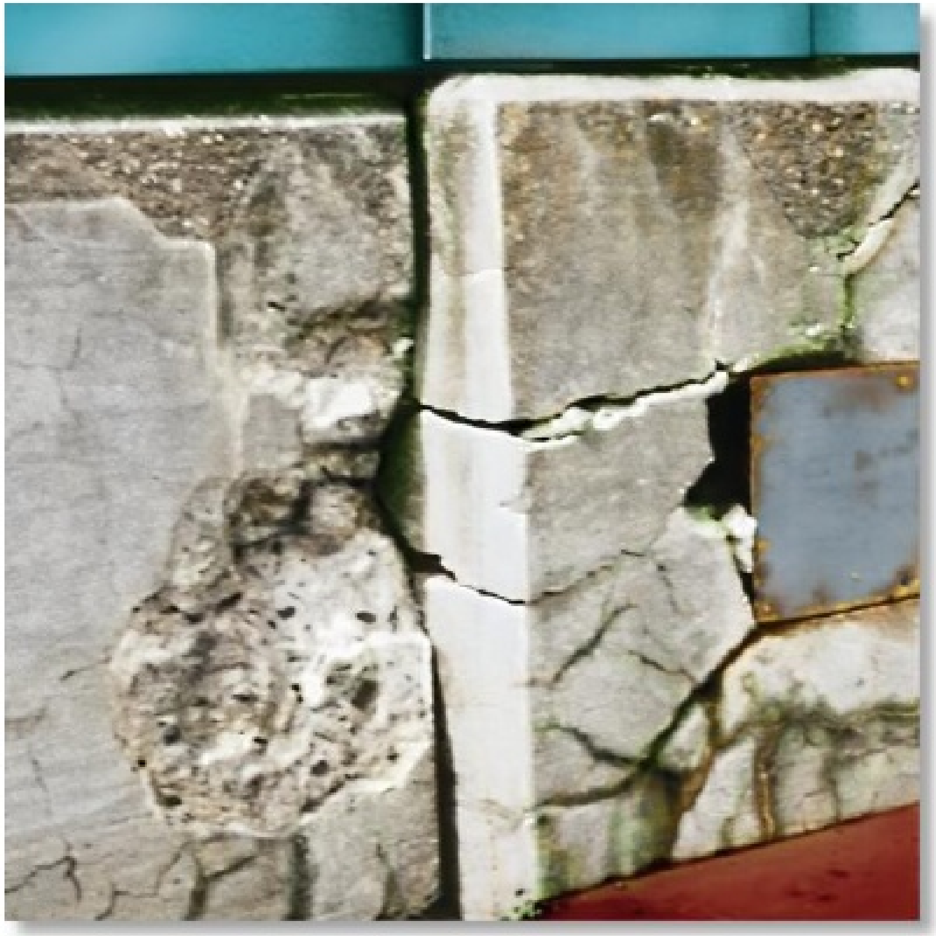
SOLID MECHANICS

2 Chapter

Strain

CHAPTER OBJECTIVES

- Define concept of normal strain
- Define concept of shear strain
- Determine normal and shear strain in engineering applications



2. Strain

1. Deformation
2. Strain

2.1 DEFORMATION

Deformation

- Occurs when a force is applied to a body
- Can be highly visible or practically unnoticeable
- Can also occur when temperature of a body is changed
- Is not uniform throughout a body's volume, thus change in geometry of any line segment within body may vary along its length

2.1 DEFORMATION

- When a force is applied to a body, it will change the body's shape and size.
- These changes are ***deformation***.

Note the positions of 3 line segments before and after where the material is subjected to tension.

2. Strain

Normal strain

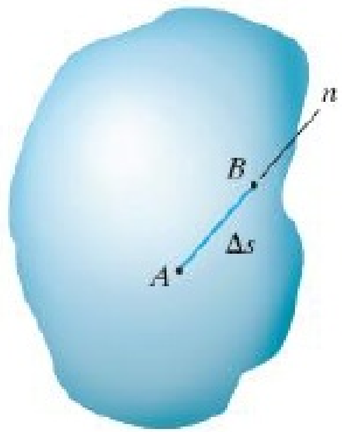
- Defined as the elongation or contraction of a line segment per unit of length
- Consider line *AB* in figure below
- After deformation, Δs changes to $\Delta s'$

2.2 STRAIN

Normal strain

- Average **normal strain** is defined as ε_{avg} (epsilon)

