10. Types of Current (Voltage) and their Characteristics

10.1 Types of Current (Voltage)

The discussions up to now, have dealt exclusively with electrical processes based on the flow of a **direct current** (DC) A property of DC is the **constancy with respect to time of the magnitude of the current**. This causes a **continual flow of current in one direction** in an electrical circuit. The magnitude and direction of the DC current is caused by a constant **DC voltage**. At various instants of time (t_1, t_2, etc.), stable conditions can be measured at the terminals of a DC voltage source, in the circuit and at the consumer[1]. Fig. 10.1.1 shows the voltage and flow of current over a period of several seconds.

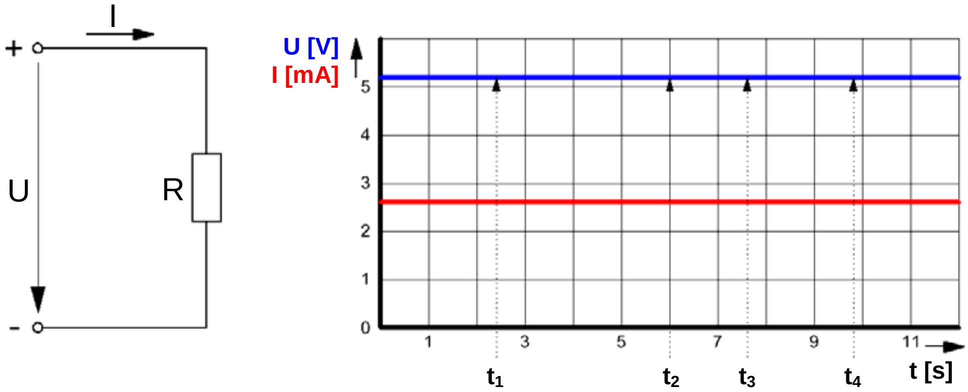
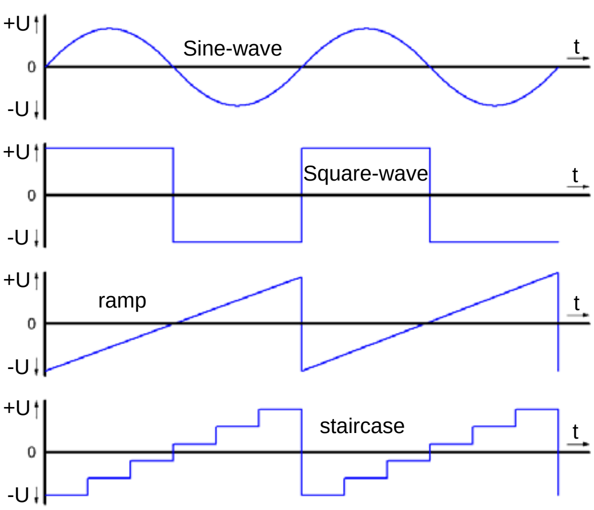


Fig. 10.1.1: Time characteristic of DC voltage and current

The term **alternating current** is used when the current in a circuit periodically changes in magnitude and direction. This alternating current (AC) is caused by an alternating voltage applied to the circuit. Fig. 10.1.2 shows various forms of AC voltage that have been displayed on an oscilloscope, used in circuits depending on the required effect or function of the circuit. The voltage curves shown are known as '**sine-wave**' (or sinusoidal), '**square-wave**', '**ramp**' (or sawtooth) and '**staircase**' voltages. For all voltages above the zero axis (+U) a varying current flows according to the magnitude (or '**amplitude**') of the voltage, in the same direction. When the voltage changes to the area below the zero axis (-U), the current flows in the opposite direction.

Fig. 10.1.2: Examples of alternating voltages



[1] The slow excursions of electrical properties caused by unwanted physical influences such as the temperature variations in a resistor, are ignored.

10.2 Characteristics of Sine-wave Voltages (and Current)

10.2.1 Derivation of the Characteristics

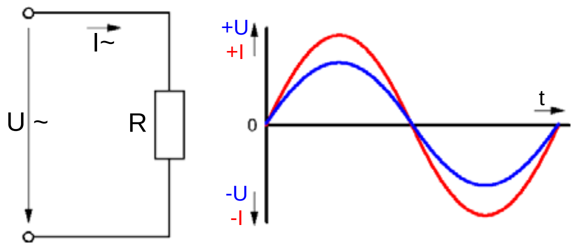
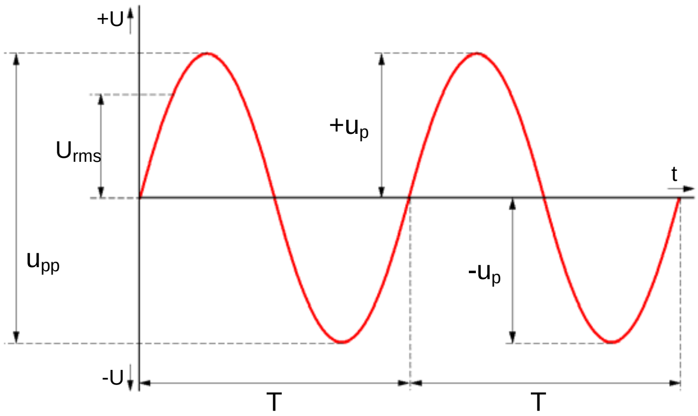


Fig. 10.2.1.1: Voltage and current, 'in phase'

At pure ohmic consumers, such as a simple resistor, the responses of voltage and current follow the same conditions with respect to time (Fig. 10.2.1.1). Voltage and current are said to be **'in phase'**. The descriptions of the characteristics that follow assume corresponding temporal conditions for voltage and current.

Fig. 10.2.1.2: Characteristics of a sine-wave voltage



The amplitude between the zero axis and maximum value, is known as the **peak value (u_p)** of the sine-wave (Fig. 10.2.1.2) and can a be positive ($+u_p$) or negative ($-u_p$) value. The voltage between the peak values (Fig. 10.2.1.2) is known as the **peak-to-peak value (u_{pp})**. These properties of the voltage can be displayed and measured on an oscilloscope. If an alternating voltage (or current), is measured with a voltmeter (or ammeter), the indicated value corresponds to the effective value of voltage (or current). This effective value is more often referred to as the **root mean square value** (U_{rms} or I_{rms}). The following relationships between rms values and peak values for sine-wave voltages or currents:

$$U_{rms} = \frac{1}{\sqrt{2}} \cdot u_p \cong 0{,}707 \cdot u_p \quad ; \quad I_{rms} = \frac{1}{\sqrt{2}} \cdot i_p \cong 0{,}707 \cdot i_p$$

From the sine-waves shown in Figs. 10.2.1.1 and 10.2.1.2, the regular recurrence of maxima, minima and crossing of the zero axis, exhibit a *periodic response*. This 'period of oscillation', or **periodic time, T** specifies the length of time after which the voltage or current wave is repeated. Using this periodic time T, the **frequency, f** of an alternating voltage can be determined. Thus:

$$f = \frac{1}{T} \qquad \left[1Hz = \frac{1}{1s} = 1s^{-1} \right]$$

1 Kilohertz	=	1 kHz	=	1.000 periods/s
1 Megahertz	=	1 MHz	=	10^6 periods/s
1 Gigahertz	=	1 GHz	=	10^9 periods/s

The unit of frequency is the Hertz (named after the German physicist *Heinrich Rudolf Hertz* in 1935). Commonly used units are also kilo-, Mega- or Gigahertz.

The same characteristic quantities (i.e. U_{rms}, u_p, T, f) are used in part, for other waveforms of voltage (square-wave, ramp, etc.). It must be remembered here, that the relationship between the effective voltage U_{rms}, and the peak voltage u_p depends on the shape of the voltage waveform.
For various other calculations, especially on non-ohmic components, the **angular frequency** ω is used, given by the periodic time T or frequency, f[2]:

$$\omega = 2 \cdot \pi \cdot \frac{1}{T} \quad \Rightarrow \quad \omega = 2 \cdot \pi \cdot f \qquad \left[1\,\frac{\text{rad}}{s}\right]$$

Occasionally in calculations, the **instantaneous value u** or **i** of a sine-wave is required. Here, the following equations are used:

$$u = u_p \cdot \sin \omega \cdot t \qquad ; \qquad i = i_p \cdot \sin \omega \cdot t$$

Current requires time to flow from one pole through a circuit, to the other pole. Assuming a sufficiently long cable, there are several minima, maxima and zero passes of an alternating current present along a connection cable, simultaneously. The longer the cable (or, the higher the frequency), the more complete periodic time intervals are formed along the cable at the same time. The distance bridged by one periodic time T is known as the **wavelength,** λ. The name stems from the wave shape of a sine-wave oscillation. The wavelength λ is given by the quotient of the velocity of propagation of a wave ν and the frequency f:

$$\lambda = \frac{v}{f} \qquad ; \qquad \lambda_{space} = \frac{c}{f}$$

Under certain conditions, electrical energy can also radiate (or propagate) in free space in the form of waves, without any conducting connection (e.g. radio waves, mobile telephone, etc.) In this case, the velocity of propagation is the same as the speed of light, c (~ 300.000 km/s)[3]. In conducting materials, the velocity of propagation of electrical waves is approximately 30% less than the speed of light in free space.

10.2.2 Characteristic Quantities of a Sine-wave Voltage in a Practical Exercise

The characteristic quantities are to be measured and their inter-relationships proved, in a circuit consisting of an AC voltage source (generator G ~) and a load resistor R_L,.

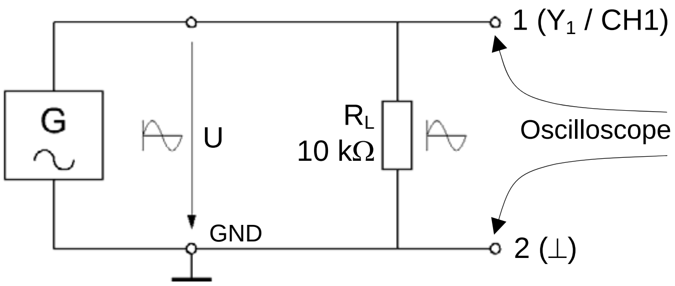


Fig. 10.2.2.1: Sine-wave generator with load resistor, R_L

- Assemble the circuit in Fig. 10.2.2.1 on the Electronic Circuits Board (notes on assembly will be found in section 10.2.3).

[2] The unit 'rad' (radian) indicates that the value is the magnitude of an angle (in circular measurements)
[3] Strictly speaking, the propagation velocity of waves of electrical energy is equal to the speed of light, only in a vacuum.

- Connect channel 1 (Y_1 or CH1) of your oscilloscope – as in Fig. 10.2.2.1 – to test terminals (outputs) 1 and 2 of the circuit.
- Set the output of the function generator on the Electronic Circuits Board, between the sockets 'Output' and 'GND', to the sine-wave voltage shown in Fig. 10.2.2.2. With the given values of timebase and amplitude, the sine-wave should be displayed on the oscilloscope as shown in Fig. 10.2.2.2.

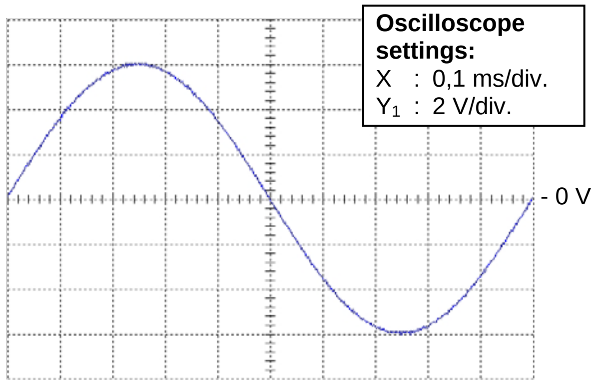


Fig. 10.2.2.2: Sine-wave voltage on the oscilloscope

- Measure the values required to complete table 10.2.2.1, from the oscilloscope display. Measure the instantaneous value of voltage u, 0,6 ms after the start of a period.

Table 10.2.2.3: Measurements on the oscilloscope

$+u_p$	$-u_p$	u_{pp}	u (after 0,6 ms)	T

- Calculate the following quantities from the values in the table: $\mathbf{i_S}$, $\mathbf{U_{rms}}$, $\mathbf{I_{rms}}$, **f**, ω, λ.

- Check the instantaneous value of voltage, u, read from the oscilloscope, by calculation.

Check the calculated value of effective voltage, U_{rms} with the multimeter.

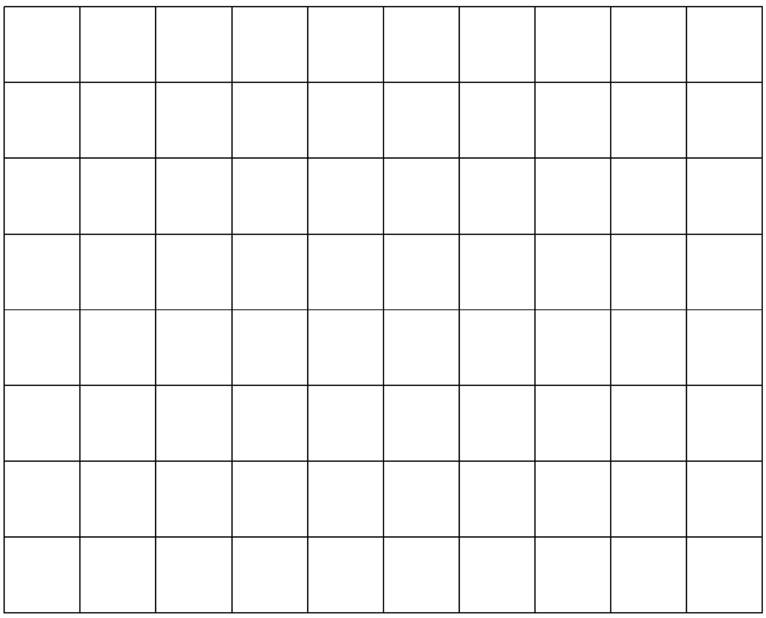
U_{rms} =

- At the output of the function generator, a sine-wave voltage of $U_{rms} = 8$ V at a frequency, f = 250 Hz should be present. First, calculate u_p, u_{pp} and T.

- Adjust the frequency of the function generator to f = 250 Hz. Use the meter on the function generator when adjusting the frequency. Adjust the effective voltage, $U_{rms} = 8$ V, whilst measuring with the multimeter at the same time.
- First, check the calculated characteristic quantities of the output AC voltage on the oscilloscope. Draw the sine-wave in the chart below (Fig. 10.2.2.4).

Fig. 10.2.2.4:
Oscilloscope display,250 Hz sine-wave, 8 V rms

Oscilloscope settings:
X : 1 ms/ div.
Y_1 : 5 V/ div.



- What time elapses after the start of a period, before the sine-wave signal reaches a voltage of 5 V? Calculate the value and check the result on the oscilloscope.

10.2.3 Exercise Assembly on the Electronic Circuits Board

The Function Generator on the Electronic Circuits Board is used for the exercises. It incorporates 4 possibilities of adjustment (Fig. 10.2.3.1):

a. The 'Waveform' of the alternating voltage can be selected by the push-button switch 'Press to change'.

b. The frequency is adjusted by way of the control marked 'Frequency' (range, 0 Hz to 210 kHz, stages depending on range).

c. By pressing the control knob 'Frequency', the frequency is immediately latched at 1 kHz ('Fixed 1 kHz').

d. The amplitude of the output voltage can be varied over the range 0 to ~ 7 V_{rms} with the 'Amplitude' control.

Fig. 10.2.3.1 shows the connections for the oscilloscope or multimeter for measuring the output AC voltage of 1 kHz.

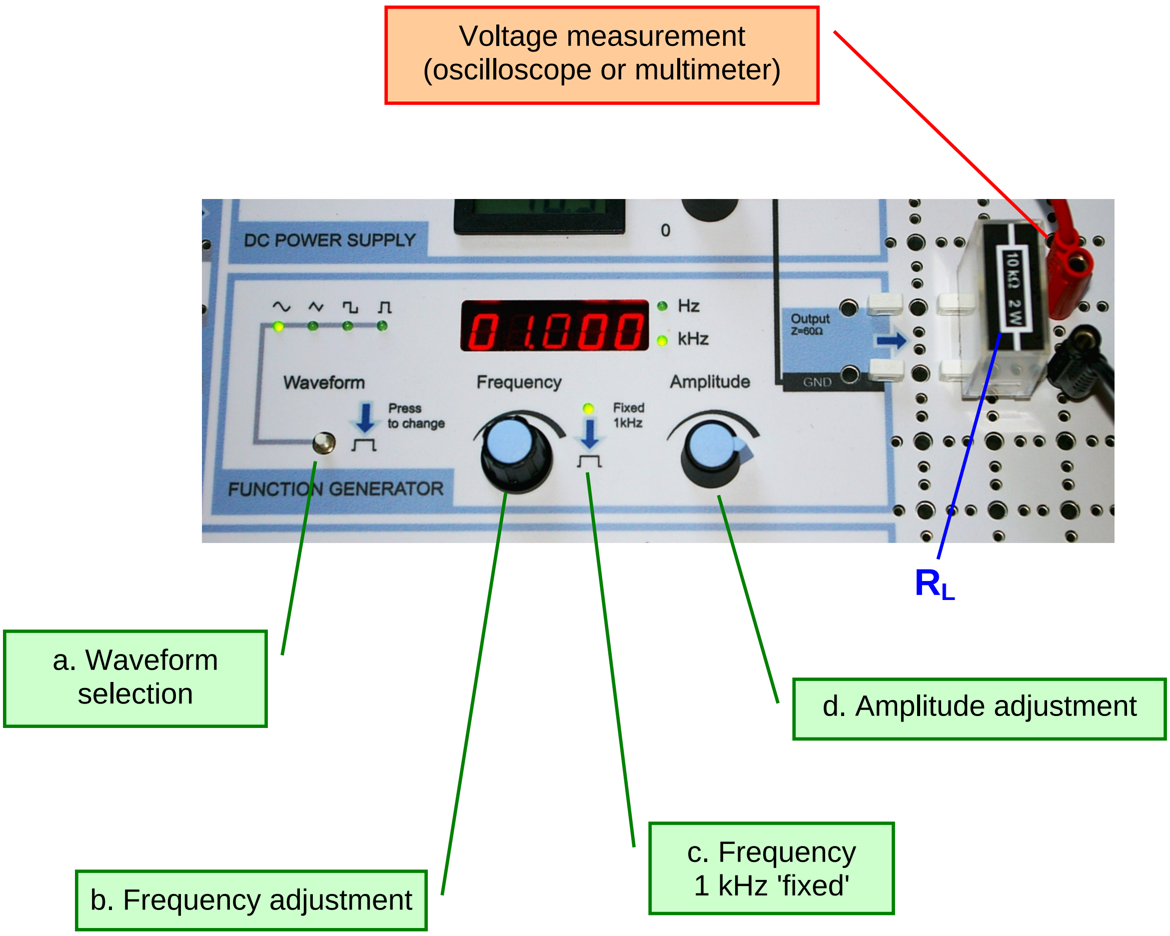


Fig. 10.2.3.1: Adjustment facilities on the function generator on the Electronic Circuits Board

10.3 Characteristics of Square-wave Voltages

10.3.1 Derivation of the Characteristics

As with sine-wave voltages, the **periodic time, T** specifies the length of time after which the voltage wave is repeated (Fig. 10.3.1.1). Thus, the same expressions applies for the frequency:

$$f = \frac{1}{T} \qquad \left[1Hz = \frac{1}{1s} = 1s^{-1}\right]$$

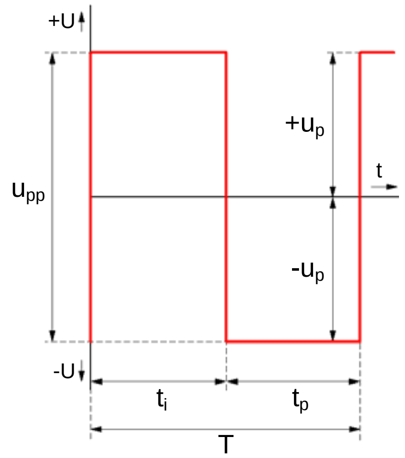


Fig. 10.3.1.1: Characteristics of a square-wave voltage

Of more interest with square-wave voltages, are the sections of the wave-form known as **pulse duration t_i** and **interpulse period t_p**. The 'pulse duration' is the time taken for the voltage to rise in a positive direction until the fall in a negative direction (Fig. 10.3.1.1). The 'interpulse period' is similarly defined in the opposite direction. The pulse duration t_i and interpulse period t_p are added to give the periodic time, T. The ratio of t_i to T is known as the **duty factor, g**. Quite often the term **duty cycle** is used although this applies more to a train of square-wave pulses:

$$T = t_i + t_p \quad ; \quad g = \frac{t_i}{T}$$

If the pulse duration t_i and interpulse period t_p are the same length, the duty factor g is then 0,5. Also, if the duration of both peak values are of the same magnitude (+/-u_p) then reference is made to a '**symmetrical square-wave voltage'** (Fig. 10.3.1.1).

Peak values (+/-u_p) and **peak-to-peak values (u_{pp})** are given as with a sine-wave voltage, between maximum, minimum and zero axis (c.f. Figs. 10.2.1.2 and 10.3.1.1).

With *symmetrical* square-wave voltages the effective value, U_{rms} corresponds to the peak value u_p. This is easier to understand if one imagines the negative section to be folded up to the positive side of the zero axis.

The same statement applies to the current flows (i_{pp}, i_p, I_{rms}) caused by square-wave voltages.

10.3.2 Characteristic Quantities of a Square-wave Voltage in a Practical Exercise

The characteristic quantities will now be derived from measurements on a symmetrical square-wave voltage circuit as shown in Fig. 10.3.2.1.

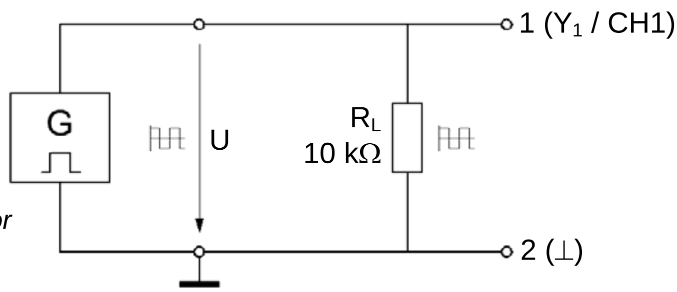


Fig. 10.3.2.1: Circuit with square-wave generator

- Assemble the circuit in Fig. 10.3.2.1 using the function generator, on the Electronic Circuits Board.
- Connect channel 1 (Y_1 or CH1) of your oscilloscope to outputs 1 and 2 of the circuit.

- Set the output of the function generator to a symmetrical square-wave voltage with the following values: f = 625 Hz, U_{rms} = 6 V. To check the settings, use the frequency meter on the Board and a voltmeter.
- Display the square-wave voltage on the oscilloscope and draw the waveform in the chart given in Fig. 10.3.2.2.

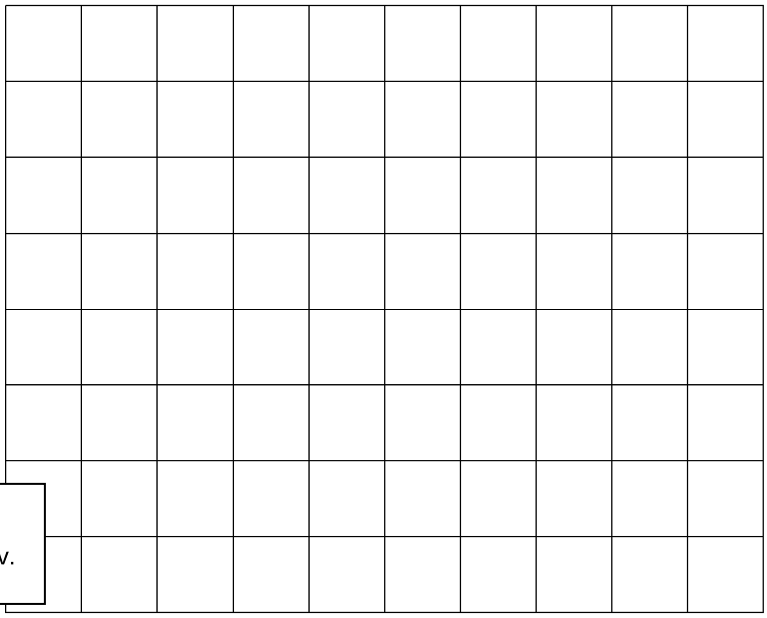


Fig. 10.3.2.2: Square-wave voltage on the oscilloscope

- Measure the values of u_{pp}, u_p, T, t_i and t_p from the oscilloscope display.

Oscilloscope settings:
X : 0,4 ms/ div.
Y_1 : 2 V/ div.

u_{pp} = ; $|+u_p|$ = $|-u_p|$ =

T = ; t_i = ; t_p =

- Calculate the peak current i_p and the duty factor of the square-wave voltage.

- Check the set frequency by calculation.

- What is the relationship between the peak value u_p read on the oscilloscope, and the value of effective voltage U_{rms} measured previously on the multimeter?

- Calculate the effective current, I_{rms}

-

- Check the rms value of current by measurement on an ammeter.

$I_{rms\ [meas.]}$ =

10.4 Characteristics of Delta Voltages

10.4.1 Derivation of the Characteristics

The main interest in a delta (or 'triangular') voltage, centres on the *linear* swing of the voltage betweeen the **peak values (+/-u_S)**. The relationships between peak values and **peak-to-peak value u_{pp}** are the same as those for sine- and square-wave. Fig. 10.4.1.1 shows a *symmetrical delta* voltage. Here, the **rise time t_{ri}** is the same as the **fall time t_{fa}**. Both add together to give the **periodic time, T**. The effective voltage U_{rms} is calculated from:

$$U_{rms} = \frac{1}{\sqrt{3}} \cdot u_p$$

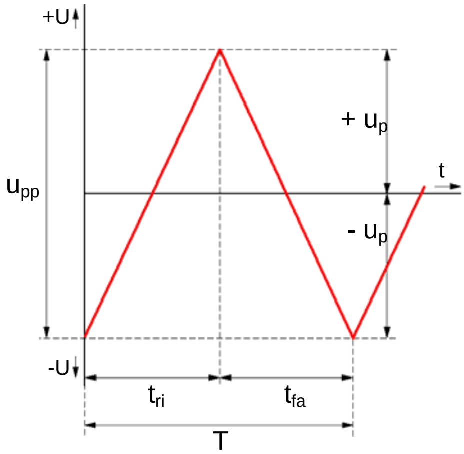


Fig. 10.4.1.1: Characteristics of a delta voltage

A common variation of a delta voltage is the ramp (or sawtooth), Fig. 10.4.1.2. The rise is made as linear as possible, with a very short fall time. One use of a ramp voltage is the deflection of the x-axis on an oscilloscope or the horizontal sweep of a TV picture. After the rise of the ramp, the fall time should be almost immediate ($t_{fa} \cong 0$) and the voltage returns to its original value.

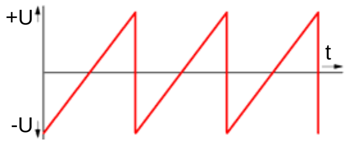


Fig. 10.4.1.2: Ramp (or sawtooth) voltage

10.4.2 Characteristic Quantities of a Delta Voltage in a Practical Exercise

- Assemble the circuit in Fig. 10.4.1.2 on the Electronic Circuits Board.
- Set the output of the function generator to a symmetrical **delta voltage**, f = 180 Hz, u_p = 3,5 V. Use the frequency meter on the Board and an oscilloscope.
- Measure the delta voltage on the oscilloscope. Draw the waveform in Fig. 10.4.2.1.

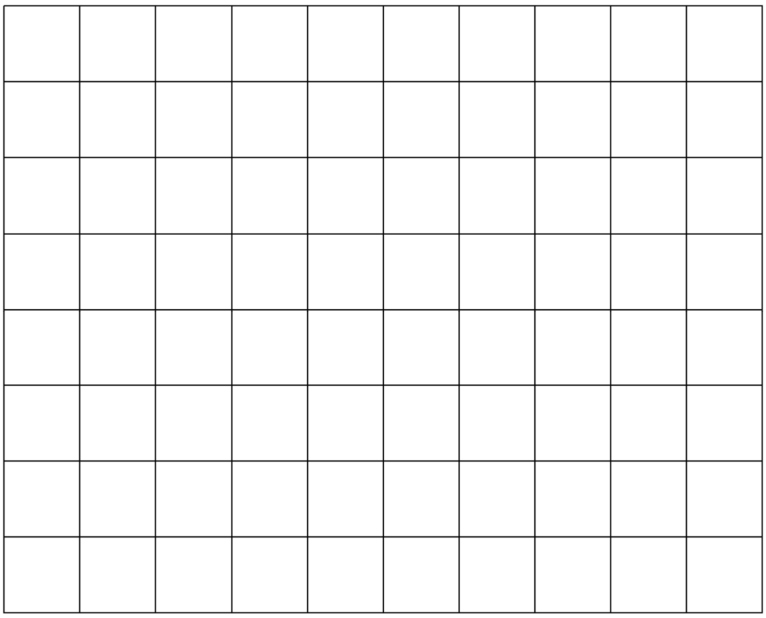


Fig. 10.4.2.1: Delta voltage on the oscilloscope

- On the waveform drawn, measure the values T, t_{ri} and t_{fa}. Check f and U_{rms} by calculation.

