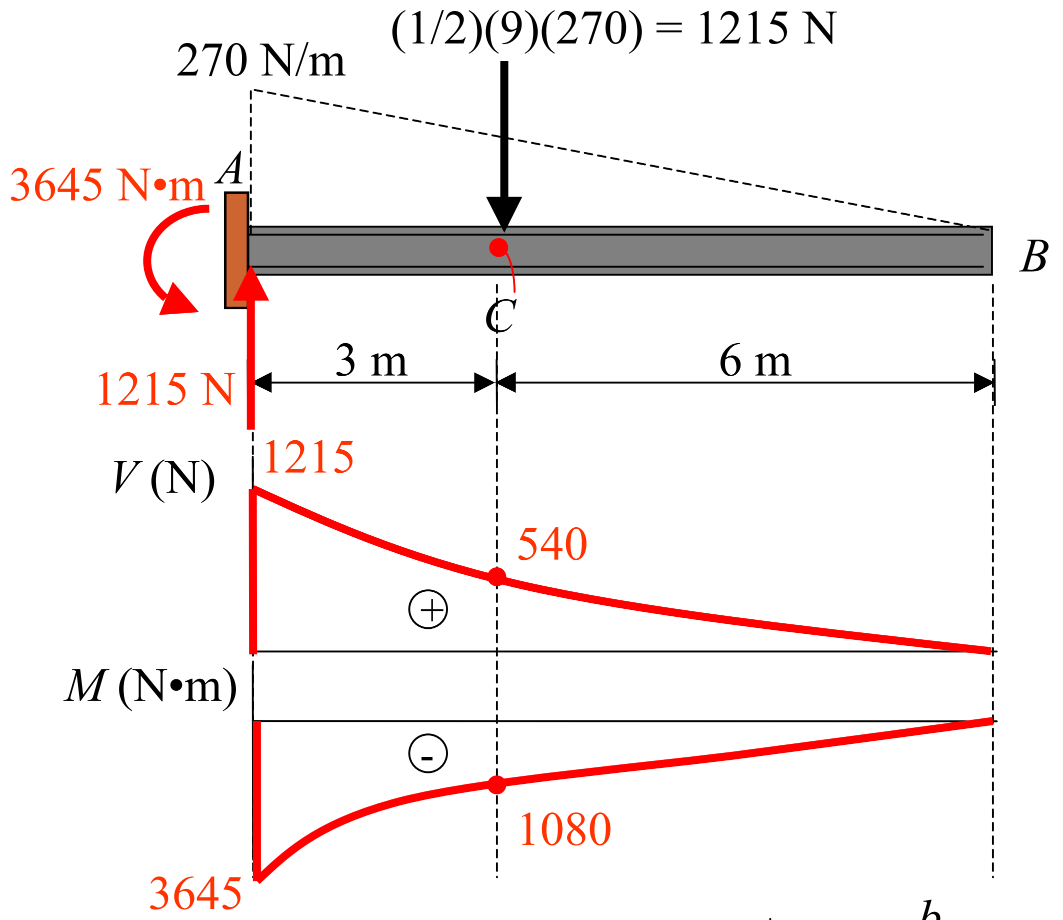
• **Support Reactions**

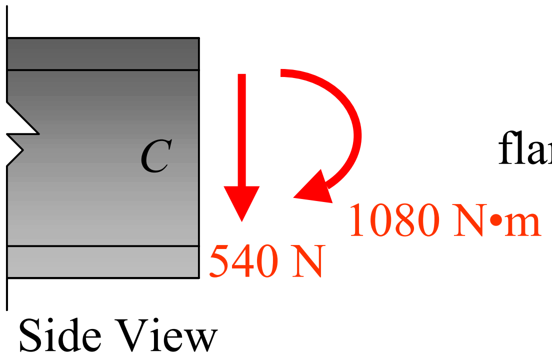


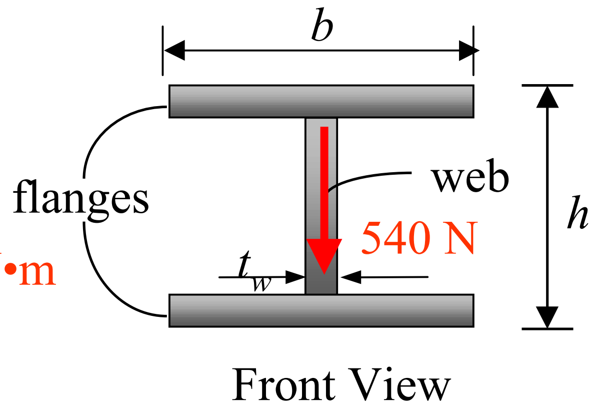
• **Internal Loading**



- **Shear and bending moment diagram**

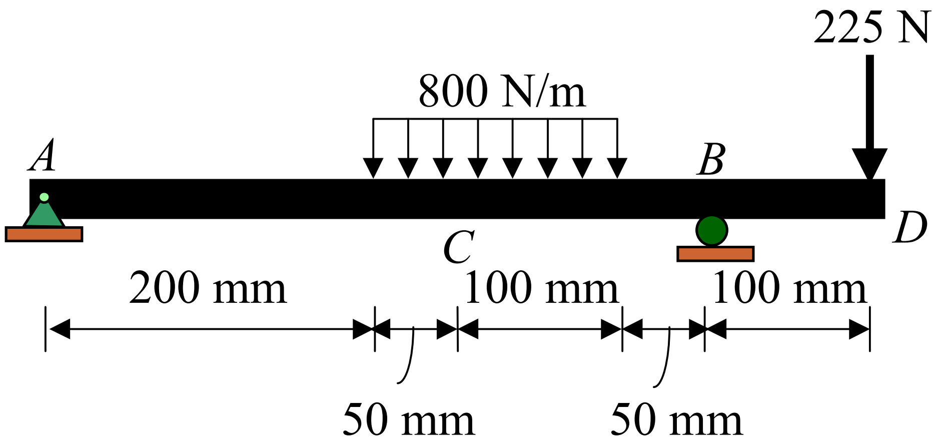




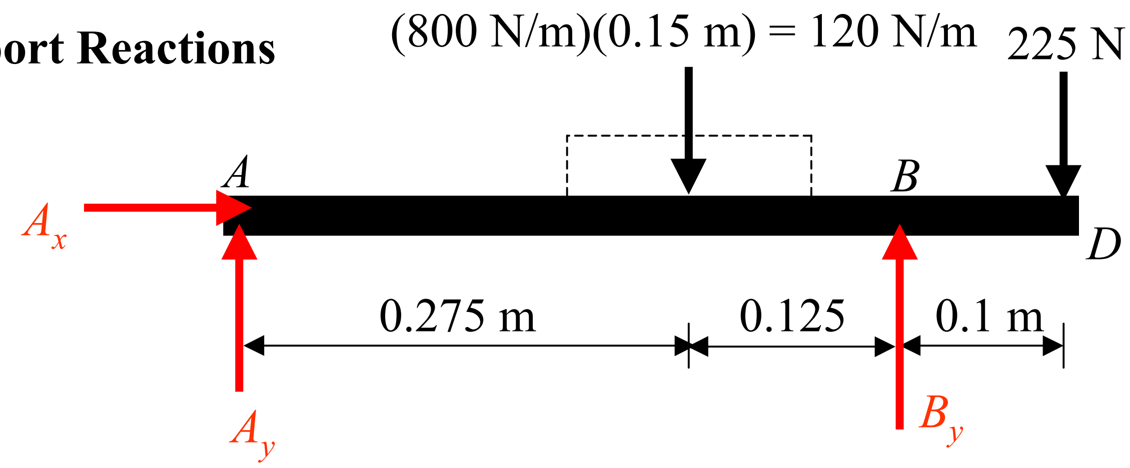


Example 2

Determine the resultant internal loadings acting on the cross section at *C* of the machine shaft shown. The shaft is supported by bearings at *A* and *B*, which exert only vertical forces on the shaft.



- **Support Reactions**

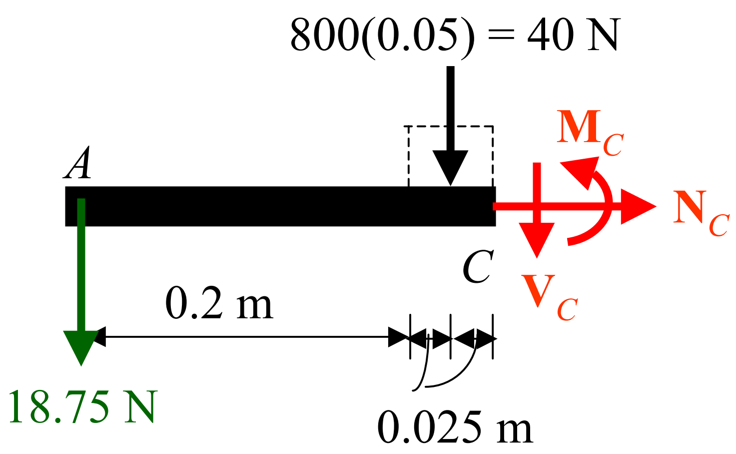


$+\circlearrowright \Sigma M_A = 0$: $-(120)(0.275) + B_y(0.4) - (225)(0.5) = 0,$ $B_y = 363.75$ N

$\xrightarrow{+} \Sigma F_x = 0$: $A_x = 0$ $A_x = 0,$

$+\uparrow \Sigma F_y = 0$: $A_y - 120 + 363.75 - 225 = 0$ $A_y = -18.75$ N, ↓

- **Internal Loading**



$\xrightarrow{+} \Sigma F_x = 0$: $N_C = 0$

$+\uparrow \Sigma F_y = 0$:

$-18.75 - 40 - V_C = 0,$

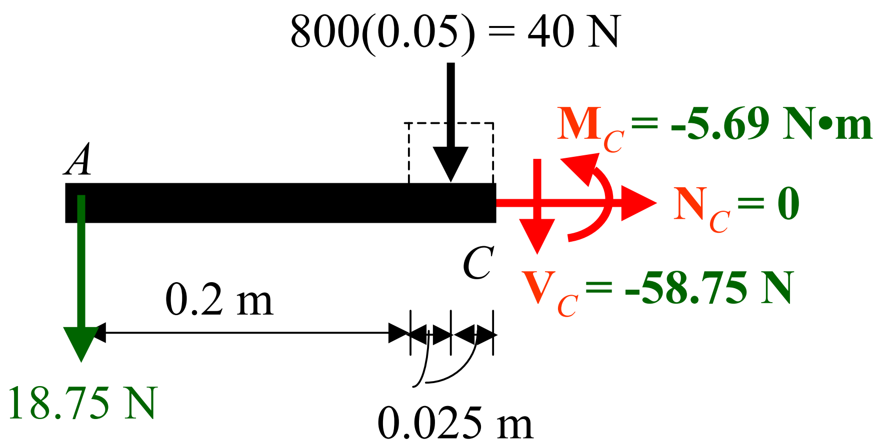
$V_C = -58.75$ N

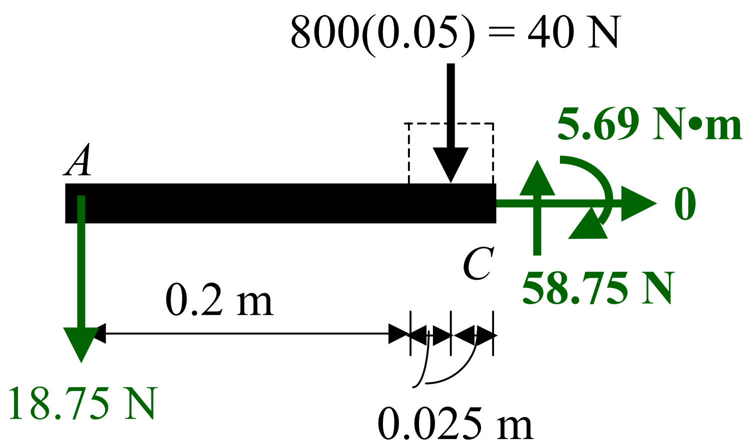
$+\circlearrowright \Sigma M_A = 0$:

$18.75(0.25) + 40(0.025) + M_C = 0$

$M_C = -5.69$ N•m

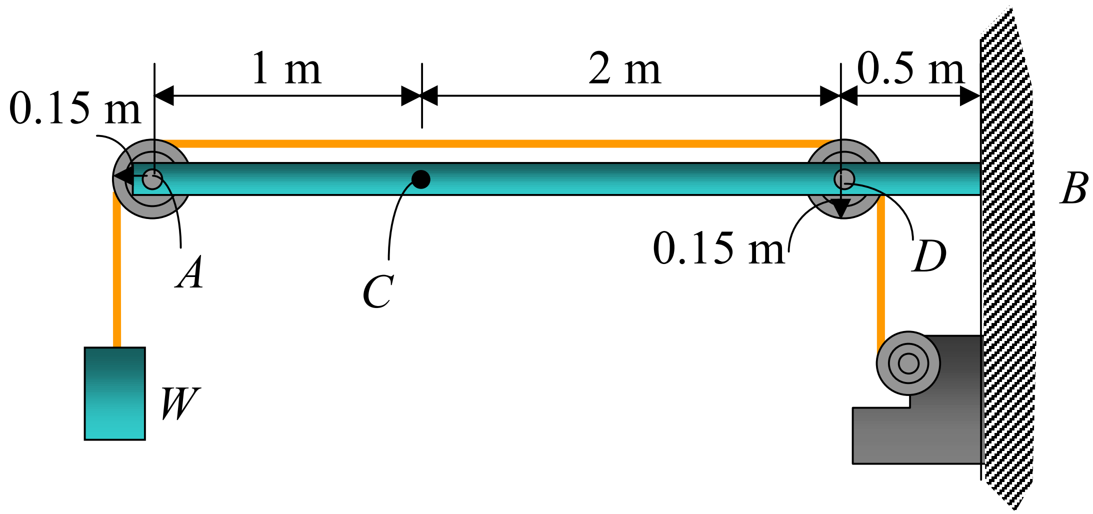
• **Internal Loading**

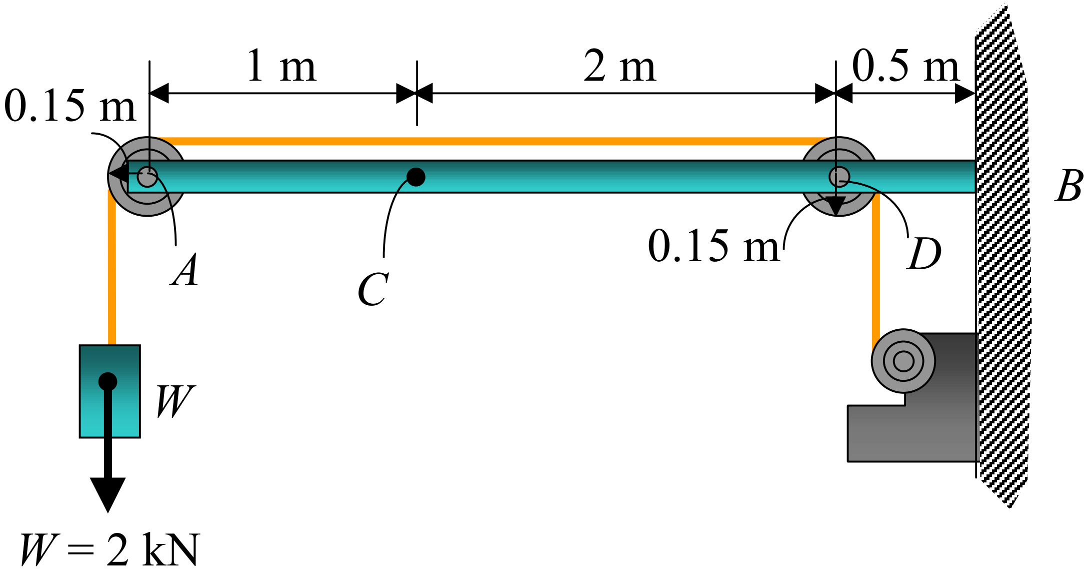


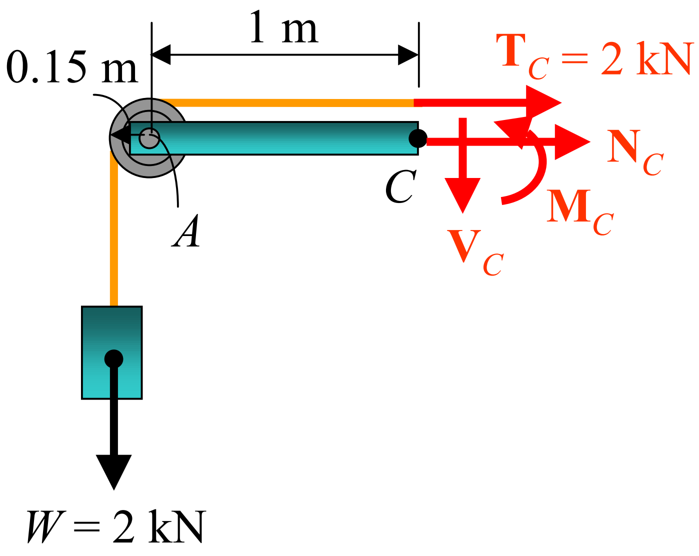


Example 3

The hoist consists of the beam *AB* and attached pulleys, the cable, and the motor. Determine the resultant internal loadings acting on the cross section at *C* if the motor is lifting the 2 kN load *W* with constant velocity. Neglect the weight of the pulleys and beam.







$\overset{+}{\rightarrow} \Sigma F_x = 0:$

$2 + N_C = 0$

$N_C = -2 \text{ kN}$

$+\uparrow \Sigma F_y = 0:$

$-2 - V_C = 0,$

$V_C = -2 \text{ kN}$

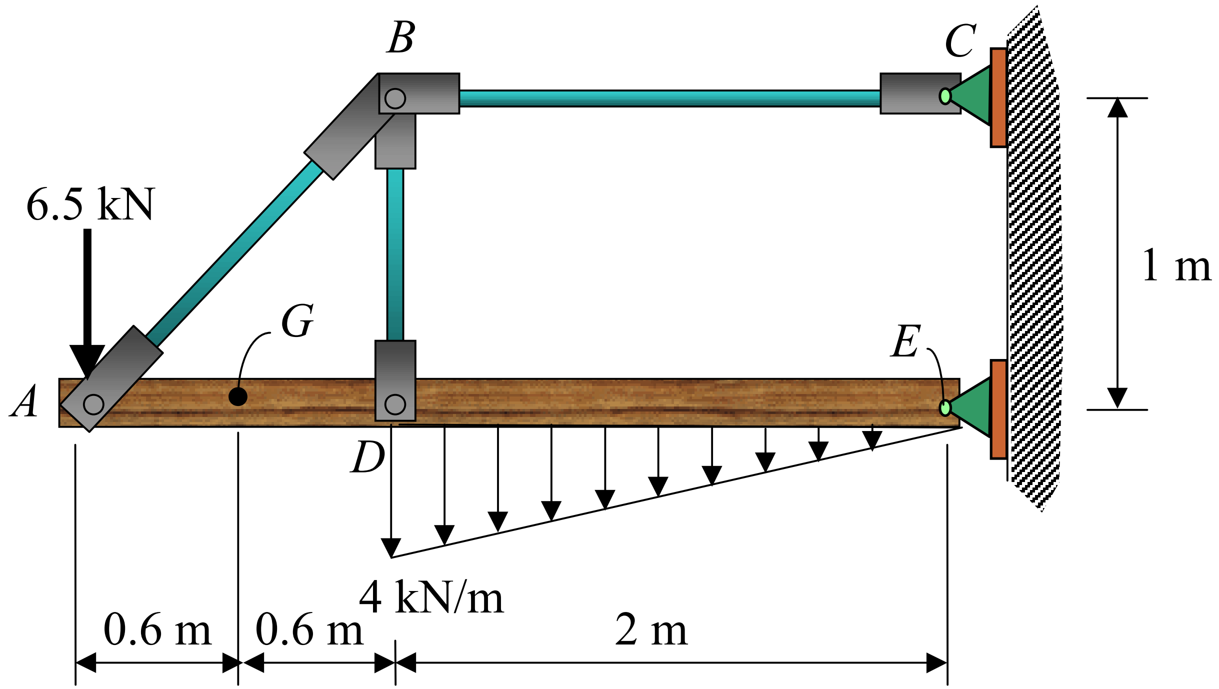
$+\circlearrowright \Sigma M_C = 0:$

$M_C - 2(0.15) + 2(1.15) = 0$

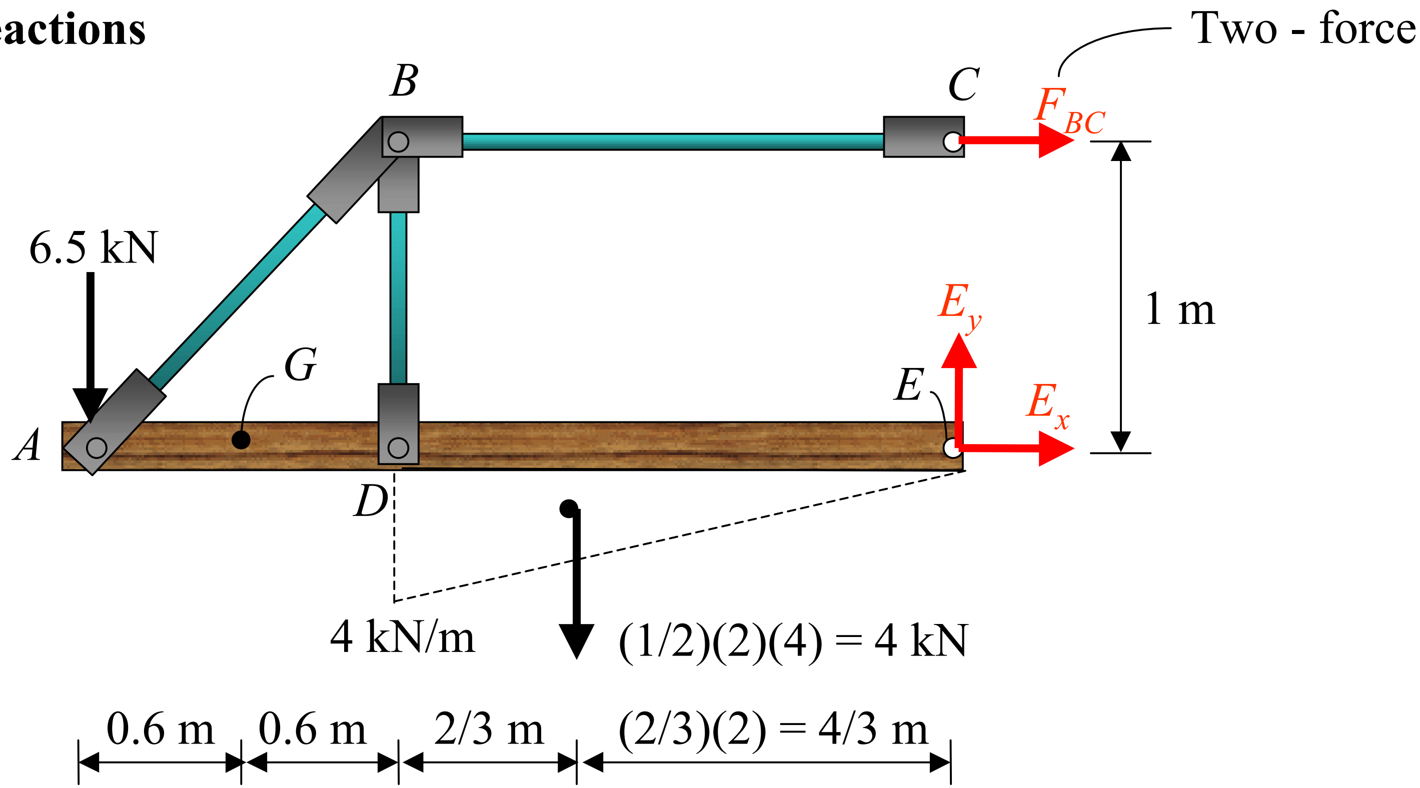
$M_C = -2 \text{ kN} \cdot \text{m}$

Example 4

Determine the resultant internal loadings acting on the cross section at *G* of the wooden beam shown . Assume the joints at *A*, *B*, *C*, *D*, and *E* are pin-connected.



• **Support Reactions**

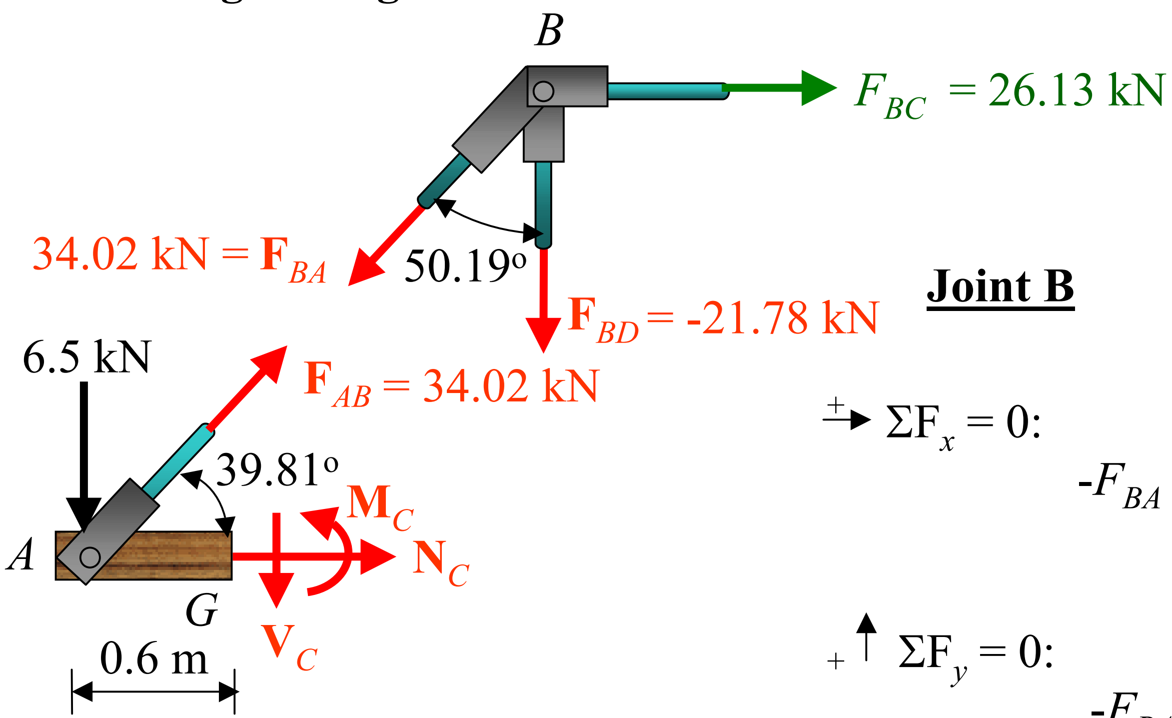


$+\circlearrowright \Sigma M_E = 0$: $4(4/3) + 6.5(3.2) - F_{BC}(1) = 0,$ $F_{BC} = 26.1$ kN

$\overset{+}{\rightarrow} \Sigma F_x = 0$: $26.13 + E_x = 0$ $E_x = -26.1$ kN, $\leftarrow$

$+\uparrow \Sigma F_y = 0$: $-6.5 - 4 + E_y = 0$ $E_y = 10.5$ kN

• **Internal loadings acting on the cross section at *G***



Joint B

$\xrightarrow{+} \Sigma F_x = 0$:

$-F_{BA} \sin 50.19° + 26.13 = 0$

$F_{BA} = 34.0$ kN, (T)

$+\uparrow \Sigma F_y = 0$:

$-F_{BA} \cos 50.19° - F_{BD} = 0$

$F_{BD} = -21.8$ kN, (C)

Member *AG*

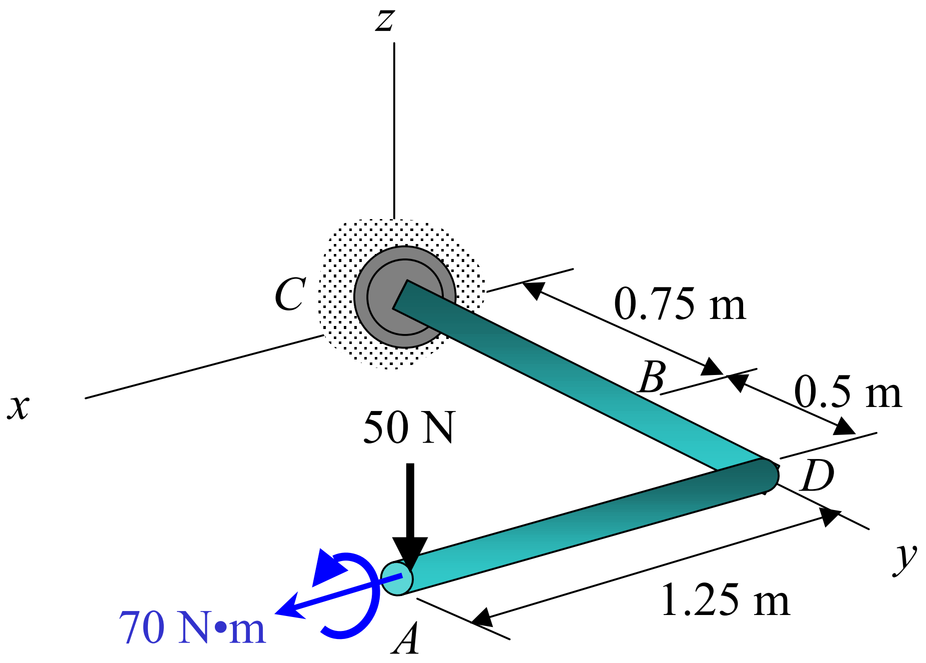
$+\circlearrowright \Sigma M_G = 0$: $\quad M_G - 34.02 \sin 39.81°(0.6) + 6.5(0.6) = 0 \quad\quad M_G = 9.17$ kN•m

$\xrightarrow{+} \Sigma F_x = 0$: $\quad 34.02 \cos 39.81° + N_G = 0 \quad\quad N_G = -26.1$ kN

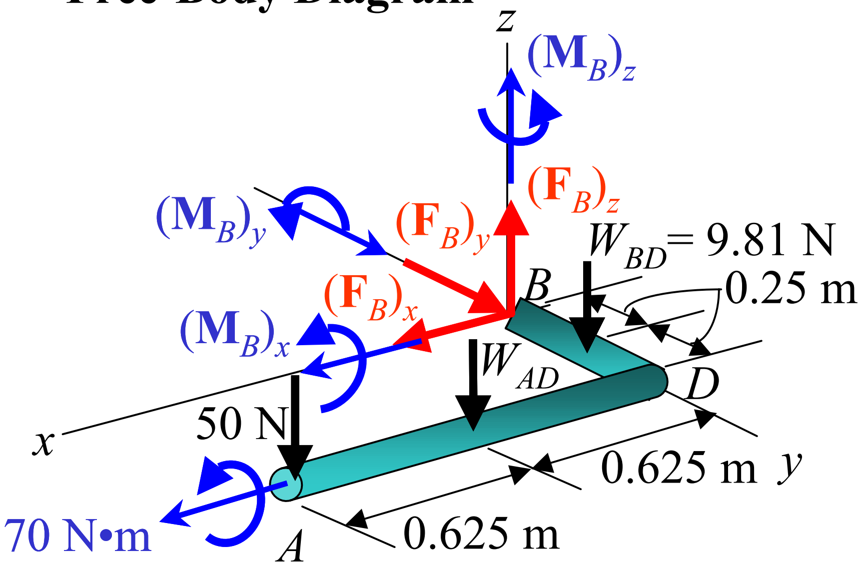
$+\uparrow \Sigma F_y = 0$: $\quad -6.5 + 34.02 \sin 39.81° - V_G = 0 \quad\quad V_G = 15.3$ kN

Example 5

Determine the resultant internal loadings acting on the cross section at *B* of the pipe shown. The pipe has a mass of 2 kg/m and is subjected to both a vertical force of 50 N and a couple moment of 70 N•m at its end *A*. It is fixed to the wall at *C*.