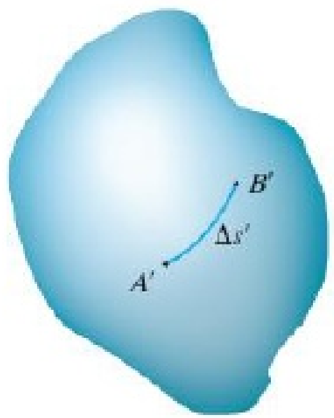
$$\varepsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s}$$



Undeformed body

- As: $\Delta s \rightarrow 0$, $\Delta s' \rightarrow 0$

$$\varepsilon = \lim_{B \rightarrow A \text{ along } n} \frac{\Delta s' - \Delta s}{\Delta s}$$



Deformed body

2.2 STRAIN

Normal strain

- If normal strain ε is known, use the equation to obtain approx. final length of a *short* line segment in direction of n after deformation.

$$\Delta s' \approx (1 + \varepsilon)\,\Delta s$$

ε is Positif (+) ➔ **line elongates**

ε is Negatif (-) ➔ **line contracts**

2.2 STRAIN

Units

- Normal strain is a *dimensionless quantity*, as it's a ratio of two lengths

- But common practice to state it in terms of meters/meter (m/m)

- ε is small for most engineering applications, so is normally expressed as micrometers per meter ($\mu m/m$) where $1\ \mu m = 10^{-6}$

- Also expressed as a percentage,
  e.g., $0.001\ m/m = 0.1\ \%$

2.2 STRAIN

Shear strain

- Defined as the *change in angle* that occurs between two line segments that were originally *perpendicular* to one another
- This angle is denoted by γ (gamma) and measured in radians (rad).

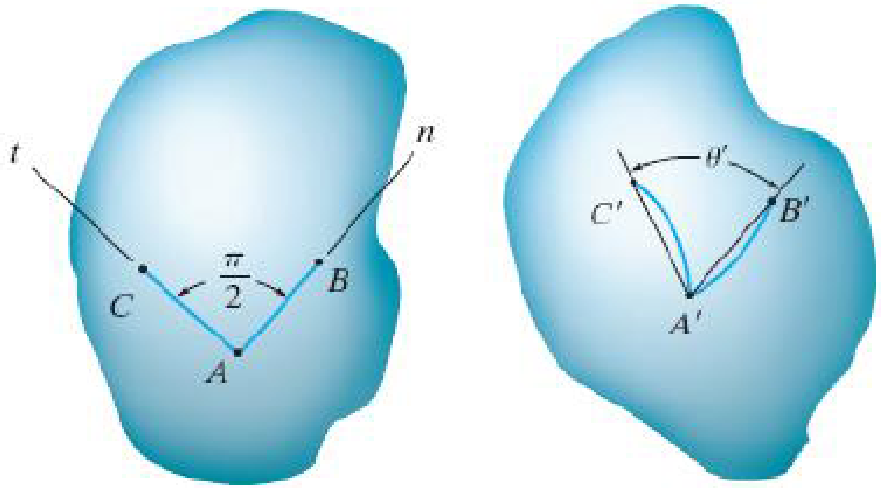
2.2 STRAIN

Shear strain

- Hence, shear strain at point A associated with $\boldsymbol{n}$ and $\boldsymbol{t}$ axes is

$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \text{ along } n \\ C \to A \text{ along } t}} \theta'$$