Oscilloscope settings:
X : 1 ms/ div.
Y_1 : 1 V/ div.

T = ; t_{ri} = ; t_{fa} =

11. Active Power of Alternating Voltages

11.1 Derivation of AC Power

When an AC voltage is applied to an ohmic consumer, the voltage produces an 'in-phase' current (Fig. 11.1.1). The power P produced is given by the product of voltage U and current I. Usually however, the useful or **active power P_{act}** is of interest:

$$P_{act} = U_{rms} \cdot I_{rms} \quad \Rightarrow \quad P = U \cdot I$$

$$\Rightarrow \quad P = \frac{U^2}{R} \quad \Rightarrow \quad P = I^2 \cdot R$$

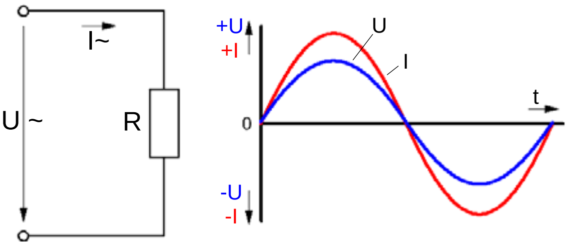


Fig. 11.1.1: Voltage and current, in phase

The active power should always be assumed when P is given without any other details. The suffix 'act' is usually omitted. The **instantaneous power p** is required only in exceptional cases. Here, lower case letters are used:

$$p = u \cdot i \quad \text{(instantaneous values)}$$

The active power of sine-wave voltages is calculated from the peak values, as follows:

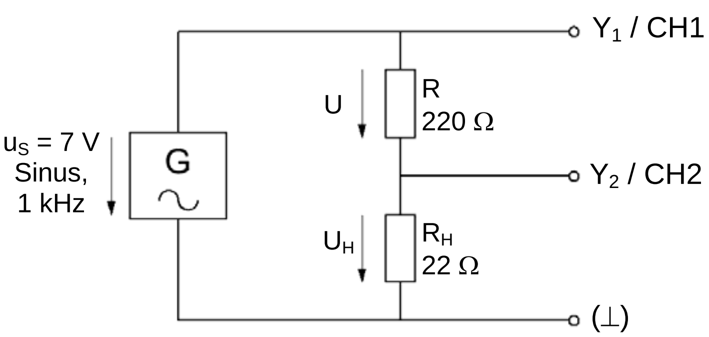
$$P_{act} = U_{rms} \cdot I_{rms} \quad \Rightarrow \quad P_{act} = \frac{1}{\sqrt{2}} \cdot u_p \cdot \frac{1}{\sqrt{2}} \cdot i_p = \frac{1}{\sqrt{2} \cdot \sqrt{2}} \cdot u_p \cdot i_p = \frac{1}{2} \cdot u_p \cdot i_p$$

The power converted at an ohmic consumer, in AC techniques, is known as the **active power**, because real energy is released and work performed. In contrast, there is the term 'reactive power' – this will be explained later.

11.2 Active Power of a Sine-wave Voltage in a Practical Exercise

The instantaneous and active values of active power will now be determined by measuring voltage and current on an oscilloscope and drawing the waveforms displayed. An oscilloscope can display only voltages present at its input. Therefore, the current measurement is made, indirectly from the voltage drop across an extra resistor (R_H).

Fig. 11.2.1: Measurement circuit for active power

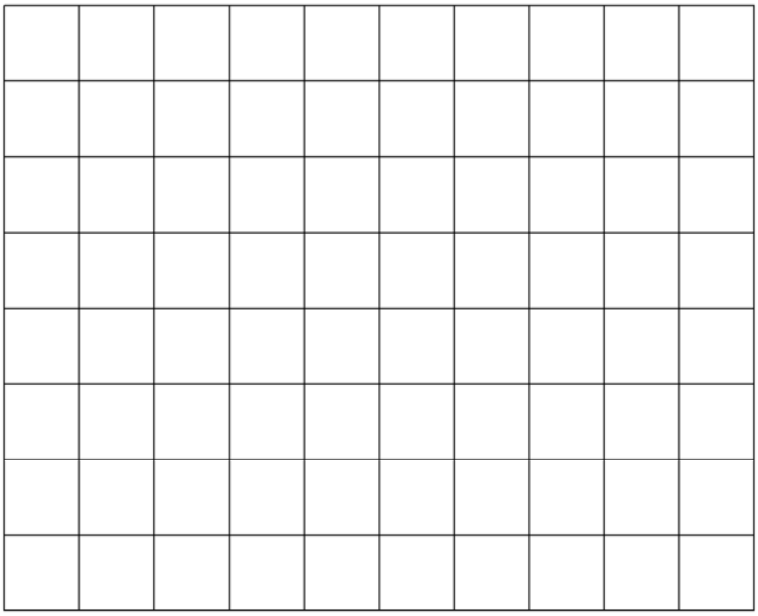


The series circuit in Fig. 11.2.1 uses the fact that the same current flows through resistors R and R_H. Thus, the voltage drop across R_H represents the magnitude of current and this can be displayed on the oscilloscope. The voltages $U+U_H$ and U_H are connected to channels Y_1 and Y_2. To allow both single voltages U and U_H to be displayed simultaneously, the reference point (Ground), must be connected between the 2 resistors. This is only possible with differential input oscilloscope or multimeter. To display the actual value of U we should subtract the value U_H from the value of $U+U_H$.

- Assemble the circuit in Fig. 11.2.1 on the Electronic Circuits Board. Connect the outputs of the circuit to the channel inputs of the oscilloscope (assembly and measurement details are in Fig. 11.3.1).
- Set the function generator to sine-wave voltage, u_p = 7 V, f = 1 kHz.
- Draw the displayed voltage waveforms $U+U_H$ and U_H (represents I) in the chart (Fig. 11.2.2).

Fig. 11.2.2: Displayed voltage and power waveforms

Oscilloscope settings:
X : 0,1 ms / div.
Y_1 : 2 V / div.
Y_2 : 0,5 V / div.



- Measure, on the waveform drawn, the instantaneous values of u and u_H. Enter the measured values in table 11.2.3.
- From the measured values, calculate the instantaneous values of current, i and power, p. Complete the table with the calculated results.
- Plot the calculated instantaneous values of the power p in the chart (Fig. 11.2.2.) and draw the power curve (extend the y-axis if necessary).

Time [ms]	$u+u_H$ [V]	u_H [V]	u [V]	i [mA]	p [mW]
0					
0,1					
0,15					
0,25					
0,35					
0,4					
0,5					
0,6					
0,65					
0,75					
0,85					
0,9					
1					

Table 11.2.3.: Instantaneous values, u, u_H, i, p

- Interpret the shape of the power curve drawn in Fig. 11.2.2.

- How much active power is dissipated at resistor, R?

- What is the active power at a resistor of R = 330 Ω, when a symmetrical square-wave voltage u_{pp} = 10 V is applied?

- How much power is dissipated in heat by an ohmic resistor R = 22 Ω, when a symmetrical delta voltage can be seen on an oscilloscope of u_p = 9 V?

11.3 Assembly and Measurements on the Electronic Circuits Board

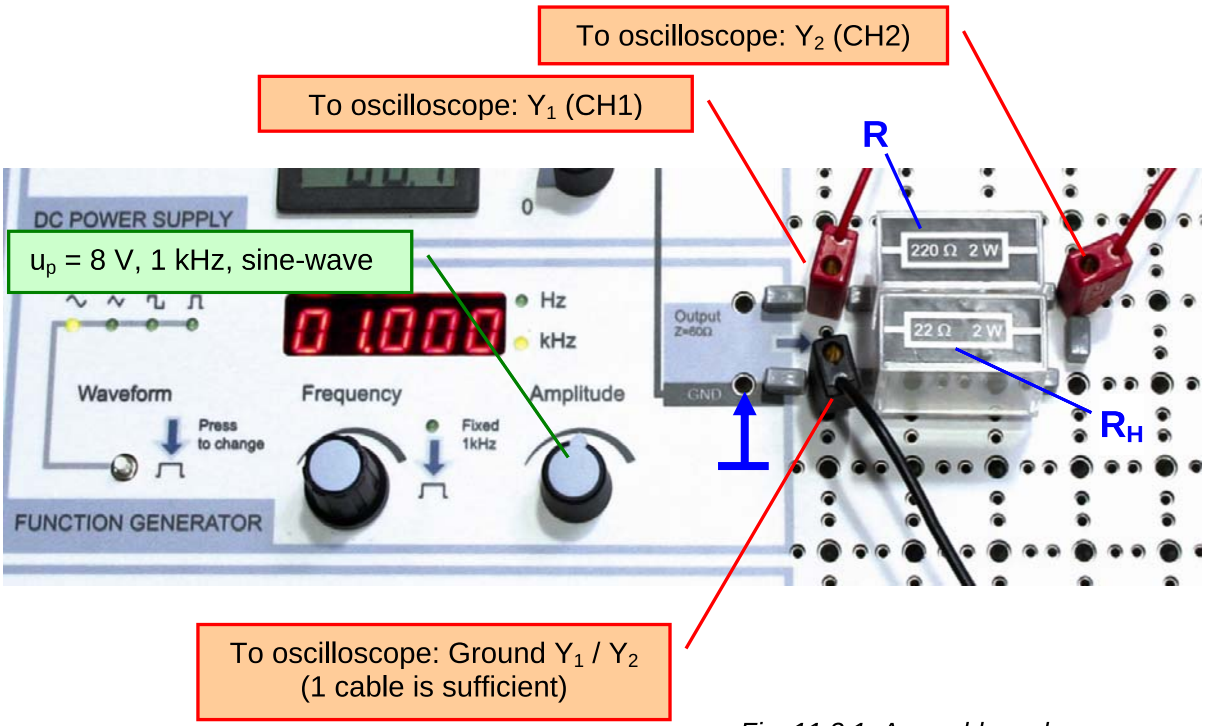


Fig. 11.3.1: Assembly and measurements, Active power, on the Electronic Circuits Board

12. Three-phase AC

12.1 Origin and Function of Three-phase AC

Electrical energy is usually fed to domestic and industrial consumers by way of a 4-wire network. This 4-wire technique originates from the energy provider (generators in an electrical power station) and a simplified diagram is shown in Fig. 12.1.1 (see also, the diagram on the front panel of the Electronic Circuits Board at the lower left. Three coils (phase windings), offset by 120° form a star circuit. The common connection of the coils is known as the star point and forms the **neutral line N** connection. The three phases, each connected to the other end of the windings, form the **phase lines L1, L2 and L3**. Between each phase line and neutral, there is a sinusoidal AC voltage of U_{rms} = 230 V. A sinusoidal AC voltage of U_{rms} = 400 V can be measured between any pair of phase lines.

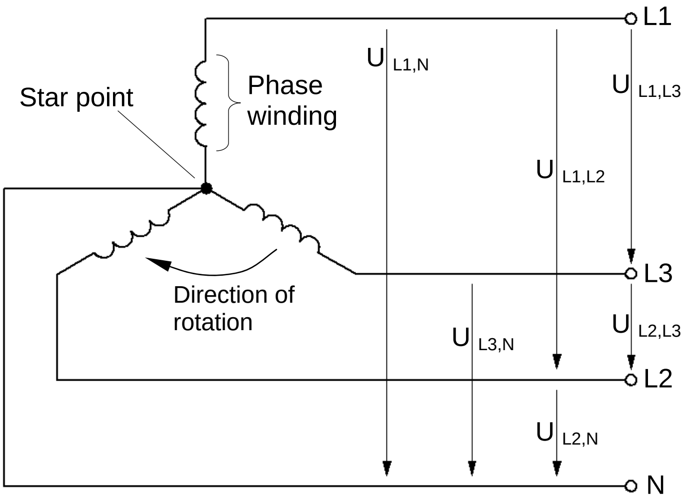


Fig. 12.1.1: Generation of three-phase AC

A three-phase network has 3 **phase voltages** and 3 **line voltages** available (Fig. 12.1.1):

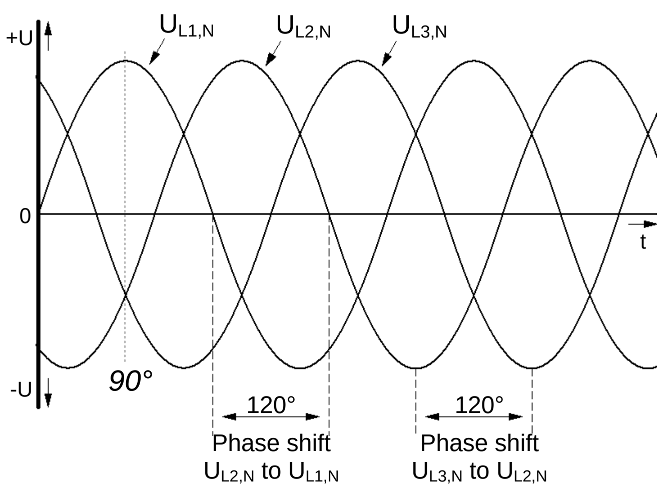
$U_{L1,N}$, $U_{L2,N}$, $U_{L3,N}$ = 230 V (phase voltage)

$U_{L1,L2}$, $U_{L2,L3}$, $U_{L1,L3}$ = 400 V (line voltage)

The following mathematical relationship exist between the rms values of line and phase voltages:

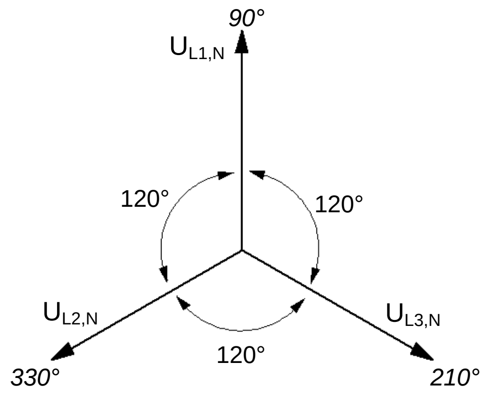
$$U_L = \sqrt{3} \cdot U_{ph} \quad ; \quad [400V \approx \sqrt{3} \cdot 230V]$$

Fig. 12.1.2: Line chart of a three-phase altenating voltage



The phase voltages $U_{Ln,N}$ have a phase shift of 120° (one-third of a period) which corresponds to the offset of the stationary generator coils. Fig. 12.1.2 shows the sinusoidal shape of the voltages whereby $U_{L1,N}$ starts at the zero axis with a positive half-wave.

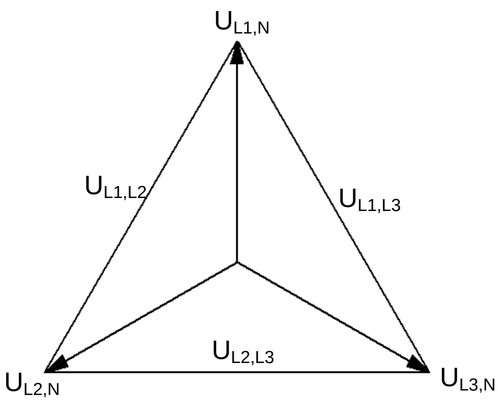
Fig. 12.1.3: Vector diagram of 3-phase AC



The **line chart** (Fig. 12.1.2) is not very often used in practice. A more straightforward representation of the phase shift between the voltages $U_{L1/2/3,N}$ can be shown using a **vector diagram** (Fig. 12.1.3). The length of each vector corresponds to its maximum amplitude. The vectors rotate at the frequency, f about the centre point. The phase voltage $U_{L2,N}$ **lags** the phase voltage $U_{L1,N}$ by 120°; in contrast, the phase voltage $U_{L3,N}$ **leads** by 120°. The instantaneous values of voltage are given by the sine value of the vector angle, multiplied by the length of the vector.

In Fig. 12.1.3, $U_{L1,N}$ has just reached its maximum amplitude (sin 90° = 1). $U_{L2,N}$ on the other hand, shows half of the negative maximum voltage (sin 330° = - 0,5), at the next fall in amplitude. Also, $U_{L3,N}$ corresponds to half of the negative maximum voltage (sin 210° = - 0,5), but the amplitude is increasing. This 'snapshot' of the phase relationship ($U_{L1,N}$ = 90°), for comparison of the 2 methods of representation, is also indicated in Fig. 12.1.2.

Fig. 12.1.4: Line voltages as a vector diagram



The relationships can also be shown easier, between phase and line voltages, using a vector diagram (Fig. 12.1.4). The lines joining the ends of the phase vectors show the line voltages. Their length is longer than the individual phase vectors, by the so-called 'concatenation' factor (the interlinking factor, √3). Thus:

$$U_L = \sqrt{3} \cdot U_{ph}$$

In addition to the basic form 'star', the phase windings in a 3-phase AC network can also be connected in a 'delta' circuit (Fig. 12.1.5). In this case, the line voltages $U_{Lm,Ln}$ are sometimes referred to as the 'delta voltage'. In a public network they are normally 400 V.

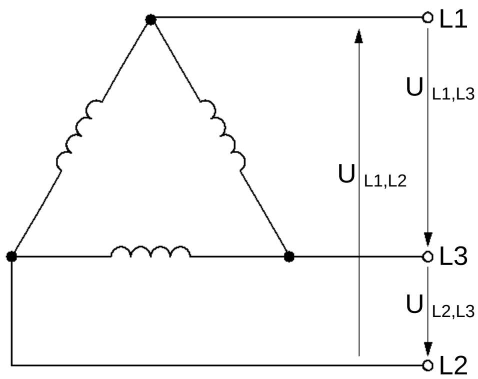


Fig. 12.1.5: Delta circuit

12.2 Measurements on a Three-phase System

The relationships between the characteristic quantities of three-phase systems will now be examined by way of various measurements at the three-phase generator on the Electronic Circuits Board (Fig. 12.2.1).

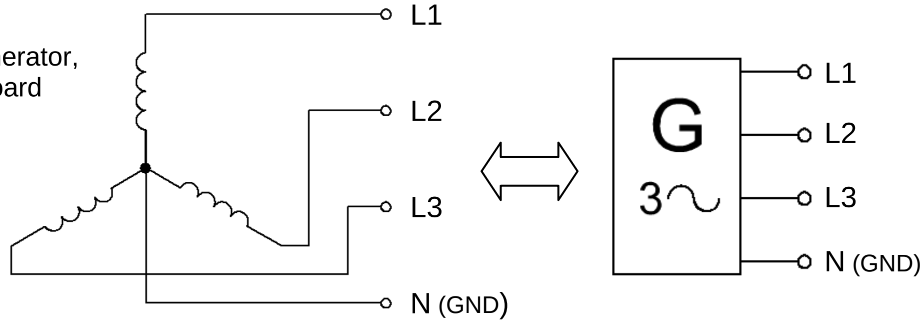
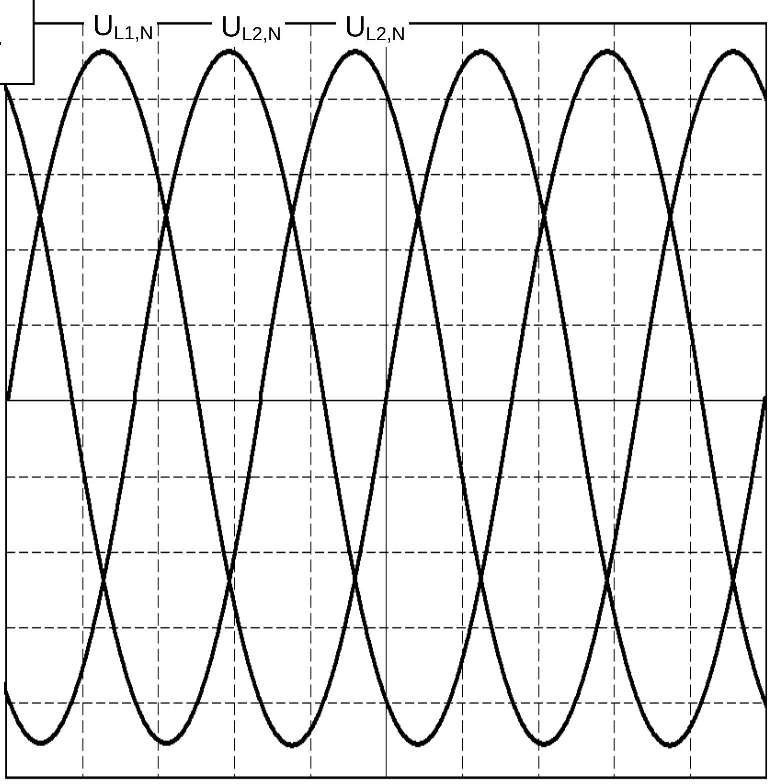


Fig. 12.2.1: 3-phase generator, Electronic Circuits Board

- Display the phase voltages L1 to L3 ($U_{L1,N}$ to $U_{L3,N}$) of the 3-phase generator on the oscilloscope and draw the voltage waveforms in the chart below (Fig. 12.2.2). Pay attention to the correct phase relationships between the individual voltages. (You will need 2 measurement processes when using a 2-channel oscilloscope).

Fig. 12.2.2: 3-phase voltage waveforms

Oscilloscope settings:
X : 4 ms/ div.
$Y_{1,2}$: 2 V/ div.



- On the waveforms drawn, measure the phase shift of the voltages. Give the difference as angle φ and in milliseconds.

- Calculate the frequency, f of the phase voltages.

- Measure the phase and line voltages with the multimeter.

Phase voltage	Line voltage
$U_{L1,N}$ =	$U_{L1,L2}$ =
$U_{L2,N}$ =	$U_{L2,L3}$ =
$U_{L3,N}$ =	$U_{L1,L3}$ =

- What is the peak value u_p of the phase voltage $U_{L1,N}$?

- Calculate the peak value u_p of the line voltage $U_{L1,L3}$.

- Calculate the interlinking factor.

12.3 Consumers in a Star Circuit

12.3.1 Current and Power Distribution in a Star Circuit

Fig. 12.3.1.1 shows three consumers, connected in a star circuit to the four lines of a 3-phase network. The phase voltages L1 to L3 ($U_{L1,N}$ to $U_{L3,N}$) cause the **line currents (phase currents) I_1 to I_3** to flow in the three branches. Their value is calculated from Ohm's law, as shown:

$$I_n = \frac{U_{Ln}}{R_n}$$

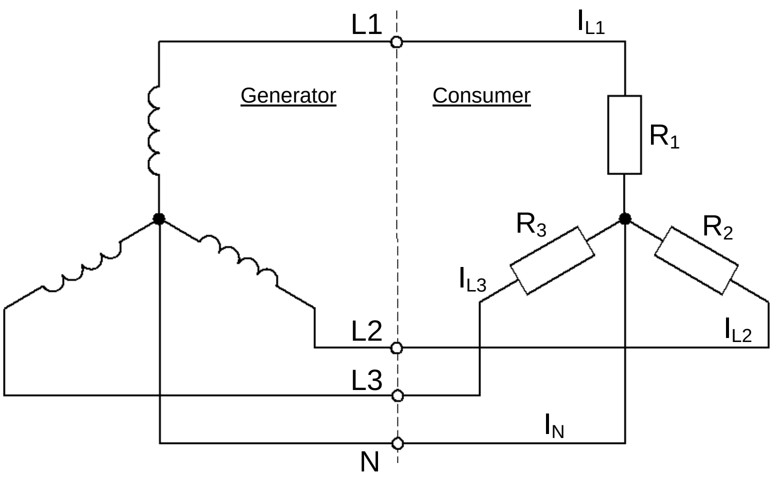


Fig. 12.3.1.1: Consumer in a star circuit

With a **symmetrical** consumer star connection, i.e. when $R_1 = R_2 = R_3$, equal line currents flow in the branches. In the line diagram of the phase voltages (see section 12.1, Fig. 12.1.2) it can be seen that the instantaneous sum of the 3 phase voltages is equal to zero. From Ohm's law, this also applies to the resultant currents.

Also, according to Kirchhoff's 1st. law, “The algebraic sum of the currents meeting at a point in a network is zero”. For this reason, **in the neutral line N with symmetrical, or balanced loading, there is no flow of current ($I_N = 0$)**.

If one of the consumers changes in value, the star circuit then becomes **asymmetric** and a compensating current flows in the neutral line.

The total power P_{tot} dissipated at the consumers, is given by the sum of the individual powers, P_{Rn}:

$$P_{tot} = P_{R1} + P_{R2} + P_{R3}$$

The product of phase voltage U_{Ln} and phase current I_{Ln} corresponds to the power in the phase, P_{Rn}:

$$P_{Rn} = U_{Ln} \cdot I_{Ln}$$

For symmetrical loading in a star circuit, the calculation of total power is simplified to:

$$P_{tot} = 3 \cdot P_{ph} = 3 \cdot U_{ph} \cdot I_{ph}$$

12.3.2 Measurements on Symmetrical and Asymmetrical Star Circuits

- Load the three-phase generator on the Electronic Circuits Board with 3 resistors connected in a star circuit, as in Fig. 12.3.2.1. First, use 3 resistors of the same value, each 1 kΩ (assembly details are given in section 12.3.3).
- Measure the phase and line voltages, as well as the current flow in the start circuit. Enter the values measured in table 12.3.2.2.

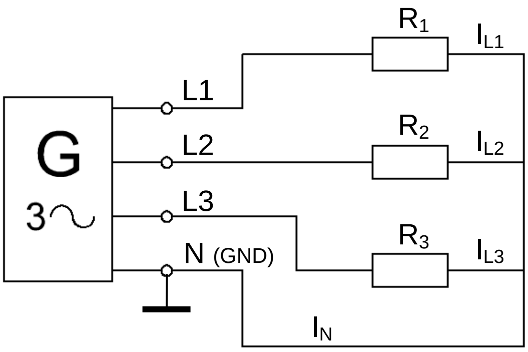


Fig. 12.3.2.1: Measurements on the star circuit

Phase voltage [V]			Line voltage [V]			Line current [mA]				
$U_{L1,N}$	$U_{L2,N}$	$U_{L3,N}$	$U_{L1,L2}$	$U_{L2,L3}$	$U_{L1,L3}$	I_{L1}	I_{L2}	I_{L3}	I_N	**Loading:**
										symmetrical ($R_1 = R_2 = R_3$)
										asymmetrical R_1 = 1 kΩ, R_2 = 470 Ω, R_3 = 100 Ω

Table 12.3.2.2: Voltage and current values measured in the star circuit

- Calculate the phase powers and the total power for the symmetrical star circuit ($R_1 = R_2 = R_3$).

Evaluate the measured values of the neutral line current, I_N.

- Replace 2 of the resistors in the star circuit: R_2 = 470 Ω, R_3 = 100 Ω.
- Repeat the voltage and current measurements on the now, asymmetrical star circuit and enter the values measured in table 12.3.2.2.
- Calculate the phase powers and the total power for the asymmetrical star circuit.

12.3.3 Star Circuit Exercise Assembly on the Electronic Circuits Board

The three-phase generator on the Electronic Circuits Board provides the 3 phase voltages L_1, L_2, L_3. The LED's at the sockets light to indicate an overload of current (limit value 150 A); normally, the LED's remain switched off (Fig. 12.3.3.1).

Fig. 12.3.3.1 shows the asymmetrical star circuit. The on-going phase voltage L1 ($U_{L1,N}$) is measured across R_1 = 1 kΩ. The connection between centre point of the star ('node') and the neutral line N of the generator is broken and an ammeter is inserted in the circuit to measure the current (Fig. 12.3.3.1).