- **Free-Body Diagram**



$$W_{BD} = (2\text{ kg/m})(0.5\text{ m})(9.81\text{ N/kg}) = 9.81\text{ N}$$

$$W_{AD} = (2\text{ kg/m})(1.25\text{ m})(9.81\text{ N/kg}) = 24.52\text{ N}$$

- **Equilibrium of Equilibrium**

$\Sigma F_x = 0$: $(F_B)_x = 0$

$\Sigma F_y = 0$: $(F_B)_y = 0$

$\Sigma F_z = 0$: $(F_B)_z - 9.81 - 24.525 - 50 = 0$, $(F_B)_z = 84.3$ N

$\Sigma (M_B)_x = 0$: $(M_B)_x + 70 - 50(0.5) - 24.525(0.5) - 9.81(0.25) = 0$, $(M_B)_x = -30.3$ N•m

$\Sigma (M_B)_y = 0$: $(M_B)_y + 24.525(0.625) + 50(1.25) = 0$, $(M_B)_y = -77.8$ N•m

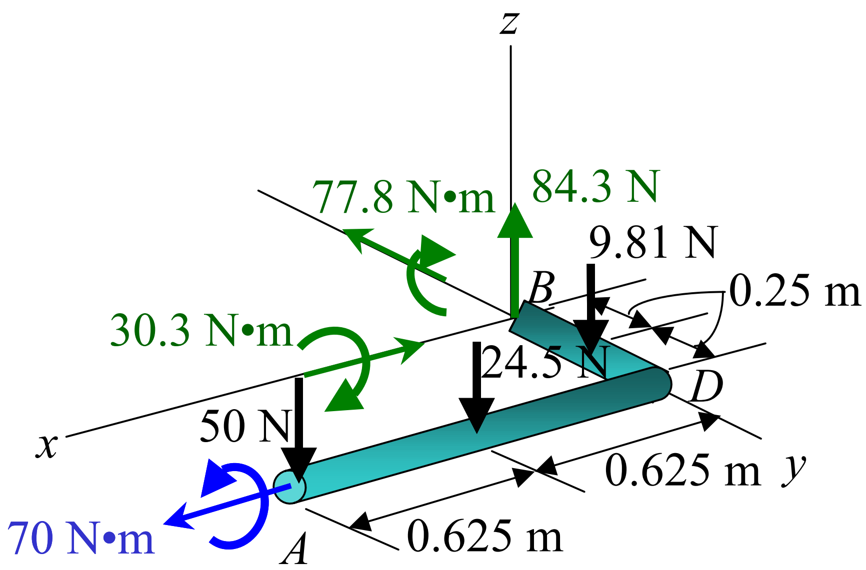
$\Sigma (M_B)_x = 0$: $(M_B)_z = 0$

• **Free-Body Diagram**

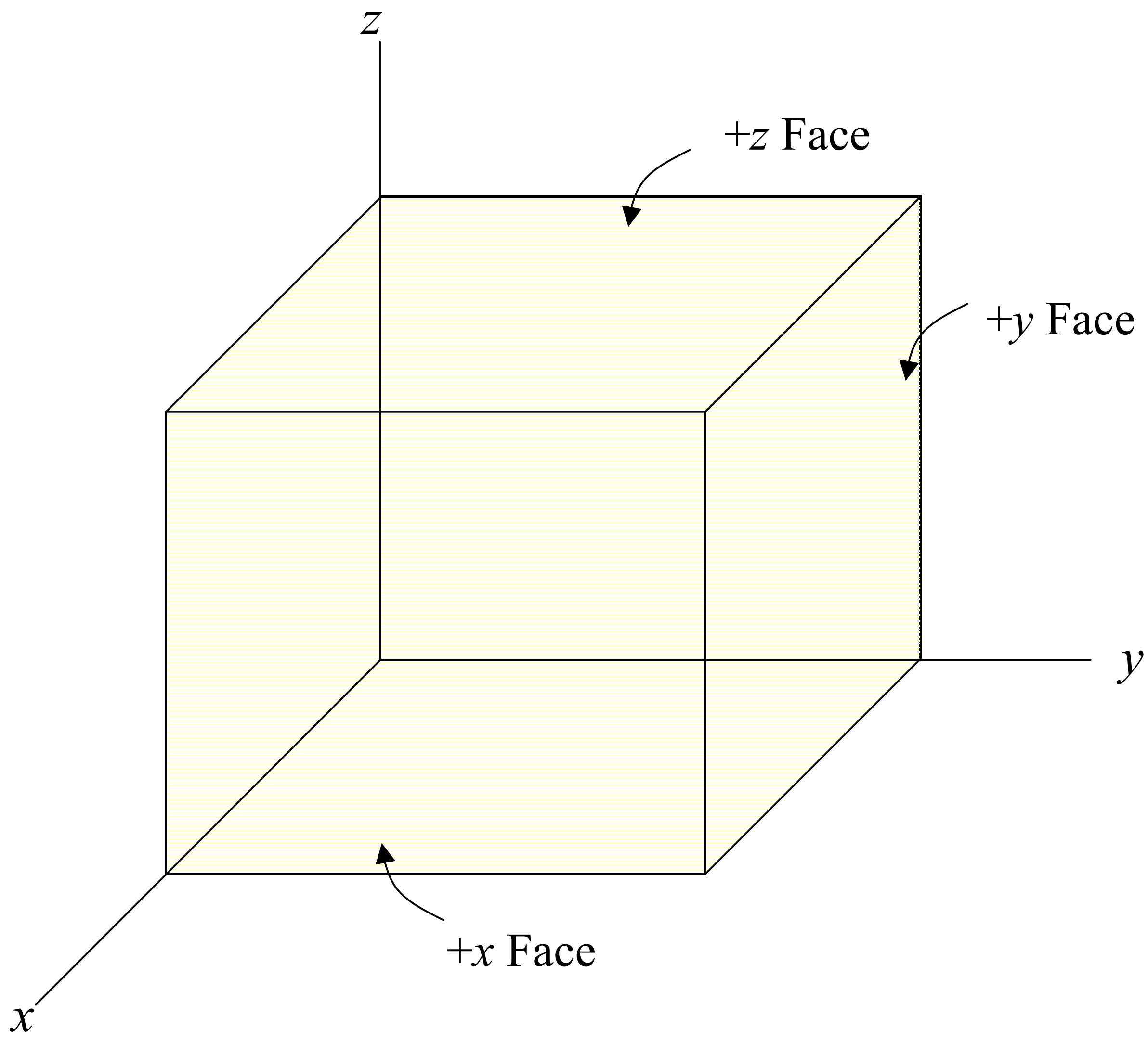
$(F_B)_x = 0$ $(M_B)_x = -30.3$ N•m

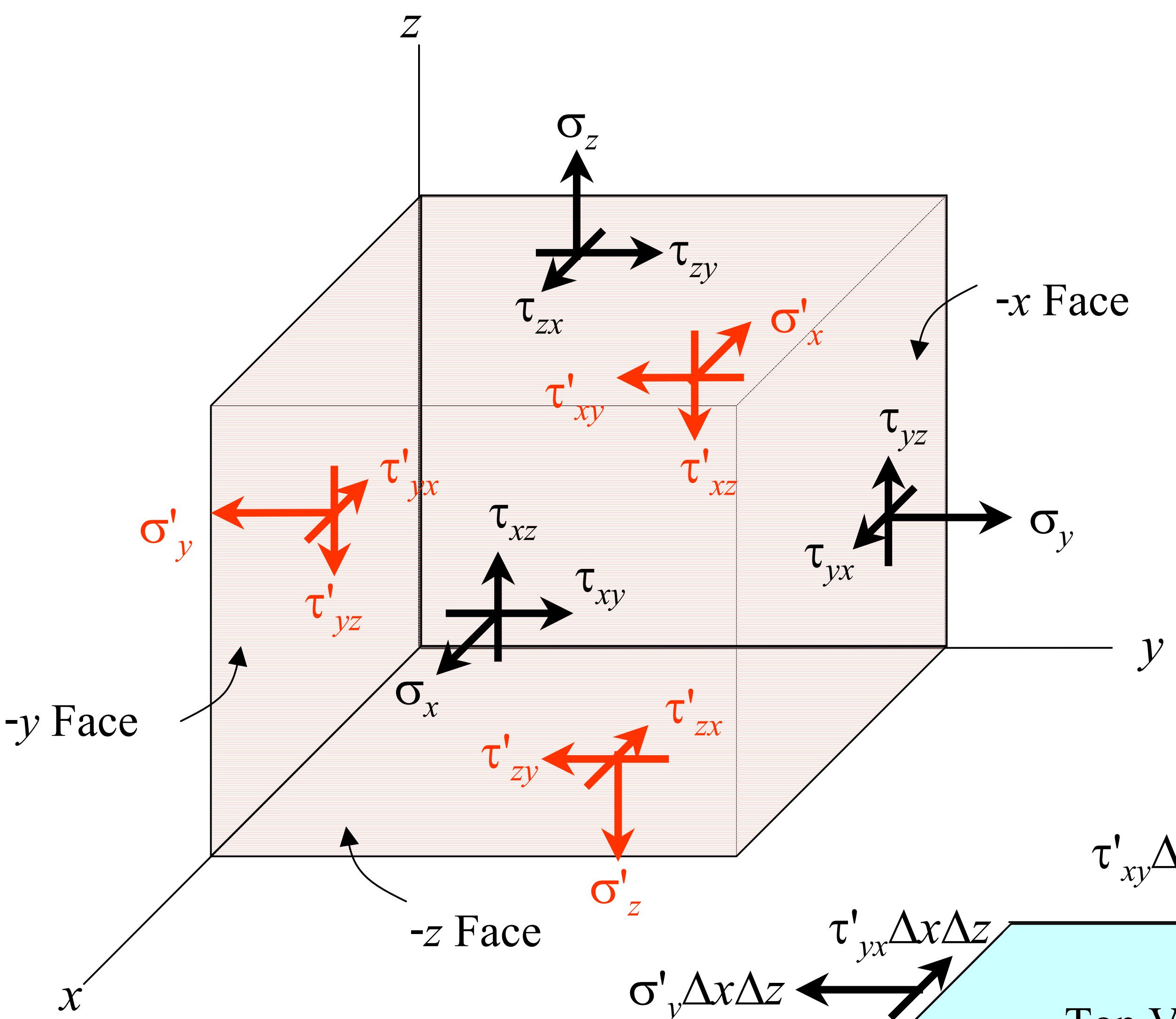
$(F_B)_y = 0$ $(M_B)_y = -77.8$ N•m

$(F_B)_z = 84.3$ N $(M_B)_z = 0$



Stress





By compatibility,

$$\sigma_x = \sigma'_x$$

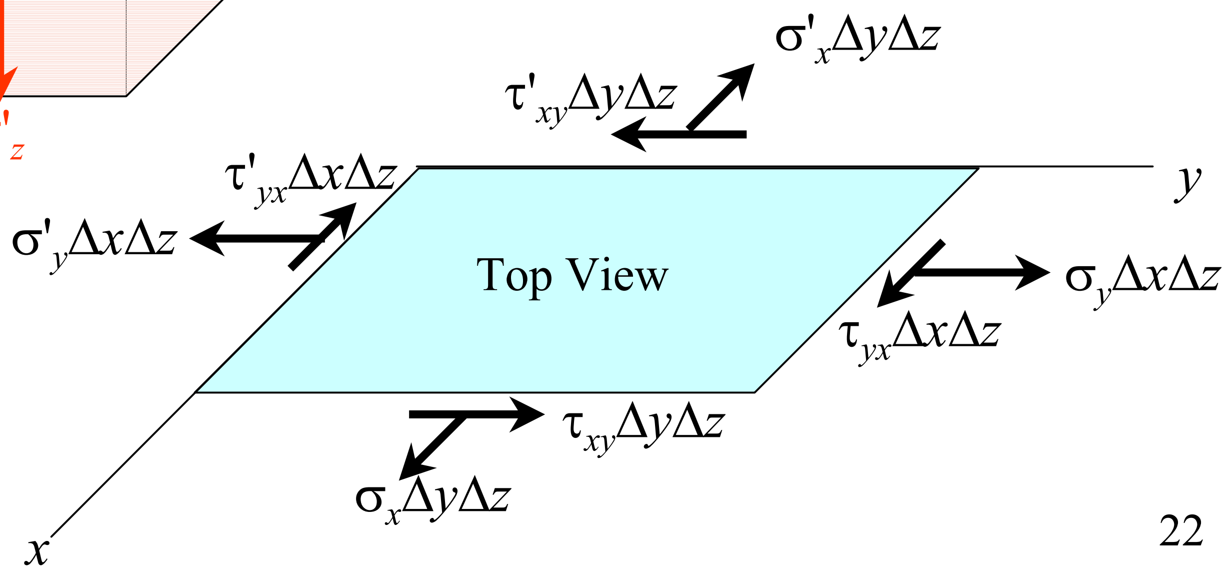
$$\sigma_y = \sigma'_y$$

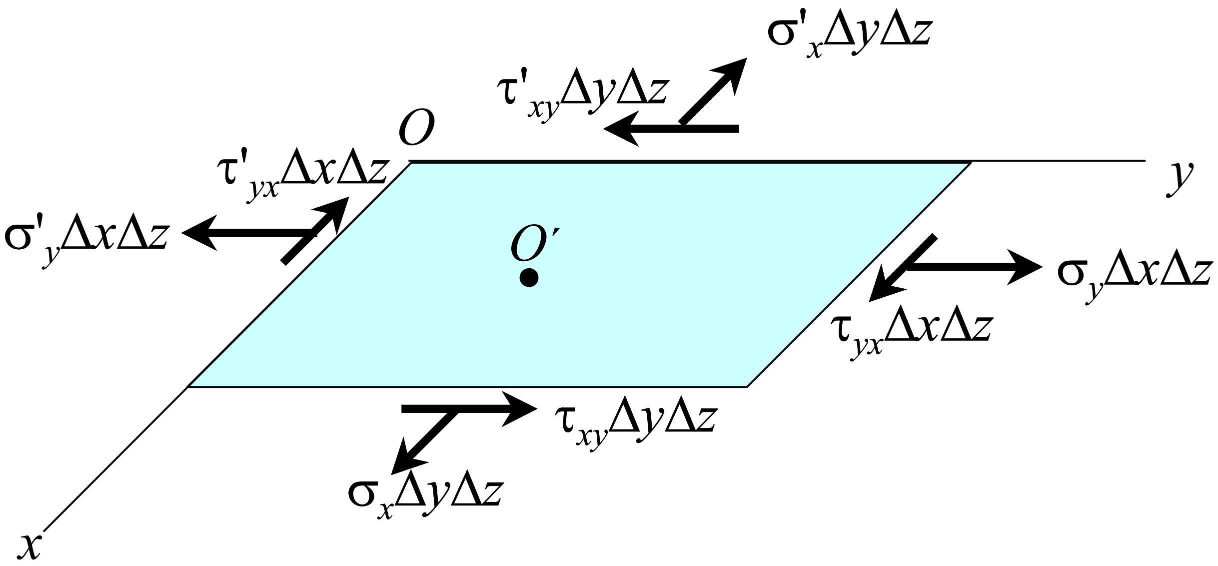
$$\sigma_z = \sigma'_z$$

$$\tau_{xy} = \tau'_{xy}$$

$$\tau_{yx} = \tau'_{yx}$$

$$\tau_{zx} = \tau'_{xz}$$





$\xrightarrow{+} \Sigma F_y = 0$: $\quad -\sigma'_y \Delta x \Delta z + \sigma_y \Delta x \Delta z = 0$

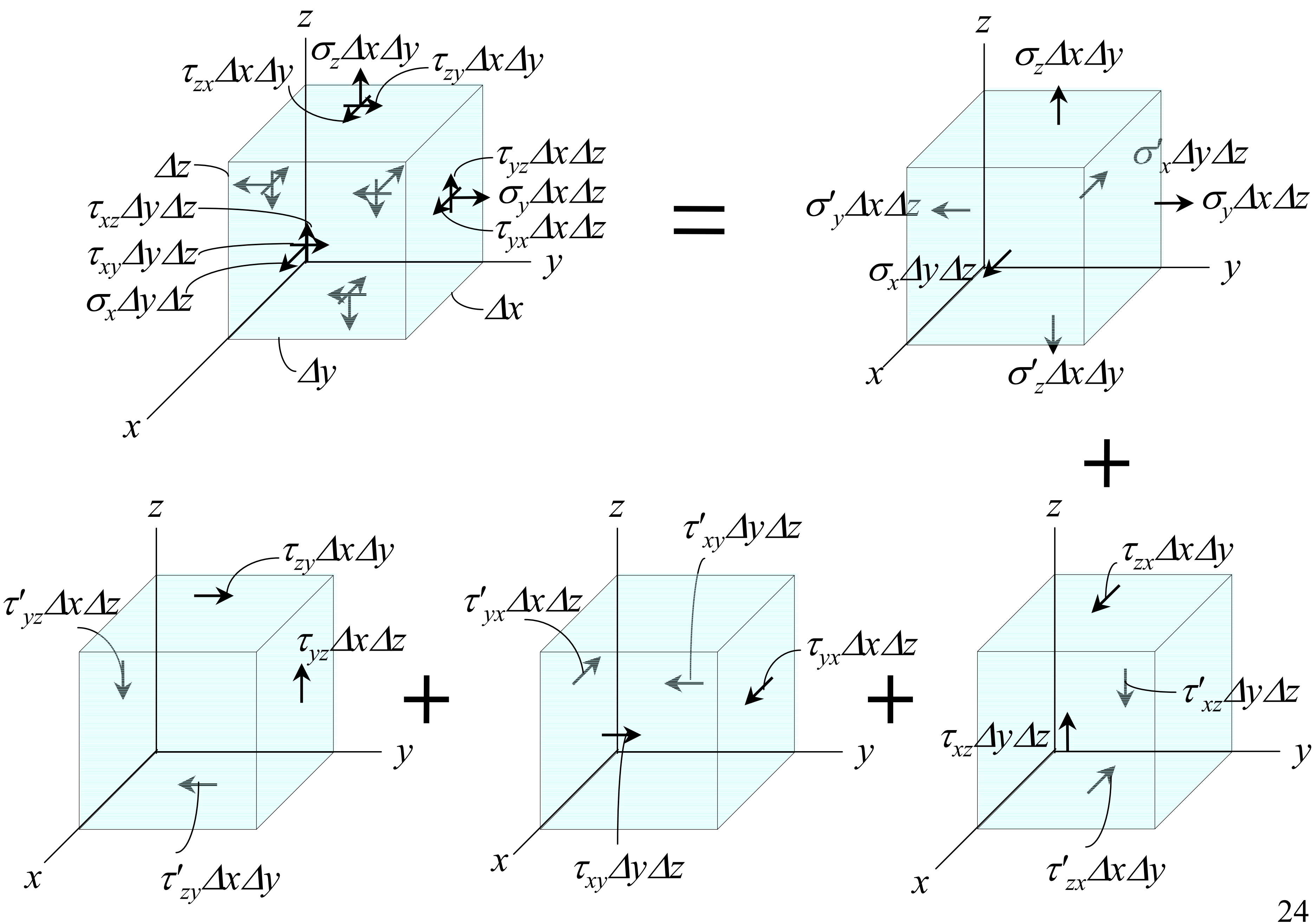
$$\sigma'_y = \sigma_y$$

$\overset{+}{\swarrow} \Sigma F_x = 0$: $\quad \sigma_x \Delta y \Delta z - \sigma'_x \Delta y \Delta z = 0$

$$\sigma_x = \sigma'_x$$

$+\circlearrowright \Sigma M_{O'} = 0$: $\quad (\tau_{xy} \Delta y \Delta z)(\Delta x) - (\tau_{yx} \Delta x \Delta z)(\Delta y) = 0$

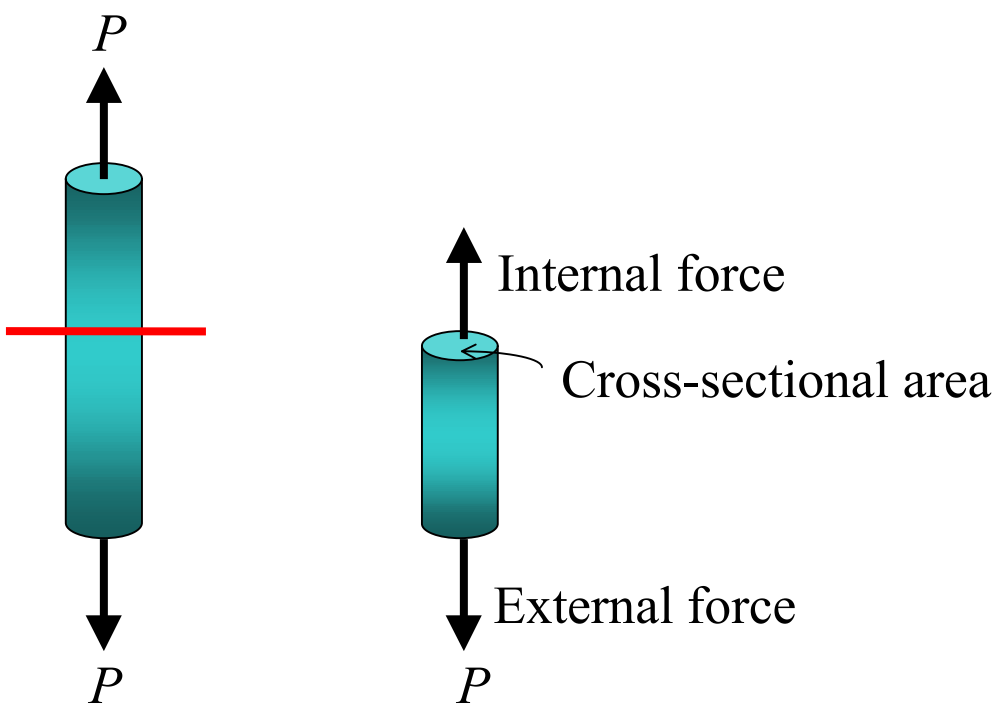
$$\tau_{xy} = \tau_{yx}$$

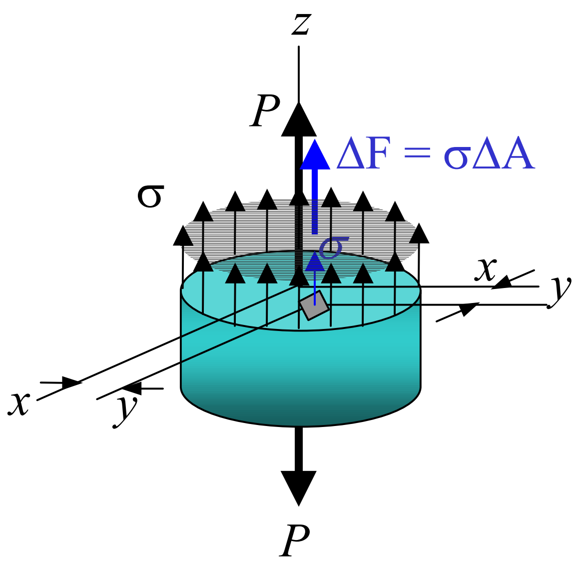

z
$\sigma_z \Delta x \Delta y$
$\tau_{zx} \Delta x \Delta y$
$\tau_{zy} \Delta x \Delta y$
Δz
$\tau_{yz} \Delta x \Delta z$
$\sigma_y \Delta x \Delta z$
$\tau_{yx} \Delta x \Delta z$
$\tau_{xz} \Delta y \Delta z$
$\tau_{xy} \Delta y \Delta z$
$\sigma_x \Delta y \Delta z$
y
Δx
Δy
x
=
z
$\sigma_z \Delta x \Delta y$
$\sigma'_x \Delta y \Delta z$
$\sigma'_y \Delta x \Delta z$
$\sigma_y \Delta x \Delta z$
$\sigma_x \Delta y \Delta z$
y
x
$\sigma'_z \Delta x \Delta y$
+
z
$\tau_{zy} \Delta x \Delta y$
$\tau'_{yz} \Delta x \Delta z$
$\tau_{yz} \Delta x \Delta z$
y
x
$\tau'_{zy} \Delta x \Delta y$
+
z
$\tau'_{xy} \Delta y \Delta z$
$\tau'_{yx} \Delta x \Delta z$
$\tau_{yx} \Delta x \Delta z$
y
x
$\tau_{xy} \Delta y \Delta z$
+
z
$\tau_{zx} \Delta x \Delta y$
$\tau'_{xz} \Delta y \Delta z$
$\tau_{xz} \Delta y \Delta z$
y
x
$\tau'_{zx} \Delta x \Delta y$

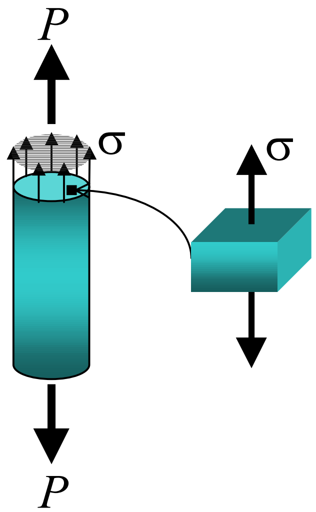
Average Normal Stress in an Axially Loaded Bar

- **Assumptions**

The material must be
- Homogeneous material
- Isotropic material







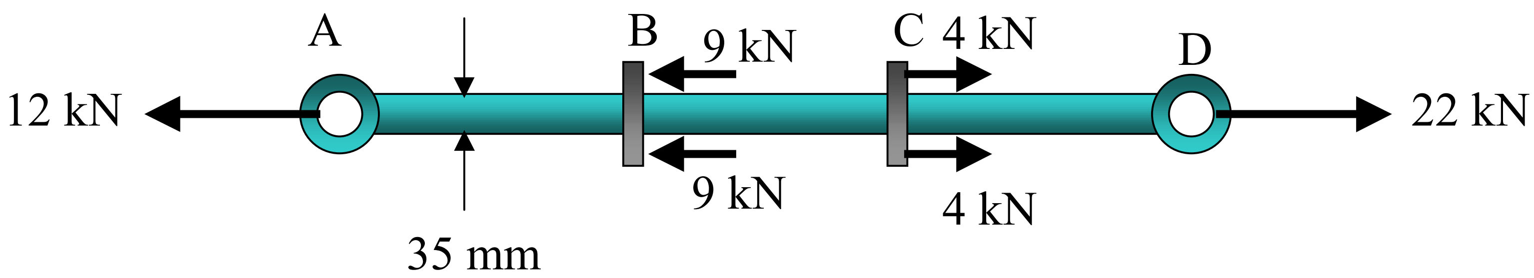
$$+\uparrow F_{Rz} = \Sigma F_z; \qquad \int dF = \int_A \sigma dA$$

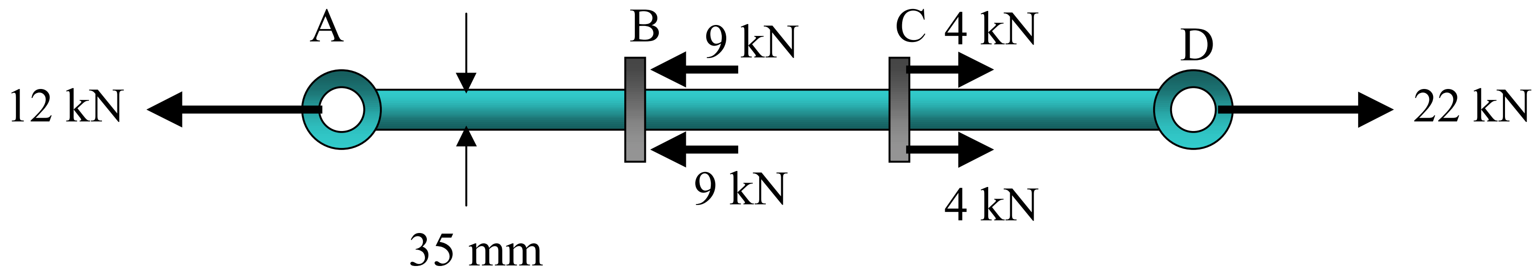
$$P = \sigma A$$

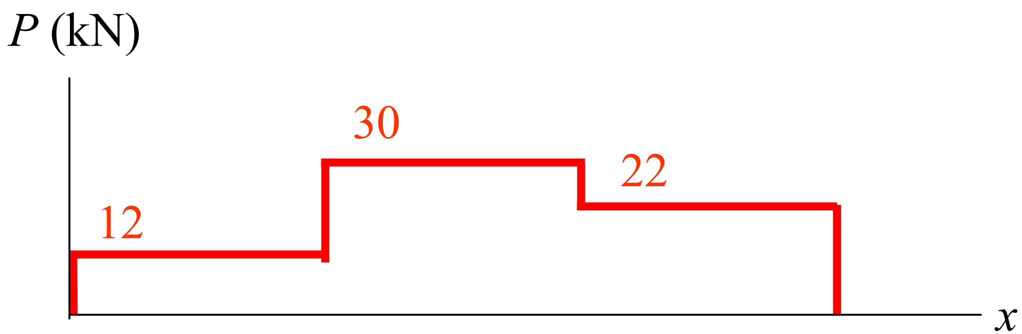
$$\sigma = \frac{P}{A}$$

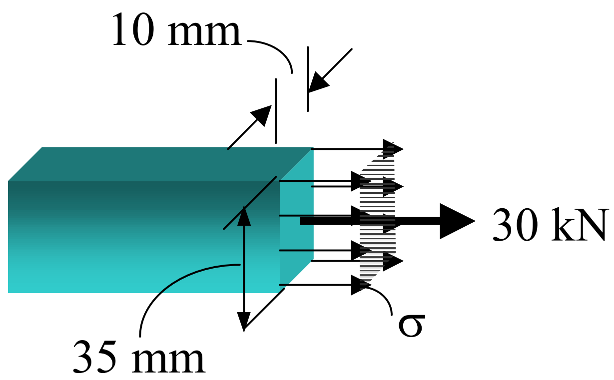
Example 6

The bar shown has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.