Θ < 90 ➔ positive (+) shear strain
Θ > 90 ➔ negative (-) shear strair



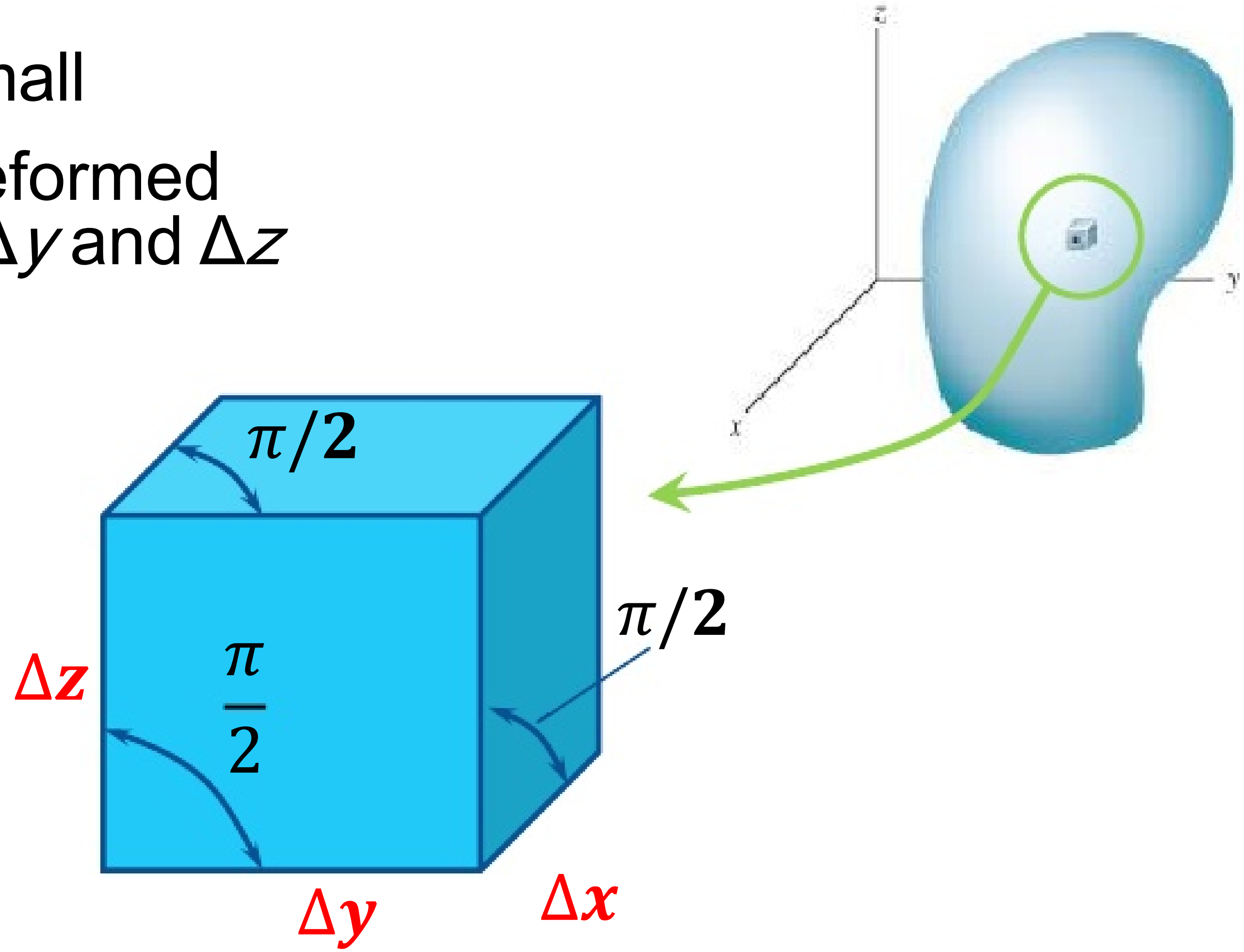
Undeformed body Deformed body

2.2 STRAIN

Cartesian strain components

- Using above definitions of normal and shear strain, we show how they describe the deformation of the body