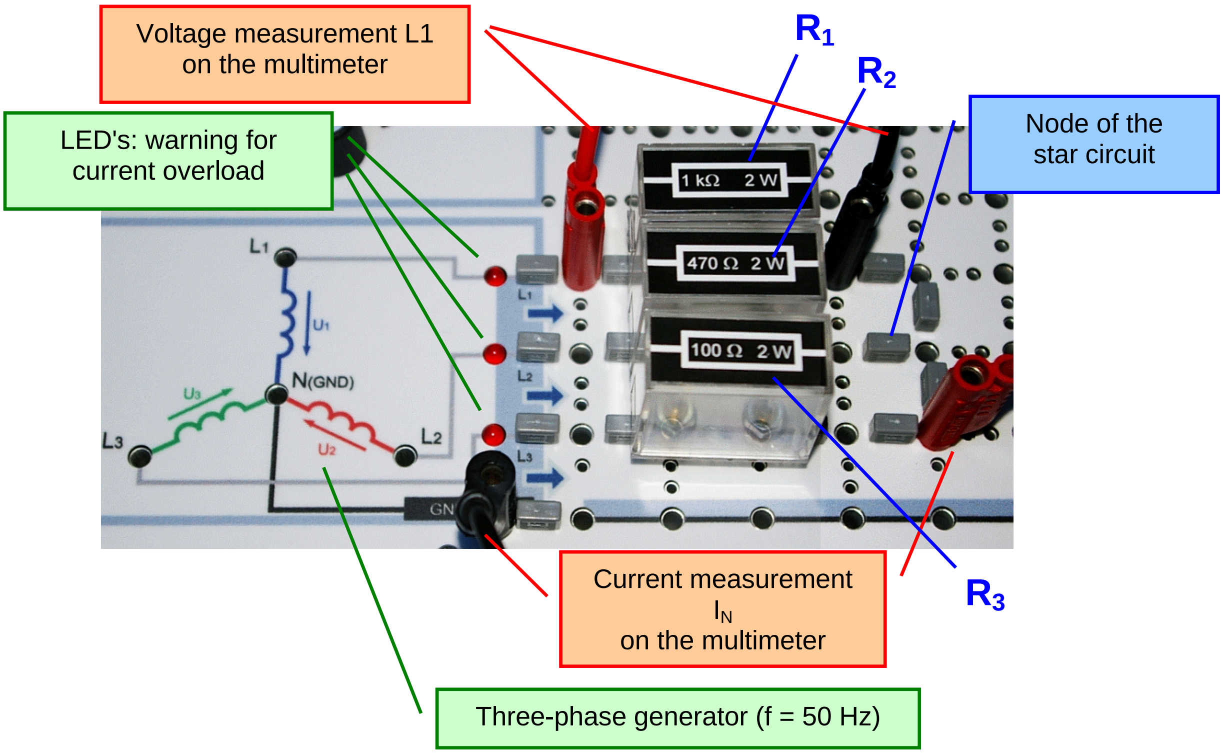
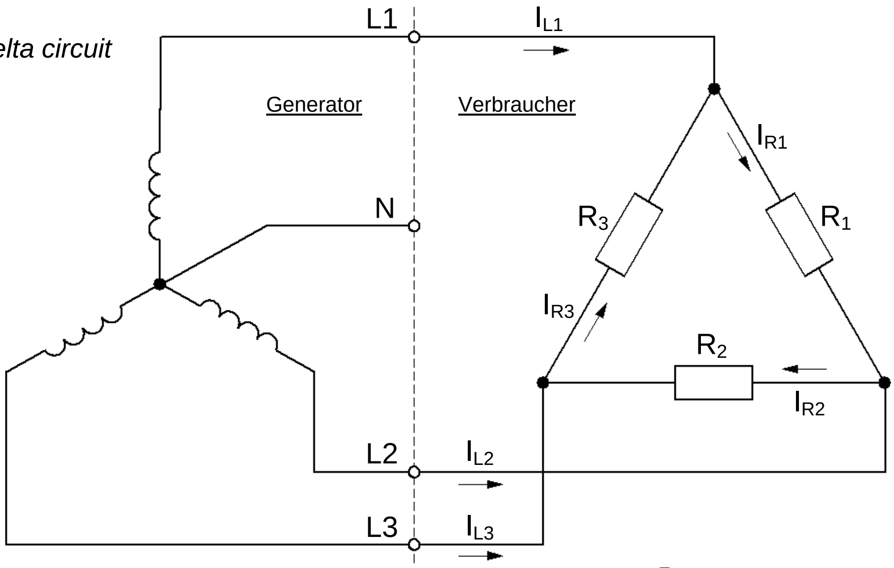


Fig. 12.3.3.1: Measurements at the 3-phase generator on te Electronic Circuits Board

12.4 Consumers in a Delta Circuit

12.4.1 Current and Power Distribution in a Delta Circuit

Fig. 12.4.1.1: Consumer in a delta circuit



When 3 consumers are connected in a delta circuit (Fig. 12.4.1.1), the loads are supplied directly from the line voltages. The neutral line is no longer required. Thus, phase voltage (across the consumer) and the line voltage are identical.

The line currents I_{Ln} branch at the delta nodes as phase currents I_{ph} (I_{Rn}). With identical line voltages and symmetrical loading :

$$I_{ph}\ (I_{Rn}) = \frac{I_{Ln}}{\sqrt{3}}$$

The total power P_{tot} in the delta circuit is calculated as the sum of the individual powers dissipated at the consumers (phase power, P_{Rn}):

$$P_{tot} = P_{R1} + P_{R2} + P_{R3}$$

In the case of symmetrical loading and equal line voltages (= phase voltages), the equation for calculating the total power, P_{tot} simplifies to:

$$P_{tot} = 3 \cdot P_{ph} = 3 \cdot U_{ph} \cdot I_{ph}$$

According to the general equation for power, the phase power is:

$$P_{Rn} = U_{Ln} \cdot I_{Rn}$$

12.4.2 Measurements on Symmetrical and Asymmetrical Delta Circuits

- Load the three-phase generator on the Electronic Circuits Board with 3 resistors connected in a delta circuit, as in Fig. 12.4.2.1. First, use 3 resistors of the same value, each 1 kΩ (assembly details are given in section 12.4.3).
- Measure the line voltages, phase and line currents. Enter the values measured in table 12.4.2.1.

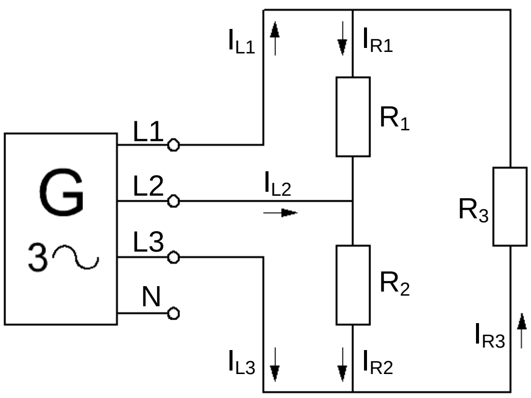


Fig. 12.4.2.1: Measurements on the delta circuit

Line voltage [V]			Line current [mA]			Phase curent [mA]			
$U_{L1,L2}$	$U_{L2,L3}$	$U_{L1,L3}$	I_{L1}	I_{L2}	I_{L3}	I_{R1}	I_{R2}	I_{R3}	**Loading:**
									symmetrical ($R_1 = R_2 = R_3$)
									asymmetrical R_1 = 1 kΩ, R_2 = 470 Ω, R_3 = 100 Ω

Table 12.4.2.2: Voltage and current values measured in the delta circuit

- Calculate the total and phase powers for the symmetrical delta circuit ($R_1 = R_2 = R_3$).

- Replace 2 of the resistors in the delta circuit: R_2 = 470 Ω, R_3 = 100 Ω.
- Repeat the voltage and current measurements on the now, asymmetrical delta circuit and enter the values measured in table 12.4.2.2.
- Calculate the total power and phase powers for the asymmetrical delta circuit.

12.4.3 Delta Circuit Exercise Assembly on the Electronic Circuits Board

Fig. 12.4.3.1 shows the asymmetrical delta circuit ($R_1 \neq R_2 \neq R_3$). A large area should be used for the assembly to allow easy access for measuring the current. As shown, in the R_1 = 1 kΩ phase, an ammeter is connected in series for measuring the phase current I_{R1}. A voltmeter measures the line voltage $U_{L1,L3}$ across R_3 (Fig. 12.4.3.1).

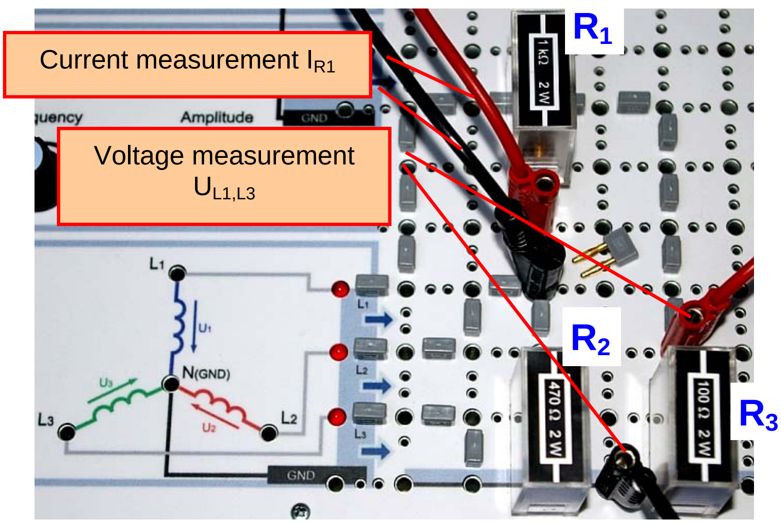


Fig. 12.4.3.1: Recording the measurements in a delta circuit

12.5 Faults in Three-phase Circuits

12.5.1 Measurements on Faulty Star Circuits

Considering the exercise assembly of a **symmetrical star circuit** in Fig. 12.5.1.1, there are 3 possible faults with a characteristic effect. Table 12.5.1.2 summarises these faults.

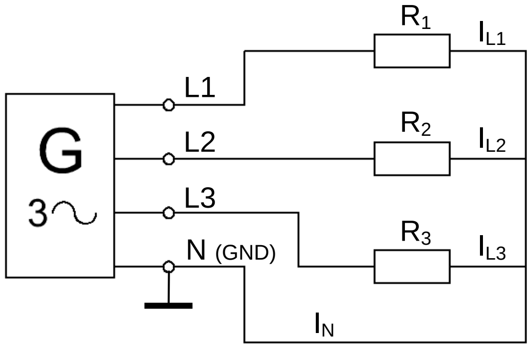
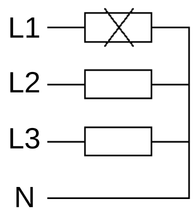
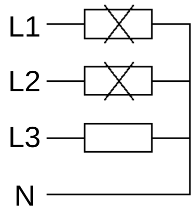
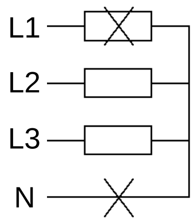


Fig. 12.5.1.1: Star circuit

Table 12.5.1.2: Possible faults in a star circuit

	Fault	Effect	Resulting circuit
1	Break in **one** line, L1 to L3 <u>or</u> one of the consumers, R_1 to R_3 <u>or</u> a failure of one of these voltages.	Power consumption P_{tot} falls by **one-third** (example shown: $I_{L1} = 0$)	
2	Failure of **two** circuit branches as described in 1.	No flow of current in two of the 3 lines (example shown: $I_{L1} = 0$ and $I_{L2} = 0$). Power consumption P_{tot} falls by **two-thirds**	
3	Break in the **neutral line** N <u>and</u> break in **one** line as described in 1.	2 consumers in series are fed by **one line voltage** (example shown: $U_{L2,L3}$ effectively a series circuit of 2R); P_{tot} falls by **half**.	

- Measure the voltages and currents as listed in table 12.5.1.3, in the faulty star circuit.

Phase voltage [V]			**Line voltage [V]**			**Line current [mA]**				
U_{R1}	U_{R2}	U_{R3}	$U_{L1,L2}$	$U_{L2,L3}$	$U_{L1,L3}$	I_{R1}	I_{R2}	I_{R3}	I_N	**Fault:**
										Break in supply line L2
										Break in L2 and L3
										Break in neutral line and L3

Table 12.5.1.3: Values measured in a faulty star circuit

- Calculate P_{tot} for the 3 faulty star circuits and compare the values with the power consumption of the fully functional circuit in 12.3.2.1.

  P_{tot} of the faultless star circuit:

  Break in supply line L2:

  Break in supply lines L2, L3:

  Break in supply line L3 and neutral line:

12.5.2 Measurements on Faulty Delta Circuits

If a 3-phase generator is loaded with 3 consumers in a **symmetrical delta circuit** (Fig. 12.5.2.1), then the faults listed in table 12.5.2.2 can occur:

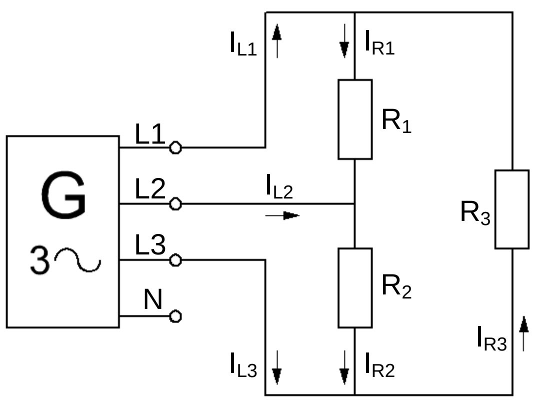


Fig. 12.5.2.1

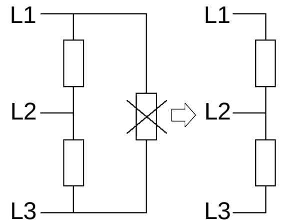
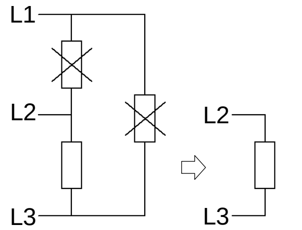
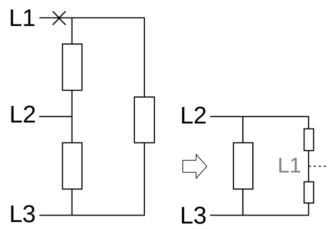
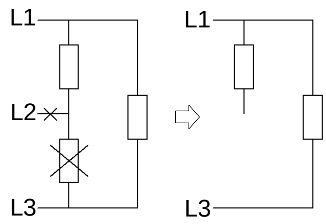
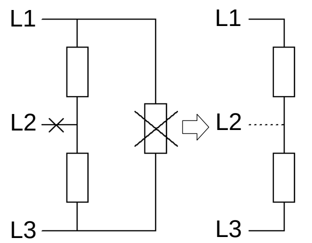
	Fault	Effect	Resulting circuit
1	Break in **one** phase or one of the consumers R_1 to R_3	Power consumption P_{tot} falls by **one-third** (example shown: $I_{R3} = 0$) compared to the maximum possible power.	
2	Break in **two** phases or 2 of the consumers R_1 to R_3	No flow of current in two of the 3 branches (example shown: $I_{R1} = 0$ and $I_{R3} = 0$), Power consumption P_{tot} falls by **two-thirds**	
3	Break in **one** supply line <u>or</u> failure in **one** phase voltage, L1 to L3	The resulting resistor network R // 2R is supplied by **one line voltage** (example shown: $U_{L2,L3}$ effective at R_2 // $R_1 + R_3$; P_{tot} falls by **half**	
4	Break / failure of **one** consumer R_1 to R_3 <u>and</u> break in **one *direct*** supply line L1 to L3 to this phase	The **one** remaining line voltage supplies the **one** remaining consumer; Power consumption P_{tot} falls by **two-thirds** (example shown: line voltage $U_{L1,L3}$ produces I_{R3})	
5	Break / failure of **one** consumer R_1 to R_3 <u>and</u> break in **one** supply line L1 to L3, that does not ***directly*** feed this phase	The **one** remaining line voltage supplies the **series circuit 2R**; **P_{tot} falls to one-sixth** (example shown: line voltage $U_{L1,L3}$ produces the current in R' = $R_1 + R_2$)	

Table 12.5.2.2: Possible faults in a delta circuit

Measure the voltages and currents as listed in table 12.5.2.3, in the faulty star circuit.

Phase voltage [V]			Line current [mA]			Phase current [mA]			
$U_{L1,L2}$	$U_{L2,L3}$	$U_{L1,L3}$	I_{L1}	I_{L2}	I_{L3}	I_{R1}	I_{R2}	I_{R3}	**Fault:**
									Break in phase R_3
									Break in phases R_1 and R_3
									Break in supply line L1
									(direct) supply line L2 and R_2 defect
									(indirect) supply line L2 and R_3 defect

Table 12.5.2.3: Values measured in a faulty delta circuit

- Calculate P_{tot} for the 5 faulty delta circuits and compare the values with the power consumption of the fully functional circuit in 12.4.2.1.

  P_{tot} of the faultless delta circuit:

  R_3 defect:

  R_1 and R_3 defect:

  Break in supply line L1:

(direct) supply line L2 and R_2 defect:

(indirect) supply line L2 and R_3 defect:

13. Capacitor in an AC Circuit

13.1 Construction and Characteristics of Capacitors

In its simplest form, a capacitor consists of 2 parallel, electrically conductive plates (Fig. 13.1.1, left). An air gap between the plates, acts as an insulator. When a constant voltage U is applied to this capacitor, charge carriers flow between the plates and one plate becomes positively charged, the other negatively charged. As the level of charge increases, an electric field is created between the plates (Fig. 13.1.1, centre). When the voltage across the plates has reached the same as the applied voltage, the flow of current stops.

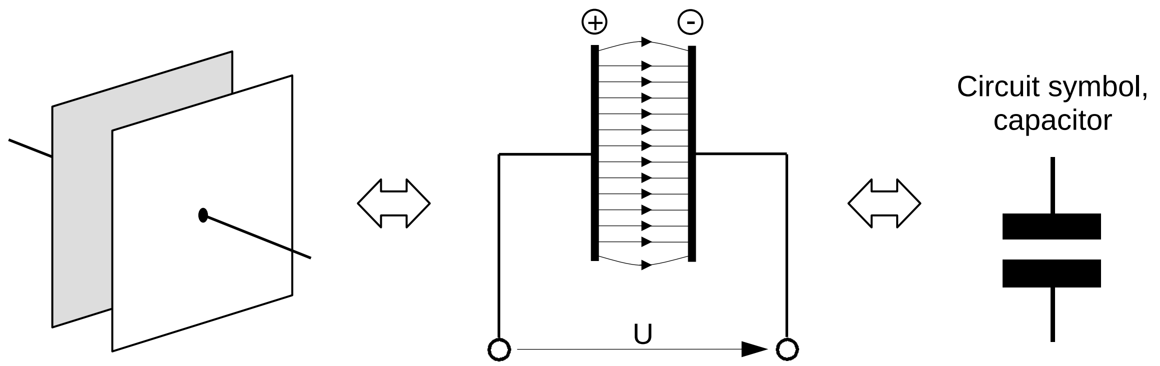


Fig. 13.1.1: Basic construction of a capacitor

The capacitor retains its charge, even when the external voltage is removed. This voltage can be measured at the external connections, on a voltmeter[1]. The amount of the **stored charge Q** depends on the magnitude of the applied **charging voltage U** and the **capacitance C** of the capacitor, i.e.:

$$Q = U \cdot C$$

The **capacitance, C** of the capacitor is determined by its form of construction. The larger the **area of the plates**, the more charge it can store. The smaller the **distance** between the plates, the stronger is the electric field and more charge can be drawn to the plates and stored. Also, the type of insulation between the plates is very significant. This insulation in known as the '**dielectric**'. Some materials support the formation of an electric field between the plates better than others, or than air (free space); these materials have a larger value of dielectric constant ε_r.

The **capacitance, C** is given by:

$$C = \varepsilon_0 \cdot \varepsilon_r \cdot \frac{A}{l}$$

C : Capacitance (Farad)
ε_0 : Absolute permittivity of free space ($8{,}85 \cdot 10^{-12}$ As/Vm)
ε_r : Relative permittivity of the dielectric (dielectric constant)
A : Area of the plates [m^2]
l : Distance between the plates [m]

[1] This measurement is quite difficult using commercial multimeters, because their internal resistance is too small. When attempting to measure the voltage, the capacitor quickly discharges through the R_i of the voltmeter.

When a DC voltage is applied, a charging current flows for only a short time, until the capacitance C of the capacitor is 'filled' with charge carriers. The insulation between the capacitor plates functions as a *break in the DC circuit*. However, if an AC voltage is applied to the capacitor, then the continuous reversal of the polarity of the external voltage results in a continual charge exchange between the plates. This exchange of charge follows the electric field between the plates, that rhythmically changes in strength and direction. An ammeter in the external circuit would indicate the flow of current due to the charge exchange. *For alternating voltages a capacitor does not present any break in the circuit*. However, the flow of alternating current in a capacitor is different to that through an ohmic resistor. There is no dissipation of power at a capacitor! The flow of alternating current in a capacitor is given the term 'reactive current'.

It is logical then, that the resistance offered by a capacitor to the flow of an AC is given the name **capacitive reactance, X_C**. The higher the frequency, f of an applied AC voltage and the larger the capacitance C of the capacitor, the smaller is the value of X_C. Equation:

$$X_C = \frac{1}{\omega \cdot C} = \frac{1}{2 \cdot \pi \cdot f \cdot C}$$

As shown, the capacitance of a capacitor increases with a smaller distance between the plates. In industrially manufactured capacitors the thickness of the dielectric is microscopic. From this it follows that high demands are placed in the insulation resistance R_p of the dielectric so that any residual current, and thus ohmic losses in the capacitor, is kept as small as possible. In practice, the value of R_p is greater than 1 GΩ.

The thin dielectric also limits the *dielectric strength* of the component. In this respect, a differentation is made between two characteristic quantities: The **nominal voltage** is the maximum permissible continuous voltage. The **peak voltage** is the maximum permissible short-term transient value of voltage that the capacitor can withstand without causing any damage to the component.

13.2 Types and Tasks of Capacitors

In electrical engineering and electronics, capacitors have a wide variety of uses:

- Storing energy
- Storing data
- Frequency-dependent resistance
- Introducing a phase shift between voltage and current
- Isolating DC and AC voltages
- Smoothing rectified AC
- Construction of delay elements
- Construction of oscillating circuits and filters

Depending on their use, capacitors are required with special properties or distinct characteristic quantities. Table 13.2.1 summarises common types of capacitor construction.

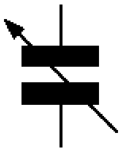
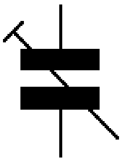
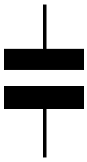
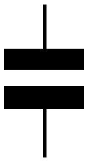
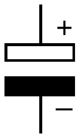
Capacitance	Description	Circuit symbol	Construction	
Variable	Variable capacitor		Capacitance is varied either by: - Plate area - Plate separation or - Dielectric	Precision engineered, intended for frequent, repeated adjustment
	Trimmer			Used seldom or only once, for alignment or tuning processes
Fixed	Ceramic capacitor		Dielectric consists of various ceramic materials (e.g. titanium dioxide, barium titanate) with high dielectric strength and sometimes a higher dielectric constant ε_r (up to 14000)	
	Wound capacitor		Metal and insulating foil (dielectric) are laid out on top of each other and rolled up to give more plate area and thus, a larger capacitance	
	Electrolytic capacitor		The anode (+) is made of metal coated with the insulating dielecric by way of electrolysis. The cathode (-) consists of a paste-like electrolyte. When electrolytic capacitors are used in circuits having a DC component, the correct polarity **must** be observed. **With incorrect connection, there is a danger of explosion!**	

Table 13.2.1: Types of capacitors

13.3 Charge and Discharge of a Capacitor

13.3.1 Principles of Charge and Discharge Processes

By charging / discharging, a capacitor attempts to reach the same value as the external voltage potential present at its terminals. This external voltage could be the output of a voltage source, the output from a votage divider, zero volts (in which case, the capacitor discharges), or any other potential. Charge and discharge current always flow via an ohmic resistor[2], that limits the charge current. In Fig. 13.3.1.1 when S_1 is closed, the capacitor charges via resistor R ($U_C = U$). When S_1 is opened, the capacitor retains the charge (U_C = constant). When S_2 is closed, the capacitor discharges via resistor R. The capacitor now functions itself as a voltage source. Therefore, the discharge current ($I_{dis.}$) flows in the opposite direction (Fig. 13.3.1.1).

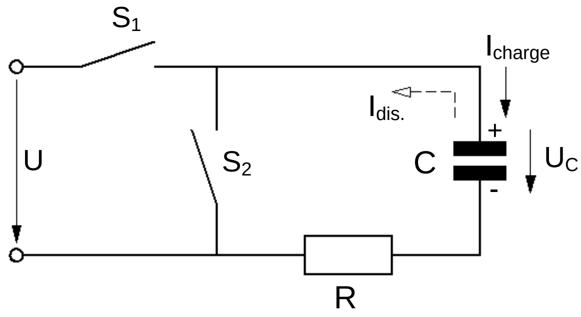


Fig. 13.3.1.1: DC circuit with capacitor C

With a constant charge voltage, the current flow in the circuit and the voltage across the capacitor, follow an **exponential function** (e-function, Fig. 13.3.1.2). The effective **time constant** τ of the curve is given by the product of resistance R in the charge circuit and capacitance C:

$$\tau = R \cdot C$$

C : Capacitance [F]
R : Resistance [Ω]
τ : Time constant [s]

At time 1τ after the start of charging, the voltage at the capacitor (U_C) has reached 0,63 of the maximum value (Fig. 13.3.1.2). After 5-times τ (5τ), the charging process is considered to be finished. The instantaneous value of capacitor voltage u_C is calculated by the equation:

$$u_C = U \cdot \left(1 - e^{-t/\tau}\right)$$

U : Charge voltage [V]
t : Charging time [s]
e : Euler's constant, 2,718

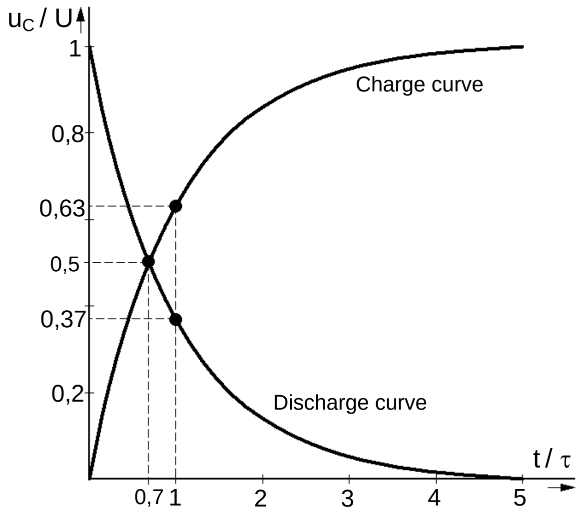


Fig. 13.3.1.2: Charge/discharge curves of a capacitor

The discharge of a capacitor follows the the same e-function (mirrored, Fig. 13.3.1.2). The same time relationships apply as for the charge process: Discharge by 50% after 0,7τ, by 63% (to 0,37·U_C) after 1τ, process end after 5τ. The instantaneous value of capacitor voltage u_C is calculated by the equation:

$$u_C = U \cdot e^{-t/\tau}$$

U : Capacitor voltage [V]
t : Discharging time [s]
e : Euler's constant, 2,718

[2] Even with a fast discharge using a short circuit bridge, there is still an effective minimum resistance – the resistance of the wire bridge.

The flow of current in an RC-circuit, also follows an exponential function $e^{-t/\tau}$ (Fig. 13.3.1.3). The current curves for charge and discharge, are identical. Since the flow of current changes direction during discharge, the equations for the instantaneous values of current differ in their sign, as given below:

Charge: $i_C = \frac{U}{R} \cdot e^{-t/\tau}$ U : Charge voltage [V]

Discharge: $i_C = -\frac{U}{R} \cdot e^{-t/\tau}$ U : Capacitor voltage [V]

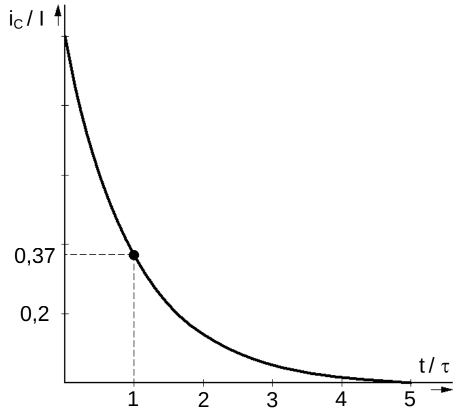


Fig. 13.3.1.3: Charge and discharge current in a capacitor

The charge and discharge response of capacitors is very significant for understanding complex circuits. Here, current, voltage and reactance X_C should be considered together. At the start of charging the flow of current is maximum, whilst at the same time, the voltage at the capacitor is minimum. Therefore, according to Ohm's law X_C is initially, very small. The current is limited by resistor R in the charging path. Towards the end of charging, at almost maximum capacitor voltage U_C and a small charging current, X_C has increased to a very high value and attempts to increase to infinity (∞). It can be seen already, that current and voltage are out of phase.

- The charging/discharging response of a capacitor will now be examined in an exercise. Assemble the circuit in Fig. 13.3.1.4 on the Electronic Circuits Board. If possible, use an analog voltmeter to enable the charging and discharging process at the capacitor to be followed from the deflection of the needle.

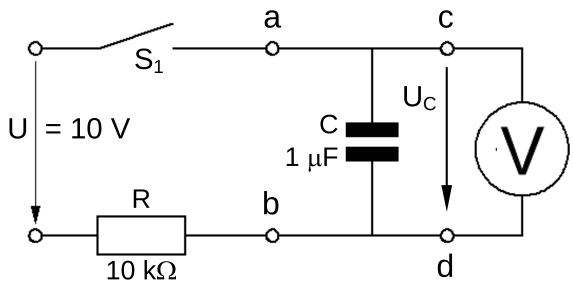


Fig. 13.3.1.4: DC circuit with R and C

- How much time is required by the capacitor C = 1 µF, to charge up to U = 10 V when switch S_1 is closed?

- Now, close the switch and observe the voltage indication. What response do you expect on the voltmeter?

- Open the switch S_1 and observe the indication of the voltmeter. What causes the slow fall of capacitor voltage?

- Assuming the discharge of the capacitor via the test meter, takes 50 s. What is the value of R_i of the voltmeter?

- Now, isolate the charge in the capacitor. Procede as follows:
1. Remove the test meter. 2. Remove the bridge between a and b. – What response is expected of the capacitor?

- Wait one or two minutes. Now, measure the capacitor voltage U_C. What initial value do you expect?

- Calculate the value of charge current at the instant of closing switch S_1 (with a fully discharged capacitor C).