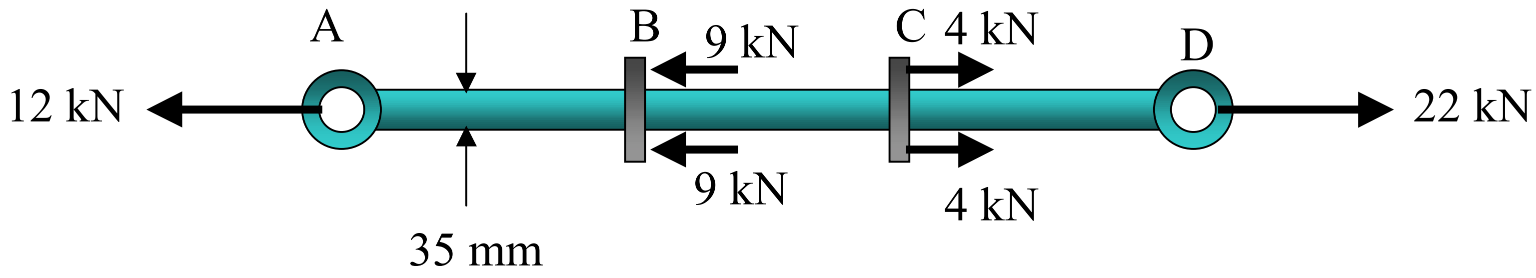


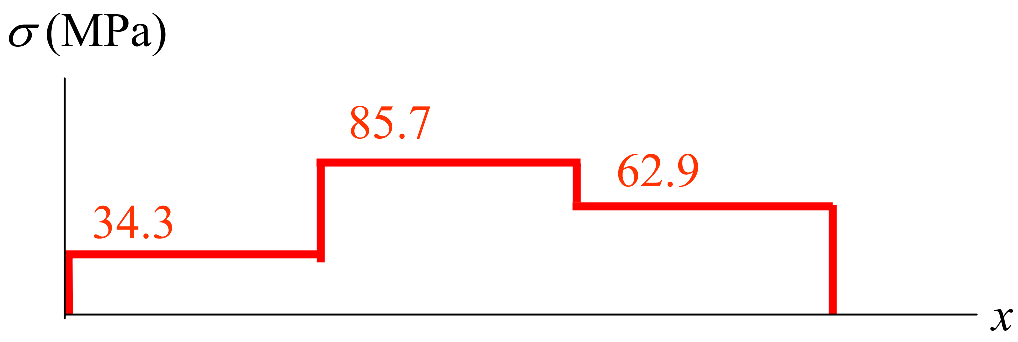


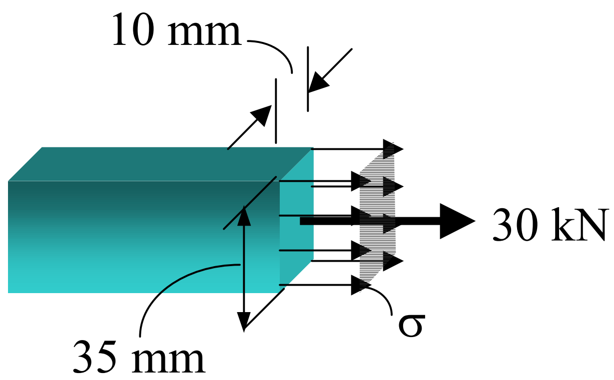




$$\sigma_{\max} = \sigma_{BC} = \frac{P_{BC}}{A} = \frac{30 \text{ kN}}{(0.035 \text{ m})(0.01 \text{ m})} = 85.7 \text{ MPa}$$



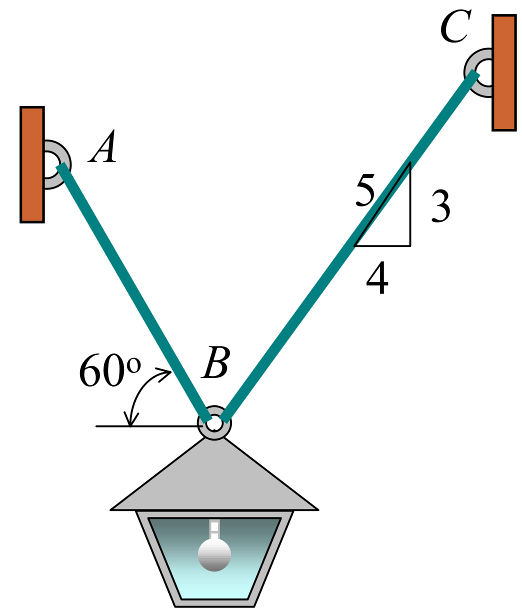




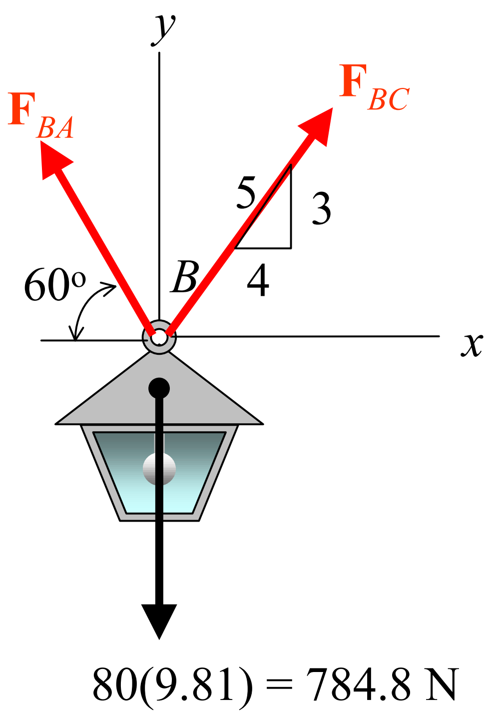
$$\sigma_{\max} = \sigma_{BC} = \frac{P_{BC}}{A} = \frac{30 \text{ kN}}{(0.035 \text{ m})(0.01 \text{ m})} = 85.7 \text{ MPa}$$

Example 7

The 80 kg lamp is supported b two rods *AB* and *BC* as shown . If *AB* has a diameter of 10 mm and *BC* has a diameter of 8 mm, determine which rod is subjected to the greater average normal stress.



• Internal Loading

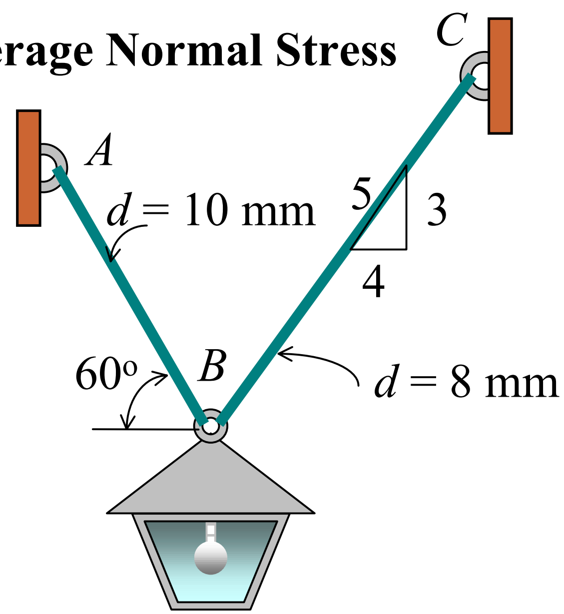


$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{BC}\left(\frac{4}{5}\right) - F_{BA}\cos 60^o = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\left(\frac{3}{5}\right) + F_{BA}\sin 60^o - 784.8 = 0$$

$$F_{BC} = 395.2 \text{ N}, \qquad F_{BA} = 632.4 \text{ N}$$

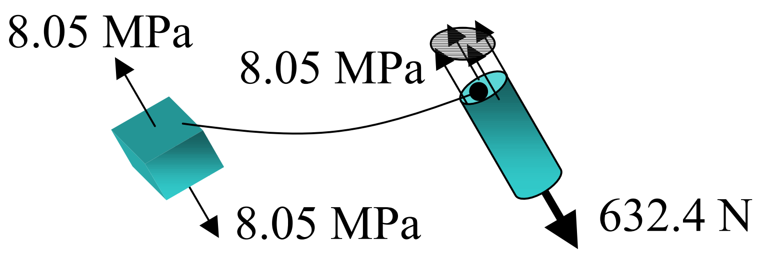
• Average Normal Stress



$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{395.2 \text{ N}}{\pi(0.004 \text{ m})^2} = 7.86 \text{ MPa}$$

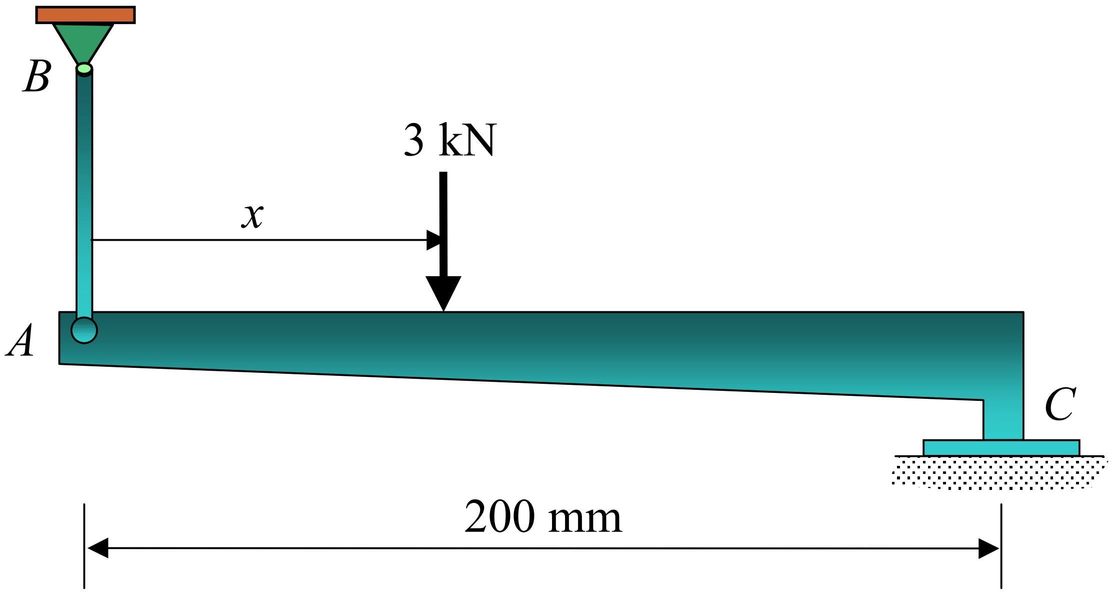
$$\sigma_{BA} = \frac{F_{BA}}{A_{BA}} = \frac{632.4 \text{ N}}{\pi(0.005 \text{ m})^2} = 8.05 \text{ MPa}$$

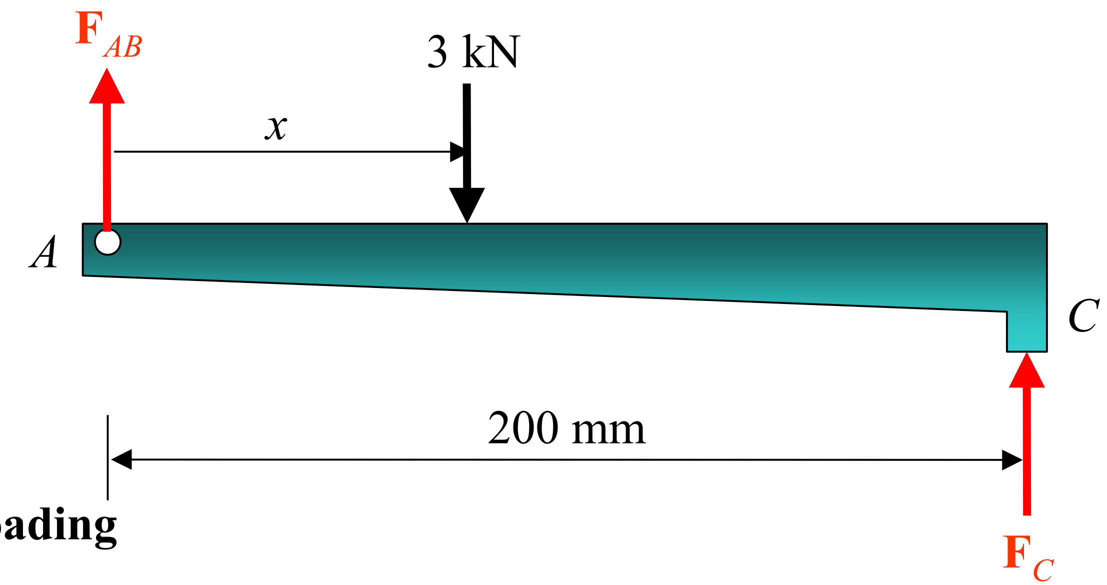
The average normal stress distribution acting over a cross section of rod *AB*.



Example 8

Member *AC* shown is subjected to a vertical force of 3 kN. Determine the position *x* of this force so that the average compressive stress at *C* is equal to the average tensile stress in the tie rod *AB*. The rod has a cross-sectional area of 400 mm^2 and the contact area at *C* is 650 mm^2.





- **Internal Loading**

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + F_C - 3 = 0 \qquad ----(1)$$

$$\circlearrowright + \; \Sigma M_A = 0: \qquad -3(x) + F_C(200) = 0 \qquad ------(2)$$

- **Average Normal Stress**

$$\sigma = \frac{F_{AB}}{400} = \frac{F_C}{650}$$

$$F_C = 1.625 F_{AB} \qquad ----(3)$$

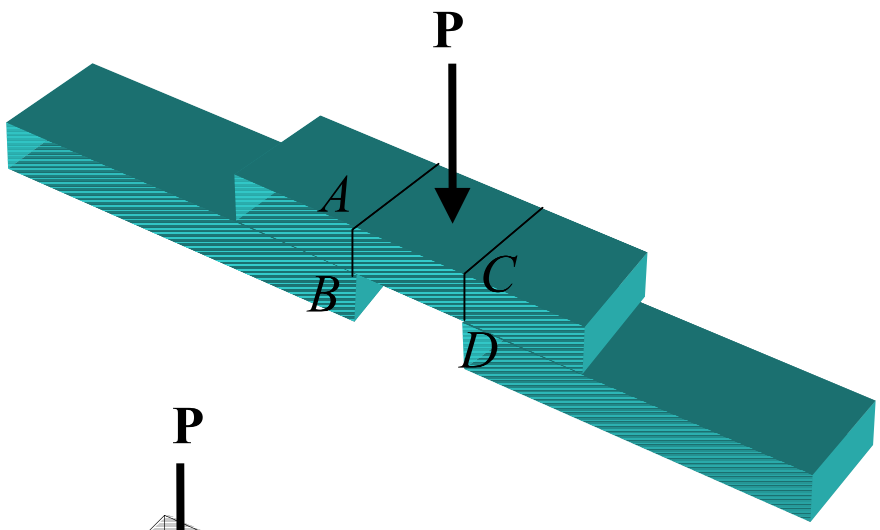
Substituting (3) into (1), solving for F_{AB}, the solving for F_C, we obtain

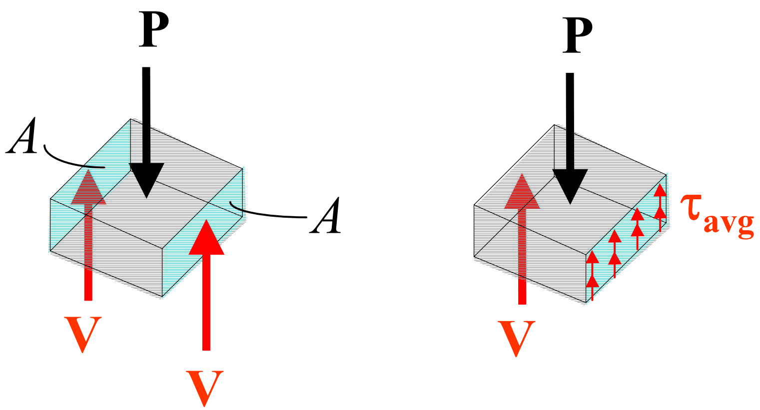
$$F_{AB} = 1.143 \text{ kN} \qquad F_C = 1.857 \text{ kN}$$