- Divide body into small elements with undeformed dimensions of Δx, Δy and Δz

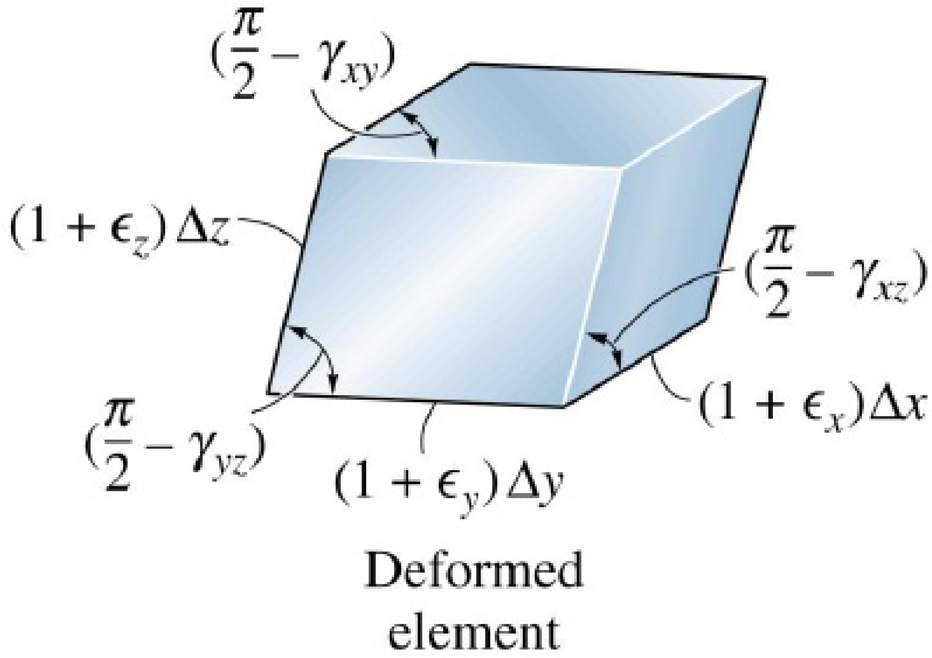


Undeformed Element

2.2 STRAIN

Cartesian strain components

- Since element is very small, deformed shape of element is a parallelepiped



- Approx. lengths of sides of parallelepiped are

$(1 + \varepsilon_x)\ \Delta x$	$(1 + \varepsilon_y)\ \Delta y$	$(1 + \varepsilon_z)\ \Delta z$

2.2 STRAIN

Cartesian strain components

- Approx. angles between the sides are

$$\frac{\pi}{2} - \gamma_{xy} \qquad \frac{\pi}{2} - \gamma_{yz} \qquad \frac{\pi}{2} - \gamma_{xz}$$

- Normal strains cause a change in its *volume*
- Shear strains cause a change in its *shape*

- To summarize, state of strain at a point requires specifying: 3 normal strains : ε_x , ε_y , ε_z

and

3 shear strains of : γ^{xy} , γ^{yz} , γ^{xz}

2.2 STRAIN

Small strain analysis

- Most engineering design involves applications for which only *small deformations* are allowed

- We'll assume that deformations that take place within a body are almost infinitesimal, so *normal strains* occurring within material are *very small* compared to 1, i.e., $\varepsilon << 1$.