- Is it possible to check this value by measurement on an ammeter (multimeter)?

13.3.2 Reaction of a Capacitor to Square-wave Voltages

A square-wave voltage with a positive amplitude can be considered as a DC voltage, periodically switched on and off (U in Fig. 13.3.2.1 right). If the pulse duration and interpulse period are both at least equal to $5{\cdot}\tau$, then the capacitor C can fully charge and discharge, via resistor R (Fig. 13.3.2.1 left). This results in the typical voltage response across the capacitor C, of a sequence of e-functions (U_C in Fig. 13.3.2.1 right).

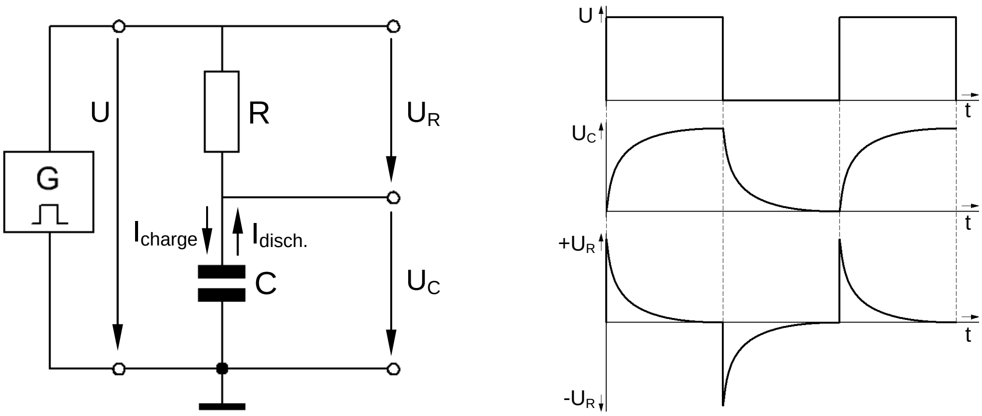


Fig. 13.3.2.1: Reaction of a capacitor to a square-wave voltage

Whilst the capacitor is charging, the voltage difference $U_R = U - U_C$ is dropped across the ohmic resistance. The reason for U_R is the initial maximum, then quickly reducing charging current I_{charge}. For the time of charging, a typical needle pulse (e-function) across R, can be displayed on an oscilloscope. The shorter the time constant $\tau = R \cdot C$, the narrower is the needle pulse.

During the interpulse period, the capacitor C functions as a voltage source, discharging via R with a current flow in the opposite direction ($I_{dis.}$). The discharge occurs with the same time constant τ, so that the second needle pulse produced has the same shape as the first pulse. Due to the current reversal, this needle pulse is negative.

The needle pulses produce heat losses at resistor R. Here, actual ohmic power is dissipated, so-called 'active power'. In contrast, at the capacitor there is only reactive power that does not produce any warming effects.

The response of a capacitor will now be examined using the components shown, together with the input voltage given in Fig. 13.3.2.2.

- Assemble the circuit in Fig. 13.3.2.2 on the Electronic Circuits Board.
- Set the square-wave generator to a peak voltage of $U_p = 4$ V at a frequency of f = 500 Hz.

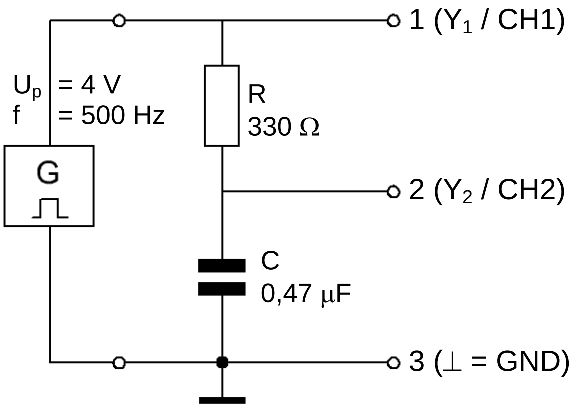
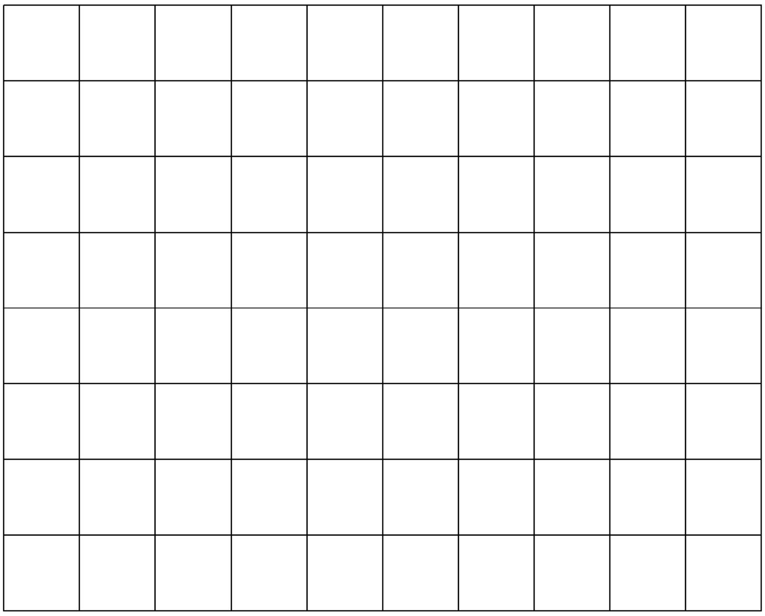


Fig. 13.3.2.2: Square-wave voltage in an RC-circuit

- Connect the oscilloscope as shown. Select the settings on the oscilloscope so that both signals are displayed (one below the other) and a timebase to display at least one complete signal period.
- Draw the signals displayed U and U_C in the chart in Fig. 13.3.2.3.

Fig. 13.3.2.3: Display, U and U_C

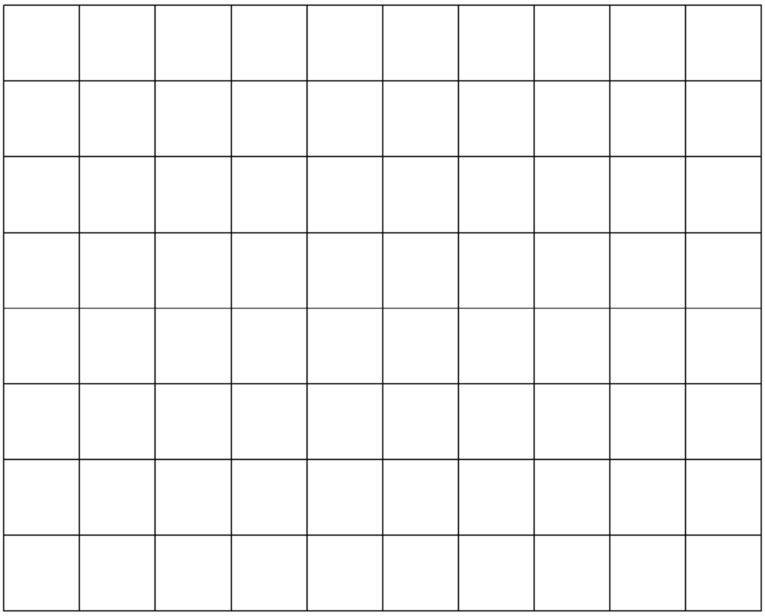


Oscilloscope settings:
X : 0,2 ms/ div.
Y_1 : 2 V/ div., DC
Y_2 : 2 V/ div., DC

- Change the connections of the oscilloscope in Fig. 13.3.2.2, to display the signal across resistor R.
- Draw the signals displayed U and U_R in the chart in Fig. 13.3.2.4.

Oscilloscope settings:
X : 0,2 ms/ div.
Y_1 : 2 V/ div., DC
Y_2 : 2 V/ div., DC

Fig. 13.3.2.4: Display, U and U_R



- From the waveforms drawn, determine the time constant τ as accurately as possible. Check your result by calculation.

τ From waveforms:

τ By calculation:

- What is the voltage at the capacitor (u_C) 0,4 ms after the start of charging? Measure the value from the waveforms or on the oscilloscope screen. Check your measurement by calculation.

  u_C Measured after 0,4 ms :

  u_C Calculated:

- What current is flowing 0,2 ms after the capacitor starts discharging? Determine the current from the waveforms drawn in Fig. 13.3.2.4 or directly from the oscilloscope screen. Check your measurements by calculation.

  $i_{dis.}$ After 0,2 ms:

  $i_{dis.}$ Calculated:

- At what time does the capacitor store its maximum charge and how large is this maximum charge?

- In the circuit of Fig. 13.3.2.2 the capacitor C is replaced with one of C = 1 μF. What effect has this change in the circuit?

- Check your statement by measurement. Display the voltage across the capacitor C = 1μF (R unchanged).

13.4 Capacitor with a Sine-wave Voltage

13.4.1 Phase Shift between Current and Voltage

It has already been seen that with a square-wave voltage applied to a capacitor, current and voltage at the capacitor, are out of phase. The current immediately reaches a fairly high value, whilst in comparison, the voltage gradually increases as the capacitor is charged. In other words, the current leads the voltage.

With a sine-wave voltage applied, the capacitor charges in rhythm with the periodic time T from a positive to a negative peak value (U_C in Fig. 13.4.1.1). The current leads this process by a quarter-period (I_C in Fig. 13.4.1.1). The current is at a maximum when the voltage has just cut the zero axis. The phase shift between current and voltage is 90°.

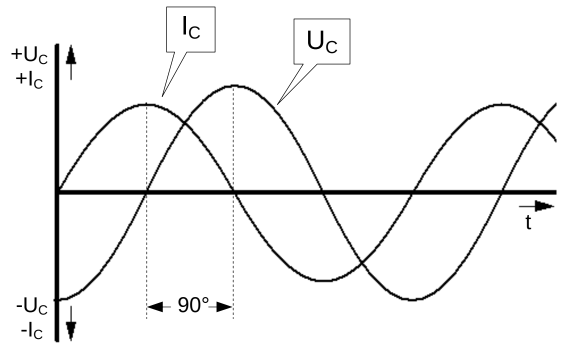
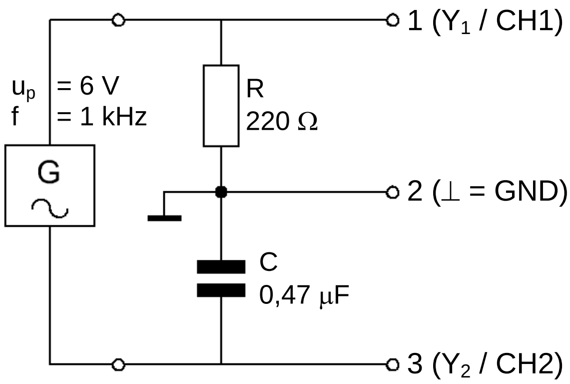


Fig. 13.4.1.1: Phase shift between current and voltage at a capacitor

The phase shift between current and voltage will now be proved in a circuit as shown in Fig. 13.4.1.2.

- Assemble the circuit in Fig. 13.4.1.2 on the Electronic Circuits Board.

Fig. 13.4.1.2: Exercise circuit to show the phase shift between I and U



Since the changes of current and voltage at an ohmic resistor are always proportional to each other, U_R (Y_1 / CH1) can be used for showing the phase of the current I_C in the circuit.

- Set the signal generator to a sine-wave voltage $u_{pp} = 12$ V at a frequency of f = 1 kHz.
- Connect the 2-channel oscilloscope as shown in Fig. 13.4.1.2.

By adjusting the 0-axis (GND) between R and C, the voltages U_R and U_C can both be displayed on the 2-channel oscilloscope. However, the negative voltage U_C (Y_2 / CH2) has a 180° phase shift. For measuring the correct phase relationship, one channel of the oscilloscope must be operated in the 'inverted' mode.

- Display the voltage waveforms U_R and U_C on the oscilloscope. Adjust the oscilloscope for a display of least 2 periods of the sine-wave.
- Draw the signal waveforms displayed in the chart, Fig. 13.4.1.3.

Oscilloscope settings:
X : 0,1 ms/ div.
Y_1 : 1 V/ div., AC
Y_2 : 2 V/ div., AC, inverted

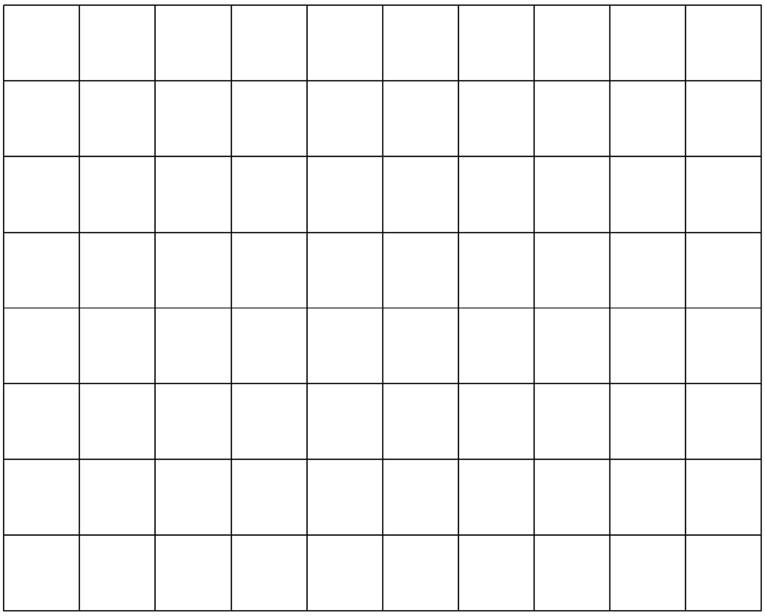


Fig. 13.4.1.3: Phase relationship between current voltage at the capacitor

- From the signal waveforms, measure the periodic time T, the frequency f and the angle of phase shift φ between current and voltage.

13.4.2 Capacitive Reactance, X_C

When an sine-wave voltage is applied to a capacitor, the capacitor is continually charged and discharged. This corresponds to a periodic build-up and decay then a build-up with the opposite polarity, of the electric field between the plates of the capacitor. The current flowing at this time, leads the voltage by 90° and determines the physical properties of the capacitor as a resistance, that limits the flow of current. Since at this resistor, there is no thermal power dissipated, it is known as '**capacitive reactance, X_C**.

The magnitude of the capacitive reactance X_C is inversely proportional to the capacitance C of the capacitor and the frequency f of the applied sine-wave voltage. Equation:

$$X_C = \frac{1}{2 \cdot \pi \cdot f \cdot C}$$

C : [F]
f : [1/s]
X_C : [Ω]

With a given capacitor current I_C and a known capacitor voltage U_C, Ohm's law can be used for calculation. Equation:

$$X_C = \frac{U_C}{I_C}$$

U_C : [V]
I_C : [A]
X_C : [Ω]

The response of the capacitive reactance X_C will now be examined using the circuit shown in Fig. 13.4.2.1. The current flowing in the capacitor can be calculated from the voltage drop U_R across the resistor R = 1 kΩ (U_R and I_C in phase).

By adjusting the 0-axis (GND) between R and C, the voltages U_R and U_C can both be displayed on the 2-channel oscilloscope.

- Assemble the circuit in Fig. 13.4.2.1 on the Electronic Circuits Board.
- Set the signal generator to a sine-wave voltage u_{pp} = 12 V at an initial frequency of f = 1 kHz.
- Connect the 2-channel oscilloscope as shown in Fig. 13.4.2.1.

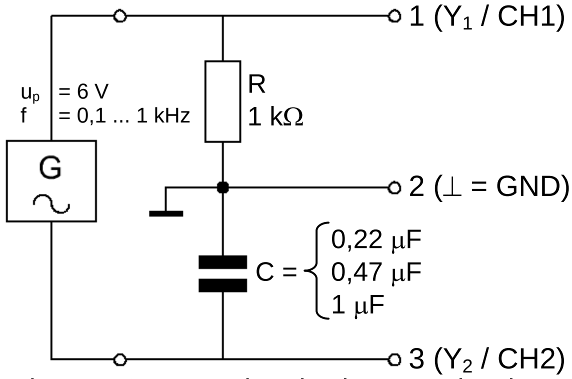


Fig. 13.4.2.1: Exercise circuit to examine the relationship between X_C, f and C

- Measure the peak-to-peak values of the voltages U_C and U_R for the 3 capacitors listed in table 13.4.2.2, at the given frequencies. Enter the measured values in table 13.4.2.2.

Table 13.4.2.2: Capacitive reactance X_C for different capacitors at various frequencies

f [kHz]		0,1	0,2	0,3	0,4	0,6	1
U_C [V_{pp}]	0,22 µF						
	0,47 µF						
	1 µF						
U_R [V_{pp}]	0,22 µF						
	0,47 µF						
	1 µF						
I_C [mA_{pp}]	0,22 µF						
	0,47 µF						
	1 µF						
X_C [kΩ]	0,22 µF						
	0,47 µF						
	1 µF						

- Calculate the peak-to-peak currents I_C and enter the values in table 13.4.2.2.
- Calculate the values of X_C and enter the values in table 13.4.2.2.
- Plot the calculated values of capacitive reactance X_C in the chart (Fig. 13.4.2.3). Join the points plotted and draw the characteristics X_{Cn} = f(f) for the 3 capacitors.

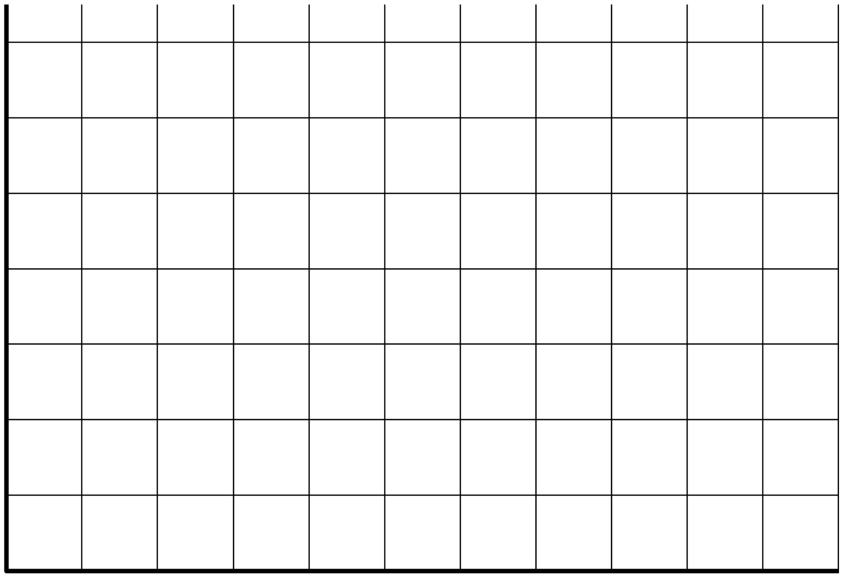


Fig. 13.4.2.3:
Characteristics $X_C = f(f)$

- Check the values measured for X_C = f (100Hz) for the capacitor C = 0,47 µF by calculation.

- How do you explain the deviation between measured and calculated values for X_C = f(100 Hz ; 0,47 µF)?

- What tendency is shown by the capacitive reactance X_C of a capacitor C = 0,01 µF (= 10 nF) at very high (> 1 MHz) and very low (< 100 Hz) frequencies?

- What value must a capacitor have, to present a capacitive reactance of X_C = 50 Ω at a frequency of f = 14,5 kHz? Check your calculated result by measurements, using the circuit in Fig. 13.4.2.1.

13.4.3 Active and Reactive Power at a Capacitor

In an *ideal capacitor,* there is no **active power** in the form of dissipated heat. But, there is a flow of energy between the capacitor plates in the form of charge carriers that the capacitor stores as a voltage which can be measured, or as an electric field between the plates (Fig. 13.1.1). With a later discharge or a reversal of charge, this energy in the capacitor, is again available. The electric field decays and drives the discharge current. Thus, current and voltage at a capacitor, produce only a **reactive power.**

In real capacitors however, there are *ohmic losses* that must be taken into account. On the one hand, there is the frequency-dependent **leakage current** through the dielectric, which like all insulators, does not have an infinitely large resistance. Leakage currents are responsible for a slow discharge of the energy stored in the capacitor. In practice, these losses are more significant in electrolytic capacitors that have a measurable insulation resistance due to their form of construction. This is more apparent on old electrolytic capacitors, where the leakage current is higher. In applications where a capacitor is used for storing information, the very minute leakage currents must also be taken into account, because they limit the length of time that the data can be stored. For all other circuits, the second form of ohmic loss, the frequency-dependent **displacement current**, is significant. The dielectric supports the build-up of the electric field and in this way, increases the capacitance of the capacitor (see section 13.1). This is achieved whereby the polarity of the molecules in the dielectric material is reversed in time with the frequency. The higher the frequency, the more often are the polarity changes in the dielectric and the higher the flow of displacement current, and the greater is the power consumed by the electric field. Thus, at high frequencies, the warming of a capacitor can be physically felt.

The active power losses are combined and represented by the **power loss resistance R_p** imagined to be connected in parallel to the capacitor (Fig. 13.4.3.1), through which the sum of all leakage currents flow I_{Rp}. Since the capacitor current I_C leads the leakage current I_{Rp} by 90°, the relevant vectors show a loss angle, δ (Fig. 13.4.3.1).

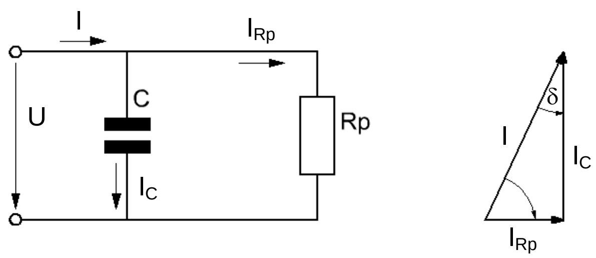


Fig. 13.4.3.1: Losses at a capacitor

The active power losses give the **loss factor d**, that should be as small as possible (< 1):

$$d = \tan\delta = \frac{I_{Rp}}{I_C} = \frac{X_C}{R_p}$$

d : Loss factor, [no dimensions]
δ : Loss angle [°]
I_{Rp} : Leakage current [A]
I_C : Reactive current [A]
X_C : Reactance [Ω]
R_p : Loss resistance [Ω]

The active power consumed by a capacitor is the result of unwanted but unavoidable losses. They must be accepted within reason, due to the physical limits in the manufacture of capacitors.

The **reactive power Q** at a capacitor is given by the reactive current and the capacitor voltage. It can be represented as a multiplication of the instantaneous values of i_C and u_C in a line chart, with the phase relationships (Fig. 13.4.3.2).

The reactive power Q is calculated from:

$$Q_C = U_C \cdot I_C \quad or$$

$$Q_C = \frac{U_C^2}{X_C} \quad or$$

$$Q_C = I_C^2 \cdot X_C$$

Q_C: Reactive power, [Ω]
U_C: [V]
I_C : [A]
X_C: [Ω]

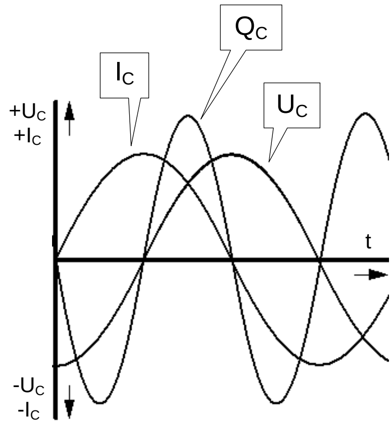


Fig. 13.4.3.2: Reactive power

In an example circuit, the response of current and voltage will be displayed on an oscilloscope, for one complete period of a sinusoidal voltage. The waveforms displayed will then be drawn in a chart. Finally, the waveform of the reactive power curve will be plotted from the values measured and the curve drawn in the chart.

Assemble the circuit in Fig. 13.4.3.3 on the Electronic Circuits Board.

The current I_C is determined from the voltage U_R measured on Y_1 / CH1 of the oscilloscope. For measuring the correct phase relationship, one channel of the oscilloscope must be operated in the 'inverted' mode.

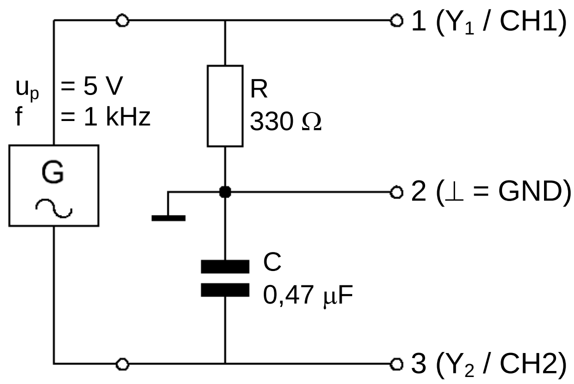


Fig. 13.4.3.3: Exercise circuit for measuring the capacitive reactance, Q_C

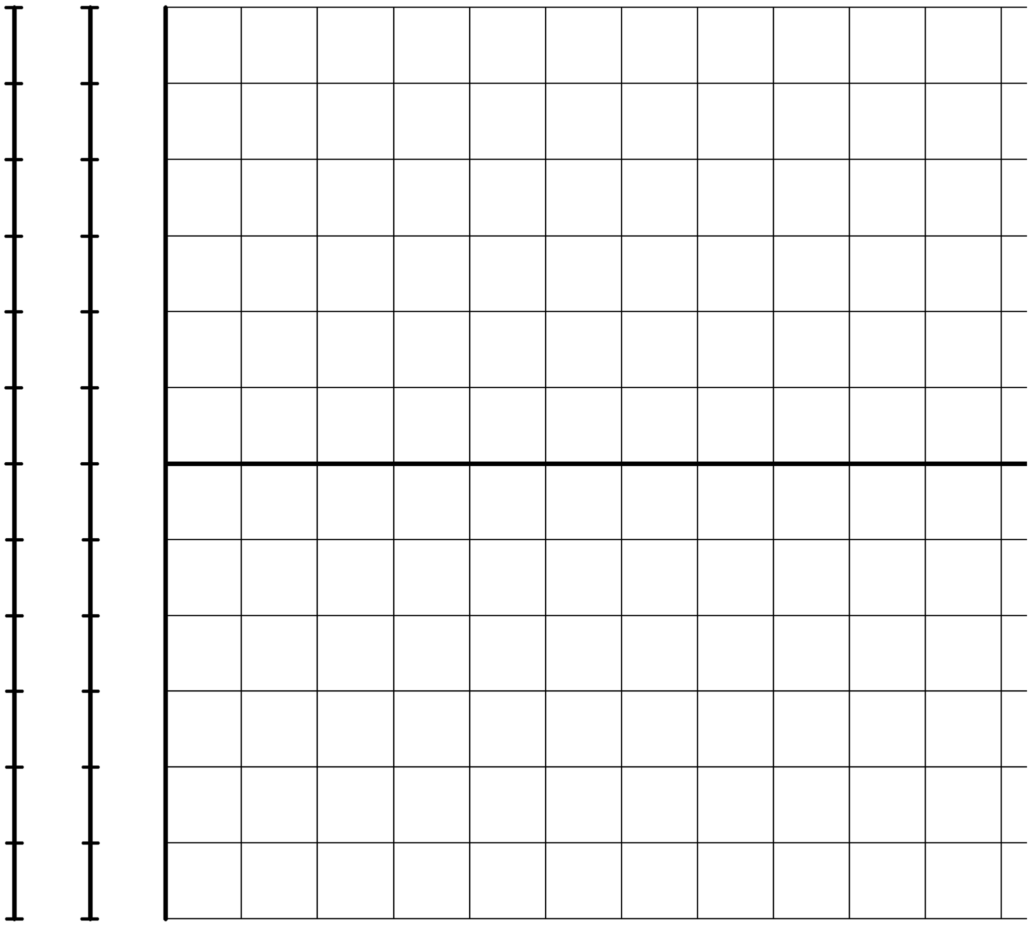
- Set the signal generator to a sine-wave voltage u_{pp} = 10 V at a frequency of f = 1 kHz.
- Connect the 2-channel oscilloscope as shown in Fig. 13.4.3.3.
- Display the voltage waveforms U_R and U_C on the oscilloscope.
- Measure the instantaneous values of the voltages u_R and u_C at the times given in table 13.4.3.4. Enter the values in the table.
- Calculate the instantaneous values of capacitor current i_C from u_R and enter the values in the table.
- Calculate the reactive power q_C from u_C and i_C, at the times given in table. Complete the table with your results of calculation.

Table 13.4.3.4: Instantaneous values, exercise circuit in Fig. 13.4.3.3

t [ms]	**0**	**0,1**	**0,2**	**0,25**	**0,4**	**0,5**	**0,6**	**0,75**	**0,8**	**0,9**	**1**
u_R [V]											
u_C [V]											
i_C [mA]											
q_C [mW]											

- Sketch the current curve I_C = f(t), the voltage curve U_C = f(t) and the power curve Q_C = f (t), as accurately as possible, in the chart given in Fig. 13.4.3.5).