The position of the applied load is determined from (2); $x = 124$ mm

Connections

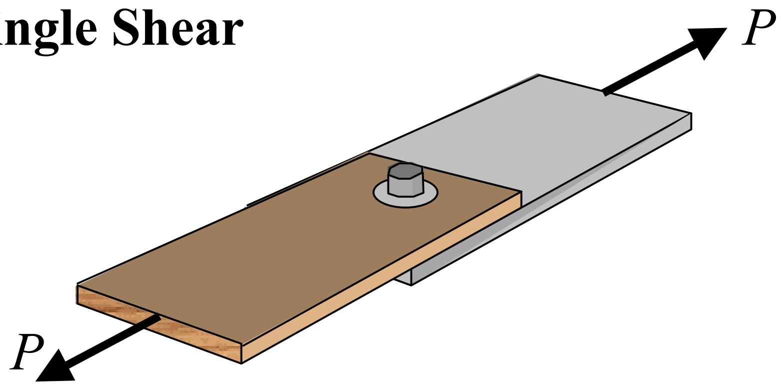
- **Simple Shear**

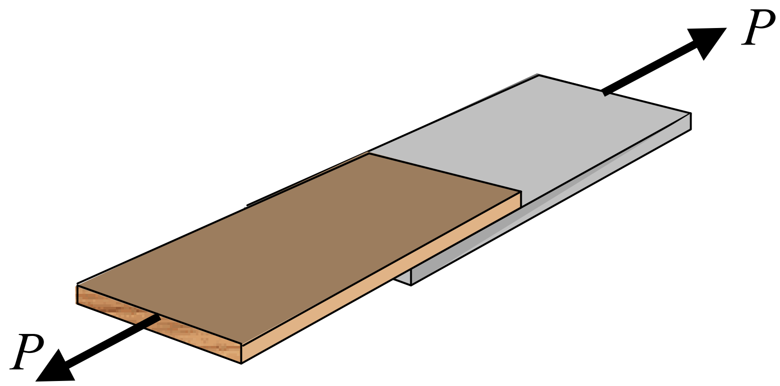


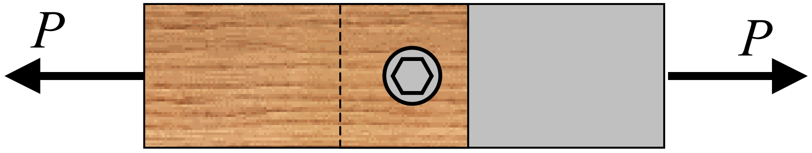


$$\tau_{avg} = \frac{V}{A}$$

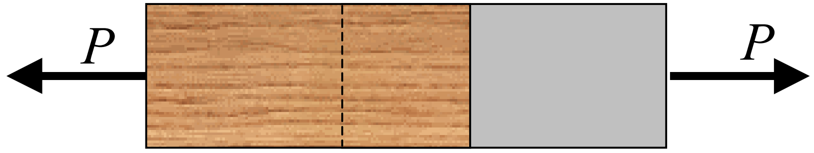
- **Single Shear**



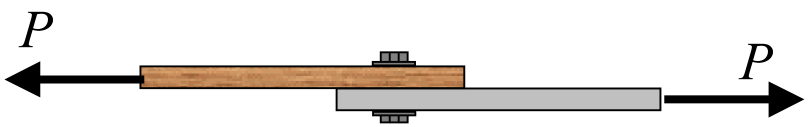




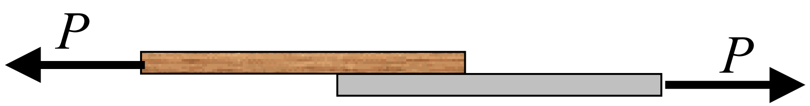
Top View



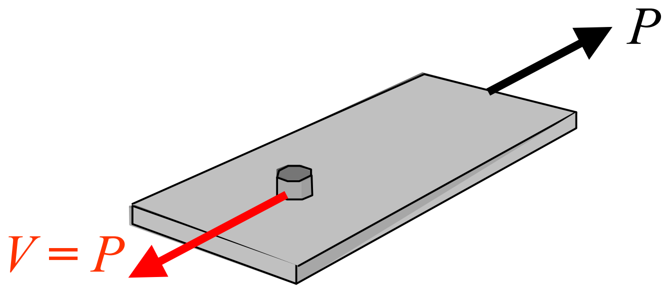
Top View

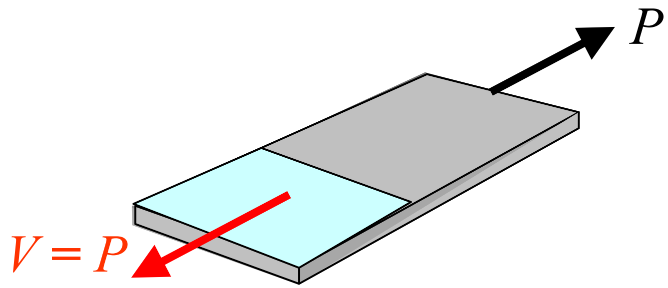


Side View

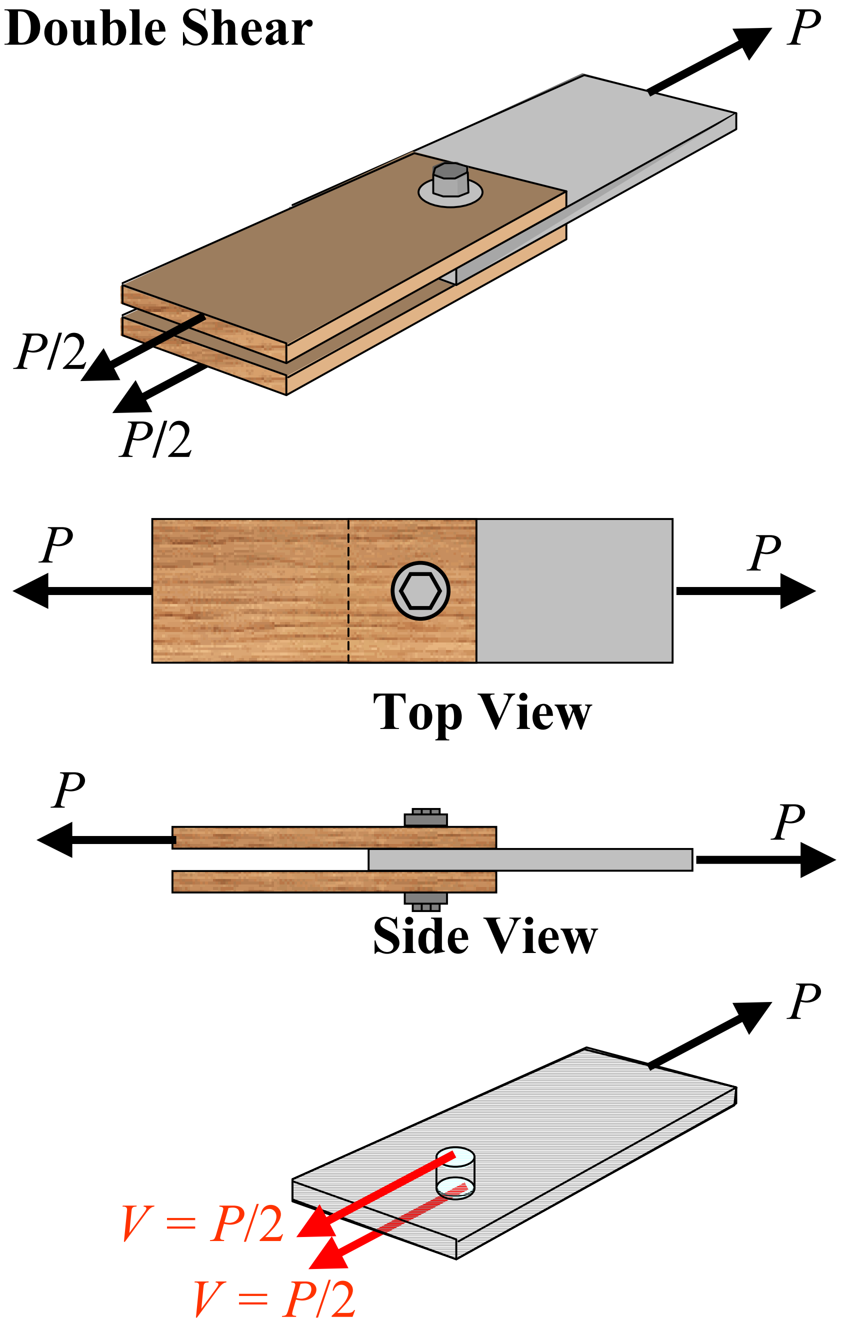


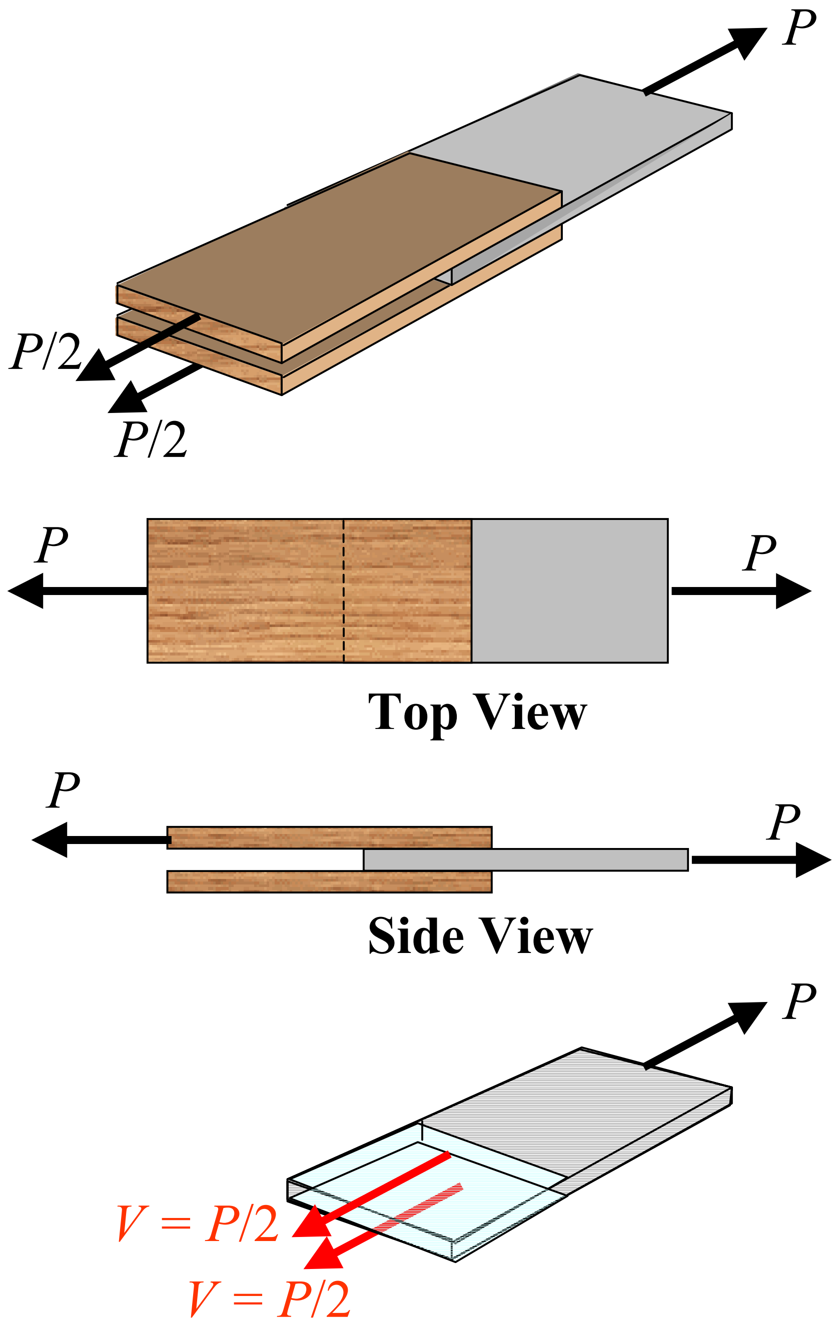
Side View





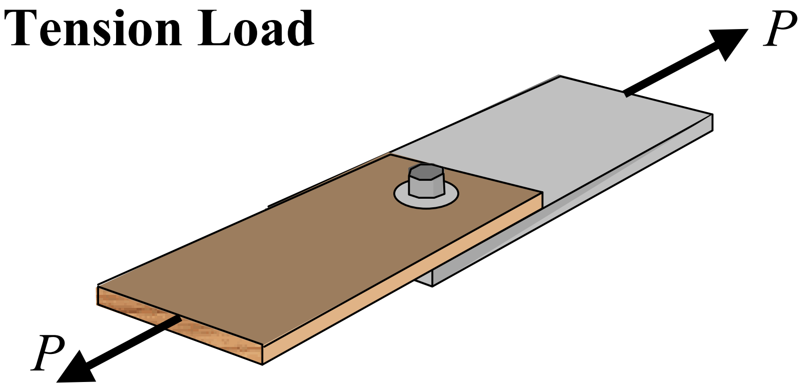
• **Double Shear**

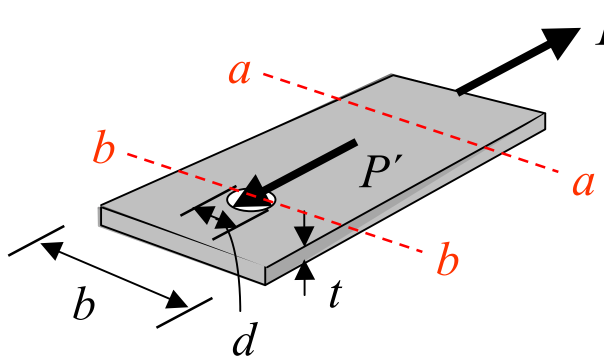


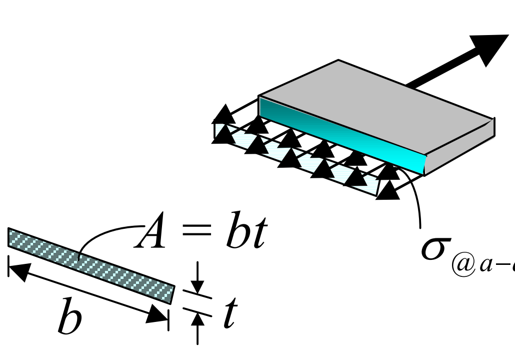


Normal Stress: Compression and Tension Load

- **Tension Load**

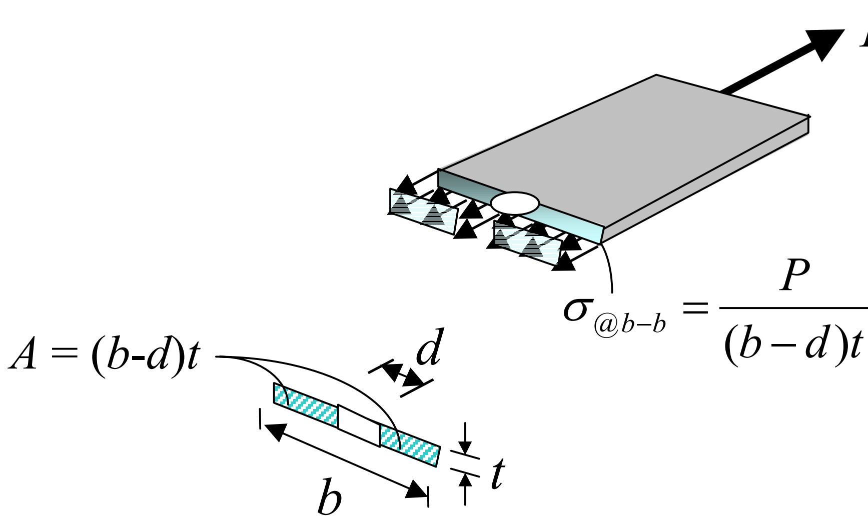






$$\sigma_{@a-a} = \frac{P}{bt}$$

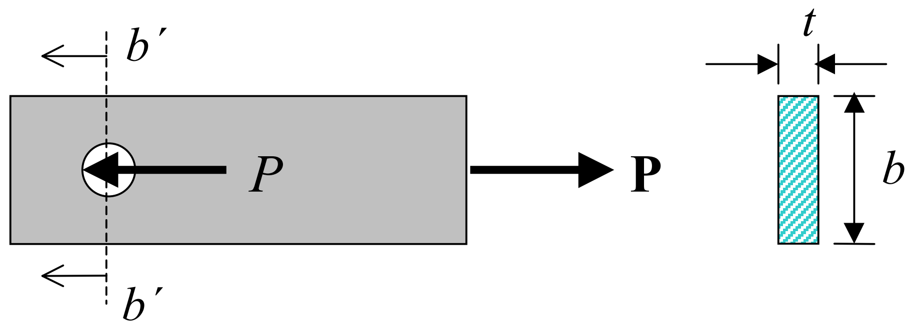
Section *a-a*

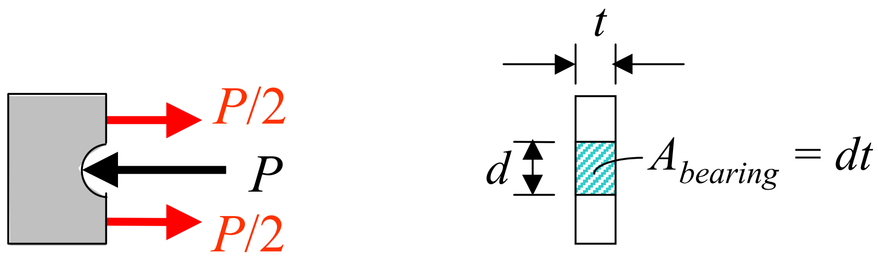


$$\sigma_{@b-b} = \frac{P}{(b-d)t}$$

Section *b-b*

• **Bearing Stress**

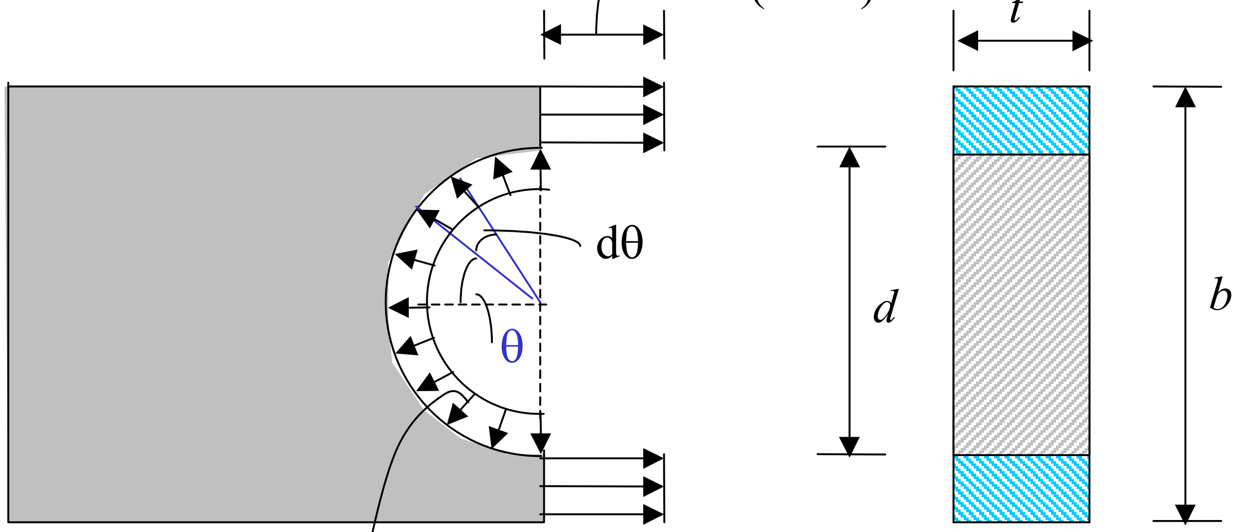




Bearing Stress

$$\sigma_{bearing} = \frac{P}{dt}$$

$$\sigma_{normal} = \frac{P}{(b-d)t}$$



$$\sigma_{bearing} = \frac{P}{dt}$$

$\xrightarrow{+}$ $\Sigma F_x = 0$:

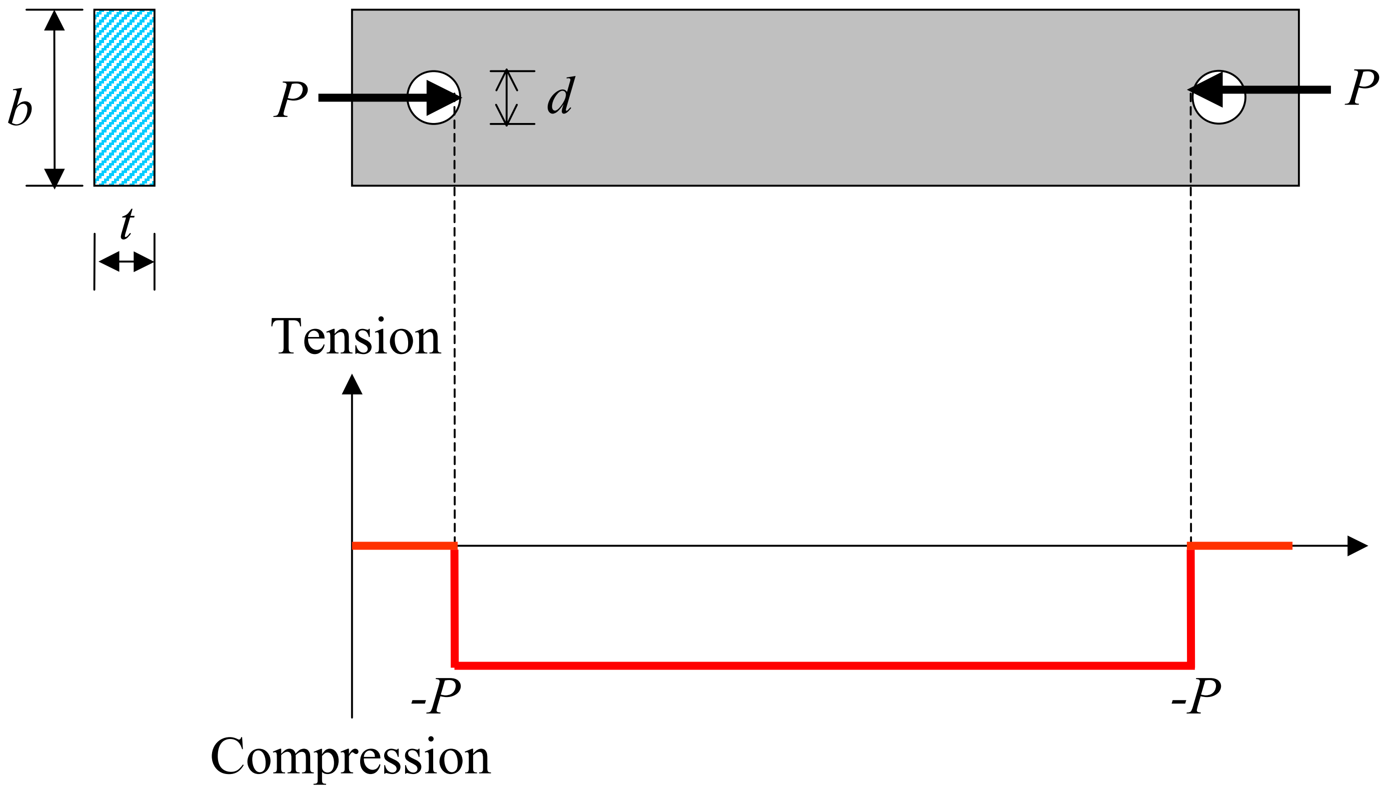
$$P - \int_{-90^\circ}^{90^\circ} \sigma_b \left(\frac{d}{2}\right) t \cos\theta d\theta = 0$$

$$\left(\frac{d}{2}\right) t \sigma_b \sin\theta d\theta \Big|_{-90^\circ}^{90^\circ} = P$$

$$\sigma_{bearing} = \frac{P}{td \sin 90^o}$$

$$\sigma_{bearing} = \frac{P}{td} \qquad ------*$$

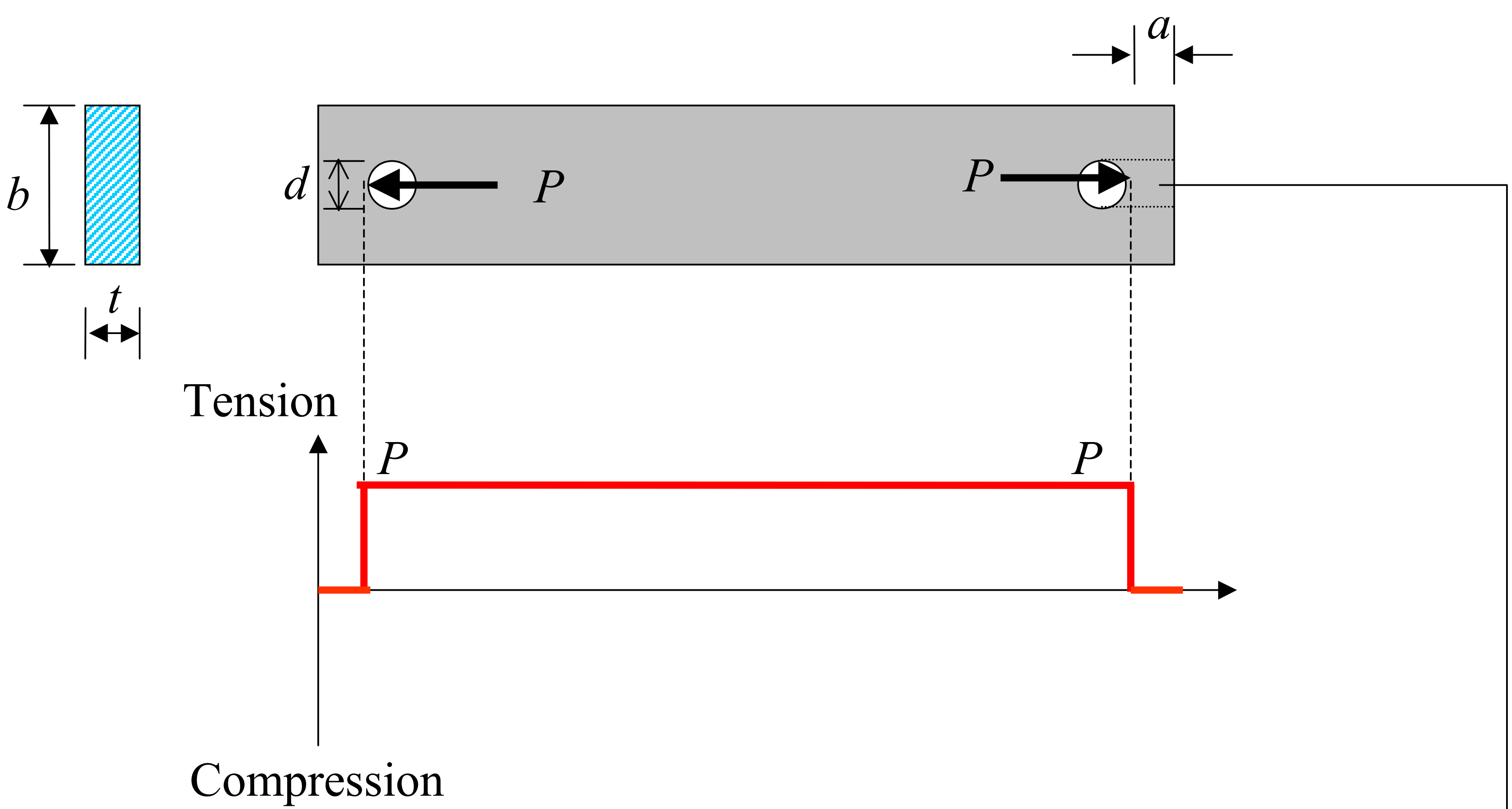
• **Axial Force Diagram for Compression Load on Plate**



$$- Normal \quad Stress: \quad \sigma = \frac{-P}{(b-d)t}, \quad compression$$

$$- Bearing \quad Stress \quad \sigma_{bearing} = \frac{P}{dt}$$

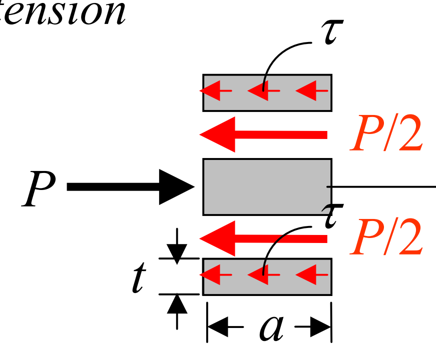
- **Axial Force Diagram for Tension Load on Plate**



$$-\,Normal \quad Stress: \max \qquad \sigma = \frac{+P}{(b-d)t}, \qquad tension$$

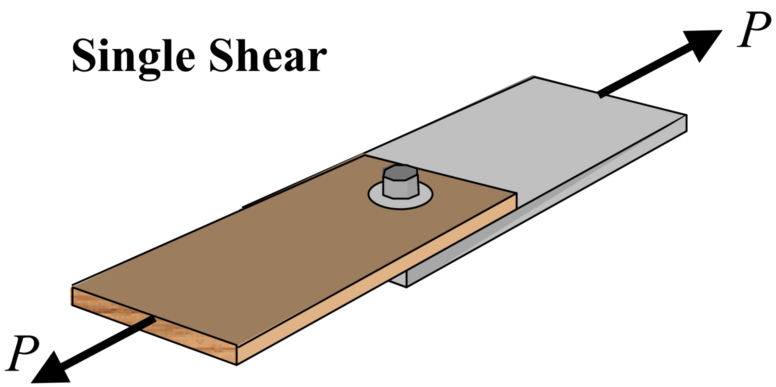
$$-\,Bearing \quad Stress \qquad \sigma_{bearing} = \frac{P}{dt}$$

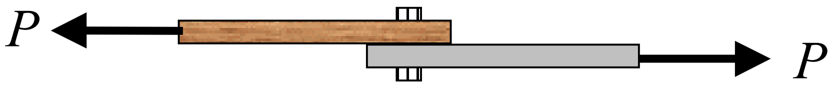
$$-\,Shearing \quad Stress \qquad \tau = \frac{P}{2at}$$

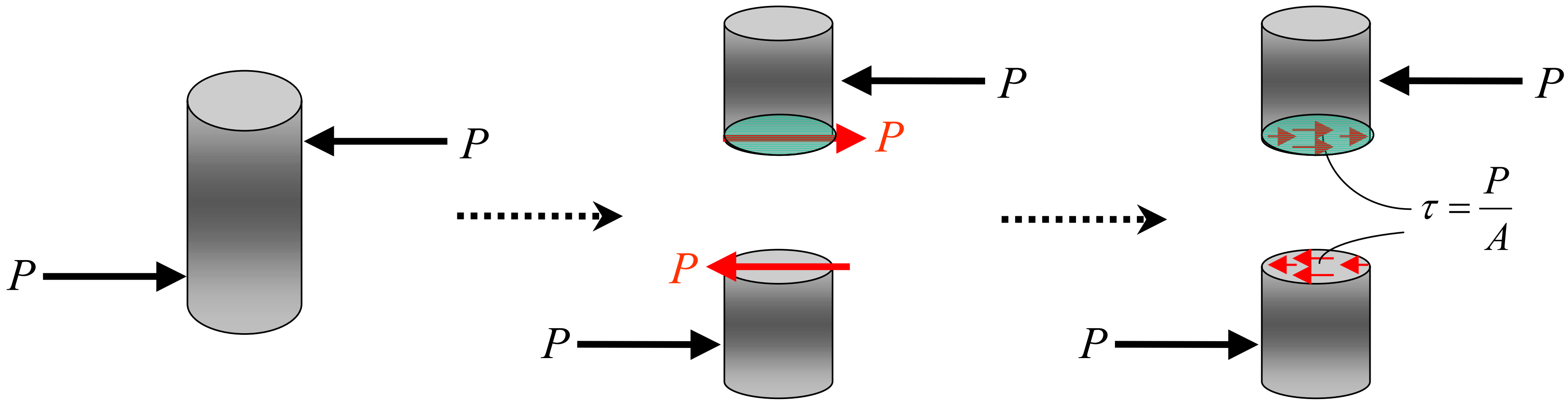


- **Shearing Stress on pin**

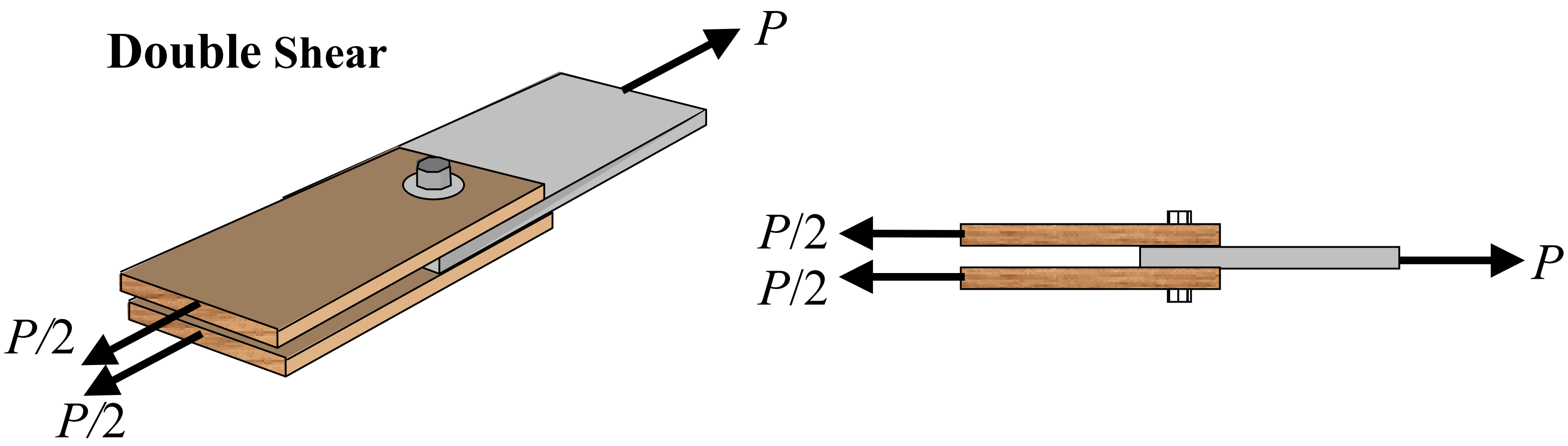
Single Shear

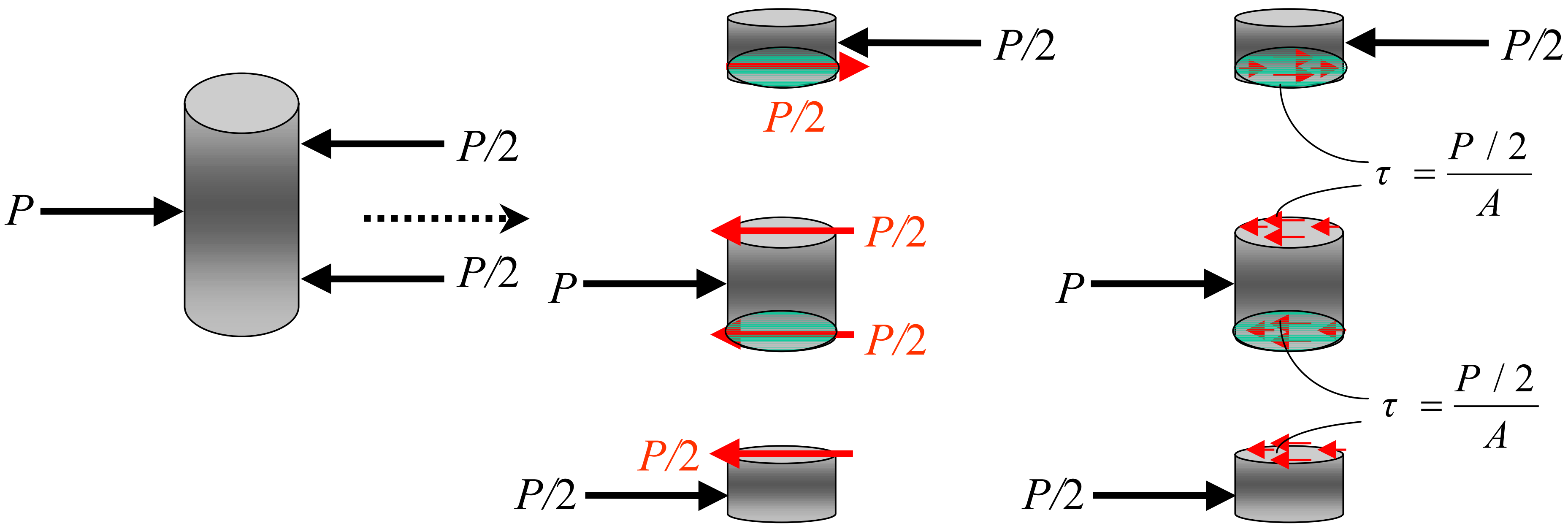




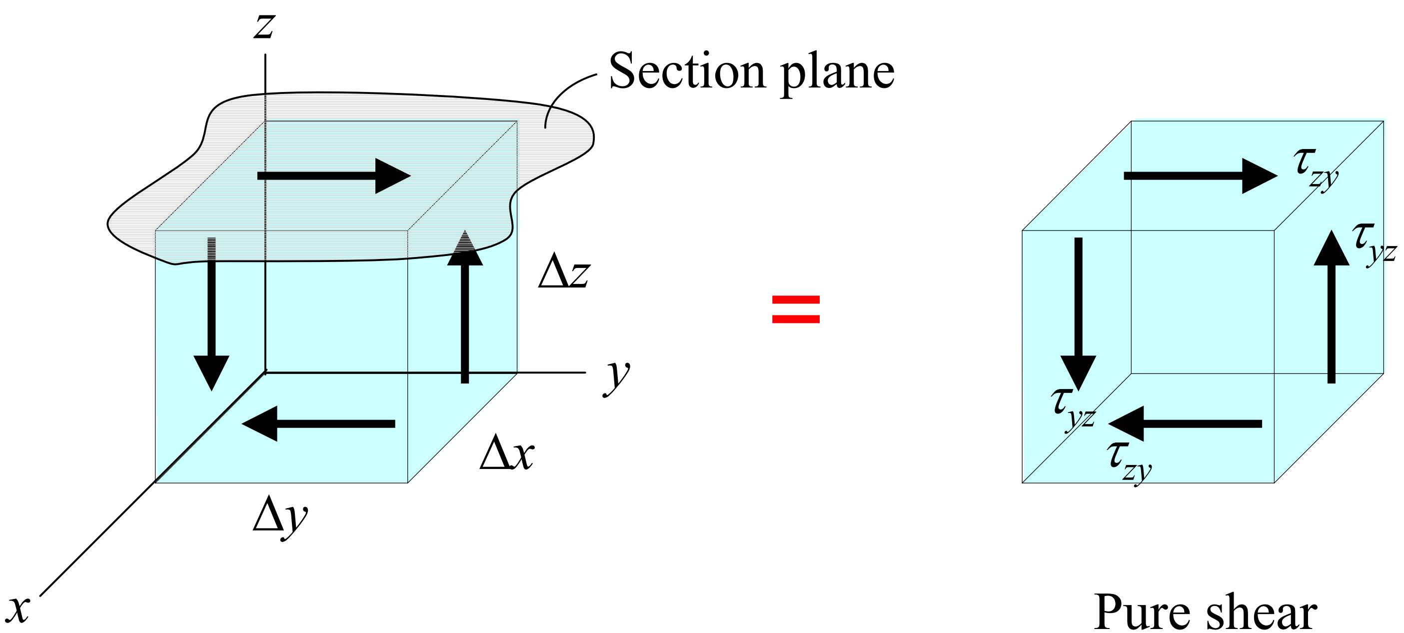


Double Shear





Pure Shear

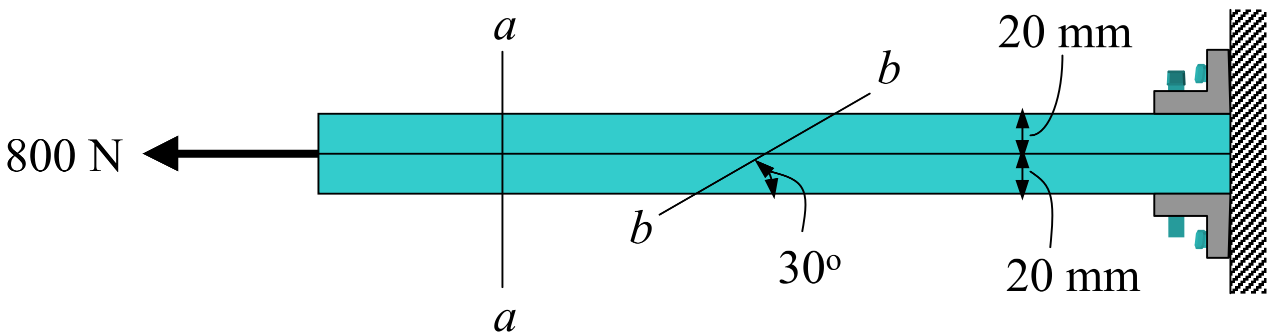


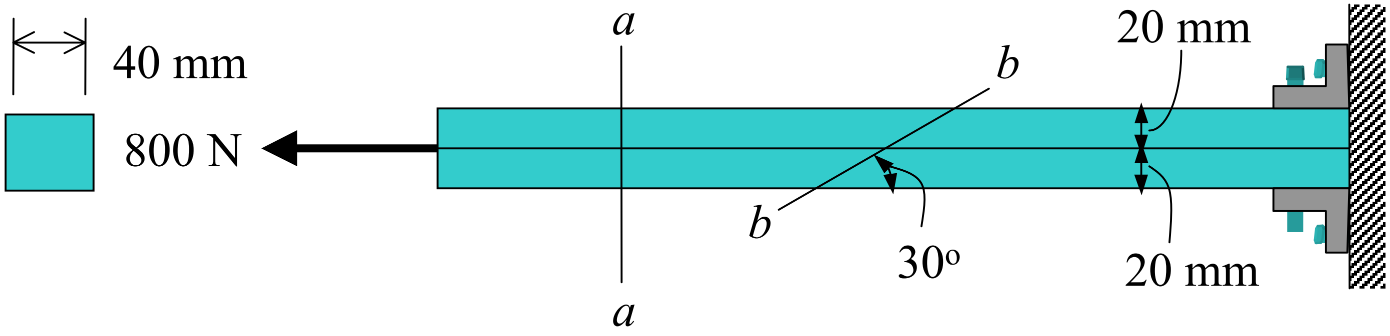
Pure shear

$$\tau_{zy} = \tau_{yz}$$

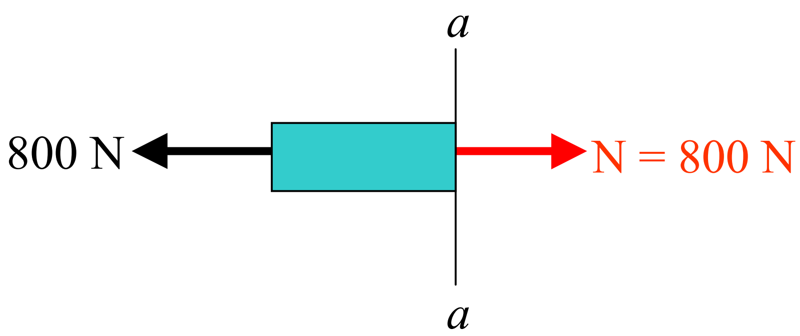
Example 9

The bar shown a square cross section for which the depth and thickness are 40 mm. If an axial force of 800 N is applied along the centroidal axis of the bar's cross-sectional area, determine the average normal stress and average shear stress acting on the material along (*a*) section plane *a-a* and (*b*) section plane *b-b*.