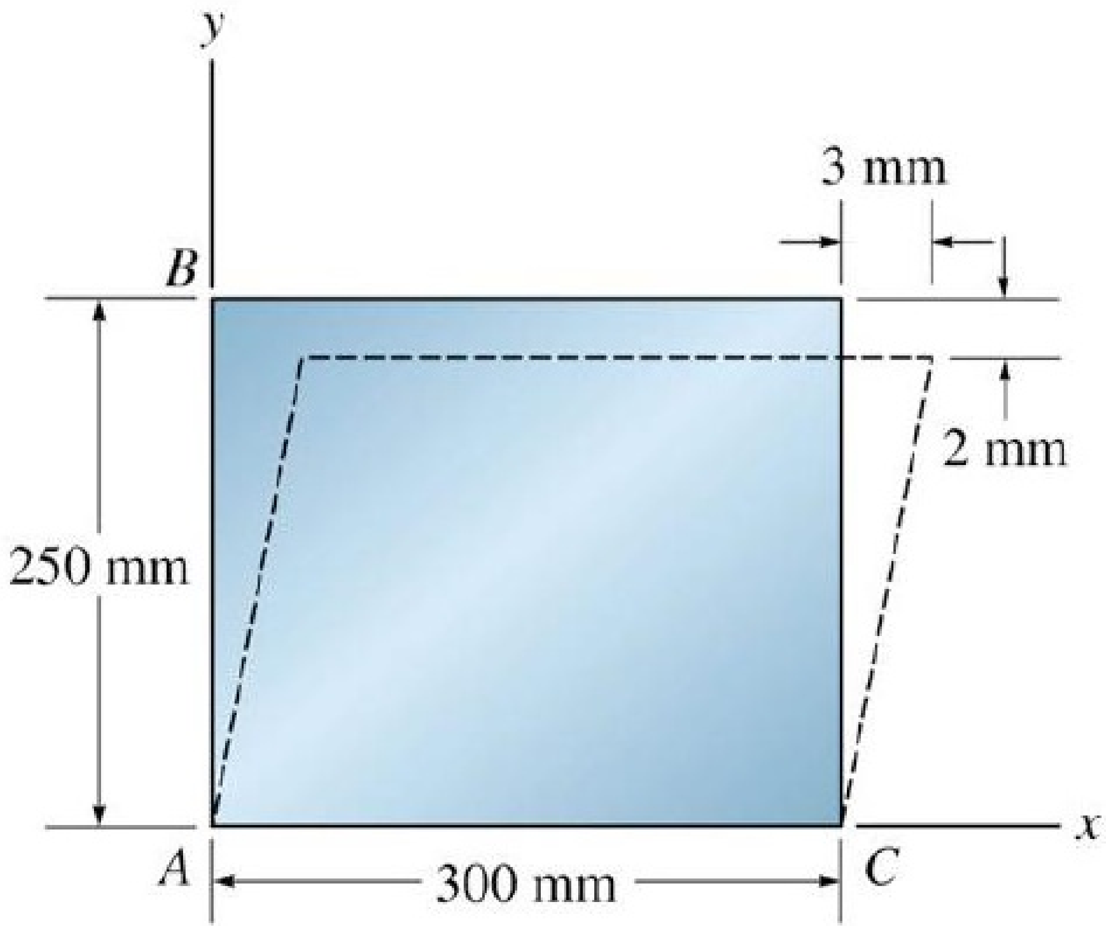
EXAMPLE 2.3

Plate is deformed as shown in figure. In this deformed shape, horizontal lines on the on plate remain horizontal and do not change their length.

Determine

(a) average normal strain along side *AB*,

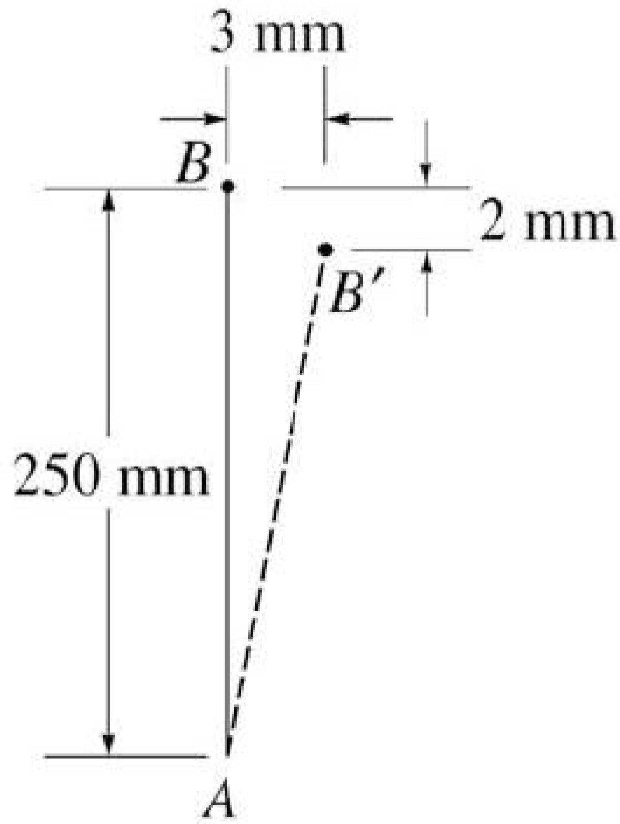
(b) average shear strain in the plate relative to *x* and *y* axes



EXAMPLE 2.3 (SOLN)

(a) Line *AB*, coincident with *y* axis, becomes line *AB'* after deformation. Length of line *AB'* is

$$AB' = \sqrt{(250-2)^2 + (3)^2} = 248.018 \text{ mm}$$



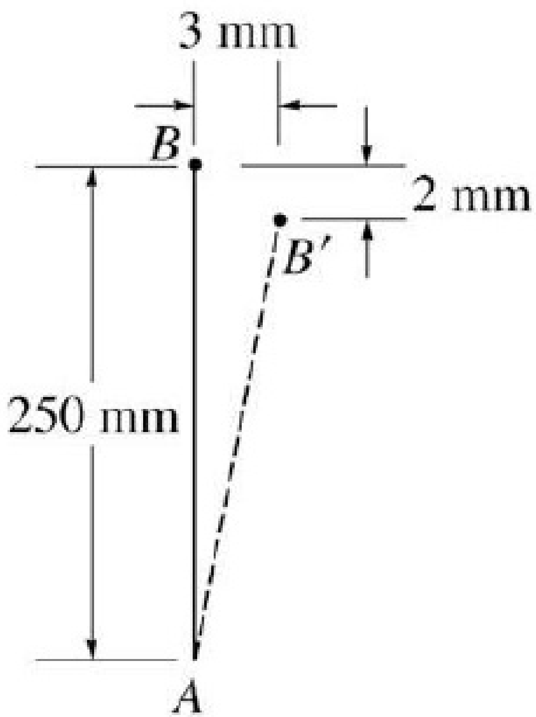
EXAMPLE 2.3 (SOLN)