Fig. 13.4.3.5: Voltage U_C, reactive current I_C and reactive power Q_C waveforms at the capacitor



13.5 Capacitors Connected in Series

13.5.1 Response of Capacitors Connected in Series

In a **series connection** of capacitors, the *plate separation* l (Fig. 13.5.1.1) is effectively increased. The total capacitance is therefore, less than the smallest single capacitor (Fig. 13.5.1.2). Equation:

$$C_{tot} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}}$$

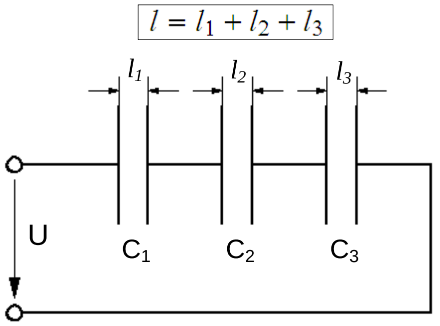
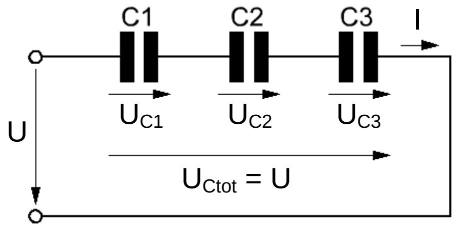


Fig. 13.5.1.1: Series connection of capacitors

With only 2 capacitors connected in series, the equation simplifies to:

$$C_{tot} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

Fig. 13.5.1.2: Series connection of capacitors



The same current flows through all capacitors and the individual voltages across the capacitors, add to give the total voltage U. In some applications, this is referred to as a **capacitive voltage divider** (Fig. 13.5.1.2).

$$U = U_{tot} = U_{C1} + U_{C2} + U_{C3} + \ldots + U_{Cn}$$

Similarly, at a given freqency f, the total reactance X_{Ctot} is given by:

$$X_{C\,tot} = X_{C1} + X_{C2} + X_{C3} + \ldots + X_{Cn}$$

13.5.2 Practical Proof of the Capacitor Response in a Series Circuit

The statement, 'the total capacitance of a series circuit is always less than the smallest single capacitor', will now be proved by voltage and indirect current measurements on a multimeter.

- Assemble the circuit in Fig. 13.5.2.1 on the Electronic Circuits Board.
- Set the function generator to U_{rms} = 4 V, f_{Sine} = 1 kHz.
- Measure the values listed in table 13.5.2.2 with a voltmeter and enter the values in the table.

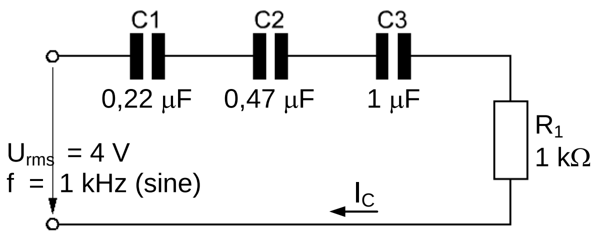


Fig. 13.5.2.1: Measurements on a series circuit of capacitors

Table 13.5.2.2: Measurements on a series circuit of capacitors

All voltages in [V]					
U_{rms}	U_{C1}	U_{C2}	U_{C3}	U_{Ctot}	U_{R1}

- From the measured values and using Ohm's law, calculate first the capacitor current I_C, and then the reactances X_{Cn} and X_{Ctot}.

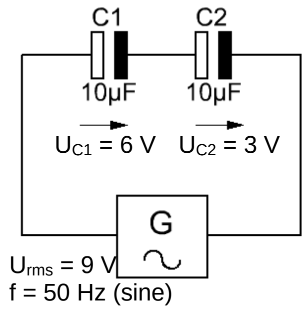
- Calculate the individual capacitances C_n and the total capacitance C_{tot}. Use the calculated values of reactance.

- Check the flow of alternating current I_C on an ammeter. Compare the measured and calculated values.

- Calculate C_{tot} as a check, using the nominal values of the 3 capacitors.

In the circuit of Fig. 13.5.2.3, a fault has occurred in the voltage distribution between the capacitors. What could have been the cause?

Fig. 13.5.2.3: Series circuit, 2 equal value electrolytic capacitors



- What is the value of C_{tot} in a faulty circuit as in Fig. 13.5.2.3?

13.5.3 Exercise Assembly of a Capacitor Series Circuit on the Electronic Circuits Board

An output voltage is set on the function generator of the Electronic Circuits Board of $U_{rms} = 4$ V at a frequency of f = 1 kHz (Fig. 13.5.3.1). The voltage divider of R_1 and capacitors C_1, C_2, C_3 is arranged so that the voltmeter cam measure each individual voltage, without any hinderance. The voltage $U_{Ctot} = 3{,}07$ V is measuredacross the 3 capacitors (Fig. 13.5.3.1). For a check measurement of the current, the ammeter can be inserted in the circuit in place of one of the bridges to the generator connections.

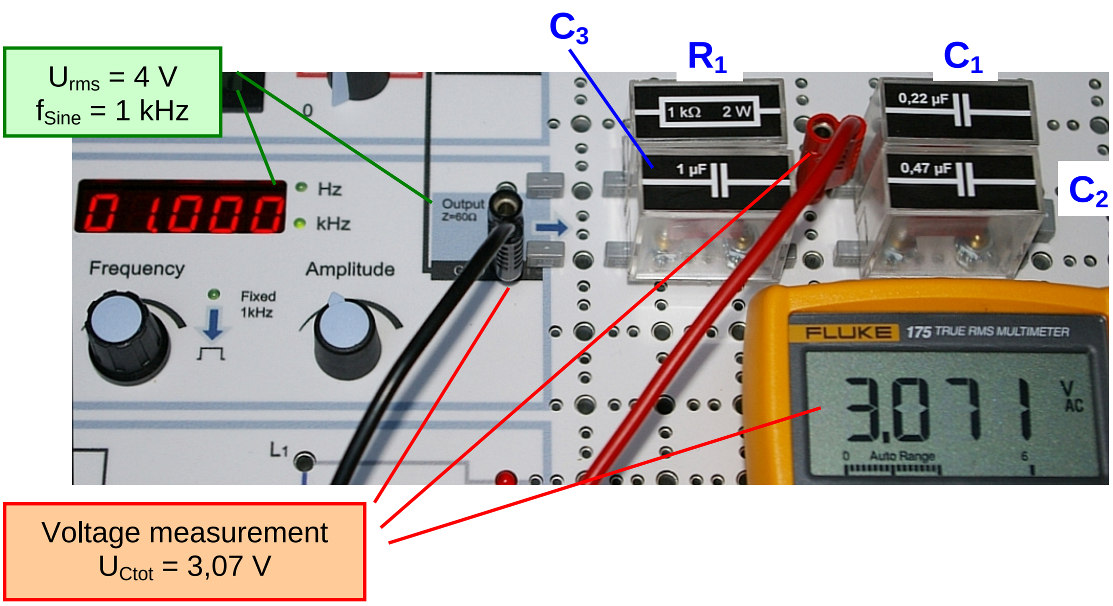


Fig. 13.5.3.1: Exercise assembly for the series connection of capacitors

13.6 Capacitors Connected in Parallel

13.6.1 Response of Capacitors Connected in Parallel

In a **parallel circuit** of capacitors, the *plate area A* becomes larger. Therefore, the capacitance C is given by the sum of all single capacitors (Fig. 15.6.1.1). Equation:

$$C_{tot} = C_1 + C_2 + C_3 + \ldots + C_n$$

The total current I_{Ctot} is divided between the individual capacitor branches, according to the values of the individual capacitors.

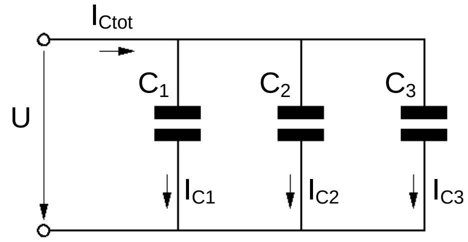


Fig. 13.6.1.1: Parallel circuit of capacitors

The capacitor voltage U_C across all parallel connected capacitors, is the same. The capacitive reactance of the whole circuit, X_{Ctot} is less than the smallest single reactance, X_{Cn}.

$$X_{C_{tot}} = \frac{1}{\frac{1}{X_{C1}} + \frac{1}{X_{C2}} + \frac{1}{X_{C3}} + \ldots + \frac{1}{X_{Cn}}}$$

13.6.2 Practical Proof of the Capacitor Response in a Parallel Circuit

The statement, 'the total capacitance of a parallel circuit of capacitors is equal to the sum of the single capacitances' will now be proved by voltage and current measurements on a multimeter.

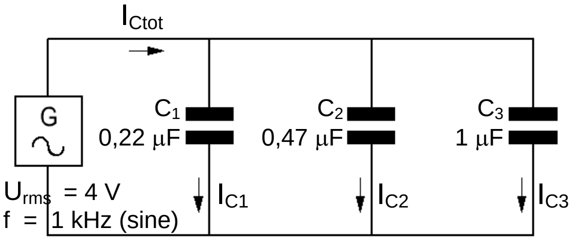


Fig. 13.6.2.1: Measurements on a parallel circuit of capacitors

- Assemble the circuit in Fig. 13.6.2.1 on the Electronic Circuits Board.
- Set the function generator to U_{rms} = 4 V, f_{Sine} = 1 kHz.
- Measure the voltage U and the total current I_{Ctot}.

U = ... ; I_{Ctot} = ...

- Calculate the total capacitance of the parallel circuit using the measured values.

- As a check, calculate the capacitor values from their nominal valus and compare the result with the value of C_{tot} calculated from the measured values.

14. Coil in an AC Circuit

14.1 Construction and Characteristics of Coils

Current flowing through a conducting material (e.g. copper wire), generates a *magnetic field*, the *lines of force* of which can be considered as concentric circles about the centre of the wire (Fig. 14.1.1). The term 'magnetic field' is often used (and is also used in this handbook), but strictly speaking, the real term is **magnetic flux density B** or **magnetic induction**. The direction of the magnetic flux ('lines of force') around the wire is given by the *right hand rule*: If the thumb of the right hand points in the direction of the current flow, then the fingers bent around the wire, point in the direction of the magnetic flux.

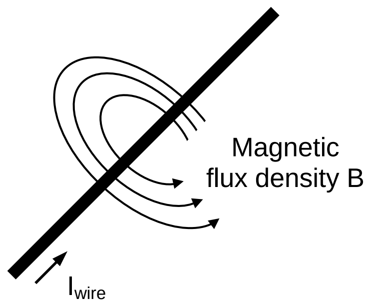


Fig. 14.1.1: Magnetic flux density B

A coil is created when a conductor is wound to form a spring-like body. The shape of the coil causes a concentration of the magnetic flux B. Fig. 14.1.2 shows the orientation of the magnetic flux in and around, a cylindrical coil of wire. In electronic circuits, either of the circuit symbols shown in Fig. 14.1.2 right, can be used. The top symbol is used mainly for high-energy, low frequency applications (e.g. electric motors or transformers); the symbol below this, is used in applications for higher frequencies with a lower power (e.g. oscillatory circuits or small coupling transformers).
Coils react to a change in current flow that causes a build-up or decay of their magnetic field. The reaction is always opposite to the cause, thus:

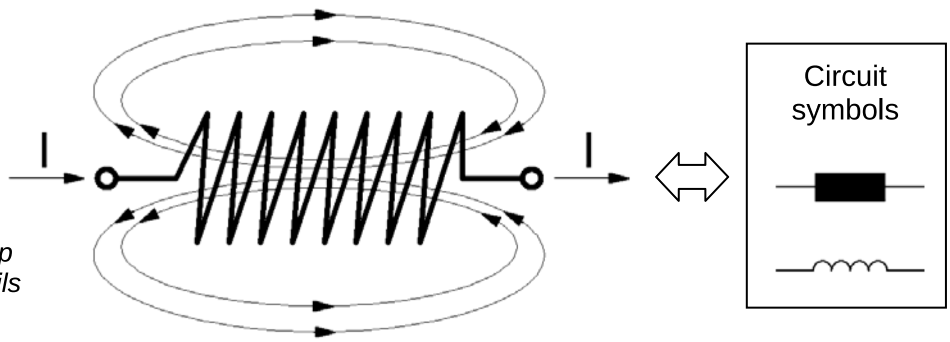


Fig. 14.1.2: Field build-up and circuit symbols of coils

- An increase in current produces a **mutual induction U_L** in the coils that opposes the external voltage. This voltage cannot be measured, but it has the effect on the current, of producing a decay of the magnetic field. The coil consumes energy from the circuit to build up the magnetic field, that the coil stores as magnetic flux.
- If the current in a circuit decreases, then the coil generates a voltage by mutual induction that attempts to maintain the current flow. In other words, it 'pushes' current into the circuit. The energy for this process originates from the magnetic field, that in turn, decays by the same amount.

The reaction of a coil to changes in the current, depends on its **inductance L**. The larger the inductance, the greater is the effect of the mutual induction of the coil in the circuit. The inductance L is given by the properties of the coil, *number of turns*, *cross sectional area and length of the coil*, *and material* of the conductor.

A decisive factor is also whether the coil is air-spaced or has a *core* in the centre, the material of which supports the magnetic flux. An example is shown in Fig. 14.1.3, a small so-called cross-wound coil for high frequency applications in the region of 300 to 3000 kHz. It is wound on a plastic former with an iron-dust core in the centre. By screwing the core in or out, the inductance of the coil can be altered for the purposes of tuning.

Fig. 14.1.3:
Example of a cross-wound coil

In the calculation of inductance, the number of windings is taken as a square of the number. The other terms or factors, in the equation can only be approximately estimated. Therefore, these factors are combined to give the **coil constant A_L.**

Inductance: $$L = N^2 \cdot A_L$$

L : Inductance, Henry [H]
N : Number of windings [no dimensions]
A_L : Coil constant [H]

Coil constant:
(cylindrical coil) $$A_L = \frac{\mu_0 \cdot \mu_r \cdot A}{l}$$

A : Cross-sectional area [m^2]
l : Length [m]
μ_0 : Magnetic field constant $1{,}257 \cdot 10^{-6}$ [Vs/Am]
μ_r : Permeability [no dimensions]

For a *cylindrical* coil, the coil constant A_L is valid with the above equation. Determining the magnitude of A (cross-sectional area) and l (length), depends on whether the coil has a core or not:

- *Without a core,* the area and length refer to the actual coil. Also, the permeability number μ_r is omitted from the equation.
- *With a core,* the area and length of the core as well as the permeability number μ_r for the material, must be inserted in the equation.

The permeability number μ_r in a vacuum is unity ('1'), with air, almost unity and increases when special core materials are used, up to a 6-figure value.

Due to the effects of mutual induction in the coil, *alternating currents* produce an **inductive reactance X_L**, where no active power is dissipated. The coil does consume energy however, from the circuit for the build-up of the magnetic field, but with the later decay of the field, feeds this energy back into the circuit.

The magnitude of the inductive reactance depends on the frequency and the inductance; equation:

$$X_L = \omega \cdot L = 2\pi \cdot f \cdot L$$

X_L : Inductive reactance [Ω]
ω : Angular frequency [1/s]
f : Frequency [1/s]
L : Inductance [H]

14.2 Types and Tasks of Coils

In electrical engineering and electronics, coils are required in a multitude of very different applications:

- Energy storage
- Electric drives (electromotors)
- Converting other forms of energy to electrical energy (generators)
- Transformation of AC voltages and current (transformers)
- Isolating electrical circuits (coupling transformers)
- Switching large currents by way of a smaller current (relays, contactors)
- Selection of frequency ranges (filters)
- Generating oscillations (oscillatory circuits)

Table 14.2.1 Summary of basic types of coils and applications.

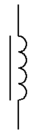
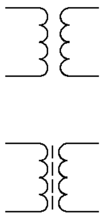
Inductance	**Description**	**Circuit symbol**	**Construction**
Variable	Adjustable inductance		For single or seldom alignemt processes (high frequency techniques)
Fixed	Fixed inductance		Coil without core, air-spaced, wound on a former or encapsulated; $\mu_r \sim 1$
	Coil with core		Coil with core for improving the magnetic flux by increasing the inductance; $\mu_r \neq 1$
	Transformer		Two coils (primary & secondary windings), coupled via a common magnetic field; the transformation of energy is determined by the ratio of the coil windings. For low-frequency uses; for energy transfer, always with core, for high-frequency uses, also without core.

Table 14.2.1: Types of coils

14.3 Reaction of a Coil to Voltage Changes

14.3.1 On and Off Switching Processes at a Coil

The current in a coil changes only when the current is switched on and off. The change produced in the magnetic flux generates a *self-induced e.m.f. (or voltage)*. Its direction is such that it maintains the existing state of the magnetic field. When the current is switched on, the self-induction effect then opposes the build-up of the magnetic flux. At the instant of switch-on, the mutually induced voltage opposes the input voltage, thus there is no flow of current. With the initial rapid increase in current flow, the magnetic field in the coil increases and the effects of mutual induction are reduced. Finally, the current in the circuit is limited by the ohmic resistance of the coil, or any other resistor in the circuit. (Fig. 14.3.1.1).

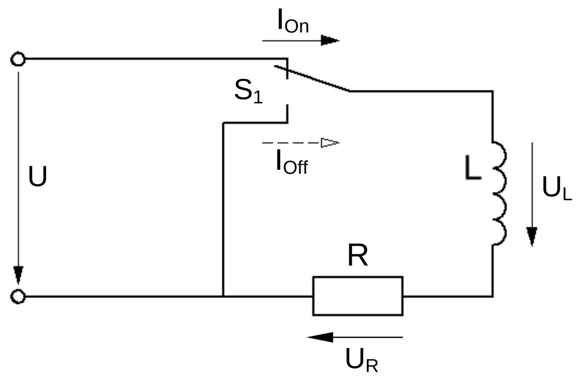


Fig. 14.3.1.1: DC circuit with coil, L

Current and voltage at the coil, both follow an *e-function* (Fig. 14.3.1.2 shows the current curve). The same applies to the switch-off process. At the instant of switching off, an opposing self-induced voltage delays the decay of current. The field energy in the coil, drives the current in the same direction, through the circuit. The decay of current follows a similar e-function as before (Fig. 14.3.1.2). Finally, both coil and resistor are without voltage and there is no flow of current; the field energy in the coil is converted at the resistor, to heat.

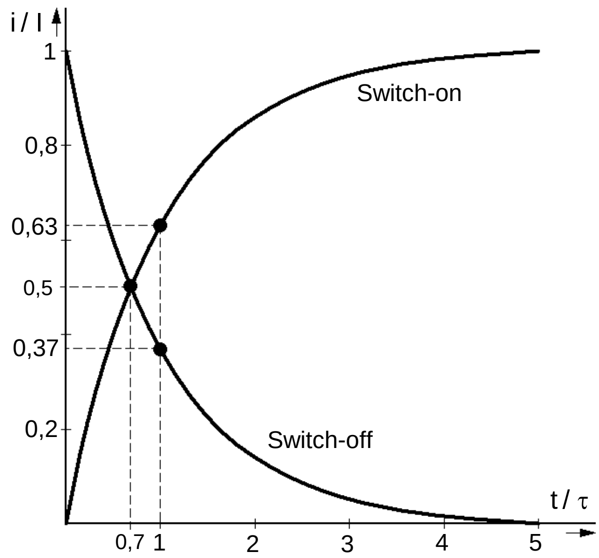


Fig. 14.3.1.2: On and Off switching curves at the coil

The time response of a coil, corresponds to that of a capacitor. The time constant τ is given by the ratio of inductance L to resistance R:

$$\tau = \frac{L}{R}$$

L : Inductance [H]
R: Resistance [Ω]
τ : Time constant [s]

At 1·τ after switch-on, the current in the coil has reached 0,63-times its maximum value (Fig. 14.3.1.2). After 5·τ, the field build-up is complete and the current flow is at maximum.

The instantaneous value of current i_L in the coil, is given by:

$$i_L = I_{max} \cdot \left(1 - e^{-t/\tau}\right)$$

I_{max} : U/R [A]
t : Switch on time [s]
e : Euler's number: 2,718

The decay of coil voltage U_L after switch-on, is given by:

$$u_L = U \cdot e^{-t/\tau}$$

U : Maximum coil voltage [V]

The decay of current after switch-off, follows a mirrored e-function (Fig. 14.3.1.2). The same time relationships apply as for the switch-on process: Current decays by 50% after 0,7·τ; by 63% (to 0,37·I_{max}) after 1·τ; process end after 5·τ.

The instantaneous value of current i_L in the coil, is given by:

$$i_L = I_{max} \cdot e^{-t/\tau}$$

I_{max} : U/R [A]
t : Switch off time [s]
e : Euler's number: 2,718

The negative sign should be remembered for the decay of the coil voltage U_L after switch-off:

$$u_L = -U \cdot e^{-t/\tau}$$

U : Maximum coil voltage [V]

Familiarity with the reponse of the coil to sudden changes in voltage, is important for understanding more complex circuits. The following relationships exist between current, voltage and reactance X_L of the coil : Immediately after switching on a voltage, there is only a minimum flow of current whilst the voltage across the coil reaches its maximum value. Thus, according to Ohm's law, X_L is very large. The coil blocks the flow of current. Towards the end of the build-up of the field, at almost maximum coil current I_L and a small residual voltage U_L, X_L has fallen to a very small value and is still reducing towards zero. From this description, it can be recognised the current and voltage at the coil (as seen previously for a capacitor), are out of phase.

14.3.2 Reaction of a Coil to Square-wave Voltages

A square-wave voltage can be considered as a DC voltage, periodically switched on and off. If the pulse duration t_i is at least equal to 5·τ, then the current through the coil and thus the voltage at the resistor, can increase to their maximum values, following an e-function. In the interpulse period (if tp ≥ 5·τ), the current in the coil I_L and voltage U_R again, fall to zero (I_L/U_R in Fig. 14.3.2.1 right).

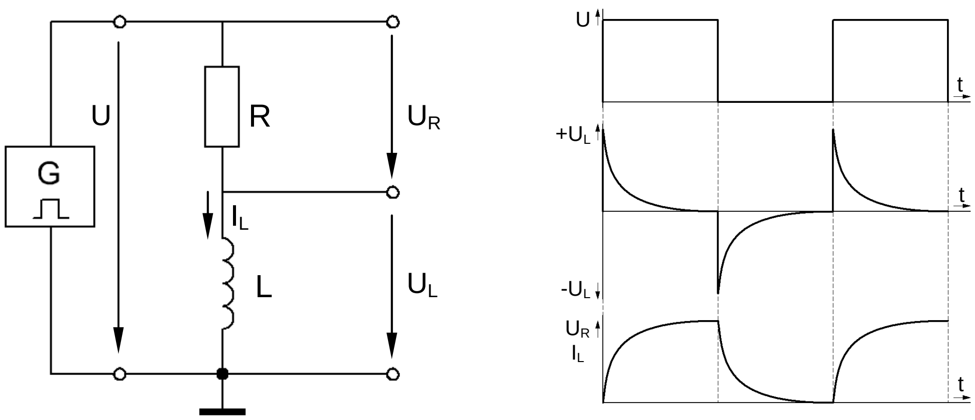


Fig. 14.3.2.1: Reaction of a coil to a square-wave voltage

The voltage drop across a coil, is given by $U_L = U - U_R$ in the form of needle pulses. The shorter the time constant $\tau = L / R$, the narrower are the needle pulses. The negative needle pulses in the interpulse period, are the result of the opposing self-induced voltage that attempts to maintain the flow of current in the coil.

The response of a coil will now be examined using the components shown, together with the input voltage given in Fig. 14.3.2.2.

- Assemble the circuit in Fig. 14.3.2.2 on the Electronic Circuits Board.
- Set the square-wave generator to a peak voltage of $U_p = 5$ V at a frequency of f = 800 Hz.
- Connect the oscilloscope as shown in Fig. 14.3.2.2. Adjust the oscilloscope so that both signals are displayed, one above the other with at least one complete period of the signal.

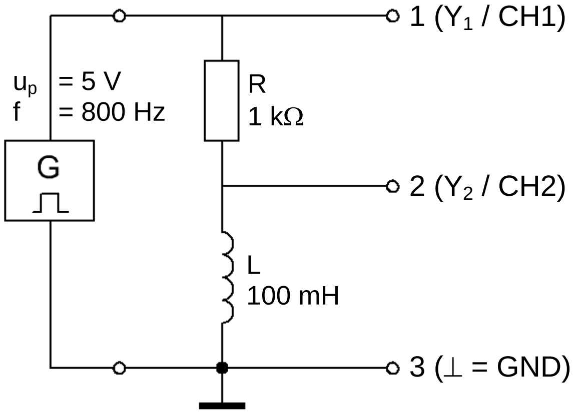
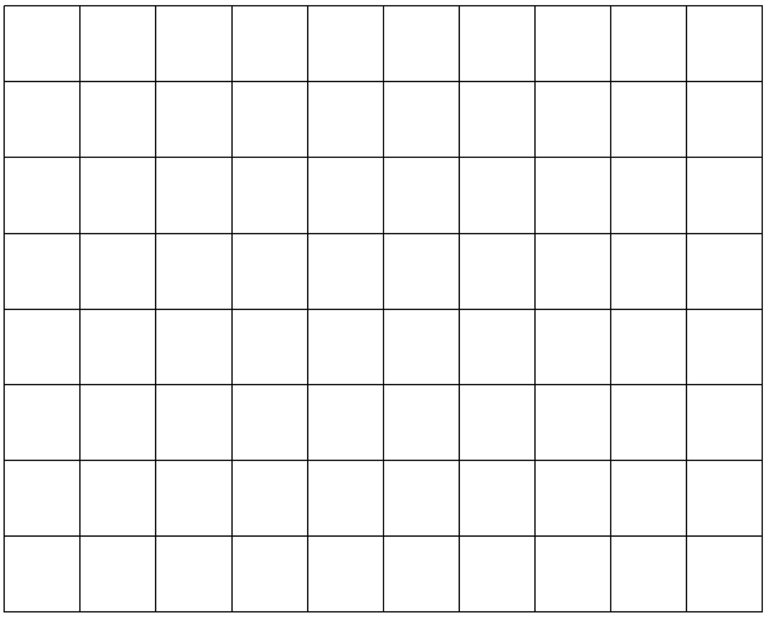


Fig. 14.3.2.2: Square-wave voltage in an LC-circuit

- Draw the signals displayed of U and U_L in the chart in Fig. 14.3.2.3.

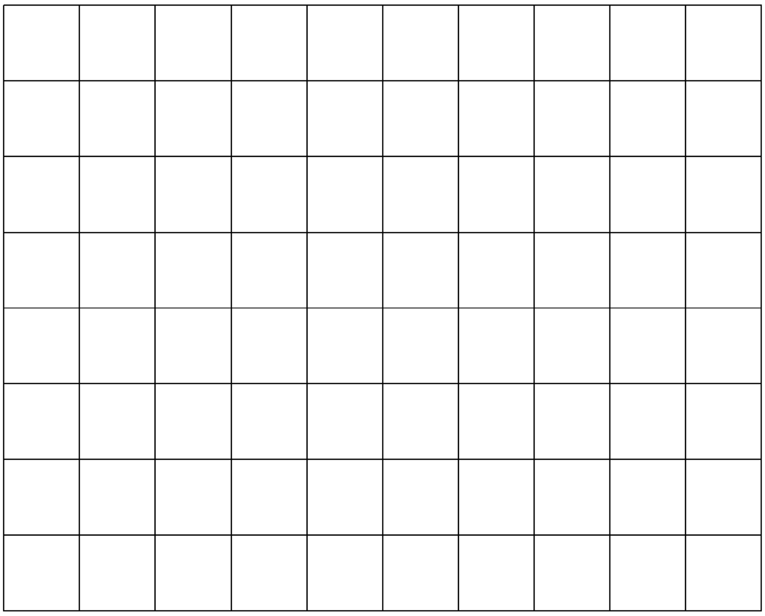
Fig. 14.3.2.3: Display, U and U_L

Oscilloscope settings:
X : 0,2 ms/ div.
Y_1 : 5 V/ div., DC
Y_2 : 2 V/ div., DC



- Exchange R and L in the circuit, to display the signal across resistor R.
- Draw the signals displayed of U and U_R ($\rightarrow I_L$) in the chart in Fig. 14.3.2.4.

Fig. 14.3.2.4:
Display, U and U_R ($\rightarrow I_L$)



Oscilloscope settings:
X : 0,2 ms/ div.
Y_1 : 2 V/ div., DC
Y_2 : 2 V/ div., DC

- From the waveforms drawn, determine the time constant τ as accurately as possible. Check your result by calculation.

  τ From waveforms:

  τ By calculation:

- What is the value of current in the coil (I_L), 0,2 ms after the start of the pulse t_i? Determine the value of U_R, by Ohm's law, using the values drawn or read from the oscilloscope ascreen. Check your measurement by calculation (use the equation from section 14.3.1).

  u_R Measured after 0,2 ms / read from screen:

  i_L by calculation:

- Explain the deviation between your calculated value and the measured value of u_R?

- Calculate the inductance L from the value measured for τ.

- What voltage can be measured across the coil, 0,2 ms after the start of the interpulse period t_p? Read the value as accurately as possible, from the oscilloscope screen. Optimise the oscilloscope settings for reading the value accurately.

  U_L = ...

- Check the value read, by calculation.

- At what time has the magnetic field of the coil, reached its full strength?

- In the circuit of Fig. 14.3.2.2, the resistor is replaced by one of R = 220 Ω. What effect has this change to the circuit have, on the time constant and the build-up of the field?