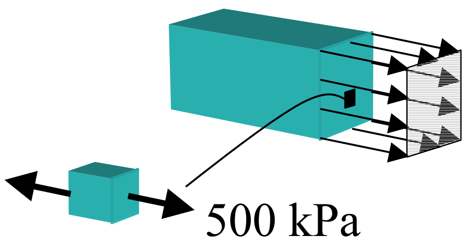


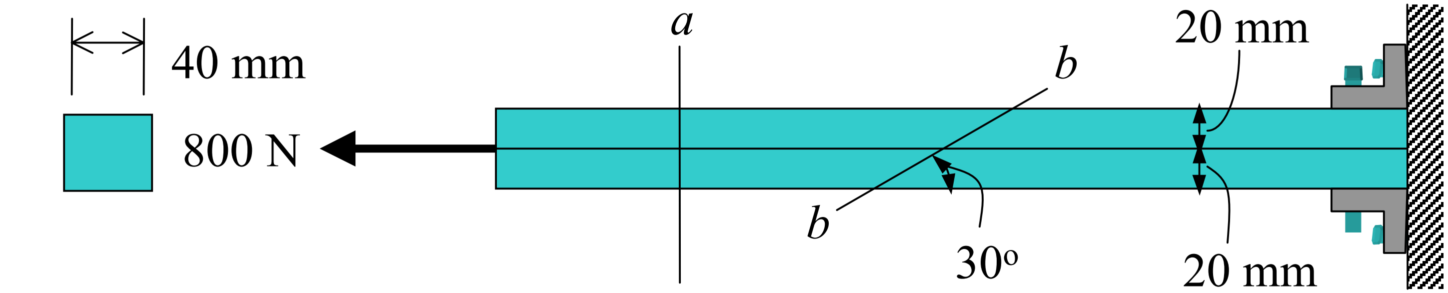
(*a*) section plane *a-a*



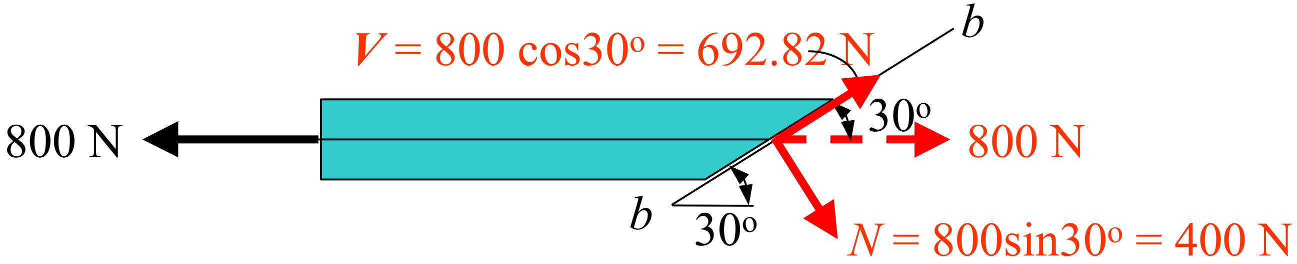
$$\sigma = \frac{N}{A} = \frac{800\ \text{N}}{(0.04\ \text{m})(0.04\ \text{m})} = 500\ \text{kPa}$$

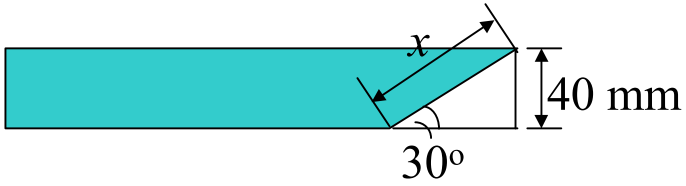
$$\tau_{avg} = 0$$



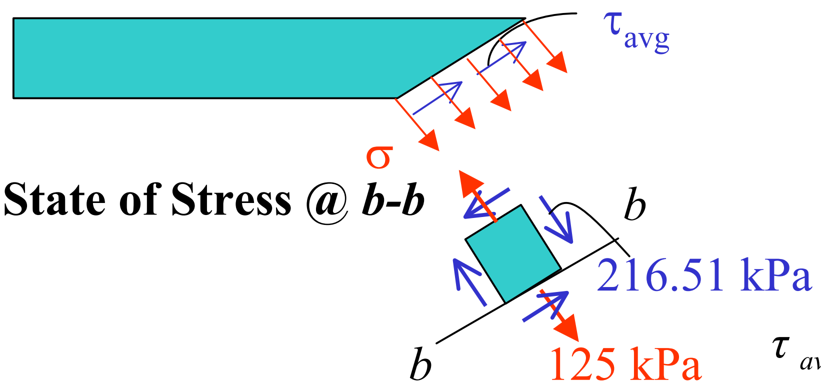


(*b*) section plane *b-b*





$$\sin 30^{o} = \frac{40}{x} \quad ; \quad x = 80 \text{ mm}$$

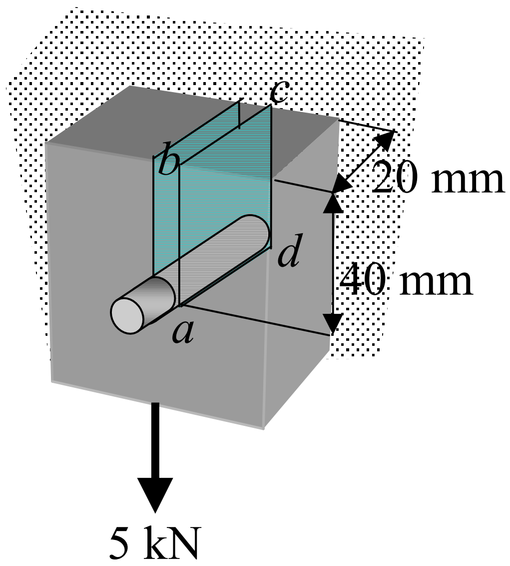


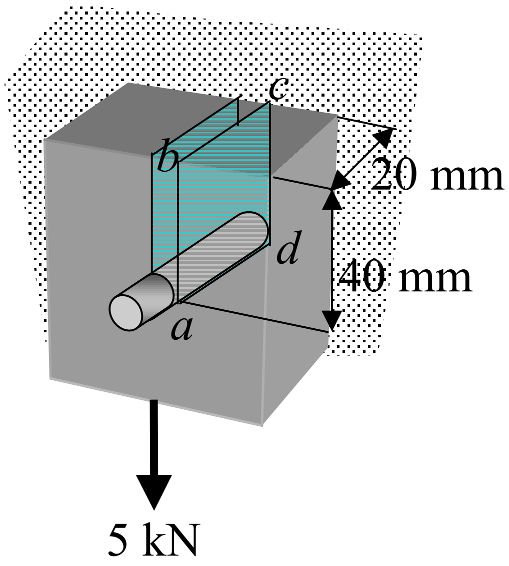
$$\sigma = \frac{N}{A} = \frac{400 \text{ N}}{(0.04 \text{ m})(0.08 \text{ m})} = 125 \text{ kPa}$$

$$\tau_{avg} = \frac{V}{A} = \frac{692.82 \text{ N}}{(0.04 \text{ m})(0.08 \text{ m})} = 216.51 \text{ kPa}$$

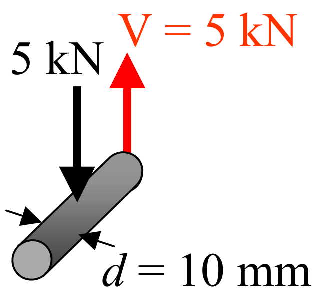
Example 10

The wooden strut shown is suspended form a 10 mm diameter steel rod, which is fastened to the wall. If the strut supports a vertical load of 5 kN, compute the average shear stress in the rod at the wall and along the two shaded planes of the strut, one of which is indicated as *abcd*.

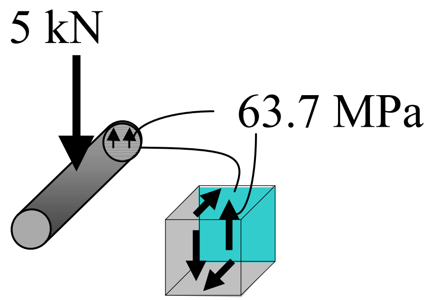




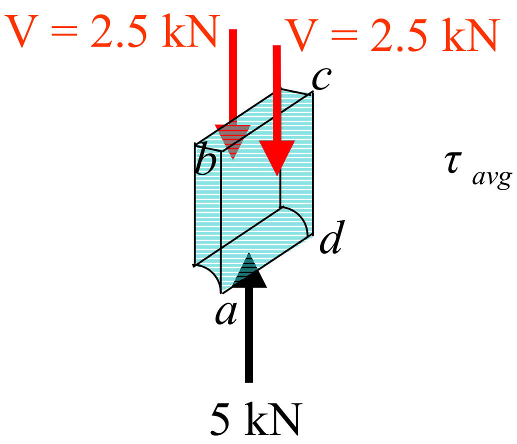
• **Average shear stress in the rod at the wall**



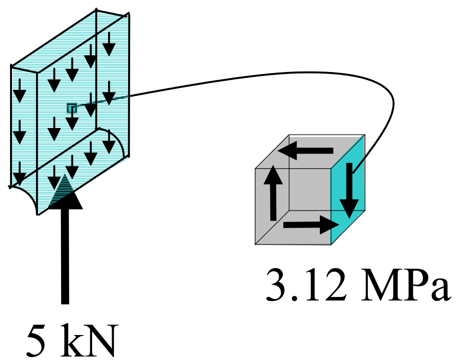
$$\tau_{avg} = \frac{V}{A} = \frac{5\ \text{kN}}{\pi (0.005\ \text{m})^2} = 63.7\ \text{MPa}$$



• **Average shear stress along the two shaded plane**

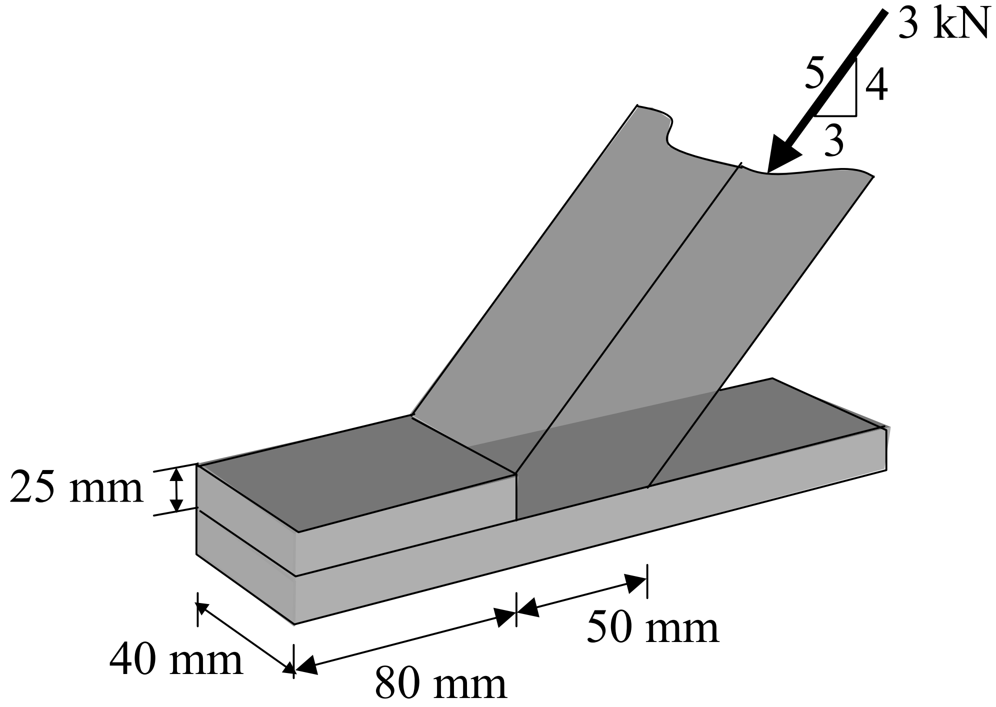


$$\tau_{avg} = \frac{V}{A} = \frac{2.5\ \text{kN}}{(0.04\ \text{m})(0.02\ \text{m})} = 3.12\ \text{MPa}$$

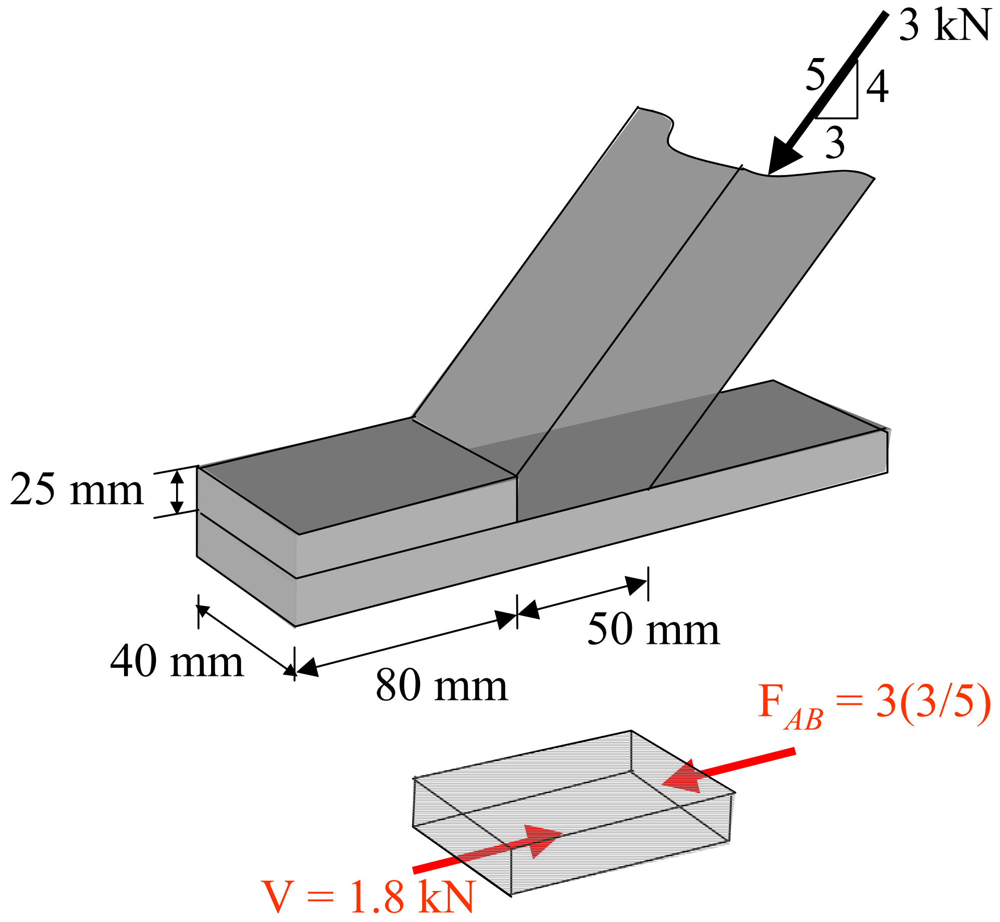


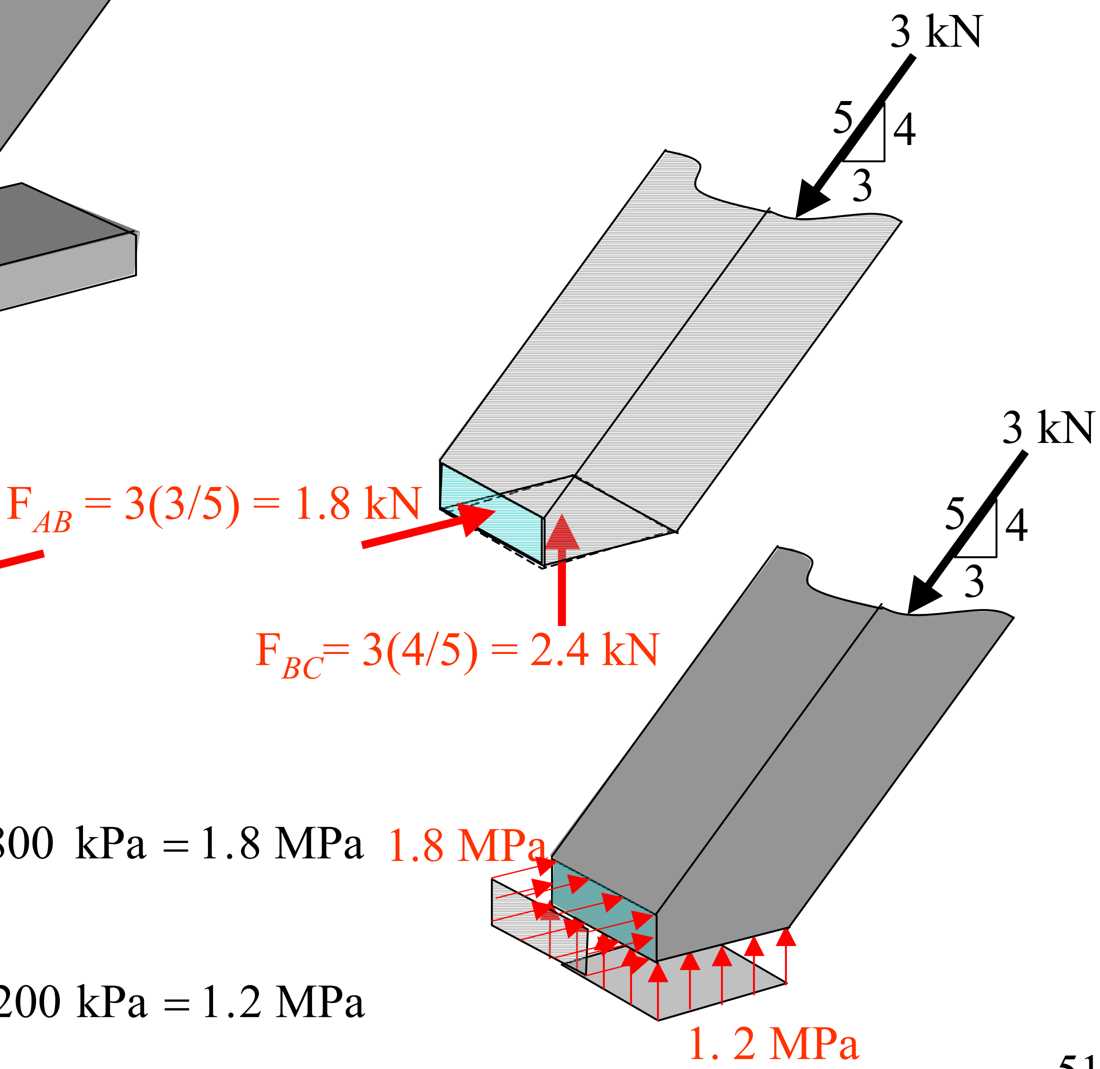
Example 11

The inclined member show is subjected to a compressive force of 3 kN. Determine the average compressive stress along the areas of contact defined by *AB* and *BC*, and the average shear stress along the horizontal plane defined by *EDB*.



• **The average compressive stress along the areas of contact defined by *AB* and *BC* :**

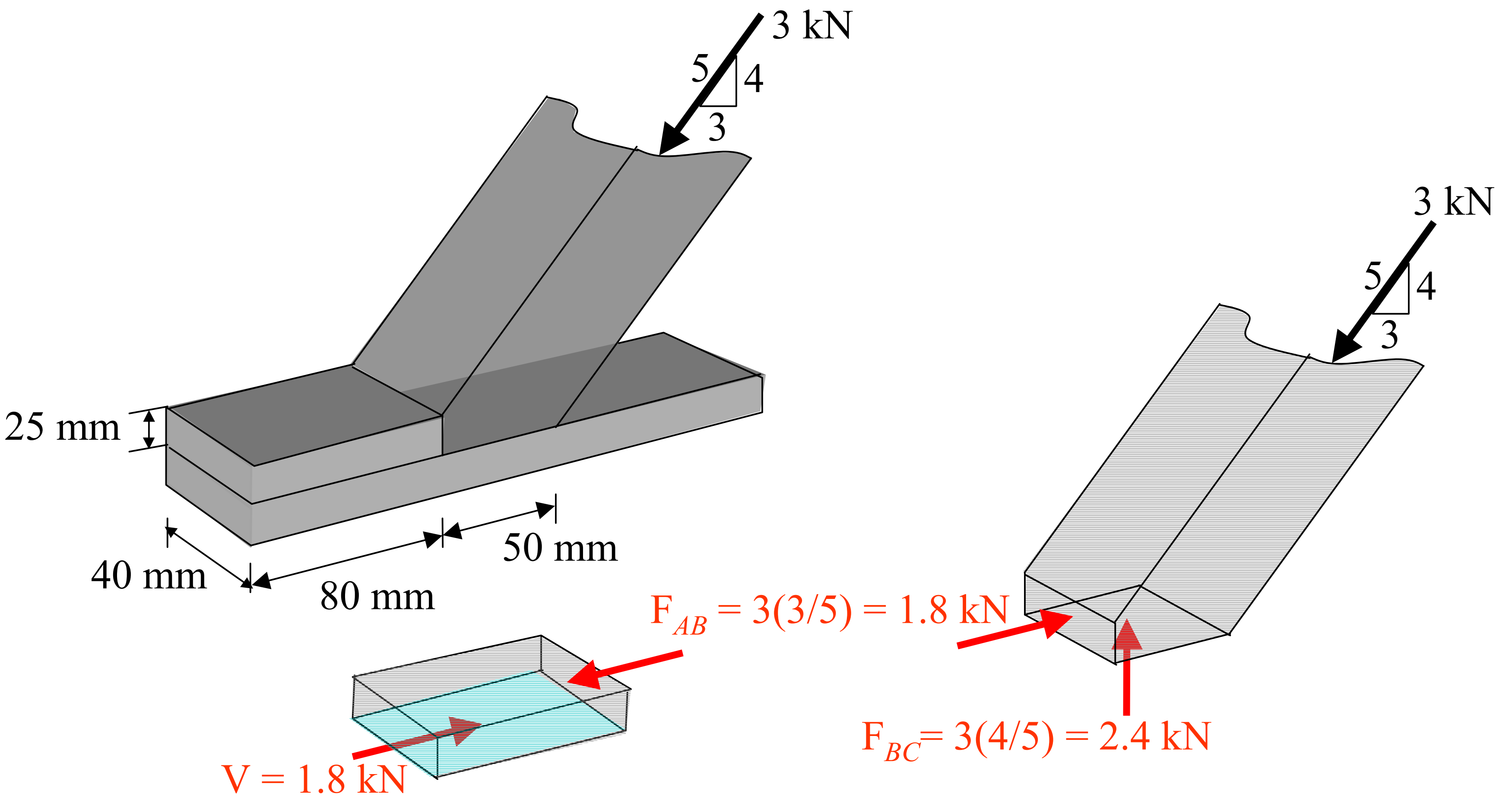




$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{1.8\ \text{kN}}{(0.025\ \text{m})(0.040\ \text{m})} = 1800\ \text{kPa} = 1.8\ \text{MPa}$$

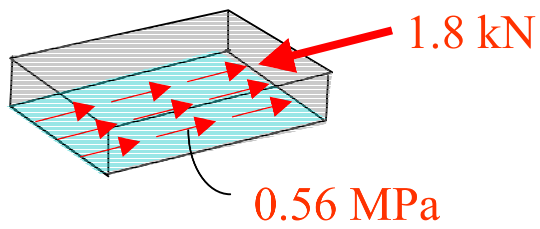
$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{2.4\ \text{kN}}{(0.050\ \text{m})(0.040\ \text{m})} = 1200\ \text{kPa} = 1.2\ \text{MPa}$$

- **The average shear stress along the horizontal plane defined by *EDB* :**



$$\tau_{avg} = \frac{V}{A} = \frac{1.8\ \text{kN}}{(0.04\ \text{m})(0.08\ \text{m})} = 562.5\ \text{kPa}$$

$$= 0.562\ \text{MPa}$$



Allowable Stress

$$F.S = \frac{P_{fail}}{P_{allow}}$$

$$F.S = \frac{\sigma_{fail}}{\sigma_{allow}}$$

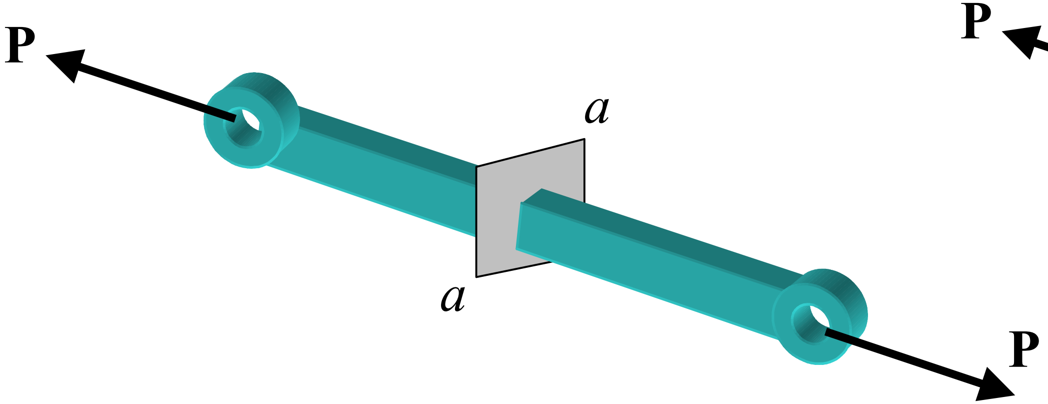
$$F.S = \frac{\tau_{fail}}{\tau_{allow}}$$

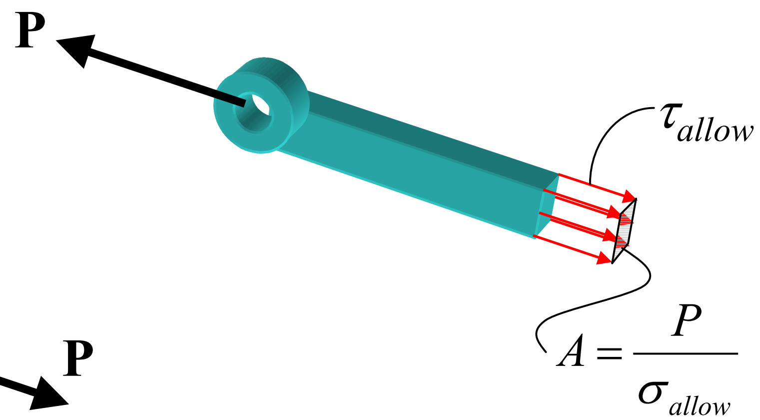
6. Design of Simple Connections

$$A = \frac{P}{\sigma_{allow}}$$

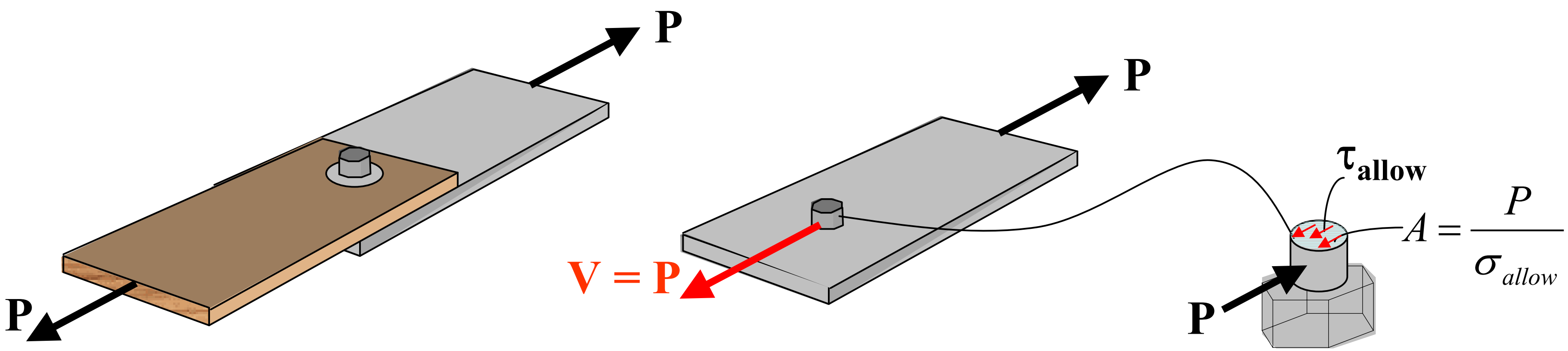
$$A = \frac{V}{\tau_{allow}}$$

- **Cross-Sectional Area of a Tension Member**

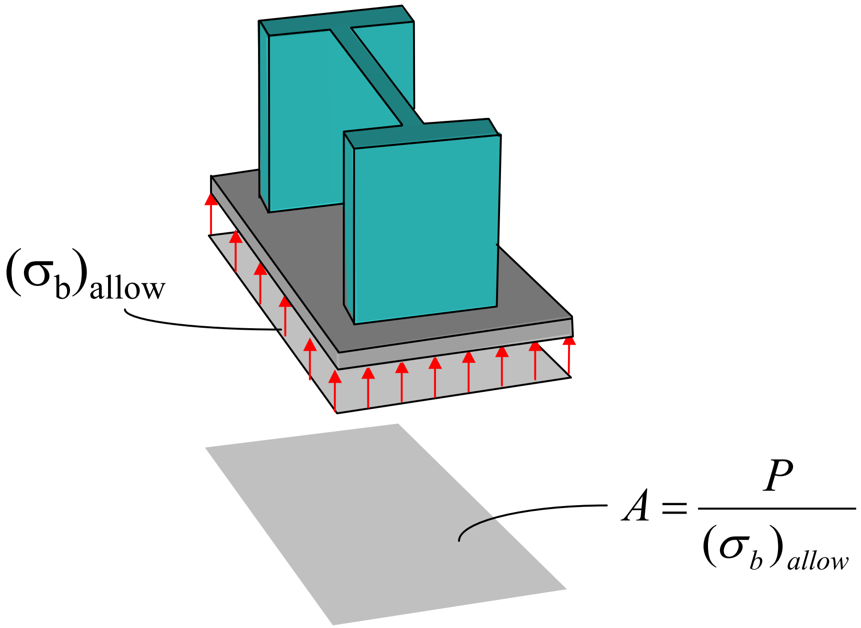




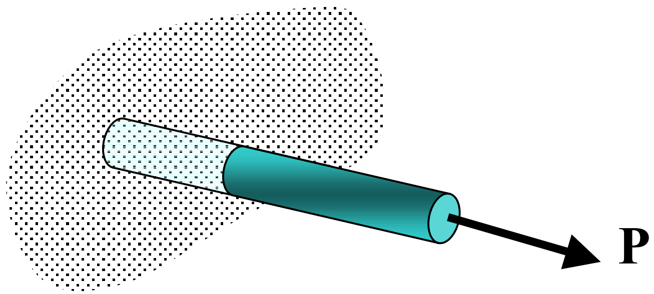
- **Cross-Sectional Area of a Connector Subjected to Shear**