(a) Therefore, average normal strain for *AB* is,

$$(\varepsilon_{AB})_{avg} = \frac{AB' - AB}{AB} = \frac{248.018\text{ mm} - 250\text{ mm}}{250\text{ mm}}$$

$$= -7.93(10^{-3})\text{ mm/mm}$$

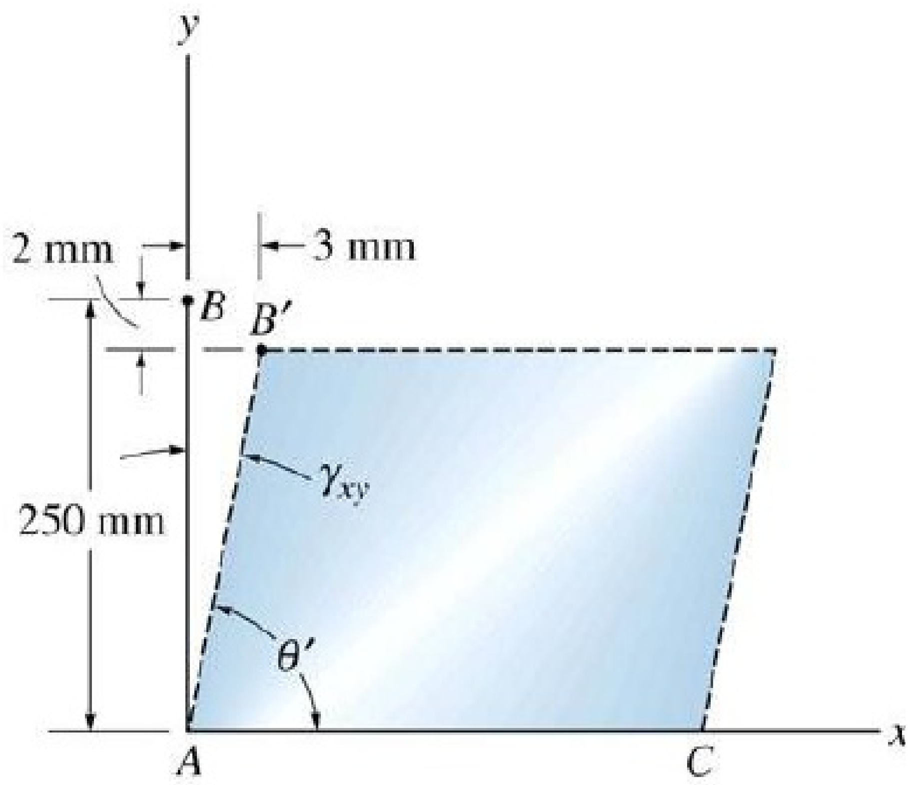
Negative sign means strain causes a contraction of AB.



EXAMPLE 2.3 (SOLN)

(b) Due to displacement of *B* to *B'*, angle BAC referenced from *x*, *y* axes changes to *θ'*.

Since $\gamma_{xy} = \pi/2 - \theta'$, thus



$$\gamma_{xy} = tan^{-1}\left(\frac{3mm}{250mm - 2mm}\right)$$

$$\gamma_{xy} = 0.0121\, rad$$

CHAPTER REVIEW

- Loads cause bodies to deform, thus points in the body will undergo *displacements or changes in position*

- *Normal strain* is a measure of elongation or contraction of small line segment in the body

- *Shear strain* is a measure of the change in angle that occurs between two small line segments that are originally perpendicular to each other

CHAPTER REVIEW

- State of strain at a point is described by six strain components:
  - a) Three normal strains: ε_x , ε_y , ε_z
  - b) Three shear strains: γ_{xy} , γ_{xz} , γ_{yz}
  - c) These components depend upon the orientation of the line segments and their location in the body
- Strain is a geometrical quantity measured by *experimental techniques*. Stress in body is then determined from material property relations

CHAPTER REVIEW

Most engineering materials undergo small deformations, so normal strain ε << 1. This assumption of "small strain analysis" allows us to simplify calculations for normal strain, since first-order approximations can be made about their size