- Check your statement by measurement. Replace the resistor in the circuit of Fig. 14.3.2.2 with one of R = 220 Ω. Display the voltages across the components on the oscilloscope. Compare the voltage waveforms with the results of the measurements in Figs. 14.3.2.3/4).

14.4 Inductance with a Sine-wave Voltage

14.4.1 Phase Shift between Current and Voltage

It has already been seen that with a square-wave voltage, voltage and current at a coil were out of phase. Due to the self-induction, the voltage immediately increases to a maximum, whilst the current increases only after the self-induced voltage has decayed: the current lags the voltage.

With a sine-wave voltage applied, the polarity of the magnetic field reverses, in rhythm with the frequency of the current through the coil (I_L in Fig. 14.4.1.1). The voltage across the coil leads on this process by a quarter-period (U_L in Fig. 14.4.1.1): The voltage is at a maximum when the current cuts the zero axis. Thus, between voltage and current, there is a phase shift of 90°.

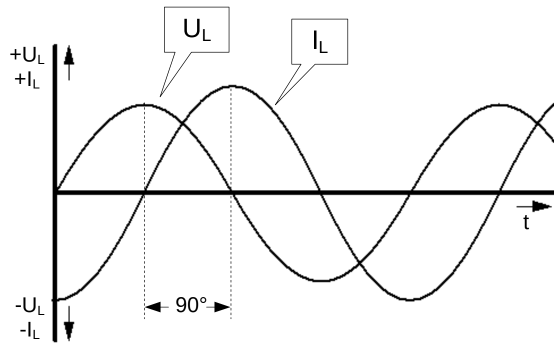
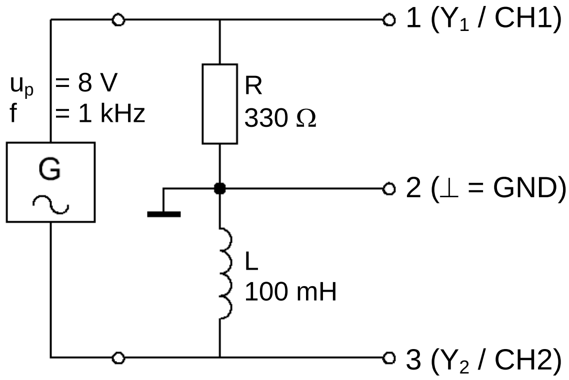


Fig. 14.4.1.1: Phase shift between voltage and current at a coil

The phase shift between current and voltage will now be proved in a circuit as shown in Fig. 14.4.1.2.

- Assemble the circuit in Fig. 14.4.1.2 on the Electronic Circuits Board.

Fig. 14.4.1.2: Exercise circuit to show the phase shift between U and I



Since the changes of current and voltage at an ohmic resistor are always proportional to each other, U_R (Y_1 / CH1) can be used for showing the phase of the current I_L in the circuit.

- Set the signal generator to a sine-wave voltage $u_{pp} = 16$ V at a frequency of f = 1 kHz.
- Connect the 2-channel oscilloscope as shown in Fig. 14.4.1.2.

By adjusting the 0-axis (GND) between R and L, the voltages U_R and U_L can both be displayed on the 2-channel oscilloscope. However, the negative voltage U_L (Y_2 / CH2) has a 180° phase shift. For measuring the correct phase relationship, one channel of the oscilloscope must be operated in the 'inverted' mode.

- Display the voltage waveforms U_R and U_L on the oscilloscope. Adjust the oscilloscope for a display of least 2 periods of the sine-wave.
- Draw the signal waveforms displayed in the chart, Fig 14.4.1.3.

Oscilloscope settings:
X : 0,2 ms/ div.
Y_1 : 2 V/ div., AC
Y_2 : 2 V/ div., AC, inverted

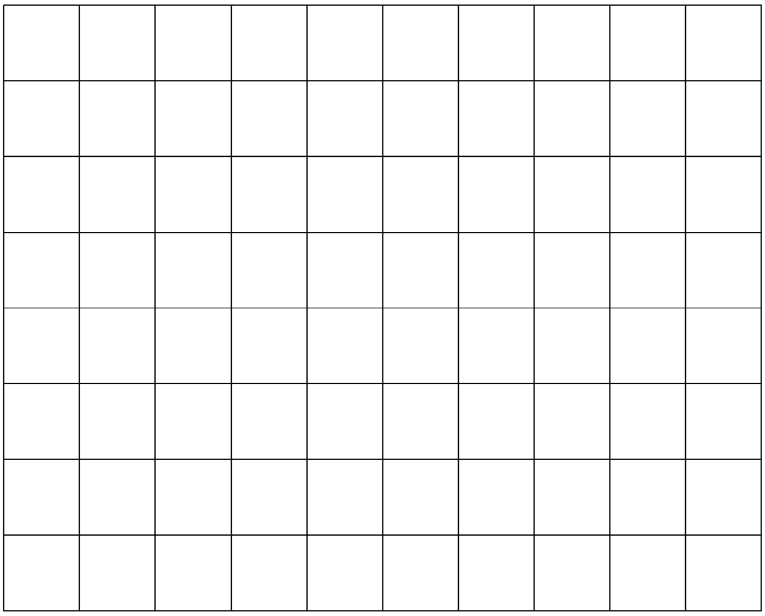


Fig. 14.4.1.3: Phase shift between voltage and current at the coil

- From the waveforms, determine the periodic time T, the frequency f and the angle of phase shift φ between voltage and current.

- In the circuit of Fig. 14.4.1.2, the resistance of 330 Ω in increased to 1 kΩ. What effects do you expect to see on the signals displayed on the oscilloscope? What is the tendency of events and check your considerations by measurement.

14.4.2 Inductive Reactance, X_L

On an inductance, a sinusoidal voltage generates a magnetic field that periodically reverses in polarity. The coil presents a limiting resistance to the current produced that lags on the voltage by 90°. At this resistance, there is no thermal (active) power dissipated, therefore the resistance is known as '**inductive reactance, X_L**'.

The magnitude of the inductive reactance X_L is proportional to the inductance L of the coil and the frequency f of the applied sinusoidal voltage :

$$X_L = 2 \cdot \pi \cdot f \cdot L$$

L : [H]
f : [1/s]
X_L : [Ω]

With a given coil current I_L and a known coil voltage U_L, Ohm's law can be used for calculating the value of X_L :

$$X_L = \frac{U_L}{I_L}$$

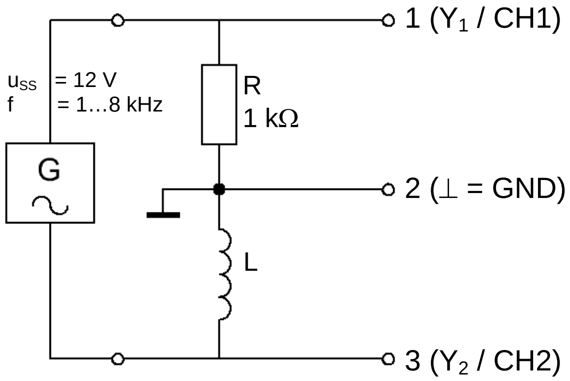
U_L : [V]
I_L : [A]
X_L : [Ω]

The response of inductive reactance X_L will now be examined using the circuit in Fig. 14.4.2.1, assembled on the Electronic Circuits Board. The current flow through the inductance will be calculated from the voltage drop U_R across the resistor R = 1 kΩ (U_R and I_L in-phase).

By adjusting the 0-axis (GND) between R and L, the voltages U_R and U_L can both be displayed on the 2-channel oscilloscope.

- Assemble the circuit in Fig. 14.4.2.1 with L = 100 mH on the Electronic Circuits Board (assembly layout notes and the measurement details, are given at the end of this section).
- Set the signal generator to a sine-wave voltage u_{pp} = 12 V at an initial frequency of f = 1 kHz.
- Connect the 2-channel oscilloscope as shown in Fig. 14.4.2.1.

Fig. 14.4.2.1: Exercise circuit to examine the relationship between X_L, f and L



- Measure the peak-to-peak values of the voltages U_L and U_R at the frequencies given in the table 14.4.2.2. Complete these measurements with 2 different coils:

  Coil 1: L = 100 mH (component in plastic housing)
  Coil 2: Transformer coil N = 900; upper half of the iron core, inserted.

- Enter the values measured in the table.

Table 14.4.2.2: Reactance X_L, inductance L and frequency f

f [kHz]		**1**	**2**	**3**	**4**	**6**	**8**
U_L [V_{pp}]	**N = 900**						
	100 mH						
U_R [V_{pp}]	**N = 900**						
	100 mH						
I_L [mA_{pp}]	**N = 900**						
	100 mH						
X_L [kΩ]	**N = 900**						
	100 mH						

- Calculate the peak-to-peak values of current I_L and enter the values in the table.
- Calculate the values for X_L and complete the table with your results.
- Plot the calculated values for the reactance X_L in the chart (Fig. 14.4.2.3).

- Draw the characteristic $X_L = f(f)$ for both coils.

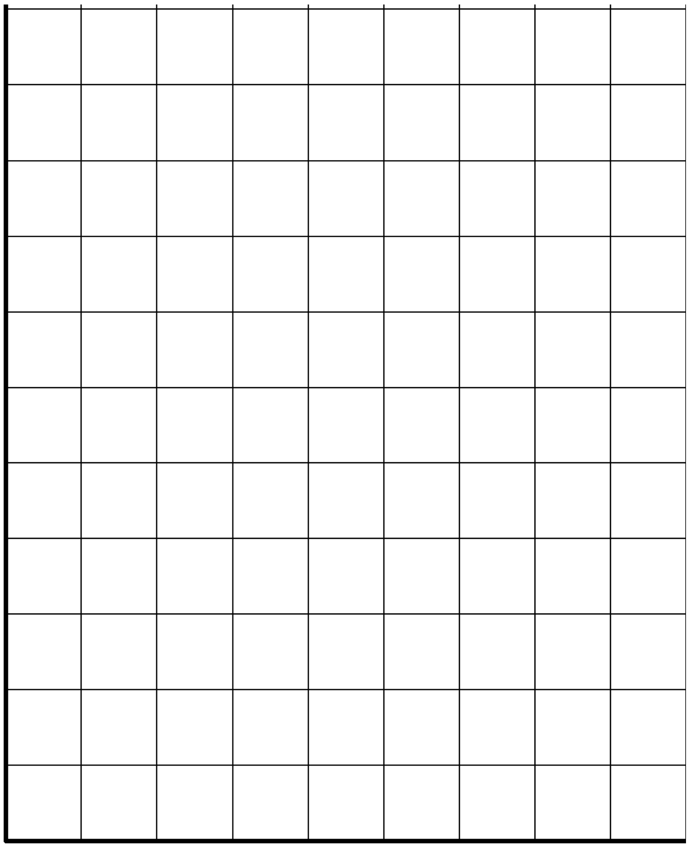


Fig. 14.4.2.3:
Characteristic $X_L = f(f)$

- Check the measured values by calculation at $X_L = f$ (6 kHz) for the coil L = 100 mH.

- Check the nominal value of the coil L = 100 mH by calculation. Use the values measured at 4 kHz.

- Explain the deviation between the two check calculations?

- Calculate the unknown inductance L of the transformer coil (N = 900) from the value measure at $X_L = f$(3 kHz).

- What rules or relationships can be deduced from the shape of the characteristics?

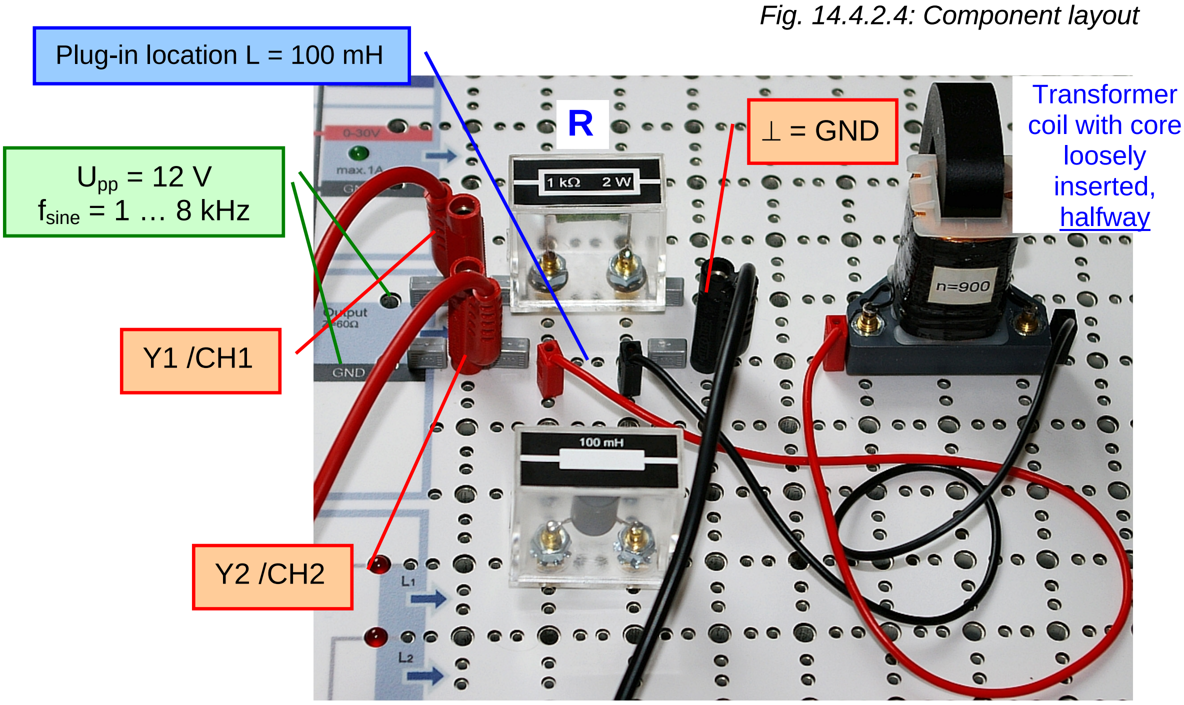
- What tendency does the reactance X_L of a coil L = 0,01 H (= 10 mH), show at very high (> 10 MHz) and very low (< 100 Hz) frequencies?

- What is the inductance L of the transformer coil (N = 900) without a core? Measure the value in a circuit corresponding to Fig. 14.4.2.1. Use a frequency of f = 8 kHz when completing the measurement.

Exercise assembly for examining the relationships between reactance X_L, inductance and frequency.

Fig. 14.4.2.4 shows one possible layout of the components on the Electronic Circuits Board. In the layout, the coil L = 100 mH, after completing the measurements, has been removed from its original position and inserted somewhere else without connections. The transformer coil is then inserted in the circuit by completing its connections.

Fig. 14.4.2.4: Component layout



14.4.3 Active and Reactive Power in a Coil

An *ideal coil* does not dissipate any **active power**. Although the build-up of the magnetic field requires energy, the coil stores the field energy and later when the field decays, the stored energy is again available. So in this respect, voltage and current in an inductance produce only **reactive power**.

In *real coils* however, *ohmic losses* are present, such as:
- Losses in the windings
- Current displacement losses in the windings
- Eddy current losses in the core of the coil
- Losses due to magnetic reversal in the core of the coil
- Eddy current losses in the windings
- Scattering (or leakage) losses.

Losses in the windings are independent of frequency and are caused by the ohmic resistance of the wire used for the windings. The losses can be in the region of a few hundred ohms when the coil consists of many turns of thin wire (the inductance L, increases with the square of the number of windings N).

Current displacement losses increase with the frequency of the AC current, that is forced from inside the wire to the surface area, or skin of the wire. This reduces the effective cross sectional area of the wire, causing the wire resistance to increase. This effect is called the **skin-effect**.

Eddy current losses are produced in the core of the coil and in the windings, by induced voltages. They cause irregular current patterns that produce warmth in the material. These losses increase with the square of the frequency.

Losses due to magnetic reversal in the core of the coil are frequency-dependent and correspond to the power that must be used to align the molecular magnetic particles in the core material.

Scattering (or leakage) losses increase with frequency. They occur when part of the magnetic field of the coil induce eddy currents in metal objects in the vicinity of the coil.

The active power losses are combined and represented by the **power loss resistance R_V** imagined to be connected in series with the inductance (Fig. 14.4.3.1). The loss current produces the voltage drop U_{Rv} across R_v. Since the voltage at the coil (U_L) leads the loss current and thus, the voltage U_{Rv} by 90°, the relevant vectors show a loss angle, δ (Fig. 14.4.3.1).

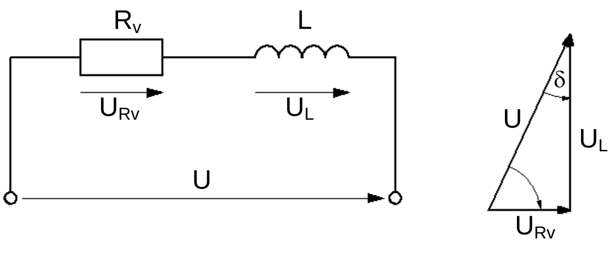


Fig. 14.4.3.1: Losses at a real coil

The active power losses give the **loss factor, d** :

$$d = \tan\delta = \frac{R_V}{X_L} = \frac{U_{Rv}}{U_L}$$

d : Loss factor, [no dimensions]
δ : Loss angle [°]
X_L : Reactance [Ω]
R_v : Loss resistance [Ω]

The **active power (P)** consumed by a coil is the result of unwanted but unavoidable losses. They must be accepted within reason, due to the physical limits in the manufacture of coils.

The **reactive power Q** at a coil is given by the product of coil voltage and reactive current. It can be represented as a multiplication of the instantaneous values of u_L and i_L in a line chart with the phase relationships (Fig. 14.4.3.2).

The reactive power Q_L is calculated from:

$$Q_L = U_L \cdot I_L \quad \text{or}$$

$$Q_L = \frac{U_L^2}{X_L} \quad \text{or}$$

$$Q_L = I_L^2 \cdot X_L$$

Q_L : Reactive power, [W]
U_L : [V]
I_L : [A]
X_L : [Ω]

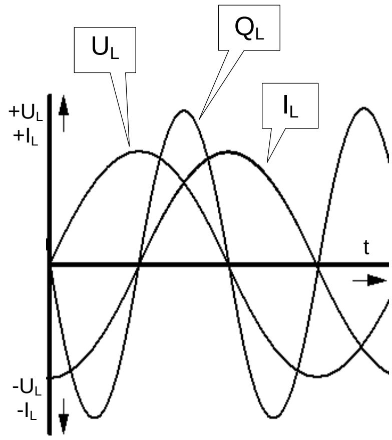


Fig. 14.4.3.2: Reactive power, Q_L

In an example circuit, the response of current and voltage will be displayed on an oscilloscope, for one complete period of a sinusoidal voltage. The waveforms displayed will then be drawn in a chart. Finally, the waveform of the reactive power curve will be plotted from the values measured and the curve drawn in the chart..

- Assemble the circuit in Fig. 14.4.3.3 on the Electronic Circuits Board.

The current I_L is determined indirectly from the voltage U_R measured on Y_1 / CH1 of the oscilloscope. Because the earth point (GND) is taken between R and L in the circuit, one channel of the oscilloscope must be operated in the 'inverted' mode to display the correct phase relationship.

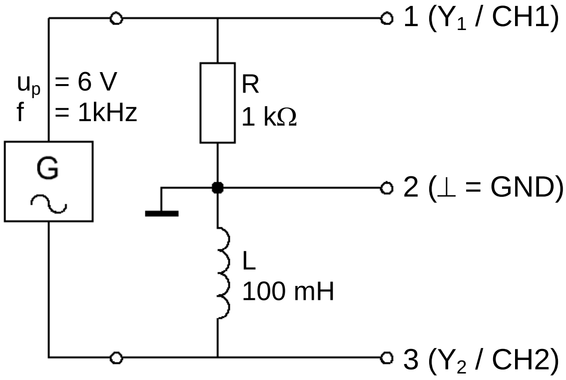


Fig. 14.4.3.3: Exercise circuit for measuring the inductive reactance, Q_L

- Set the signal generator to a sine-wave voltage u_p = 6 V at a frequency of f = 1 kHz.
- Connect the 2-channel oscilloscope as shown in Fig. 14.4.3.3.
- Display the voltage waveforms U_R and U_L on the oscilloscope.
- Measure the instantaneous values of the voltages u_R and u_L at the times given in table 14.4.3.4. Enter the values in the table.
- Calculate the instantaneous values of current in the coil i_L from u_R and enter the values in the table.

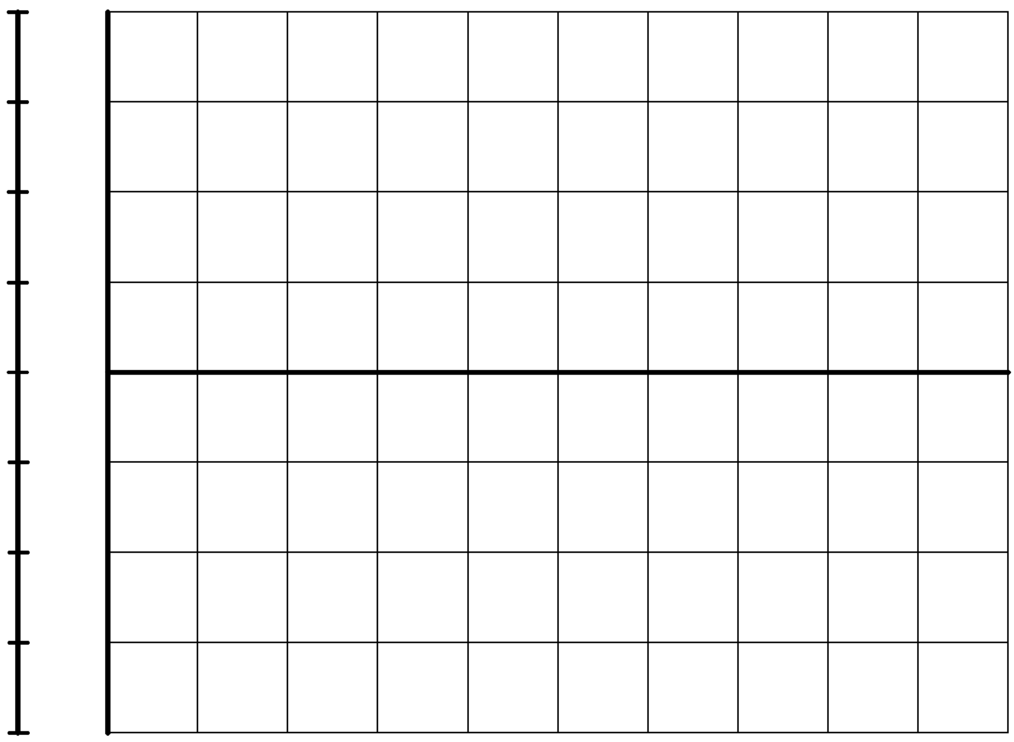
- Calculate the reactive power q_L from u_L and i_L at the times given in table. Complete the table with your results of calculation.

Table 14.4.3.4: Instantaneous values, exercise circuit in Fig. 14.4.3.3

t [ms]	**0**	**0,1**	**0,2**	**0,25**	**0,4**	**0,5**	**0,6**	**0,75**	**0,8**	**0,9**	**1**
u_R [V]											
u_L [V]											
i_L [mA]											
q_L [mW]											

- Sketch the voltage curve $U_L = f(t)$, the current curve $I_L = f(t)$ and the power curve $Q_L = f(t)$ as accurately as possible, in the chart given in Fig. 14.4.3.5.

Fig. 14.4.3.5: Waveforms of voltage U_L, reactive current I_L and reactive power Q_L at the coil



14.5 Coils Connected in Series

14.5.1 Response of Coils Connected in Series

In a **series connection** of coils (Fig. 14.5.1.1), the change of current ΔI/Δt acts in all individual coils at the same time. Thus, at each coil a self-induced voltage is produced of the same polarity, that acts in opposition to the current change. The self-induced voltages in the series connected coils, add to give the total voltage.

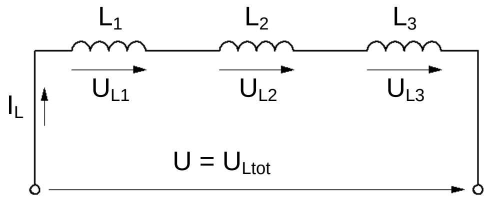


Fig. 14.5.1.1: Coils connected in series

$$U = U_{tot} = U_{L1} + U_{L2} + U_{L3} + \ldots + U_{Ln}$$

The total inductance L_{tot} is given by the sum of the individual inductances :

$$L_{tot} = L_1 + L_2 + L_3 + \ldots + L_n$$

Also, the total inductive reactance X_{Ltot} is given by the sum of the individual inductances :

$$X_{ltot} = X_{L1} + X_{L2} + X_{L3} + \ldots + X_{Ln}$$

14.5.2 Practical Proof of the Coil Response in a Series Circuit

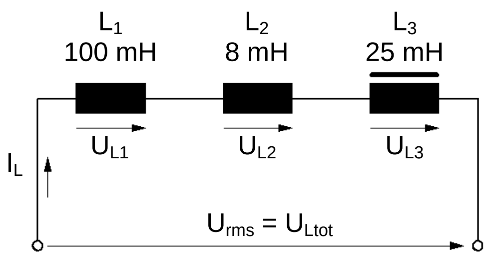
The statement, 'the total inductance of a series circuit is given by the sum of all individual inductances', will now be proved by voltage and current measurements on a multimeter.

- Assemble the circuit in Fig. 14.5.2.1 on the Electronic Circuits Board. Use the following coils:

L_1 : Component in plastic housing; L = 100mH
L_2 : Transformer coil; N = 900; without core; L = 8 mH
L_3 : Transformer coil; N = 900, half-core; L = 25 mH

- To avoid any influence of mutual magnetic coupling, the coils should be located on the Board with as much clearance as possible. Notes on the layout and measurements, will be found in section 14.5.3).

Fig. 14.5.2.1: Measurements on a series circuit of coils



- Set the function generator to U_{rms} = 6 V, f_{sine} = 1 kHz.
- Measure the values of voltage named in table 14.5.2.2 with a multimeter and enter the values in the table.

Table 14.5.2.2: Measurements on a series circuit of coils

Voltage [V]				Current [mA]
$U_{rms} = U_{Ltot}$	**U_{L1}**	**U_{L2}**	**U_{L3}**	**I_L**
6				

- From the values measured and using Ohm's law, determine the reactances X_{Ln} and X_{Ltot}. Calculate the total reactance X_{Ltot} as a check, using 2 different methods.

- Use your results from calculating the reactance, to determine the inductances, L_n and L_{tot}. Calculate the total reactance L_{tot}, using 2 different methods.

- In the circuit of Fig. 14.5.2.1, a transformer coil (N = 900) with half a core inserted. Insert the full core. Now, determine the changed inductance of the transformer coil L_3 and the total inductance of the circuit. Use as few measurements as possible.

- Assume that at coil L_1 in the circuit of Fig. 14.5.2.3, due to a manufacturing fault, a break occurred in the core in the coil. This has had an effect on the build-up and decay of the magnetic field. In such a case, how do the values change of the variables given below? Indicate the tendency in the form of up and down arrows.

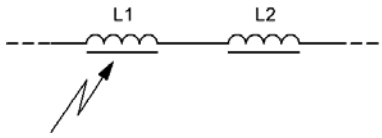


Bild 14.5.2.3: Break in the core

L_1: ; X_{L1}: ; I_L: ; U_{L1}: ; U_{L2}:

Check your assumptions for I_L and U_{Ln} by measurement in the circuit shown in Fig. 14.5.2.1. Simulate the break in the core by removing half of the core in L_3.

14.5.3 Exercise Assembly of a Series Circuit of Coils on the Electronic Circuits Board

To avoid any influence of mutual magnetic coupling between the coils, L_1, L_2 and L_3 should be located on the Electronic Circuits Board with as much clearance as possible (Fig. 14.5.3.1). The actual spacing is limited only by the connection leads (preferably, with 2 mm plugs). The ammeter is in the circuit and forms the connection between L_3 and GND of the sine-wave generator.