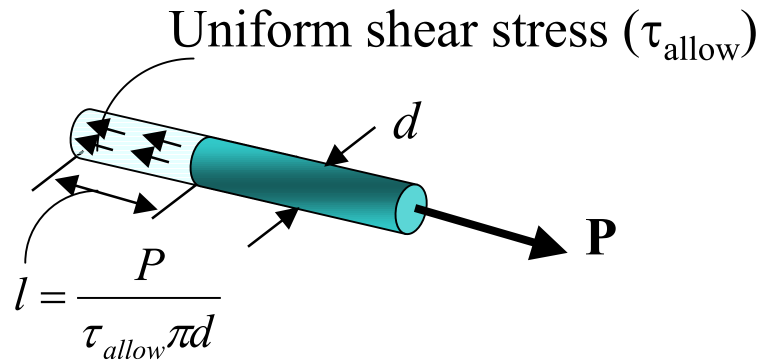


- **Required Area to Resist Bearing**



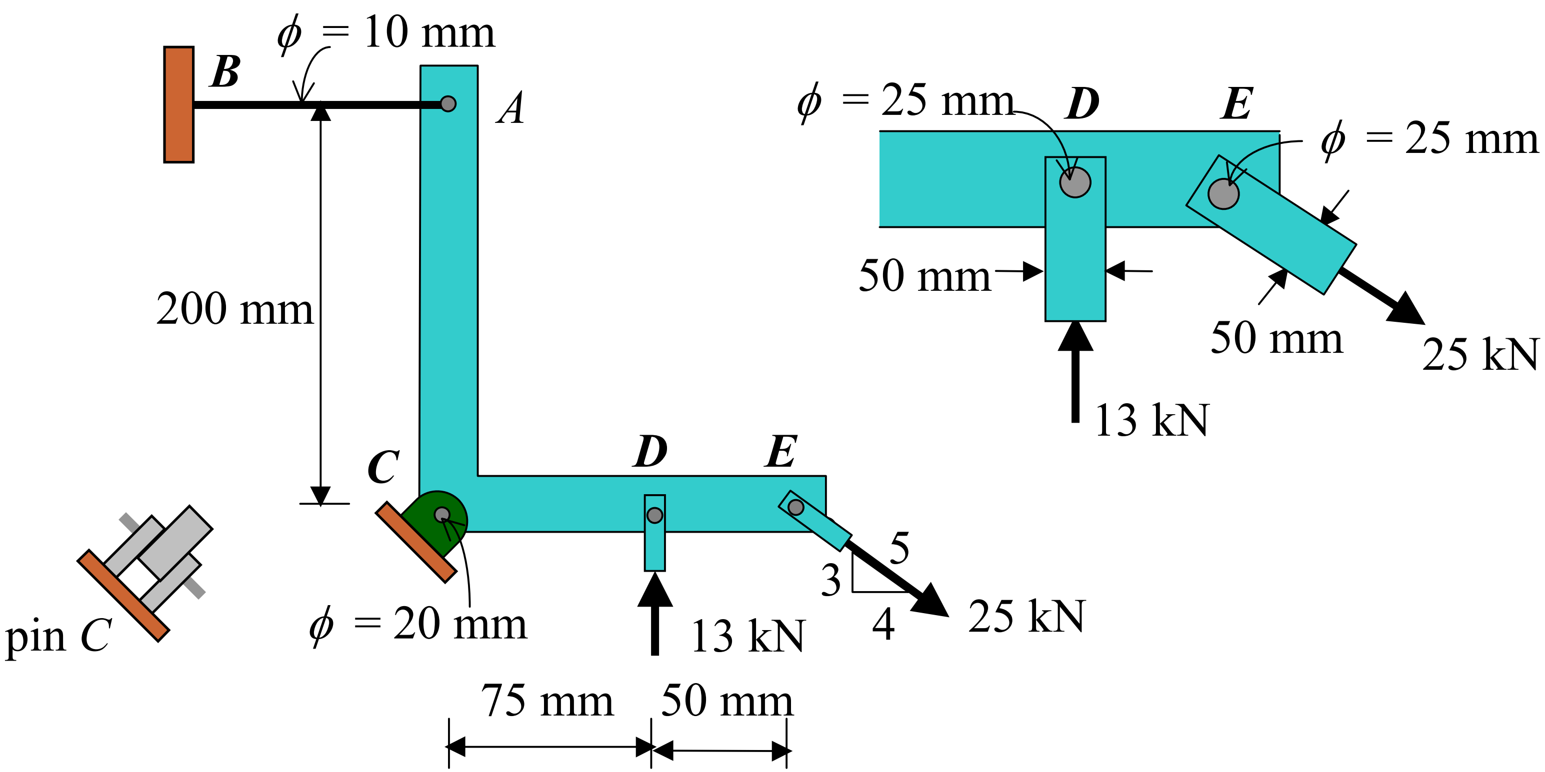
• **Required Area to Resist Shear by Axial Load**

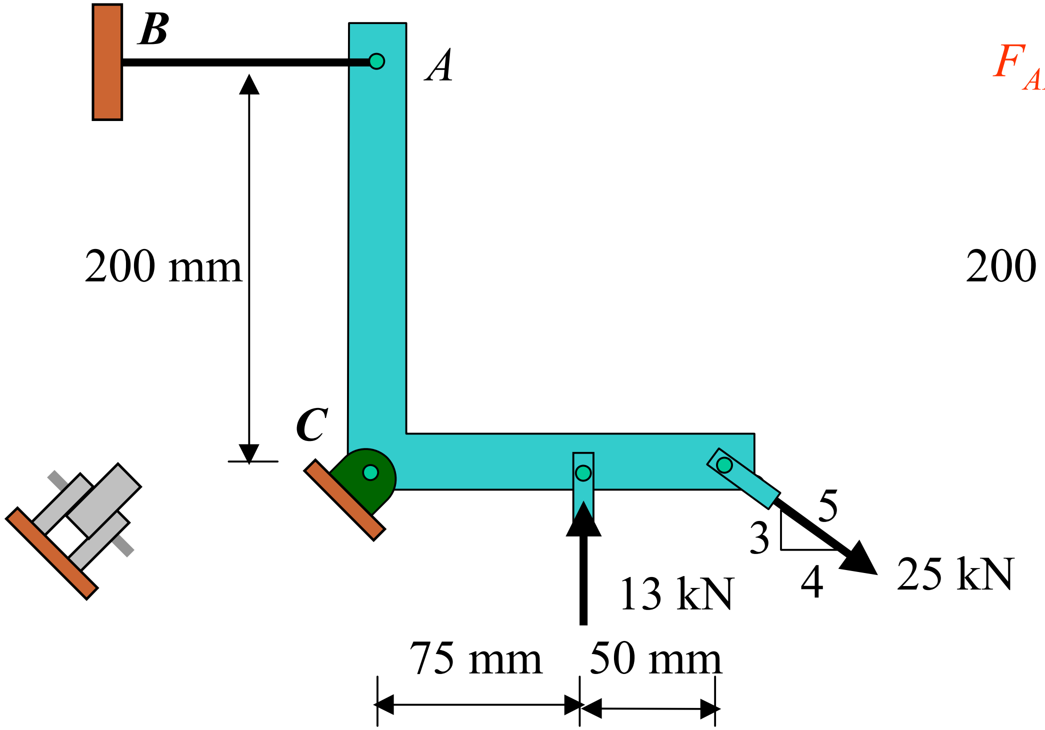


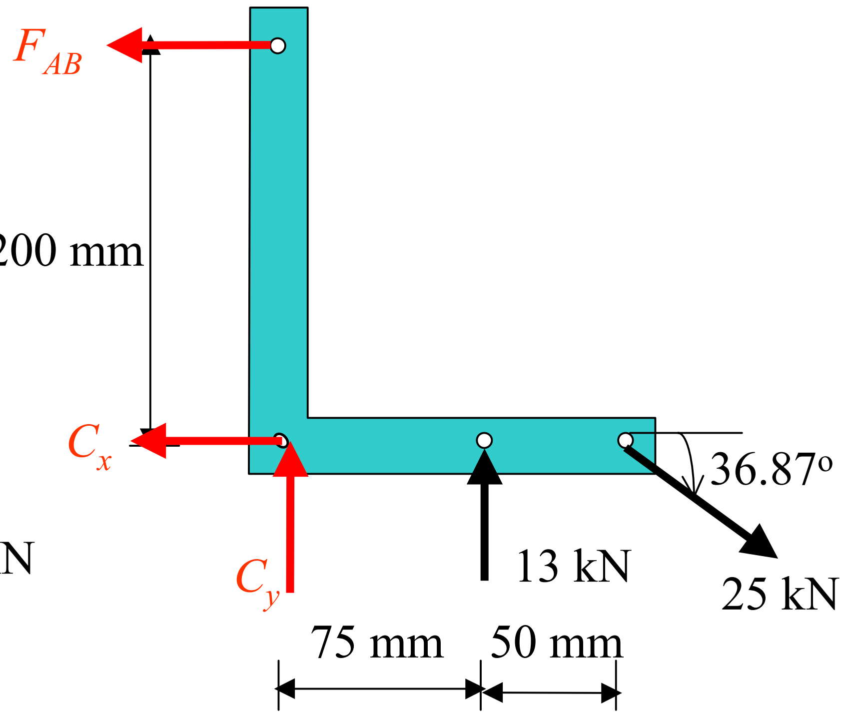


Example 12a

The control arm is subjected to the loading shown. (a) Determine the shear stress for the steel at all pin (b) Determine normal stress in rod *AB,* plate *D* and *E.* The thickness of plate *D* and *E* is 10 mm.







- **Reaction *C***

$$\circlearrowright^{+} \Sigma M_C = 0: \qquad F_{AB}(0.2) + 13(0.075) - 25\sin 36.87(0.125) = 0$$

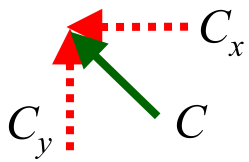
$$F_{AB} = 4.5 \text{ kN}, \leftarrow$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \qquad -4.5 - C_x + 25\cos 36.87^\circ = 0$$

$$C_x = 15.5 \text{ kN}, \leftarrow$$

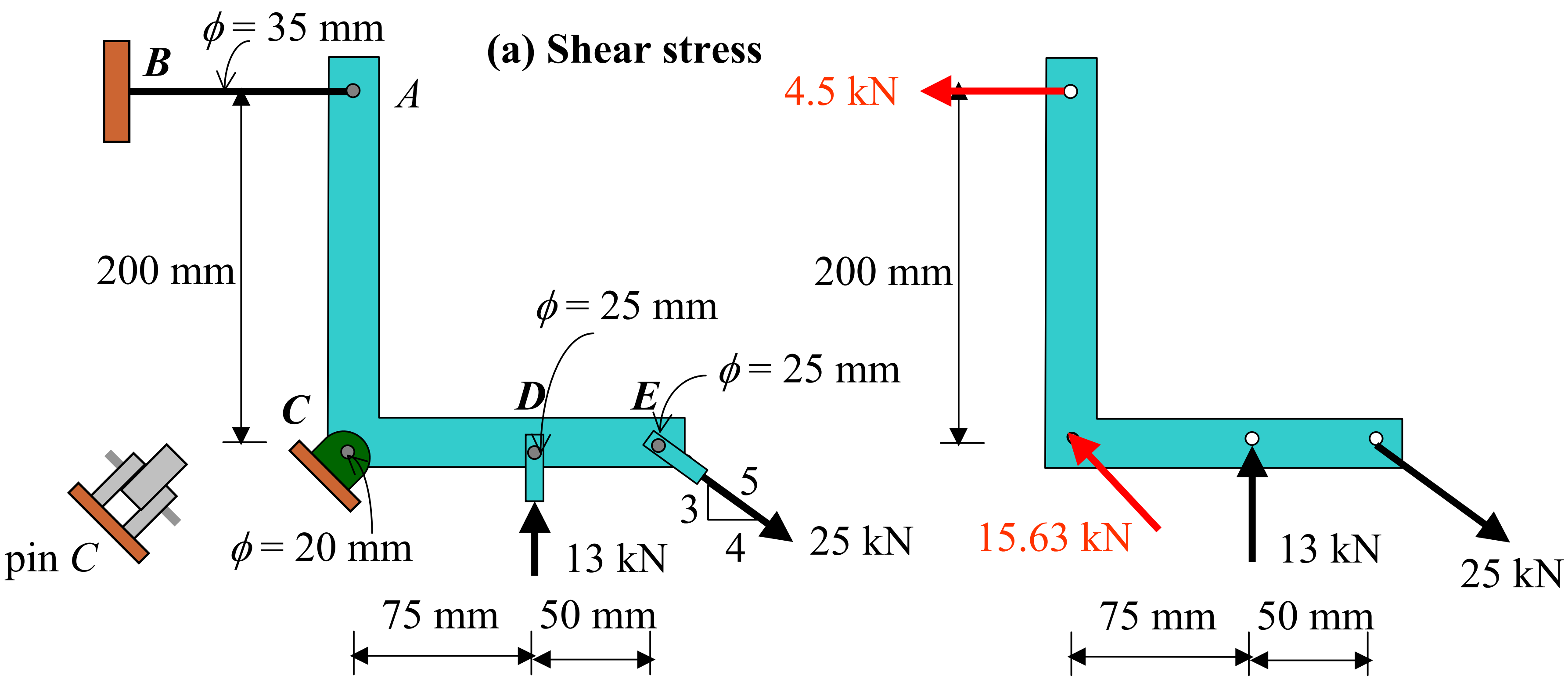
$$+\uparrow \Sigma F_y = 0: \qquad C_y + 13 - 25\sin 36.87^\circ = 0$$

$$C_y = 2 \text{ kN}, \uparrow$$

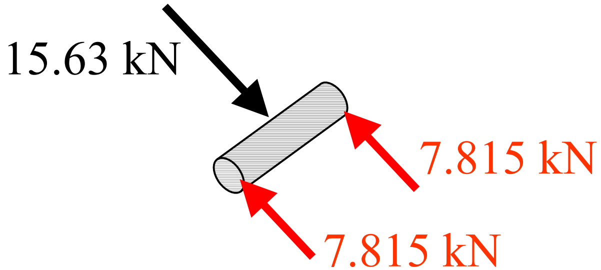


$$C = \sqrt{(15.5)^2 + (2)^2} = 15.63 \text{ kN}$$

(a) Shear stress



Pin *C* (Double shear)



$$\tau_C = \frac{V_C}{A_C} = \frac{7.815\ \text{kN}}{(\pi/4)(0.02)^2} = 24.88\ \text{MPa} \Leftarrow$$

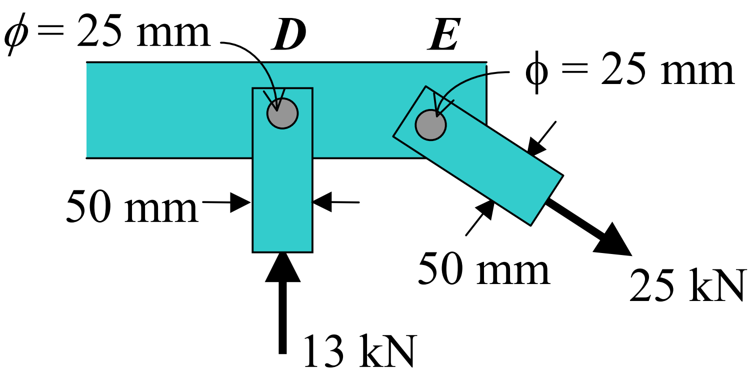
Pin *D* and *E* (Single shear)

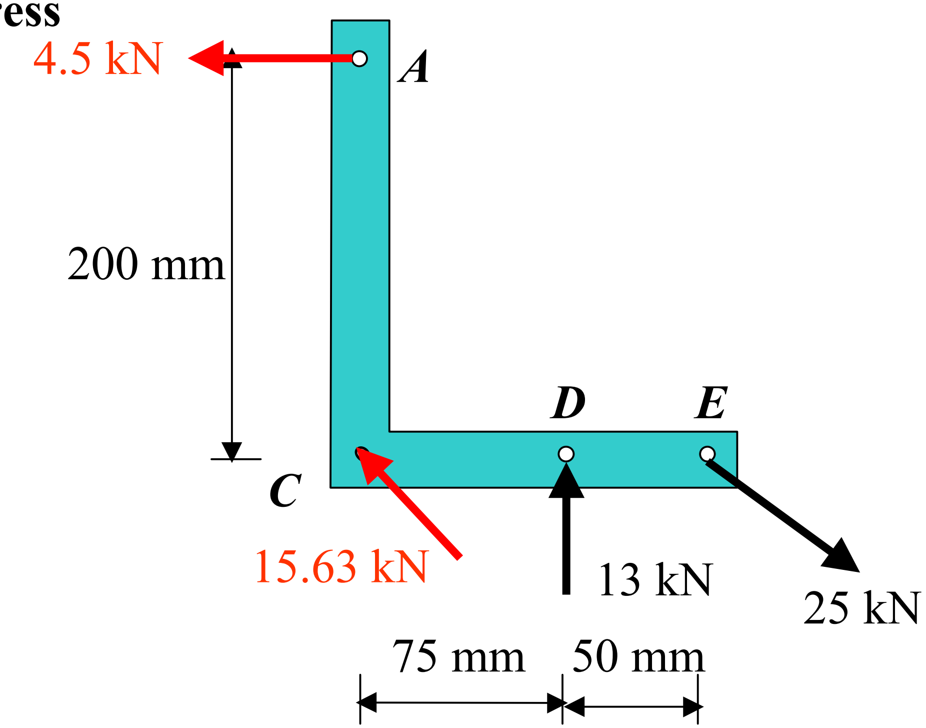
$$\tau_D = \frac{V_D}{A_D} = \frac{13\ \text{kN}}{(\pi/4)(0.025)^2} = 26.48\ \text{MPa} \Leftarrow$$

$$\tau_E = \frac{V_E}{A_E} = \frac{25\ \text{kN}}{(\pi/4)(0.025)^2} = 50.93\ \text{MPa} \Leftarrow$$

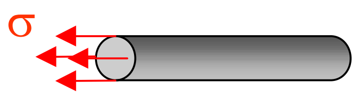
(b) Normal stress

$\phi_{rod} = 10$ mm



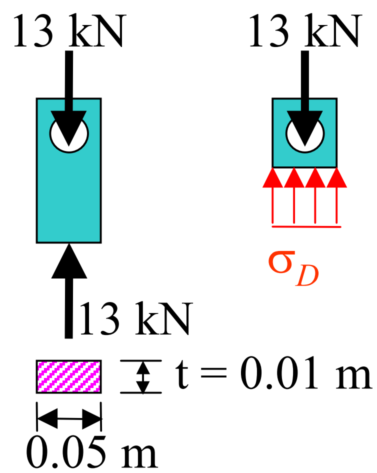


Cale *AB*



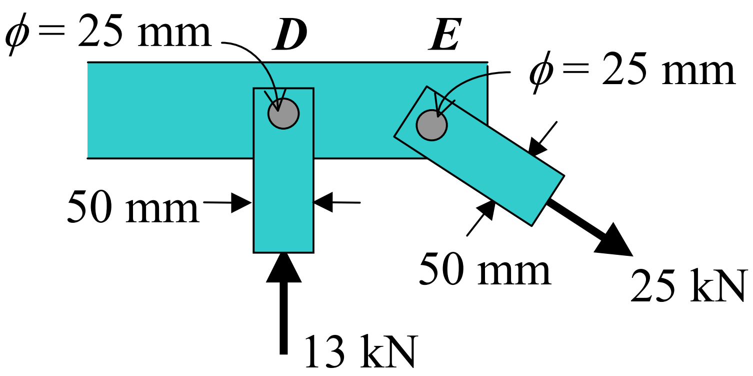
$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{4.5 \text{ kN}}{(\pi/4)(0.010)^2} = 56.7 \text{ MPa} \Leftarrow$$

Plate D



$$\sigma_D = \frac{P}{A} = \frac{13 \text{ kN}}{(0.05)(0.01)} = 26 \text{ MPa} \Leftarrow$$

$\phi_{\text{rod}} = 35$ mm



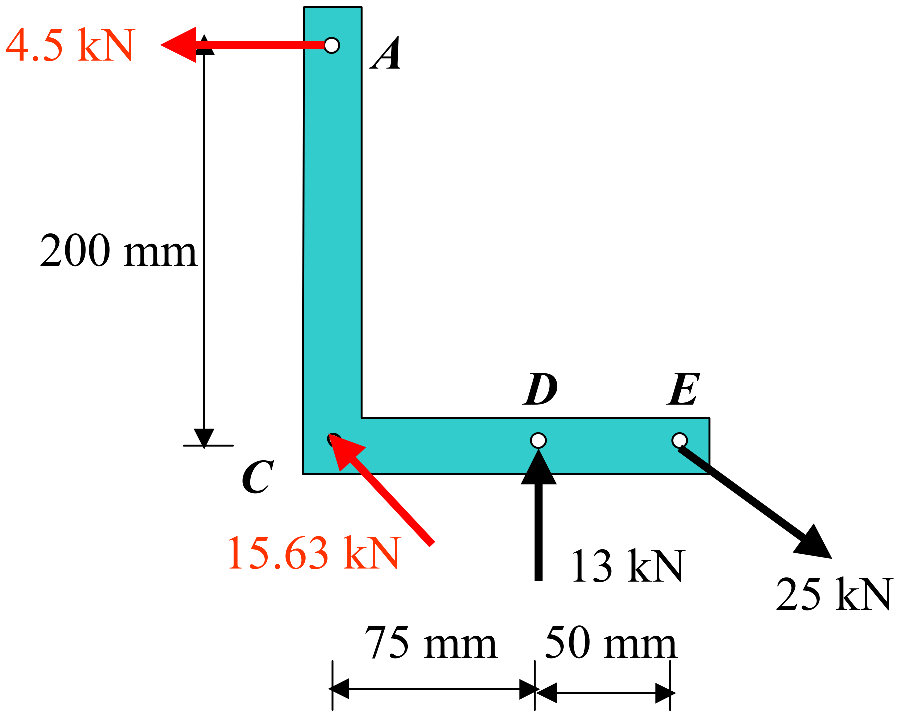
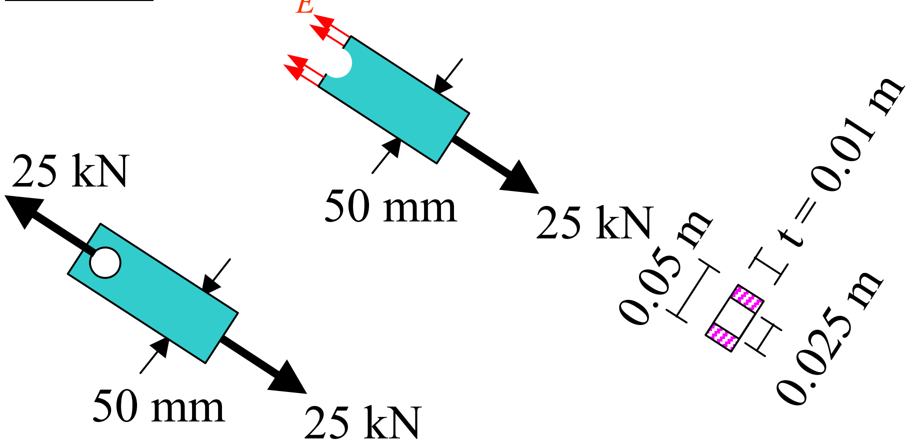


Plate *E*



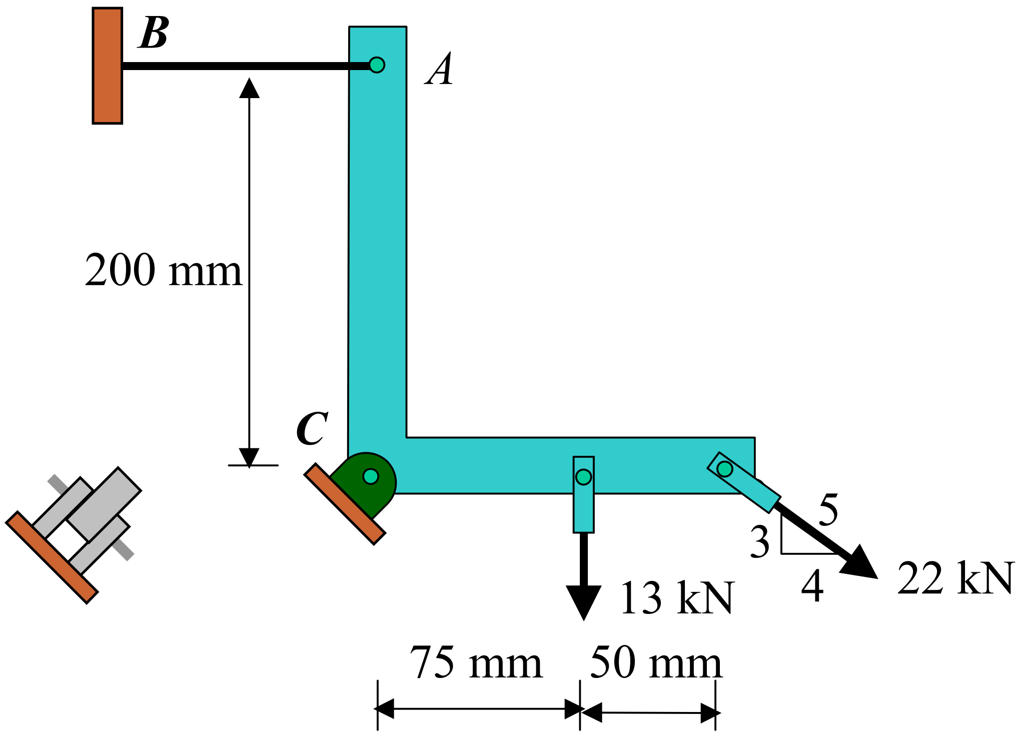
$$\sigma_E = \frac{P}{A}$$

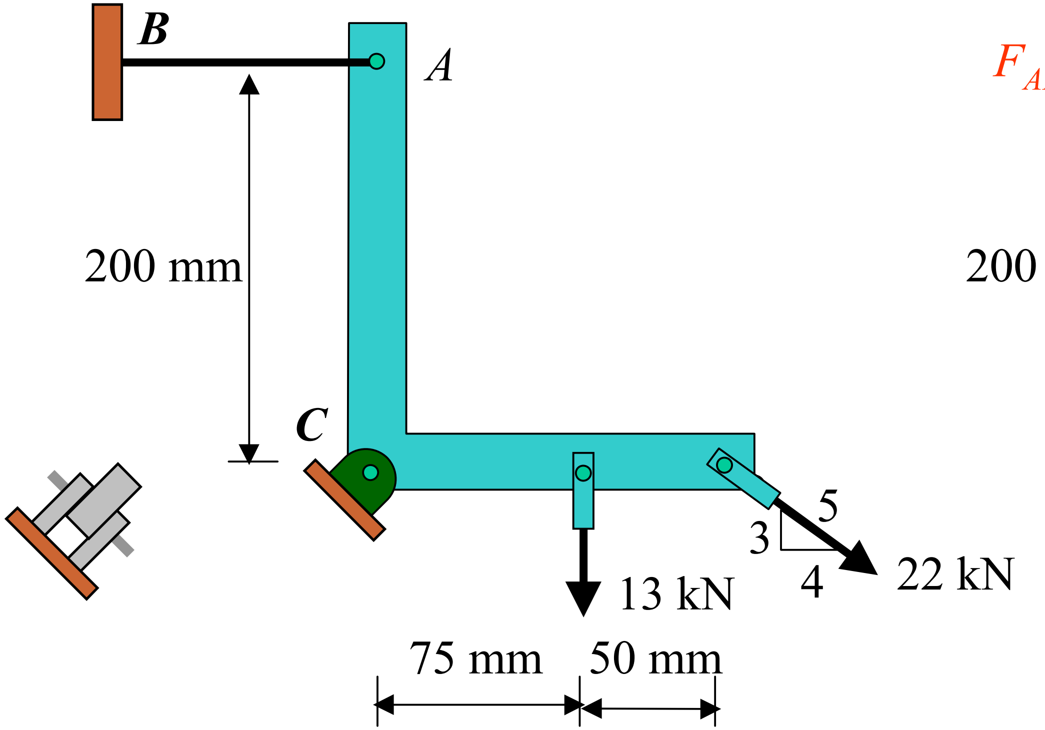
$$= \frac{25 \text{ kN}}{(0.05 - 0.025)(0.01)}$$

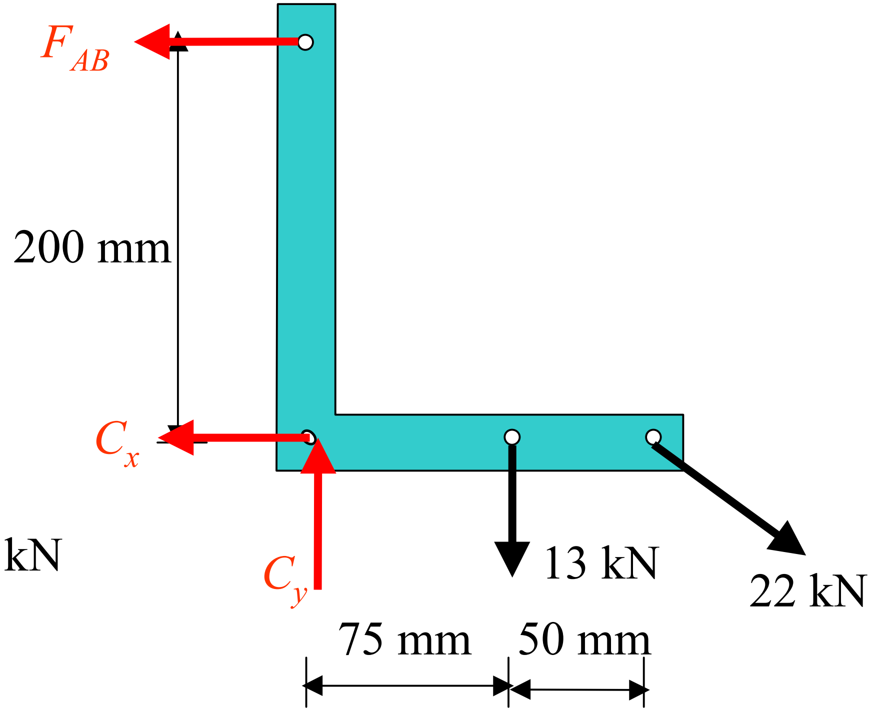
$$= 100 \text{ MPa} \Leftarrow$$

Example 12b

The control arm is subjected to the loading shown. Determine the required diameter of the steel pin at *C* if the allowable shear stress for the steel is τ_{allow} = 55 MPa. Note in the figure that the pin is subjected to double shear.







- **Internal Shear Force**

$\circlearrowright + \; \Sigma M_C = 0$: $\quad F_{AB}(0.2) - 13(0.075) - 22\sin 36.87(0.125) = 0$

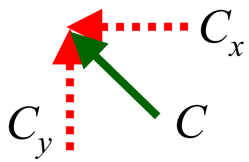
$$F_{AB} = 13.125 \text{ kN}$$

$\overset{+}{\rightarrow} \; \Sigma F_x = 0$: $\quad -13.125 - C_x + 22\cos 36.87^{o} = 0$

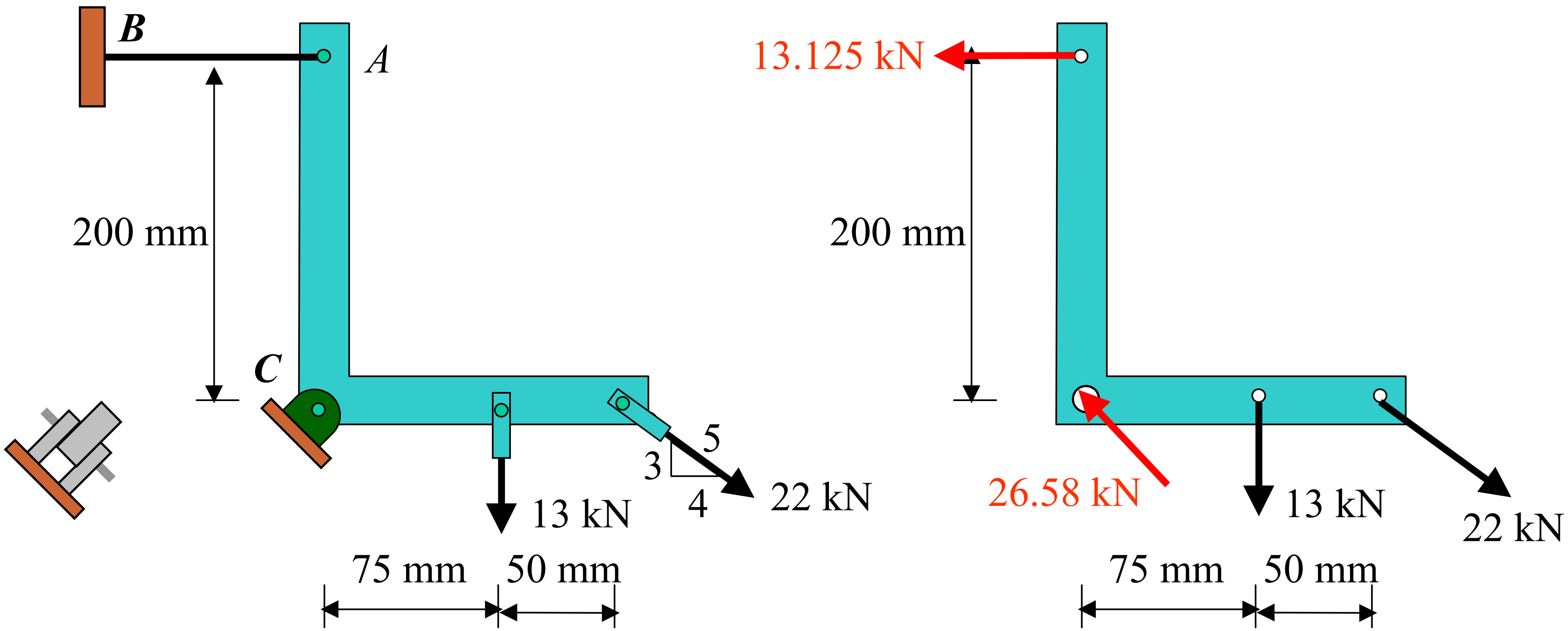
$$C_x = 4.47 \text{ kN}$$

$+\uparrow \; \Sigma F_y = 0$: $\quad C_y - 13 - 22\sin 36.87^{o} = 0$

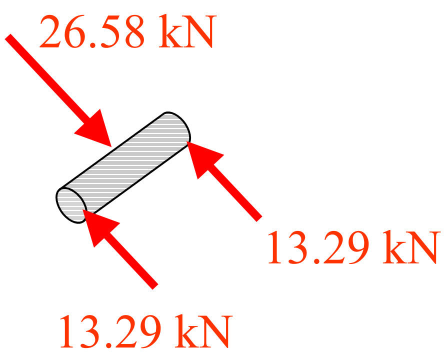
$$C_y = 26.2 \text{ kN}$$



$$C = \sqrt{(4.47)^2 + (26.2)^2} = 26.58 \text{ kN}$$



- **Required Area**



$$A = \frac{V}{\tau_{allow}} = \frac{13.29 \times 10^3}{55 \times 10^6} = 241.6 \times 10^{-6} \text{m}^2 = 242 \text{ mm}^2$$

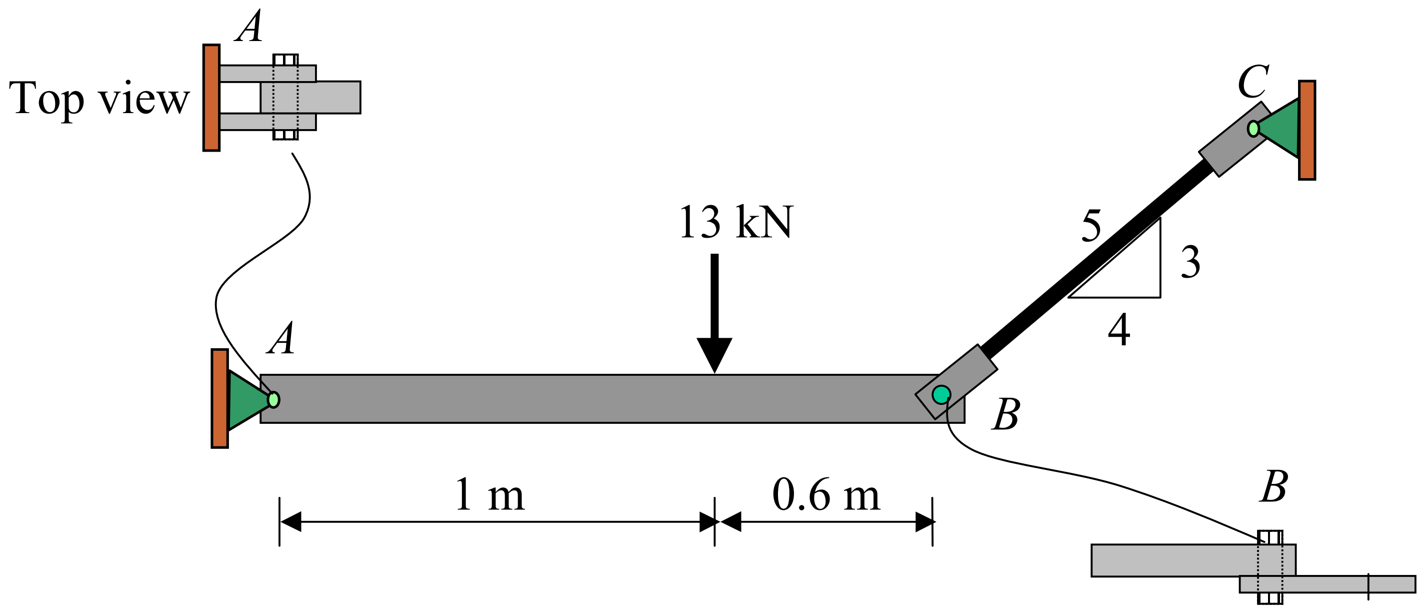
$$\pi \left(\frac{d}{2}\right)^2 = 242 \text{ mm}^2$$

$$d = 17.55 \text{ mm}$$

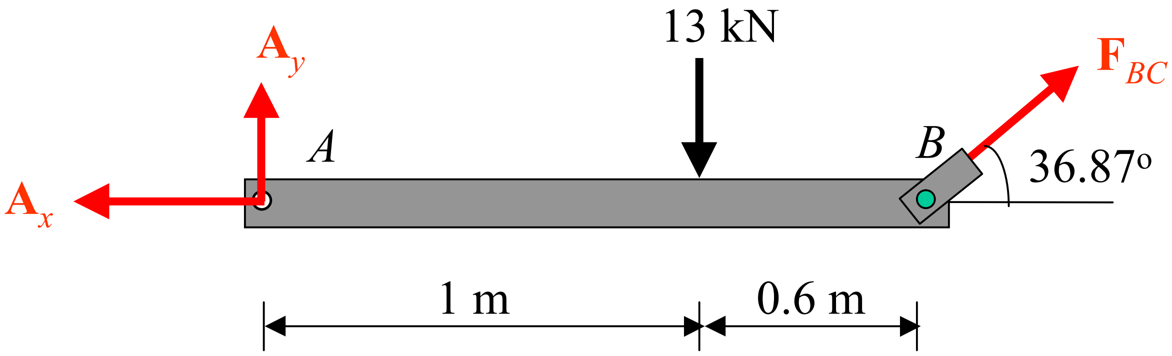
$$Use \;\; d = 20 \text{ mm}$$

Example 13a

The two members are pinned together at *B* as shown below. Top views of the pin connections at *A* and *B* are also given. If the pins have an allowable shear stress of τ_{allow} = 86 MPa , the allowable tensile stress of rod *CB* is $(\sigma_t)_{allow}$ = 112 MPa and the allowable bearing stress is $(\sigma_b)_{allow}$ = 150 MPa, determine to the smallest diameter of pins *A* and *B* ,the diameter of rod *CB* and the thickness of *AB* necessary to support the load.



- **Internal Force**



$$\circlearrowright^{+}\ \Sigma M_A = 0: \quad -13(1) + F_{BC}\sin 36.87(1.6) = 0$$

$$F_{BC} = 13.54 \text{ kN}$$

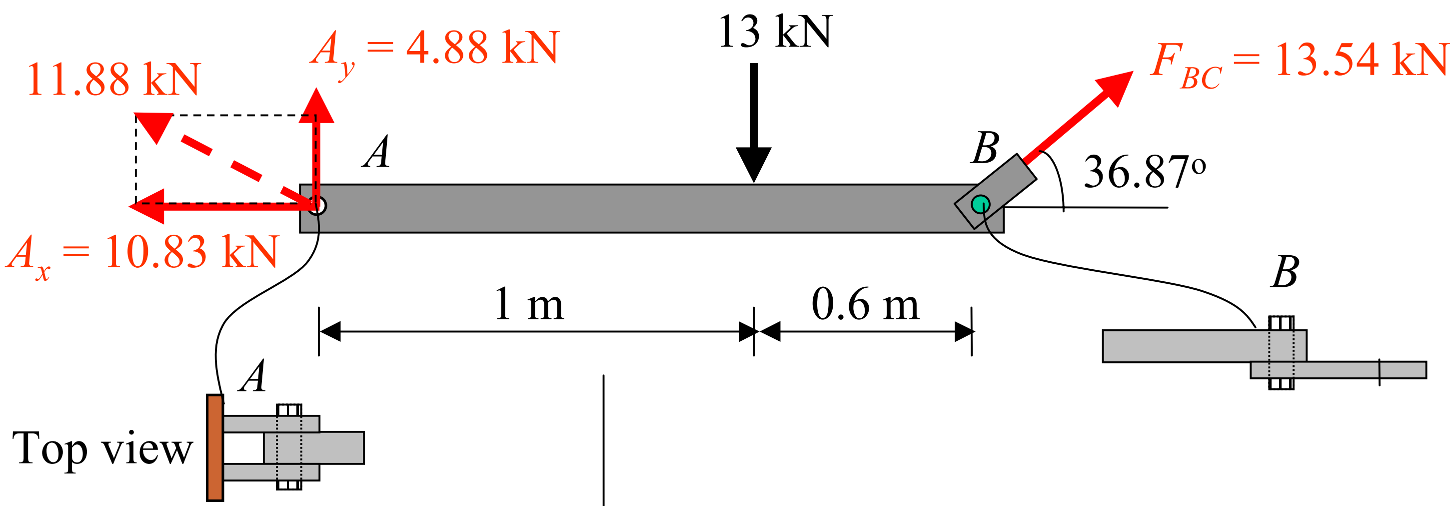
$$+\uparrow\ \Sigma F_y = 0: \quad A_y - 13 + 13.54\sin 36.87 = 0$$

$$A_y = 4.88 \text{ kN}$$

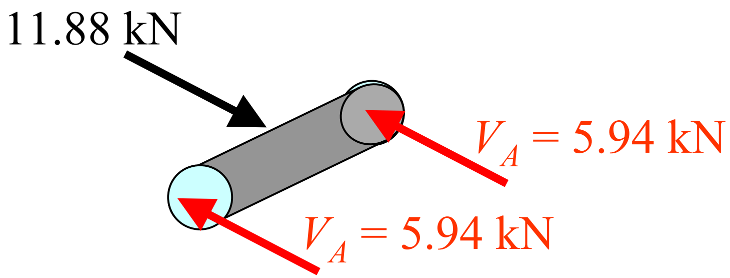
$$\overset{+}{\rightarrow}\ \Sigma F_x = 0: \quad -A_x + 13.54\cos 36.87 = 0$$

$$A_x = 10.83 \text{ kN}$$

• **Diameter of the Pins**



Pin A

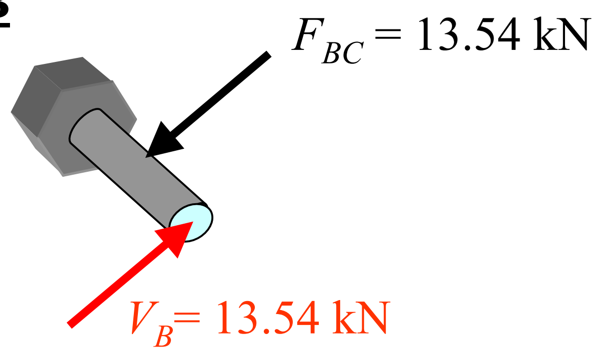


$$A_A = \frac{V_A}{\tau_{allow}} = \frac{5.94 \text{ kN}}{86 \times 10^3 \text{ kN/m}^2} = 69.07 \text{ mm}^2$$

$$\frac{\pi}{4}(d_A)^2 = 69.07 \text{ mm}^2$$

$$d_A = 9.38 \text{ mm, Use } d_A = 10 \text{ mm}$$

Pin B

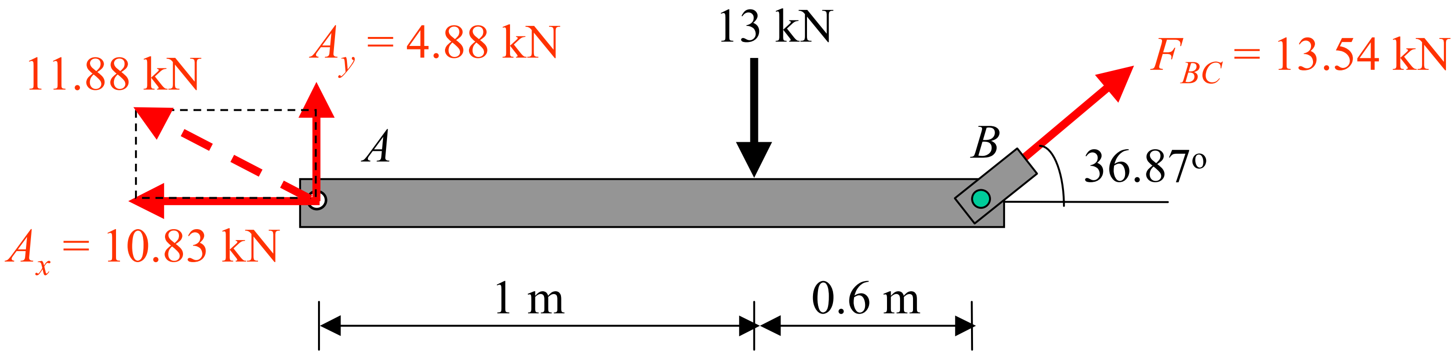


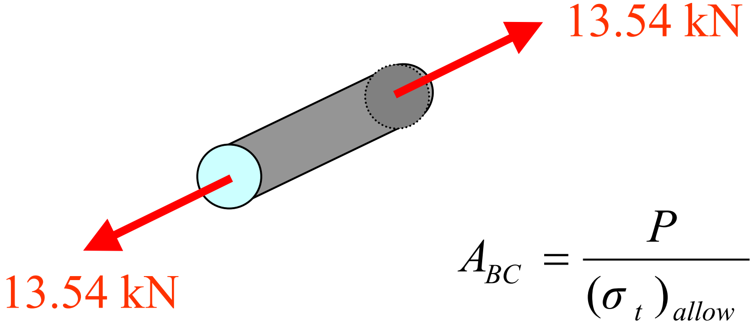
$$A_B = \frac{V_B}{\tau_{allow}} = \frac{13.54 \text{ kN}}{86 \times 10^3 \text{ kN/m}^2} = 157.4 \text{ mm}^2$$

$$\frac{\pi}{4}(d_B)^2 = 157.4 \text{ mm}^2$$

$$d_B = 14.16 \text{ mm, Use } d_B = 15 \text{ mm}$$

• **Diameter of Rod**



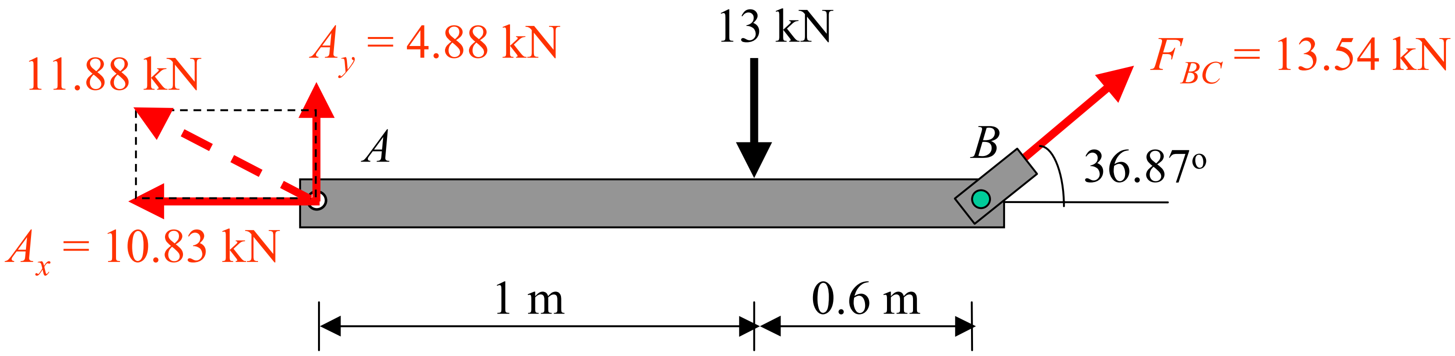


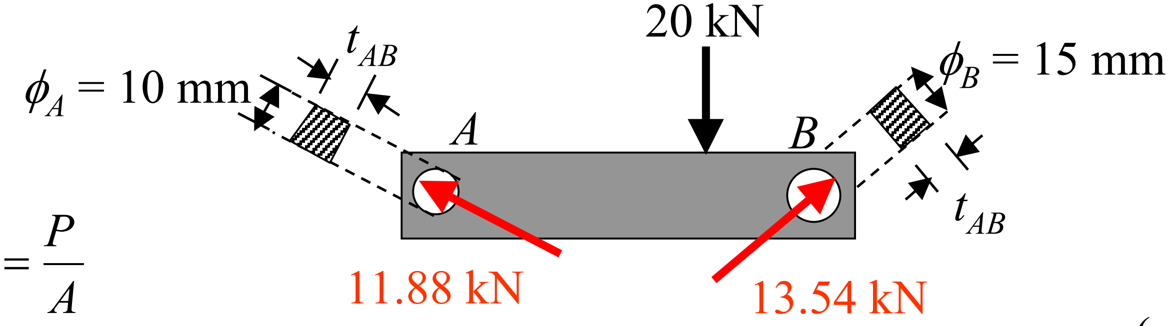
$$A_{BC} = \frac{P}{(\sigma_t)_{allow}} = \frac{13.54 \text{ kN}}{112 \times 10^3 \text{ kN/m}^2} = 120.9 \text{ mm}^2$$

$$\frac{\pi}{4}(d_{BC})^2 = 120.9 \text{ mm}^2$$

$$d_{BC} = 12.4 \text{ mm, Use } d_{BC} = 15 \text{ mm}$$

• **Thickness**

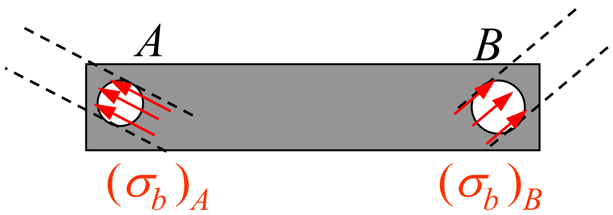




$$(\sigma_b)_{allow} = \frac{P}{A}$$

$$150 \times 10^6 = \frac{11.88 \times 10^3}{(0.010)t_{AB}}$$

$$t_{AB} = 0.00792 \text{ m}$$



$$(\sigma_b)_{allow} = \frac{P}{A}$$

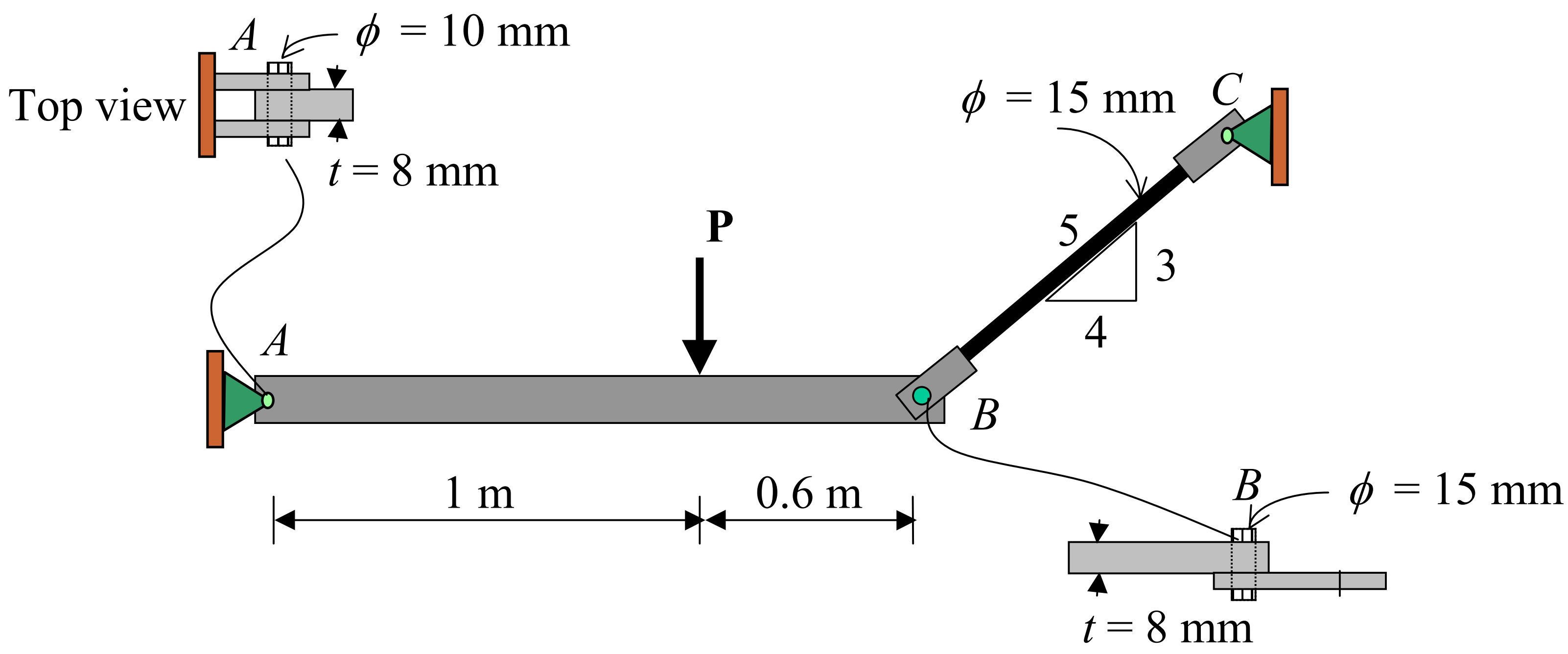
$$150 \times 10^6 = \frac{13.54 \times 10^3}{(0.015)t_{AB}}$$

$$t_{AB} = 0.00602 \text{ m}$$

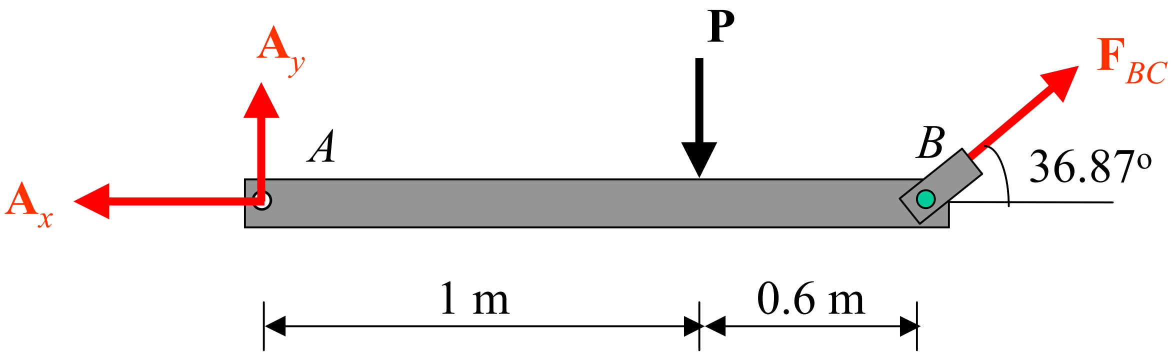
By comparison use $t_{AB} = 8$ mm

Example 13b

The two members are pinned together at *B* as shown below. Top views of the pin connections at *A* and *B* are also given. If the pins have an allowable shear stress of τ_{allow} = 86 MPa, the allowable tensile stress of rod *CB* is $(\sigma_t)_{allow}$ = 112 MPa and the allowable bearing stress is $(\sigma_b)_{allow}$ = 150 MPa, determine to the maximum load P that the beam can supported.



• **Internal Force**



$$\overset{+}{\curvearrowleft} \ \Sigma M_A = 0: \quad -P(1) + F_{BC}\sin 36.87(1.6) = 0$$

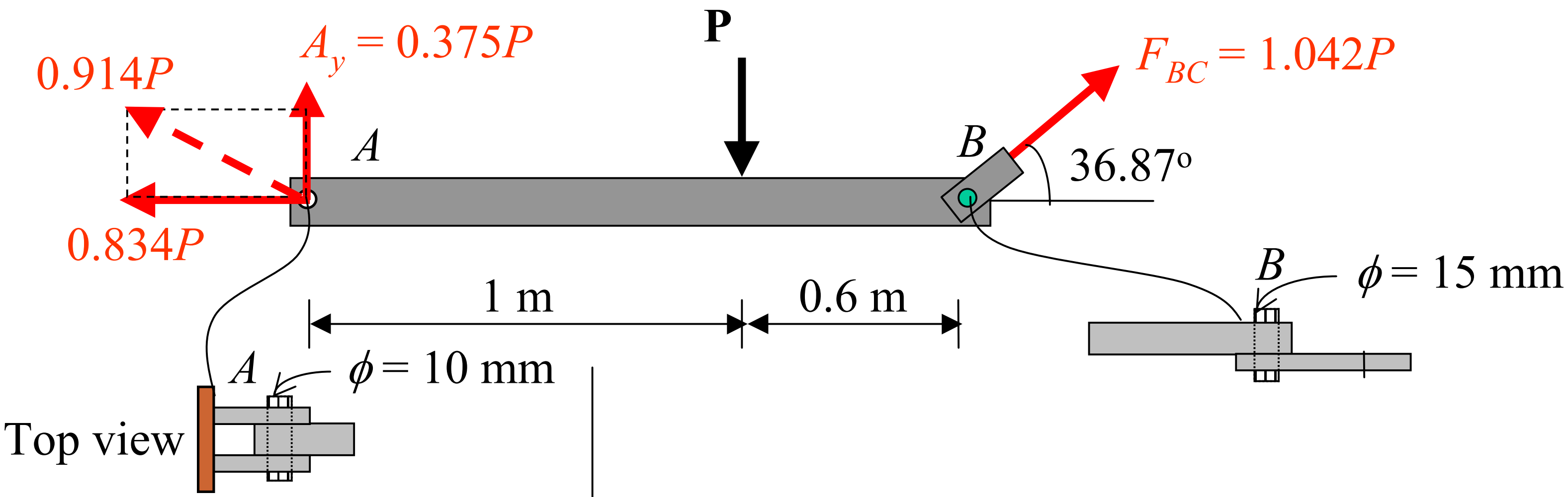
$$F_{BC} = 1.042P \text{ (T)}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - P + 1.042P\sin 36.87 = 0$$

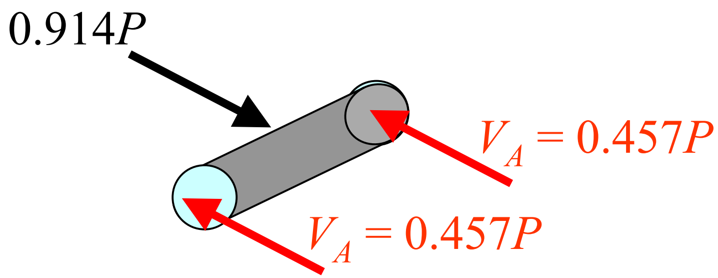
$$A_y = 0.375P$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad -A_x + 1.042P\cos 36.87 = 0$$

$$A_x = 0.834 \text{ kN}$$



Pin A (double shear)

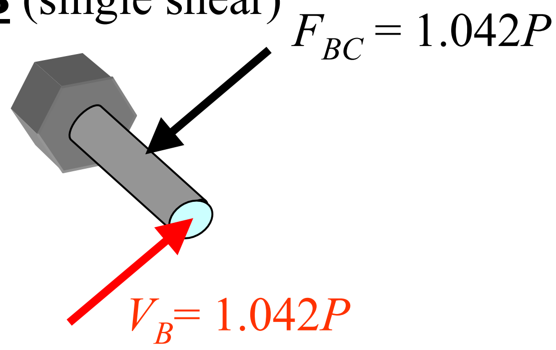


$$\tau_{allow} = \frac{V_A}{A_A}$$

$$86 \times 10^3 \text{ kPa} = \frac{0.457P}{(\pi/4)(0.01)^2}$$

$$P = 14.78 \text{ kN}$$

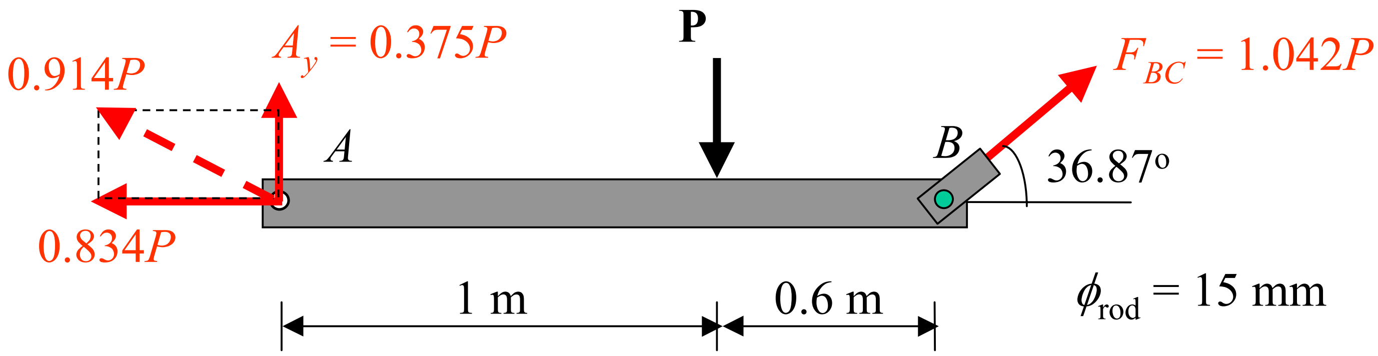
Pin B (single shear)