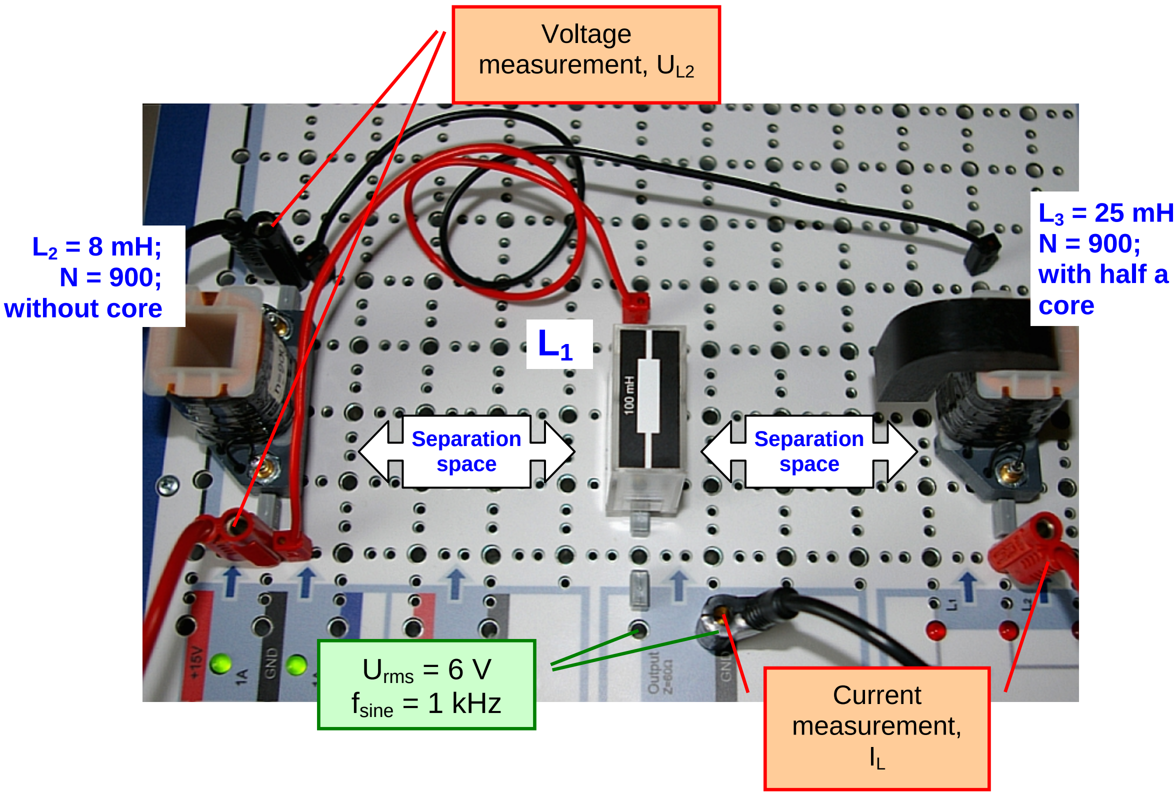


Fig. 14.5.3.1: Component layout for the series circuit of coils

14.6 Coils Connected in Parallel

14.6.1 Response of Coils Connected in Parallel

In a **parallel connection** of coils (Fig. 14.6.1.1) the total inductance L_{tot} is always less than the smallest single inductance :

$$L_{tot} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots + \frac{1}{Ln}} \quad \text{or} \quad :$$

$$\frac{1}{L_{tot}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots + \frac{1}{Ln}$$

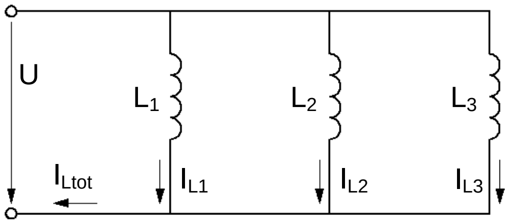


Fig. 14.6.1.1: Parallel connection of coils

The equation can be simplified for a parallel connection of only 2 coils :

$$L_{tot} = \frac{L_1 \cdot L_2}{L_1 + L_2}$$

The total current I_{tot} is divided in the individual branches of coils, inversely proportional to the corresponding inductance in each branch. The inductive reactance of the circuit X_{Ltot} is less than the smallest single reactance X_{Ln}:

$$X_{Ltot} = \frac{1}{\frac{1}{X_{L1}} + \frac{1}{X_{L2}} + \frac{1}{X_{L3}} + \ldots + \frac{1}{X_{Ln}}} \quad \text{or} \quad : \quad \frac{1}{X_{Lges}} = \frac{1}{X_{L1}} + \frac{1}{X_{L2}} + \frac{1}{X_{L3}} + \ldots + \frac{1}{X_{Ln}}$$

14.6.2 Practical Proof of the Coil Response in a Parallel Circuit

The statement, 'the total inductance of a parallel connection of coils is less than the smallest single inductance', will now be proved by voltage and current measurements on a multimeter.

- Assemble the circuit in Fig. 14.6.2.1 on the Electronic Circuits Board. Remember the spacing between individual coils (layout notes, see section 14.5.3).

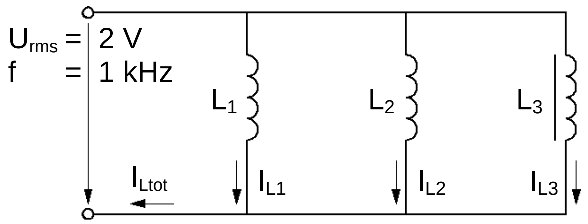


Fig. 14.6.2.1: Measurements on a parallel circuit of coils

- Use the following coils:

  L_1 : Component in plastic housing; L = 100mH

  L_2 : Transformer coil; N = 900; without core; L = 8 mH

  L_3 : Transformer coil; N = 900, with half-core; L = 25 mH

- Set the function generator to U_{rms} = 2 V; f_{sine} = 1 kHz.
- Measure the values of current in table 14.6.2.2 with a multimeter and enter the values in the table.

Table 14.6.2.2: Measurements on a parallel circuit of coils

Current [mA]				Voltage [V]
I_{Ltot}	I_{L1}	I_{L2}	I_{L3}	$U = U_{rms}$
				2

- From the values measured and using Ohm's law, determine the reactances X_{Ln} and X_{Ltot}.

- Use your results from calculating the reactance, to determine the inductances, L_n and L_{tot}. Calculate the total reactance L_{tot}, using 2 different methods.

- How do the values change of the variables given below, when half of the core in L_3 is removed? Indicate the tendency in the form of up and down arrows.

L_3: ; X_{L3}: ; I_{L3}: ; I_L: ; L_{tot}:

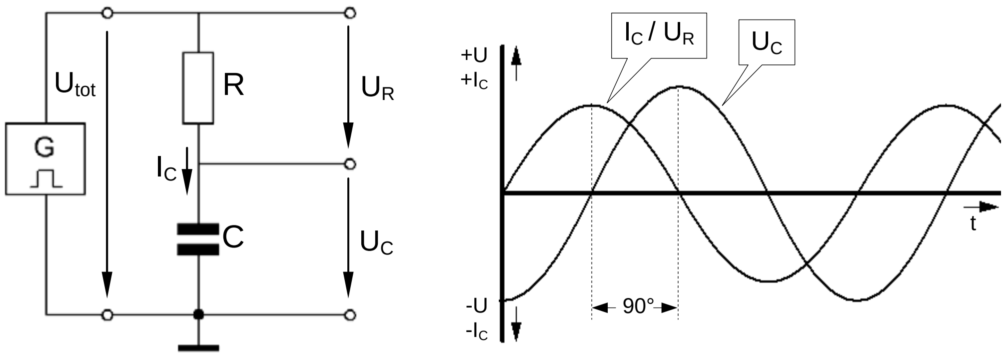
- Check your conclusions for the current changes, by measurement in the circuit of Fig. 14.6.2.1.

15. Combination of Reactive and Active Resistance

15.1 Working with Vector Diagrams

Passive components such as resistors, capcaitors and coils, are often used in a combined circuit. In such a circuit, measured alternating voltages, currents and resistances (reactances), cannot simply be added or subtracted because they exhibit a **phase shift** to one another. This fact was seen for the first time, in the study of the responses of capacitors (Fig. 15.1.1). Current and voltage are 90° out-of-phase. The voltage drop across the resistor R, produced by the current through the capacitor, must lead the capacitor voltage by 90° (Fig. 15.1.1, right).

Fig. 15.1.1: Phase shift between U_C and U_R



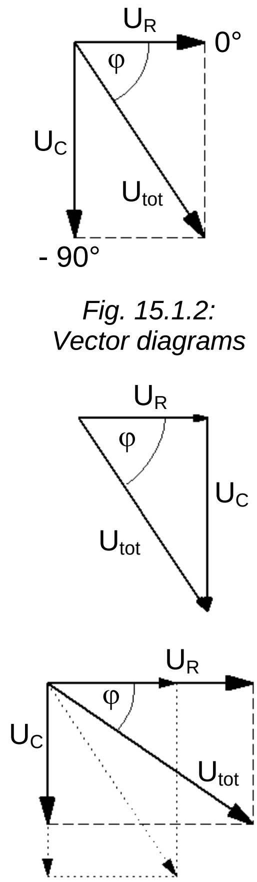


Fig. 15.1.2: Vector diagrams

The **vector diagram** has proved to be very useful (Fig. 15.1.2), for simplifying the task of understanding the relationships between the variables and as an aid in forming a picture of the resultant values. A vector diagram enables quantites to be represented as straight lines to show both magnitude and direction of the quantity. In the case here, the angle from the 360° full circle, indicates the phase relationship and the length of the line, represents the magnitude. Fig. 15.1.2 (top) shows a vector diagram for an RC combination: the vector for the voltage at resistor U_R is horizontal at 0°. The capacitor voltage U_C is drawn with –90° phase shift because it lags. The length of the vectors (= voltage values) form 2 sides of a rectanlge, the diagonal of which represent the total voltage, U_{tot}. This resultant voltage has a phase shift of φ to the 0°-axis (Fig. 15.1.2, top). The final vector diagram is shown in the middle diagram of Fig. 15.1.2.

The phase relationship of the resultant voltage also depends on the magnitude of the voltages (vector length). Due to the reactance X_C, the voltages are frequency-dependent. For example, if the frequency of the applied voltage U_{tot} in Fig. 15.1.1 is increased, the reactance X_C decreases, which is why U_C falls. U_R increases and the phase angle φ becomes smaller (Fig. 15.1.2, bottom).

Since the 2 components R and C in the circuit of Fig. 15.1.1 are *in series*, the same current I, flows through both. A vector diagram of currents, only really makes sense, when used for *parallel circuits*. In parallel circuits, the same voltage is present across all components, which is why a voltage vector diagram is not used.

Irrespective of whether it is series or parallel, all circuits have resistance. The phase angle corresponds to the relevant voltage that is dropped across the resistance. Fig. 15.1.3 shows the resistance vectors for the R-C circuit of Fig. 15.1.1. The active resistance R is on the 0°-axis. The reactance X_C, as the associated capacitor voltage U_C, lags by 90°. Both quantities add vectorially to give the **impedance Z**, the phase angle (φ) of which corresponds to that of the voltage U_{tot}.

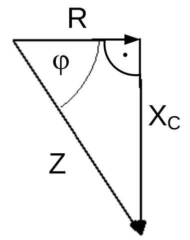


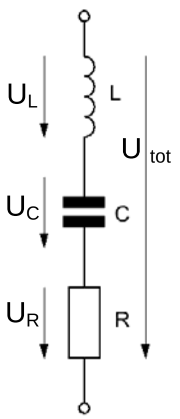
Fig. 15.1.3: Vector diagram, R, X_C, Z

Since vector diagrams form a right-angled triangle (Fig. 15.1.3), the quantities can be calculated by way of the trigonometrical functions sin φ and cos φ as well as from the theorem of Pythagoras. For example, there are several possible methods of calculating the impedance Z in Fig. 15.1.3:

$$\sin\varphi = \frac{X_C}{Z} \Rightarrow Z = \frac{X_C}{\sin\varphi} \quad \textit{or} \quad \cos = \frac{R}{Z} \Rightarrow Z = \frac{R}{\cos} \quad \textit{or}$$

$$Z = \sqrt{R^2 + X_C{}^2}$$

As shown in Fig. 15.1.4, an example of an RLC series circuit, any combination can exist of passive components such as capacitors, inductances and resistors. Since inductive and capacitive components exhibit a phase shift of 180° to each other, the vectors subtract (diagram, Fig. 15.1.4 centre). The remaining vector, together with the ohmic component, forms the resultant quantity (U_{tot} in Fig. 15.1.4, right).



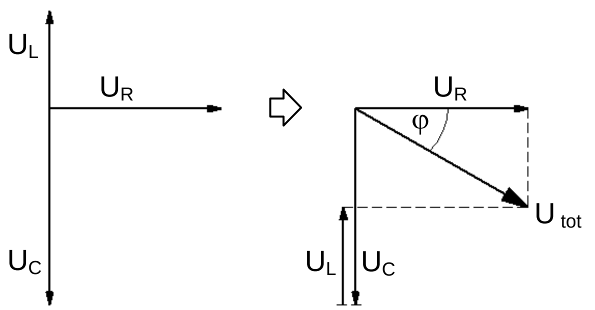


Fig. 15.1.4: Vector diagram, RLC circuit

In the consideration of parallel circuits, complicated resistance equations using calculations with the conductivity value, can be avoided. The **conductance G** is calculated with the **susceptance values B_C** (capacitive) and **B_L** (inductive) to give the **admittance value Y**. Fig. 15.1.5 clearly shows this, in an RL-parallel circuit:

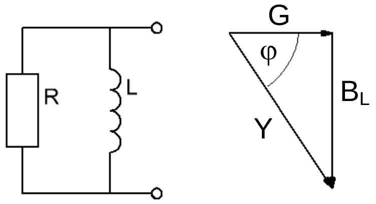


Fig. 15.1.5: Calculating with conductivity values

$$Y = \sqrt{G^2 + B_L{}^2} \quad \Leftrightarrow \quad \left(Z = \frac{1}{Y} \quad ; \quad R = \frac{1}{G} \quad ; \quad X_L = \frac{1}{B_L} \right)$$

If necessary, the resistance values Z, R, X_L and X_C can be calculated at any time, by forming the reciprocal of the relevant conductivity value.

Powers show the same phase shift as that of voltages or currents, from which the power is calculated. The **active power P** and the capacitive or inductive **reactive power (Q_C, Q_L)** are shown by the vector for **apparent power S**. Fig. 15.1.6 shows the relationships in an RL parallel circuit. Apparent power S and active power P are linked by the phase shift angle φ.

Thus, for calculating the power, the trigonometrical functions sin φ and cos φ, explained previously, can be used as well as the theorem of Pythagoras.

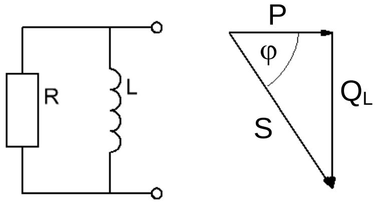


Fig. 15.1.6: Power in RLC circuits

15.2 Series Circuits of Resistor, Capacitor and Coil

The use of vector diagrams and the calculation of characteristic quantities will now be practised with examples on an RLC series circuit.

Note: A layout example for RLC circuits, will be found in section 15.4.

Exercise 1: RL Series Circuit

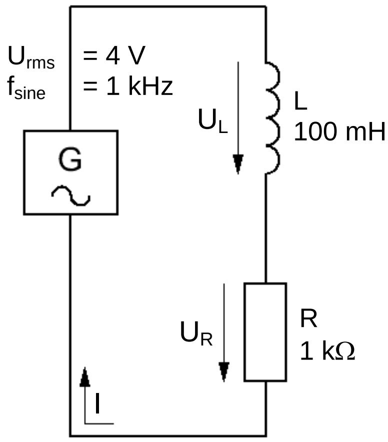
- Assemble the circuit in Fig. 15.2.1 on the Electronic Circuits Board.
- Set the function generator to an output voltage of U_{rms} = 4 V, f_{sine} = 1 kHz.
- Measure the voltages and current in the circuit.

U_{rms} = U_{tot} = ; U_L =

U_R = ; I =

- Calculate the inductive reactance X_L and the resistance R, from the values measured.

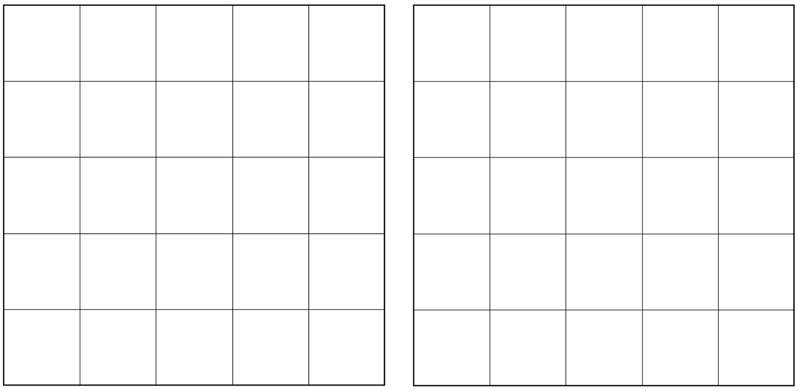
Fig. 15.2.1: RL series circuit



- Draw the vector diagram for the voltages (U_R, U_L, U_{tot}) and the resistances (R, X_L, Z) in the chart, Fig. 15.2.2.

Fig. 15.2.2: Vector diagrams

Scale: 1 cm = 1 V



Scale: 1 cm = 200 Ω

- Determine the impedance Z from the diagram and check the value by calculation, using the values measured.

From the diagram: Z =

Calculation:

- Draw the phase angle φ in the diagrams. Calculate the phase angle from your measured values.

- Connect the inputs of the oscilloscope : Y1: U_{tot} ; Y2: U_R (GND = GND function generator).
- Display the voltages $U_{rms} = U_{tot}$ and U_R. Calculate the expected peak values. Check your measurements with the calculated values.

Measurement: U_{tot} = ... ; U_R = ...

- Adjust the oscilloscope so that to ease measurements, the phase shift between the voltages is clearly seen. Draw one period of the voltage waveform in the chart below (Fig. 15.2.3).

Oscilloscope settings:
X : 0,1 ms/ div.
Y_1 : 2 V/ div., AC
Y_2 : 2 V/ div., AC

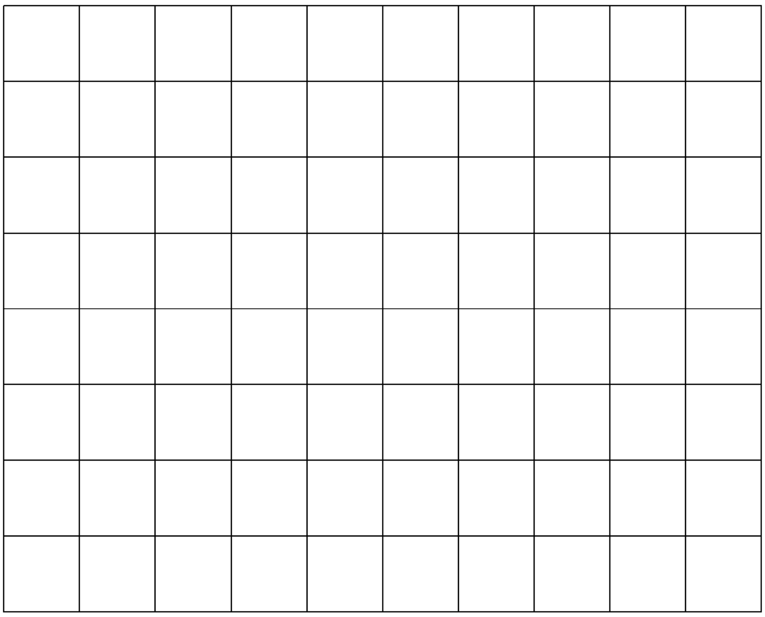


Fig. 15.2.3: Phase shift, U_{tot} to U_R

- Determine the phase shift in degrees, between the generator voltage and the voltage at the resistor from your drawing, or read the value from the oscilloscope screen.

Phase shift U_{tot} to U_R:

What angle corresponds to the measured phase shift?

Exercise 2: RC Series Circuit

- The relationships in an RC series circuit Fig. 15.2.4 are to be measured. The phase angle φ between generator voltage and the voltage across the resistor (U_R), is 46°.
- Calculate the values of U_R and U_C using the phase angle φ.

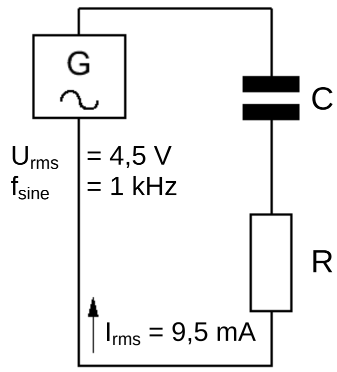


Fig. 15.2.4: RC series circuit

- Draw the vector diagram for the voltages U, U_R, U_C in Fig. 15.2.5.

Scale:
1 cm = 1 V

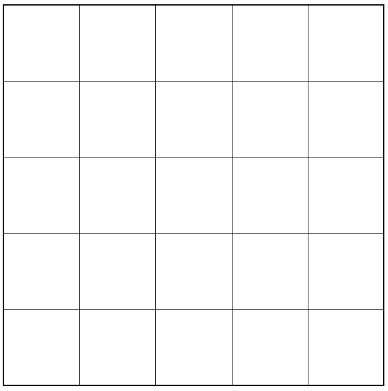


Fig. 15.2.5: U-Vector diagram

- Calculate the values of R, X_C, Z and C.

- The results from calculations will now be checked in a practical exercise. Assemble the RC series circuit in Fig. 15.2.4 on the Electronic Circuits Board. Use the following components: R = 330 Ω, C = 0,47 μF. Set the function generator to: U_{rms} = 4,5 V (f_{sine} = 1 kHz).

- Measure the voltages and current in the circuit with a multimeter. Check the measured values against the results of calculation.

U_R = U_C =   I =

- Measure the phase angle φ on the oscilloscope. Display the voltages U and U_R. Draw the voltage waveform in the chart (Fig. 15.2.6).

Oscilloscope settings:
X : 0,1 ms/ div.
Y_1 : 2 V/ div., AC
Y_2 : 2 V/ div., AC

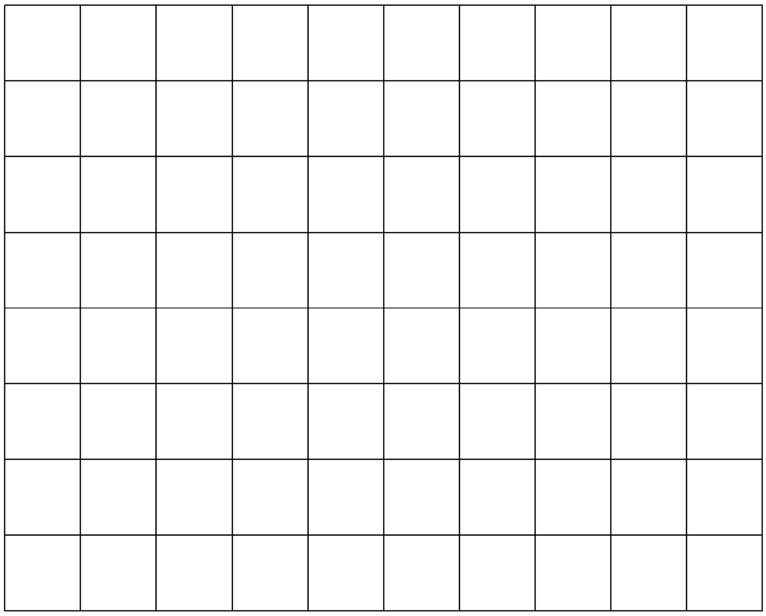
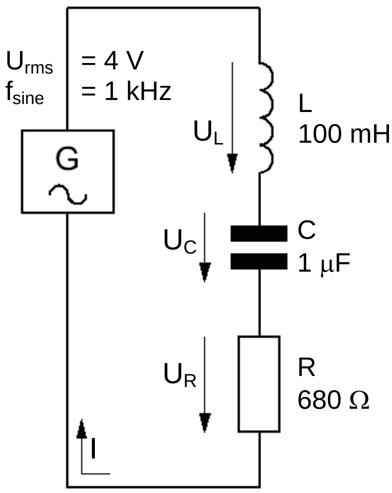


Fig. 15.2.6: Determining the phase angle

Exercise 3: RLC Series Circuit

- Assemble the circuit in Fig. 15.2.7 on the Electronic Circuits Board.
- Set the function generator to an output voltage of U_{rms} = 4 V, f_{sine} = 1 kHz.
- Measure the voltages across the components on a voltmeter.

Fig. 15.2.7: RLC-Serienschaltung



U_{rms} = U_{tot} = 4 V ; U_L =
U_C = ; U_R =

- Calculate the current flowing in the circuit. Check the calculation by measurement.

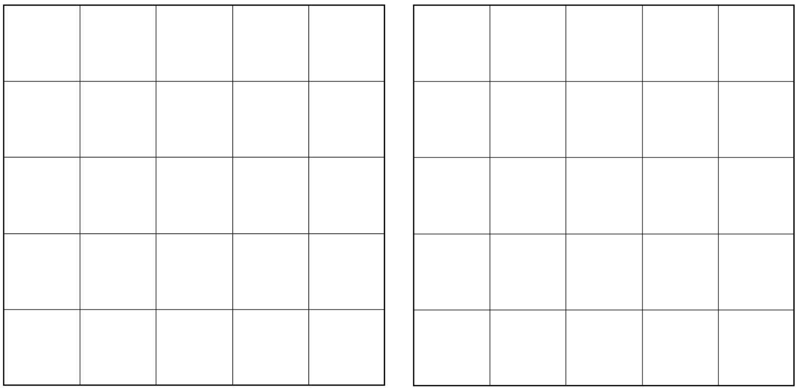
Measurement: I =

- From the measured values, calculate the reactances X_C and X_L and the resistance R.

- Draw the vector diagrams for the voltages (U_R, U_C, U_L, U_{tot}) and the resistances (R, X_C, X_L, Z) in Fig. 15.2.8.

Fig. 15.2.8:
Vector diagrams

Scale:
1 cm = 1 V



Scale:
1 cm = 200 Ω

- Determine the impedance Z from the diagram and check the value by calculation using your measured values.

From the diagram: Z =

Calculation:

- Mark the phase angle in the diagram. Calculate the phase angle from your measured values.

- What changes are seen in the variables listed in table 15.2.9, when the frequency in the circuit of Fig. 15.2.7 is reduced from 1 kHz to 800 Hz (U_{tot} = constant)?

  Indicate the tendency of each variable with arrows (except U_R and I). Then, check your considerations by measurements and calculations. Enter the values in the table.

f ↓	U_L	U_C	U_R	U_{tot} ↔	I	X_L	X_C	R	Z	φ
800				4						
[Hz]	[V]				[mA]	[Ω]				[°]

Table 15.2.9: Characteristic quantities: tendencies, measurements and calculations

15.3 Parallel Circuits of Resistor, Capacitor and Coil

The use of vector diagrams and the calculation of characteristic quantities will now be practised with examples on an RLC parallel circuit.

Note: A layout example for RLC circuits, will be found at the end of this section.

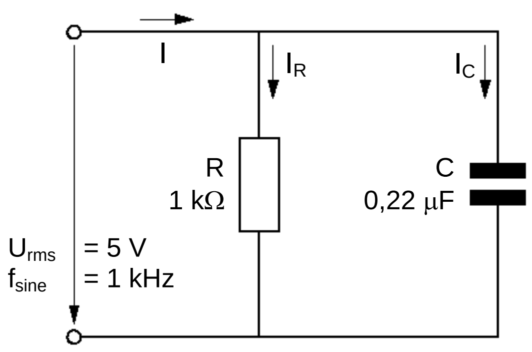
Exercise 1: RC Parallel Circuit

- Assemble the circuit in Fig. 15.3.1 on the Electronic Circuits Board.
- Set the function generator to an output voltage of U_{rms} = 5 V, f_{sine} = 1 kHz.
- Using an ammeter measure the apparent current I, the reactive current I_C and the active current I_R in the circuit.

I = ; I_C =

I_R =

Fig. 15.3.1: RC parallel circuit



- Draw the vector diagram for the currents in Fig. 15.3.2.
- Calculate the phase angle φ from your values measured.

Scale:
1 cm = 2 mA

- Calculate the susceptance B_C, the conductance G and the admittance value Y from the currents measured.

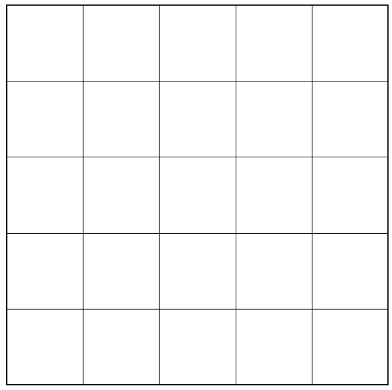


Fig. 15.3.2:
I-vector diagram

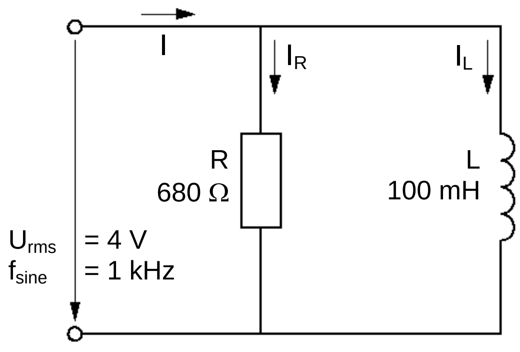
Exercise 2: RL Parallel Circuit

- Assemble the RL circuit in Fig. 15.3.3 on the Electronic Circuits Board.
- Set the function generator to an output voltage of U_{rms} = 4 V, f_{sine} = 1 kHz..
- Using an ammeter measure the currents in the parallel branches.

I_R = ... ; I_L = ...

- Calculate the total current I (apparent current) from the measured values of active current I_R and reactive current I_L.

Fig. 15.3.3: RL parallel circuit



- Check your calculated values for the apparent current I by measurement.

  $I_{meas.}$ =

- From your measured values, calculate the susceptance B_L and the conductance G.

- From the susceptance and frequency, calculate the inductance of the coil.