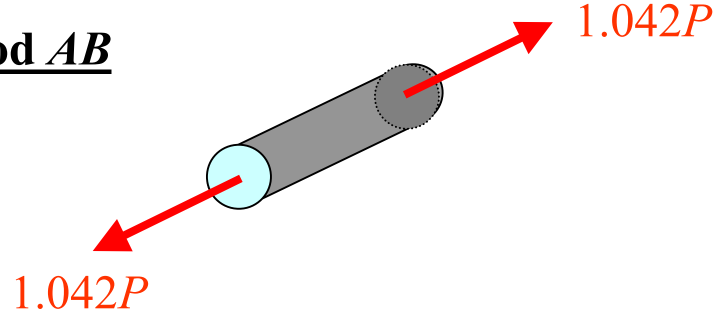
$$\tau_{allow} = \frac{V_B}{A_B}$$

$$86 \times 10^3 \text{ kPa} = \frac{1.042P}{(\pi/4)(0.015)^2}$$

$$P = 14.58 \text{ kN}$$



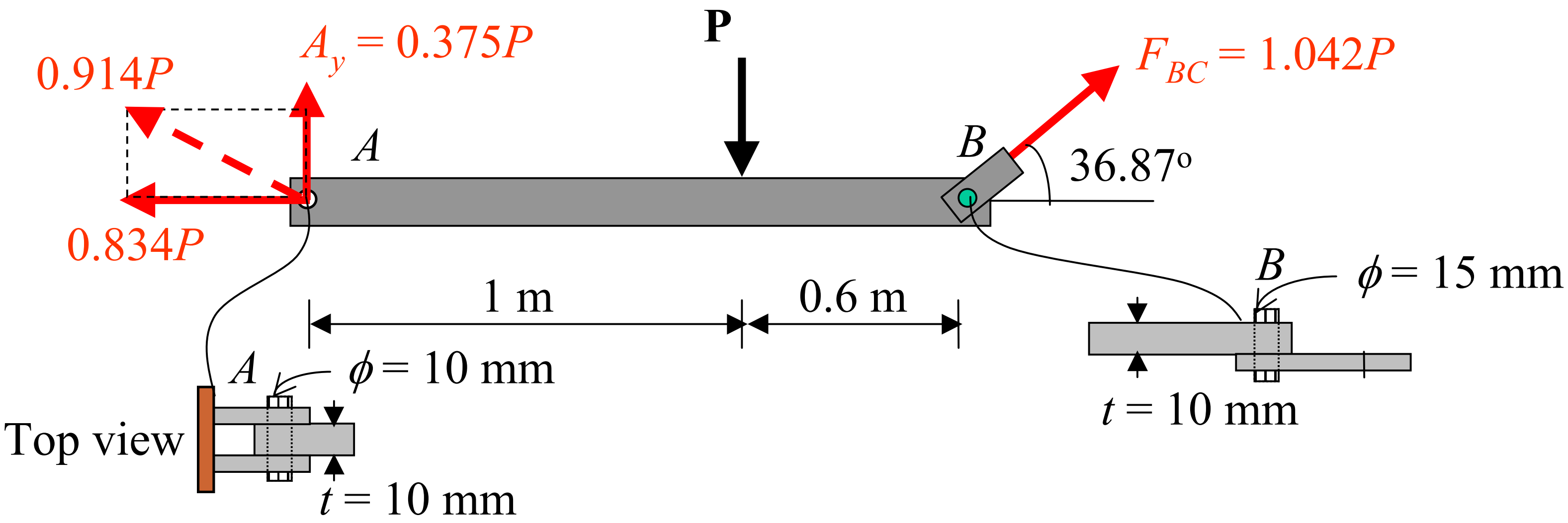
Rod *AB*

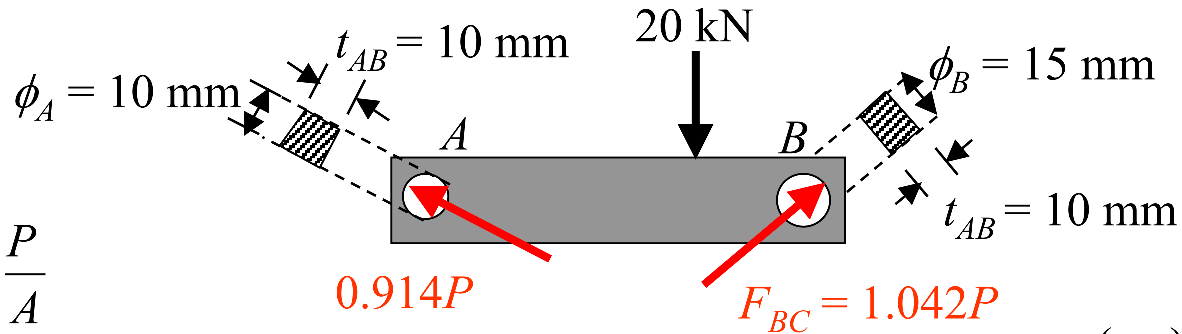


$$\sigma_{allow} = \frac{P}{A_{BC}}$$

$$112 \times 10^3 \text{ kPa} = \frac{1.042P}{(\pi/4)(0.015)^2}$$

$$P = 19 \text{ kN}$$

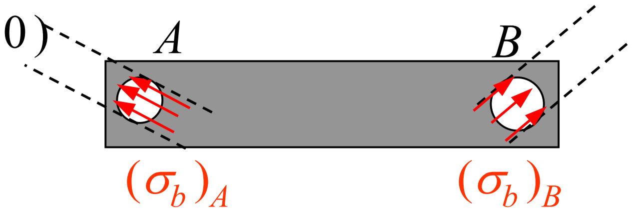




$$(\sigma_b)_{allow} = \frac{P}{A}$$

$$150 \times 10^6 = \frac{0.914P}{(0.010)(0.010)}$$

$$P = 16.41 \text{ kN}$$



$$(\sigma_b)_{allow} = \frac{P}{A}$$

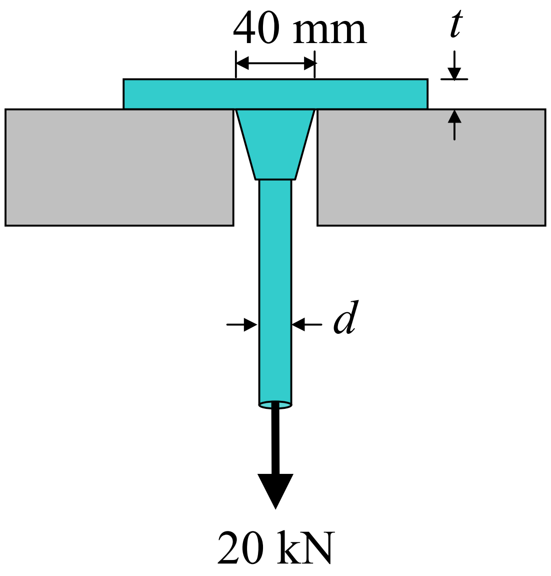
$$150 \times 10^6 = \frac{1.042P}{(0.015)(0.01)}$$

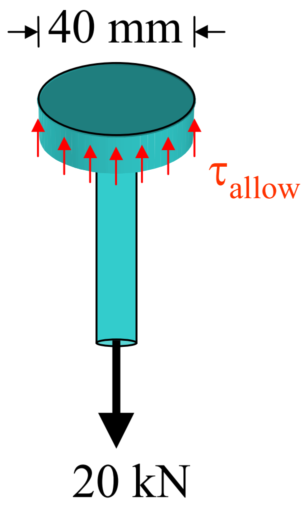
$$P = 21.60 \text{ kN}$$

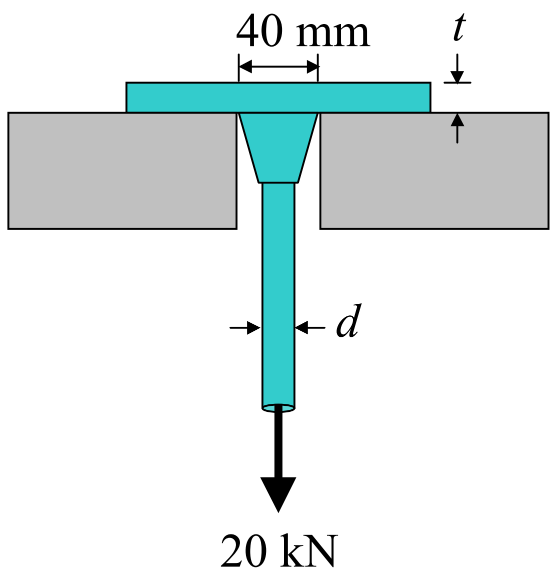
By comparison all $P = 14.58$ kN $\Leftarrow$

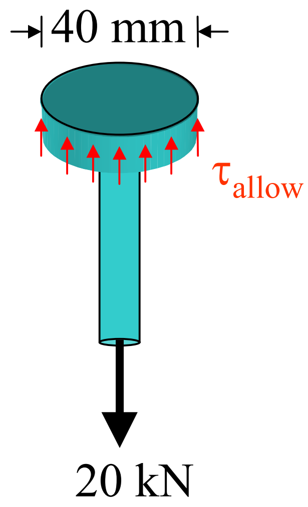
Example 14

The suspender rod is supported at its end by a fixed-connected circular disk as shown. If the rod passes through a 40-mm diameter hole, determine the minimum required diameter of the rod and the minimum thickness of the disk needed to support the 20 kN load. The allowable normal stress for the rod is σ_{allow} = 60 MPa, and the allowable shear stress for the disk is τ_{allow} = 35 MPa.









- **Diameter of Rod**

$$A = \frac{P}{\sigma_{allow}} = \frac{20\ \text{kN}}{60 \times 10^3\ \text{kN/m}^2}$$

$$A = \frac{\pi}{4} d^2 = \frac{20\ \text{kN}}{60 \times 10^3\ \text{kN/m}^2}$$

$$d = 0.0206\ \text{m} = 20.6\ \text{mm}$$

- **Thickness of Disk**

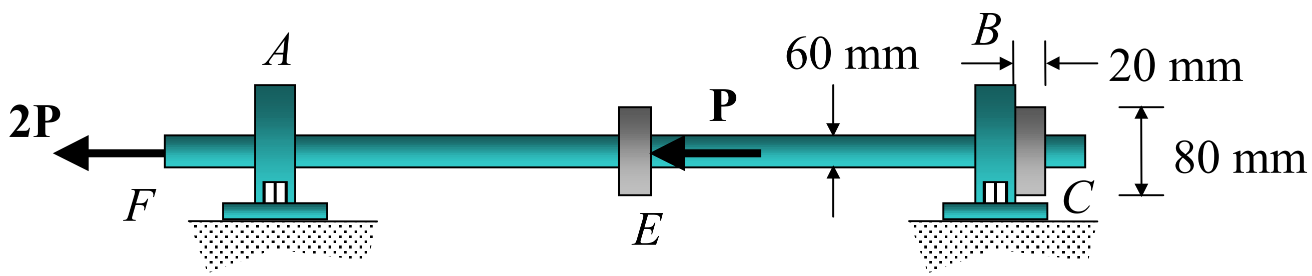
$$A = \frac{V}{\tau_{allow}} = \frac{20\ \text{kN}}{35 \times 10^3\ \text{kN/m}^2}$$

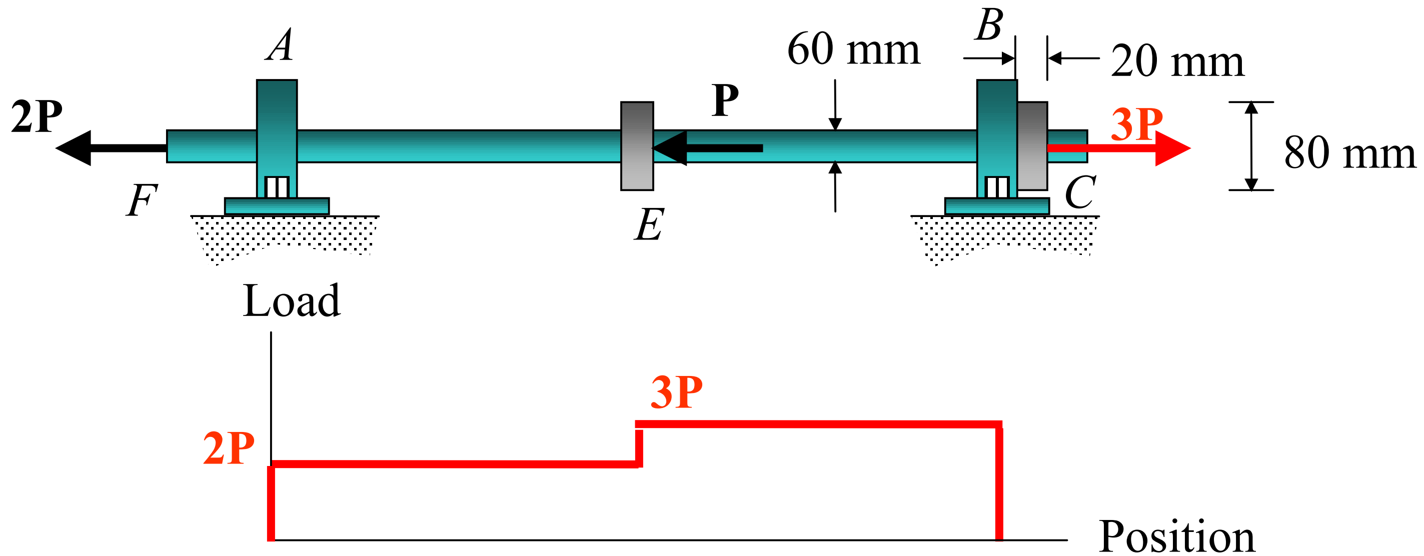
$$2\pi (0.02\ \text{m})t = \frac{20\ \text{kN}}{35 \times 10^3\ \text{kN/m}^2}$$

$$t = 4.55\text{x}10^{-3}\ \text{m} = 4.55\ \text{mm}$$

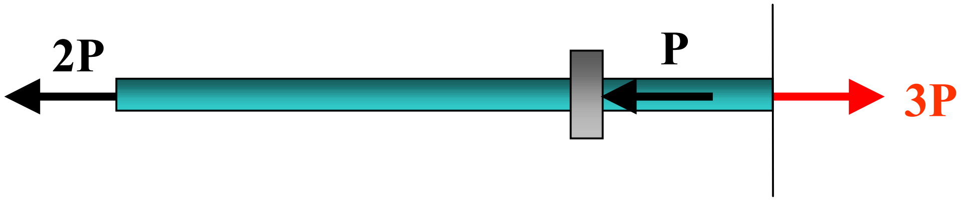
Example 15

An axial load on the shaft shown is resisted by the collar at *C*, which is attached to the shaft and located on the right side of the bearing at *B*. Determine the largest value of *P* for the two axial forces at *E* and *F* so that the stress in the collar does not exceed an allowable bearing stress at *C* of $(\sigma_b)_{allow}$ = 75 MPa and allowable shearing stress of the adhesive at *C* of τ_{allow} = 100 MPa , and the average normal stress in the shaft does not exceed an allowable tensile stress of $(\sigma_t)_{allow}$ = 55 MPa.





- **Axial Stress**

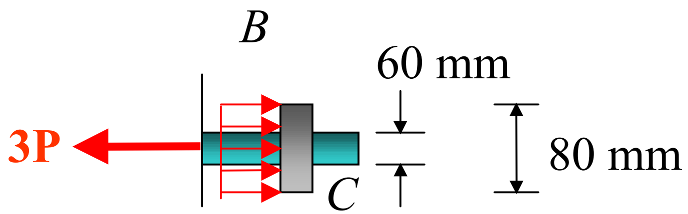


$$(\sigma_t)_{allow} = \frac{P}{A}$$

$$55 \times 10^3 \text{ kN/m}^2 = \frac{3P}{\pi(0.03 \text{ m})^2}$$

$$P_1 = 51.8 \text{ kN}$$

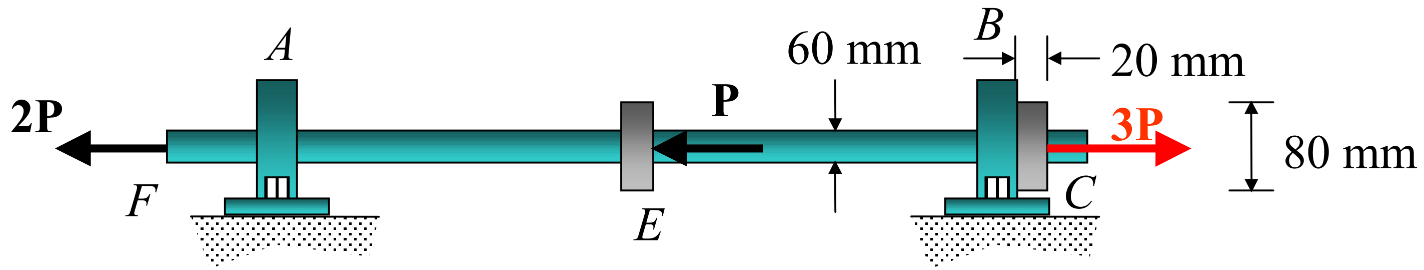
- **Bearing Stress**

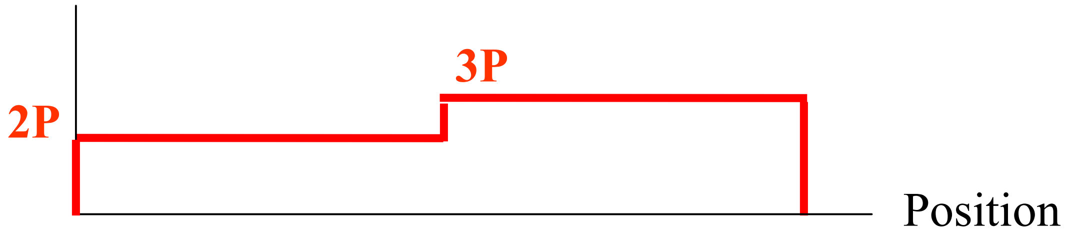


$$(\sigma_b)_{allow} = \frac{P}{A}$$

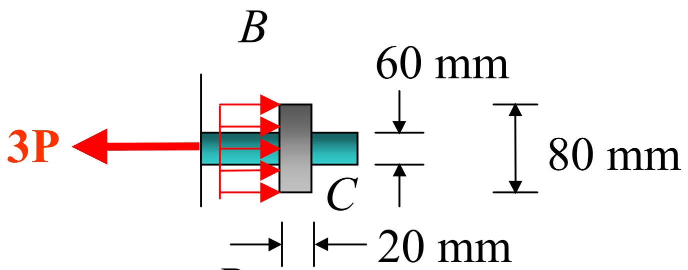
$$75 \times 10^3 \text{kN/m}^2 = \frac{3P}{[\pi(0.04 \text{ m})^2 - \pi(0.03 \text{ m})^2]}$$

$$P_2 = 55 \text{ kN}$$





- **Shearing Stress**



$$\tau_{allow} = \frac{P}{A_{shear}}$$

$$100 \times 10^3 \text{ kN/m}^2 = \frac{3P}{[2\pi(0.04 \text{ m})(.020)]}$$

$$P_3 = 55 \text{ kN}$$

- **Axial Stress**

$P_1 = 51.8$ kN

- **Bearing Stress**

$P_2 = 55$ kN

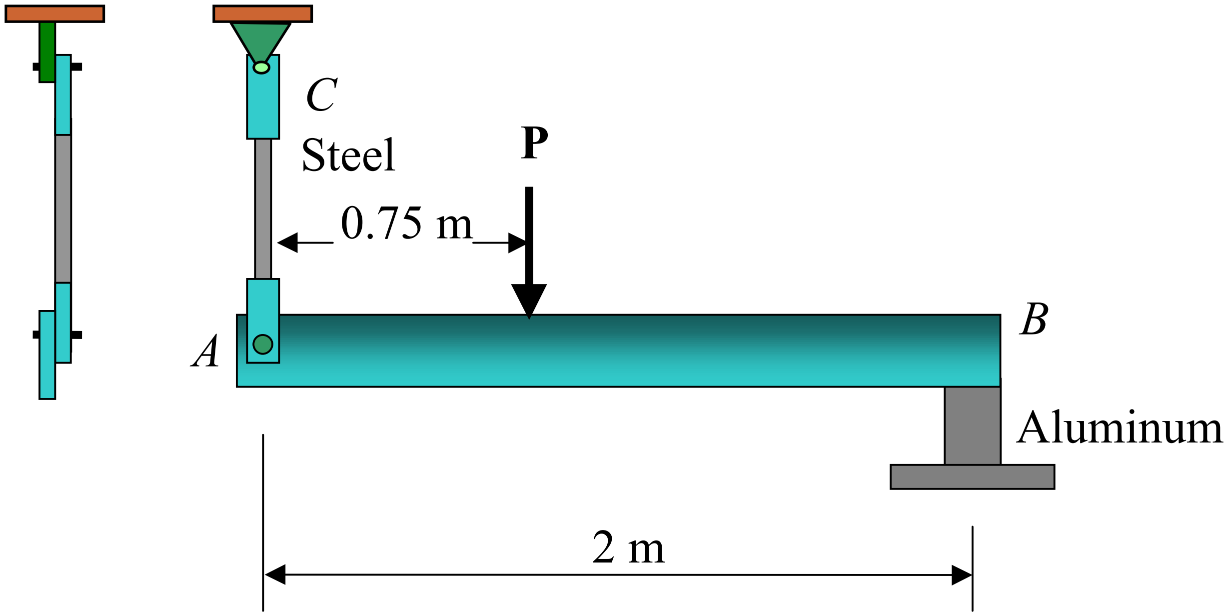
- **Shearing Stress**

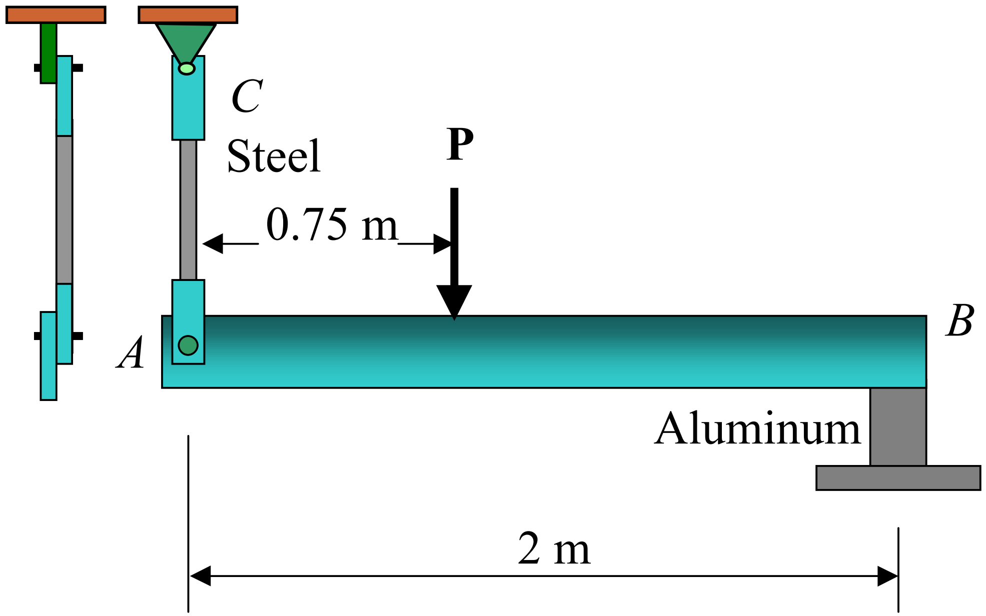
$P_3 = 55$ kN

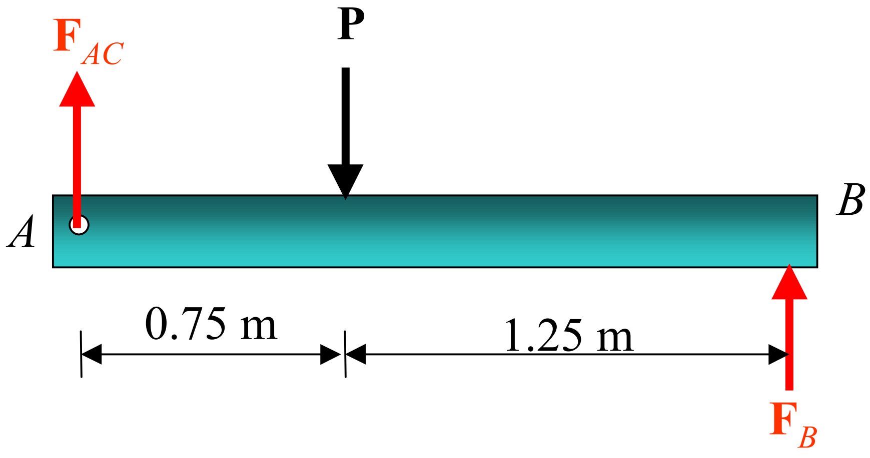
The largest load that can be applied to the shaft is $P = 51.8$ kN.

Example 16

The rigid bar *AB* shown supported by a steel rod *AC* having a diameter of 20 mm and an aluminum block having a cross-sectional area of 1800 mm^2 . The18-mm-diameter pins at *A* and *C* are subjected to single shear. If the failure stress for the steel and aluminum is $(\sigma_{st})_{\text{fail}}$ = 680 Mpa and $(\sigma_{al})_{\text{fail}}$ = 70 MPa, respectively, and the failure shear stress for each pin is τ_{fail} = 900 MPa, determine the largest load P that can be applied to the bar. Apply a factor of safety of F.S = 2.0.







$\overset{+}{\circlearrowright}\ \Sigma M_B = 0$: $P(1.25\text{ m}) - F_{AC}(2\text{ m}) = 0$ ---------> $F_{AC} = 0.625P$

$\overset{+}{\circlearrowright}\ \Sigma M_A = 0$: $P(0.75\text{ m}) - F_B(2\text{ m}) = 0$ ----------> $F_B = 0.375P$

- **Rod *AC***

$$(\sigma_{st})_{allow} = \frac{(\sigma_{st})_{fail}}{F.S} = \frac{F_{AC}}{A_{AC}}$$

$$\frac{680 \times 10^3 \text{kPa}}{2} = \frac{0.625\,P}{\pi(0.01\ \text{m})^2}$$

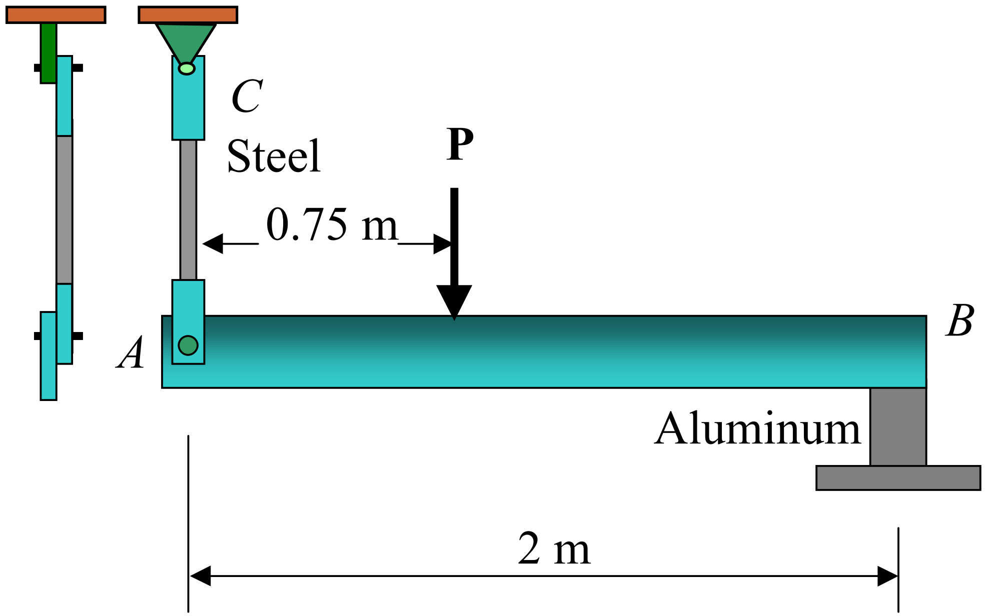
$$P_1 = 171\text{ kN}$$

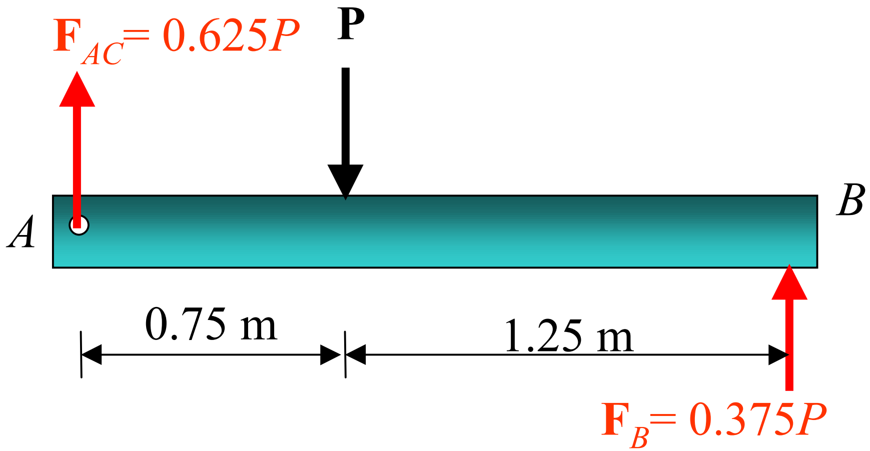
- **Block *B***

$$(\sigma_{al})_{allow} = \frac{(\sigma_{al})_{fail}}{F.S} = \frac{F_B}{A_B}$$

$$\frac{70 \times 10^3 \text{kPa}}{2} = \frac{0.375\,P}{1800 \times 10^{-6}\text{m}^2}$$

$$P_2 = 168\text{ kN}$$





- **Pin *A* or *C***

$$\tau_{allow} = \frac{\tau_{fail}}{F.S} = \frac{F_{AC}}{A_{pin}}$$

$$\frac{900 \times 10^3 \text{kPa}}{2} = \frac{0.625\,P}{\pi(0.009\ \text{m})^2}$$

$$P_3 = 183 \text{ kN}$$

Summary

- **Rod *AC*** $P_1 = 171$ kN
- **Block *B*** $P_2 = 168$ kN
- **Pin *A* or *C*** $P_3 = 183$ kN

Largest load ***P*** = 168 kN