- Draw the vector diagram of susceptance and conductance values, in Fig. 15.3.4.
- Determine the value of admittance Y from the diagram.

  Y =

- Calculate the phase angle φ.

Scale:
1 cm = 0,5 mS

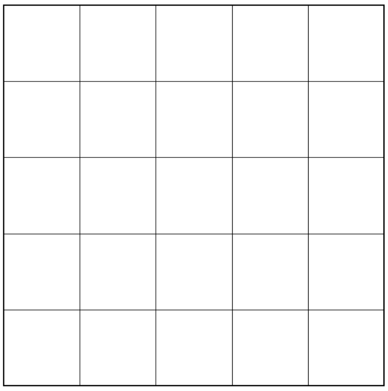


Fig. 15.3.4: Conductance vector diagram

- How much power is consumed by the circuit?

  Active power P :

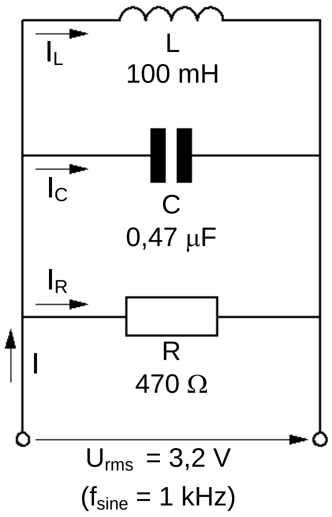
  Reactive power Q_L :

  Apparent power S :

Exercise 3: RLC Parallel Circuit

- Calculate the currents I_R, I_C and I_L in the circuit shown in Fig. 15.3.5. Use the given components.

Fig. 15.3.5: RLC parallel circuit



- Assemble the RLC parallel circuit in Fig. 15.3.5 on the Electronic Circuits Board. Set the function generator to an output voltage of U_{rms} = 3,2 V, f_{sine} = 1 kHz.
- Check your calculations by measurement on an ammeter.

I_R = ... ; I_L = ... ; I_C = ...

- Draw the vector diagram of currents in Fig. 15.3.6. From your drawing, determine the apparent current (total current) I.

Result: I =

- Check the value of I from the drawing by a calculation using the measured values of current.

Scale:
1 cm = 2 mA

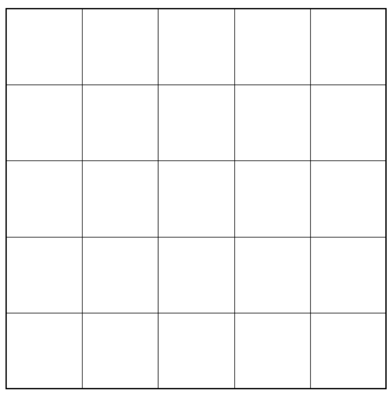


Fig. 15.3.6: Current vector diagram

- Calculate the phase shift between apparent current I and reactive current I_C.

How much active power P, is dissipated in the RLC circuit?

- Calculate the value of apparent power S, using P and the phase angle φ.

- How does the RLC circuit in Fig. 15.3.5 load the AC voltage source, capacitive or inductive? Give reasons for your answer.

15.4 Exercise Assembly – Example for an RLC Parallel Circuit

Fig. 15.4.1 shows one possible time-saving layout of the components for the exercises on the combination of active and reactive resistances. Components and the arrangement of plugs, provide an easy access for connecting the test instruments. Fig. 15.4.1 shows a current measurement in the parallel branch of a coil L = 100 mH and the voltage measurement at the output of the AC voltage source.

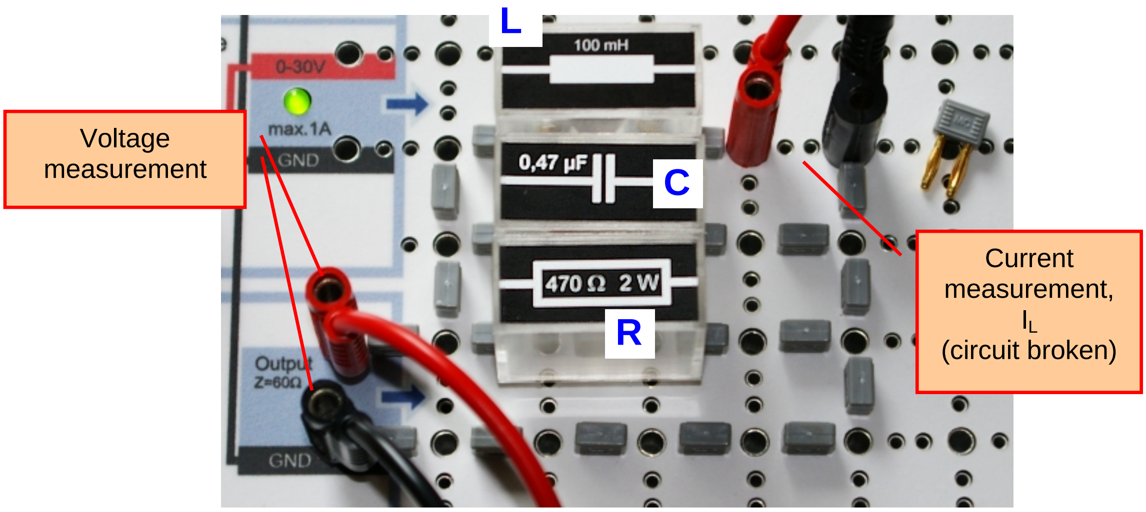


Fig. 15.4.1: Example of an exercise layout

16. Oscillating Circuit

16.1 Generation of a Sinusoidal Oscillation and Resonance

If a charged capacitor C is bridged by an active resistor, current flows for a specific time. The same applies to a coil L, in which a magnetic field has built up. Both phenomena are due to the energy that has been stored, i.e. the electrical field of the capacitor and the magnetic field in the coil. If now, both components are connected together, the capacitor is able to pass its stored energy to the coil L, and vice versa (Fig. 16.1.1). The discharge current from the capacitor generates a magnetic field in the coil. When the capacitor has fully discharged, the current flow stops and the collapse of the magnetic field in the coil, produces a current that charges the capacitor with the opposite polarity. When the magnetic field in the coil has completely decayed, the capacitor can again discharge and the process is repeated (Fig. 16.1.1). Thus at both components, a self-generated periodic oscillation is produced. Capacitor C and coil L form an **oscillating circuit**.

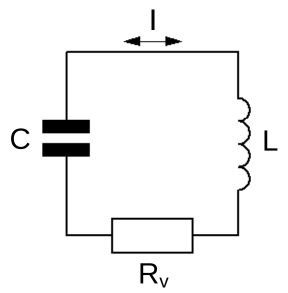


Fig. 16.1.1: L and C form an oscillating circuit

The current controlling this alternating exchange of energy – i.e. the oscillation – is sinusoidal. However, active power losses do occur in the capacitor, the coil and also with less effect, in the connecting cables (symbolised in Fig. 16.1.1 by the equivalent resistor, R_V). Therefore, power in the form of heat, is dissipated in the air surrounding the components. This causes a continual loss of energy and the amplitude of the oscillations become smaller. This is known as a **damped oscillation** (Fig. 16.1.2).

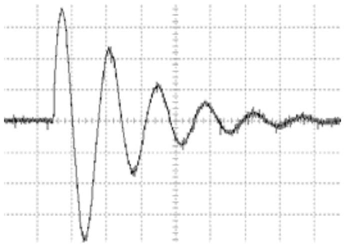


Fig. 16.1.2: Oscilloscope display of damped sinusoidal oscillation

If suitable energy is applied to the oscillating circuit, sufficient to compensation for the losses, the circuit then continues to oscillate at a constant amplitude and frequency. The circuit is then said to be at **resonance**. The **resonant frequency f_o** of the oscillating circuit depends on the capacitance of the capacitor C and the inductance of the coil L. When the circuit oscillates at its resonant frequency, both coil and capacitor must exhange the same magnitude of energy. The frequency adjusts itself, so that the two reactances X_L and X_C have the same value. At resonance, the following equation applies:

$$X_L = X_C \quad \Rightarrow \quad 2\pi \cdot f_o \cdot L = \frac{1}{2\pi \cdot f_o \cdot C} \quad \Rightarrow \quad \boxed{f_o = \frac{1}{2\pi\sqrt{L \cdot C}}}$$

16.2 Series and Parallel Oscillating Circuits

Depending on how it is connected in a circuit, the LC group in Fig. 16.1.1 can be used as a **series** or **parallel oscillating circuit**.

Series resonance: Fig. 16.2.1 shows a **series oscillating circuit**, where the coil and capacitor are connected in series. Capacitor and coil 'share' the generator voltage. The series loss resistance R_v is very small and consists mainly of the ohmic resistance of the coil windings (Fig. 16.2.1).

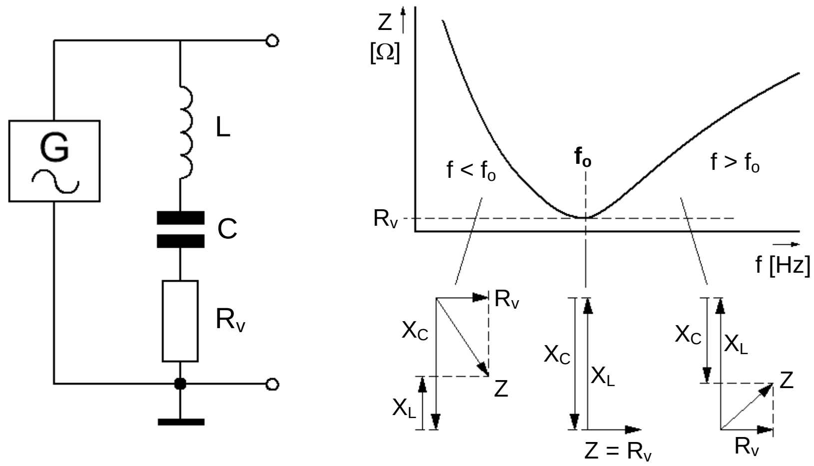


Fig. 16.2.1: Series oscillator

If the AC voltage generator supplies a sine-wave voltage below the resonant frequency ($f < f_o$), then the reactance X_C of the capacitor is the dominant factor. A series oscillating circuit is thus capacitive. The higher the frequency, the more X_L increases, whilst the impedance Z becomes smaller (Fig. 16.2.1, vector diagram, left).

At the resonant frequency f_o, the reactances cancel each other. The small, pure ohmic loss resistance R_v ($Z = R_v$) determines the flow of current in the circuit (Fig. 16.2.1, right). In the case of resonance in a series circuit, current and voltage are in phase (Fig. 16.2.1, vector diagram, centre).

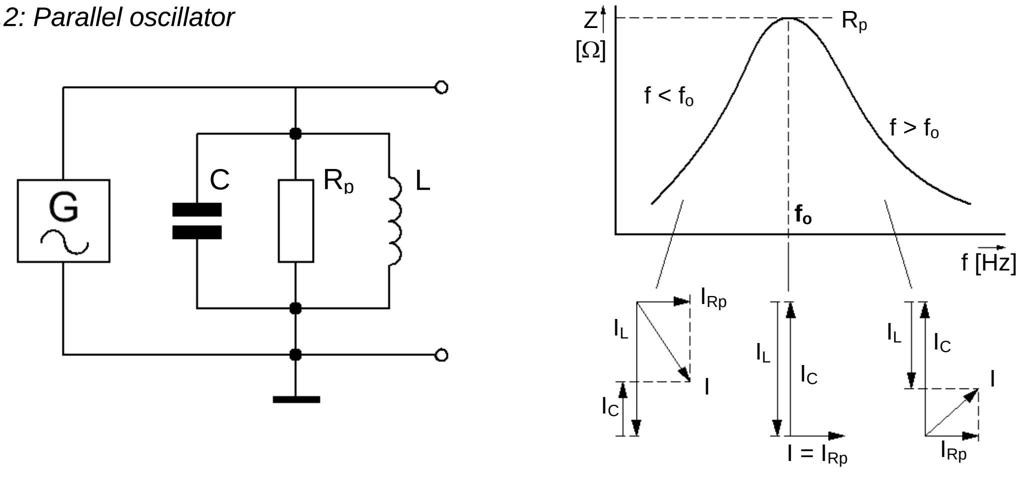
If the frequency is increased above resonance ($f > f_o$), then the series oscillating circuit is inductive, because the reactance of the coil (X_L) dominates over the reactance of the capacitor (X_C) (Fig. 16.2.1, vector diagram, right).

The curve shown in Fig. 16.2.1, $Z = f(f)$ is known as the **resonance curve** of the oscillating circuit.

Parallel resonance: In a parallel oscillating circuit, there are other electrical relationships as in a series circuit because the generator voltage supplies all components, equally. The ohmic losses at the coil and capacitor are combined in the equivalent resistance R_p that is assumed to be in parallel with the coil and capacitor (Fig. 16.2.2, left).

If the AC voltage generator supplies a sine-wave voltage below the resonant frequency ($f < f_o$), then the current I_L in the coil is greater than I_C in the capacitor. A parallel oscillating circuit in this frequency range, is inductive. If the frequency is increased, then I_L reduces and I_C increases (Fig. 16.2.2, vector diagram, left).

Fig. 16.2.2: Parallel oscillator



At the resonant frequency f_o, the reactances and reactive currents cancel each other. The high, pure ohmic loss resistance R_p ($Z = R_p$) determines the flow of current in the circuit (Fig. 16.2.2, vector diagram, centre). In the case of resonance in a parallel oscillating circuit, current and voltage are also in phase. A parallel circuit oscillating at its resonant frequency exhibits its maximum resistance.

If the frequency is increased above resonance ($f > f_o$), then a parallel oscillating circuit is inductive because the reactive current in the capacitor (I_C) dominates over the current in the coil (I_L) (Fig. 16.2.2, vector diagram, right).

The resonance curves show clearly the significant difference between series and parallel resonance: If the circuit is excited at a frequency close to the resonant frequency, then the impedance Z of a series circuit falls to a minimum. On the other hand, in a parallel oscillating circuit, the impedance reaches its maximum value in the region of the resonant frequency.

The response of the resonance curve for an oscillating circuit, is determined mainly by the active power losses occurring in the circuit. Fig. 16.2.3 shows the tendency, when R_p in a parallel oscillating circuit increase as a result of increased ohmic losses.

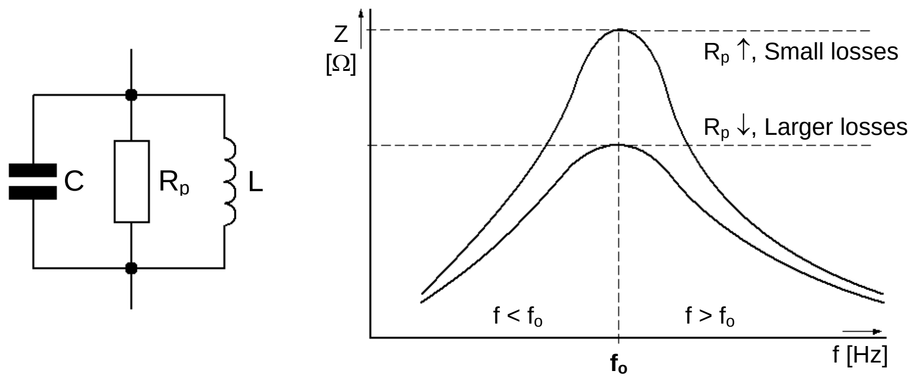


Fig. 16.2.3: Effect of active power losses on the resonance curve

16.3 Practical Proof of Resonance Curves

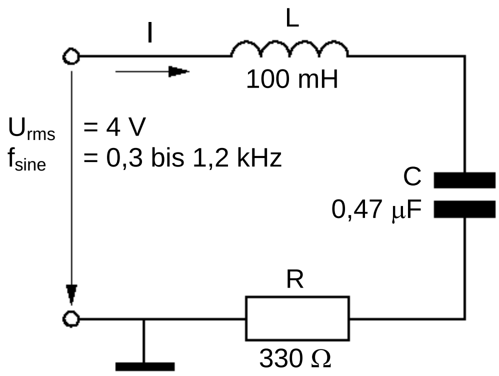
The resonance response of oscillating circuits will now be proved with practical exercises.

Note: Layout examples on the Electronic Circuits Board for the measurements in oscillating circuits, will be found after exercise 2.

Exercise 1: Series Oscillating Circuit

- Assemble the oscillating circuit in Fig. 16.3.1 on the Electronic Circuits Board.
- Calculate the resonant freqency of the series oscillating circuit.

Fig. 16.3.1: Series oscillating circuit



- Enter the calculated value of resonant frequency f_o in the empty square of frequency values in table 16.3.2.
- Set the function generator to an output voltage of U_{rms} = 4 V, at a starting frequency of f_{sine} = 300 Hz. Measure the voltage on a voltmeter.
- At the given frequencies, measure the current flow on an ammeter. Enter the values in the table.

Table 16.3.2: Measured values, series oscillating circuit

	f [kHz]	**0,3**	**0,4**	**0,6**		**0,8**	**0,9**	**1**	**1,2**
330 Ω	**I [mA]**								
	Z [kΩ]								
220 Ω	**I [mA]**								
	Z [kΩ]								

- Calculate the values of impedance Z and enter the results in the table.
- Draw the resonance curve $Z_{330\,\Omega}$= f(f) in the chart (Fig. 16.3.3) for this series oscillating circuit.
- Exchange the resistor R in the circuit of Fig. 16.3.1 with one of R = 220 Ω.
- Repeat the series of measurements for table 16.3.1 and enter the new values in the table.
-

- Draw the resonance curve $Z_{220\,\Omega} = f(f)$ in the chart (Fig. 16.3.3) for the modified series oscillating circuit.

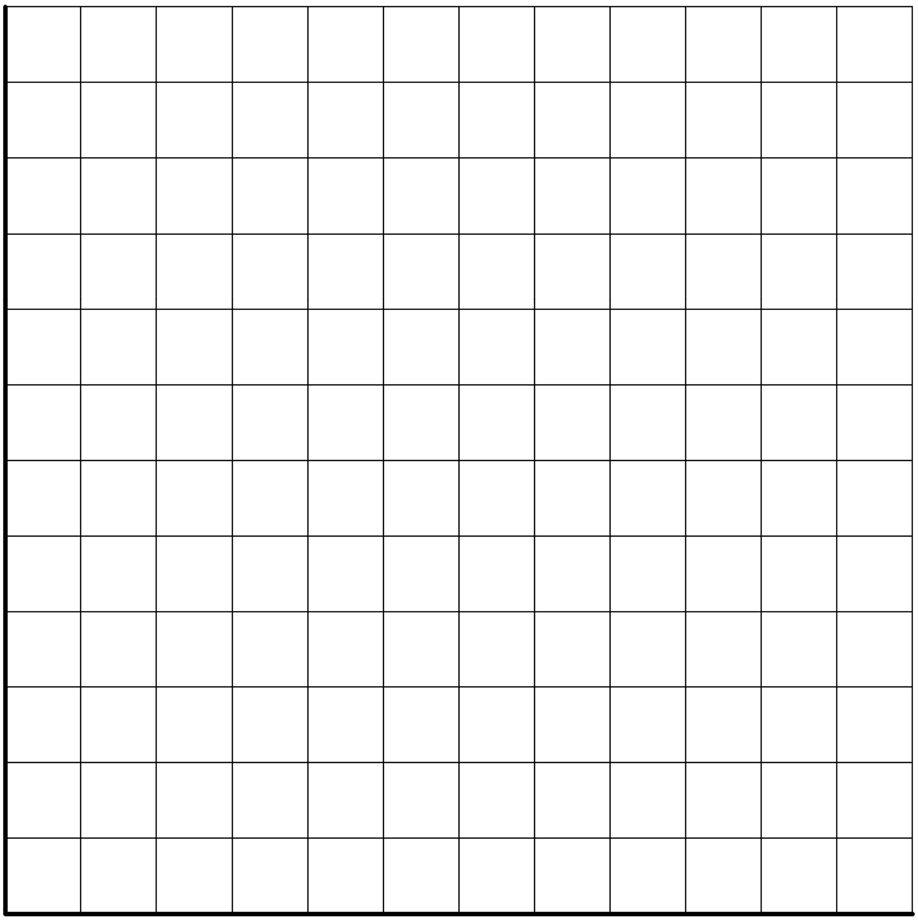


Fig. 16.3.3: Resonance curves, series oscillating circuit

- What properties has the impedance Z at the resonant frequency f_o?

- The series oscillating circuit of Fig. 16.3.1 includes a resistor (330 or 220 Ω) for limiting the current flow. What is the actual value of loss resistance R_v in the circuit?

- Indicate on the curves you have drawn, the areas where the oscillating circuit acts as a capacitance and as an inductance.

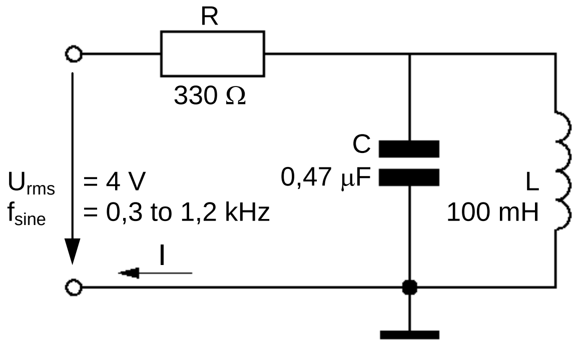
Exercise 2: Parallel Oscillating Circuit

- Assemble the circuit in Fig. 16.3.4 on the Electronic Circuits Board.

Note: Resistor R = 330 Ω is used only for current limiting and is nor part of the parallel oscillating circuit.

- Set the function generator to an output voltage of U_{rms} = 4 V, at a starting frequency of f_{sine} = 300 Hz. Measure the voltage on a voltmeter.

Fig. 16.3.4: Parallel oscillating circuit



- At the given frequencies, measure the current flow on an ammeter. Enter the values in table 16.3.5.

Table 16.3.5: Measured values, parallel oscillating circuit

f [kHz]	0,3	0,4	0,5	0,6	0,734	0,8	0,9	1	1,2
I [mA]									
Z [kΩ]									

- Calculate the values of impedance Z and enter the results in the table.
- Draw the resonance curve Z = f(f) in the chart (Fig. 16.3.6) for this parallel oscillating circuit.

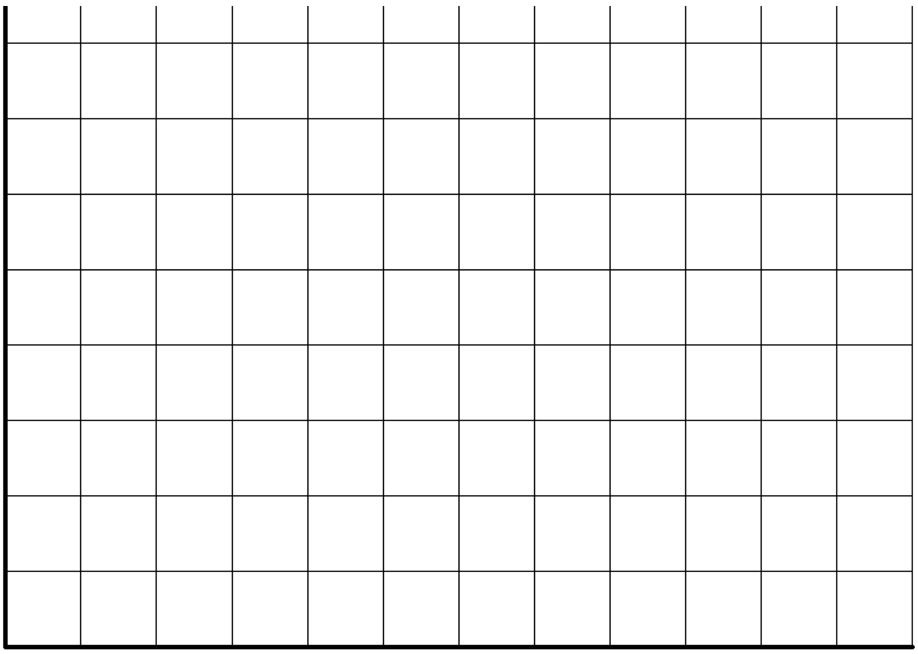


Fig. 16.3.6: Resonance curve, parallel oscillating circuit

- How do the currents I_L and I_C respond at resonance of the parallel oscillating circuit?

- Measure the currents I_L and I_C at resonance.

$I_{L\ Resonance}$ = ... ; $I_{C\ Resonance}$ = ...

Exercise Layout, Series Oscillating Circuit

Fig. 16.3.7 shows the exercise layout for the series resonant circuit. The illustration shows measurement of the current I at resonance. For this, the circuit is broken.

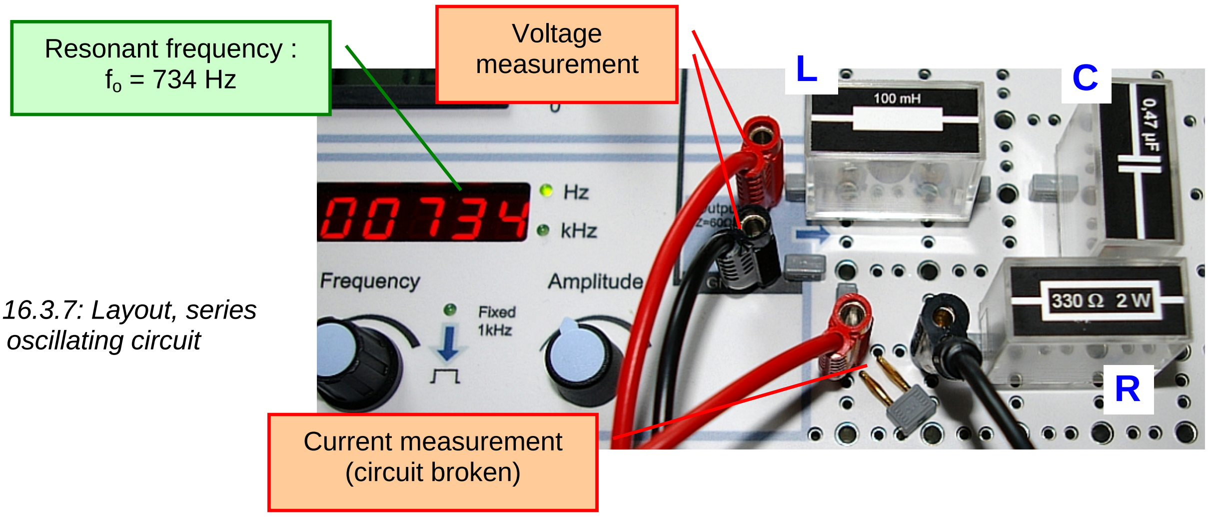


Fig. 16.3.7: Layout, series oscillating circuit

Exercise Layout, Parallel Oscillating Circuit

Fig. 16.3.8 shows a suggested layout for the exercise on a parallel resonant circuit. The capacitor and coil branches can be isolated for the current measurements at resonance.

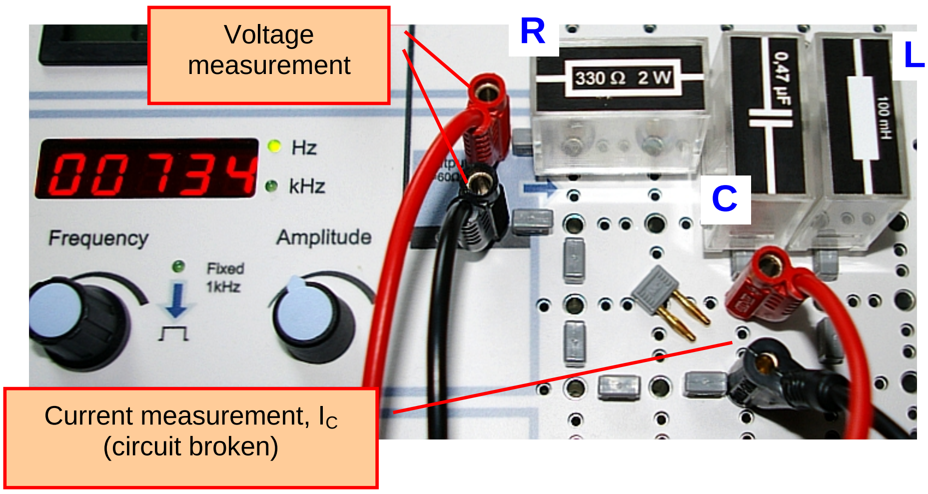


Fig. 16.3.8: Layout, parallel oscillating circuit

Exercise 3: Generating Damped Oscillations

Circuit description: The circuit shown in Fig. 16.3.9 for generating an oscillation, consist of 3 parts:

The capacitors $C_2 = C_3 = 10$ nF together with the coil L = 100 mH, form a parallel oscillating circuit, where the damped sinusoidal oscillation can be measured. To initiate oscillation, the resonant circuit must periodically be 'excited' by feeding energy to the circuit.

This excitation is the task of the RC-element C_1/R_1. It converts the positive square-wave pulse ($U_S = 6$ V, f = 250 Hz) applied to the input of the circuit, to needle pulses that can be measured as U_{R1} (c.f. section 13.3.2).

Only part of the low-energy needle pulses (U_{R1}) is effective at the oscillating circuit due to the relatively high-value coupling resistor $R_2 = 22$ kΩ (R_2 and the oscillating circuit form a voltage divider for U_{R1}).

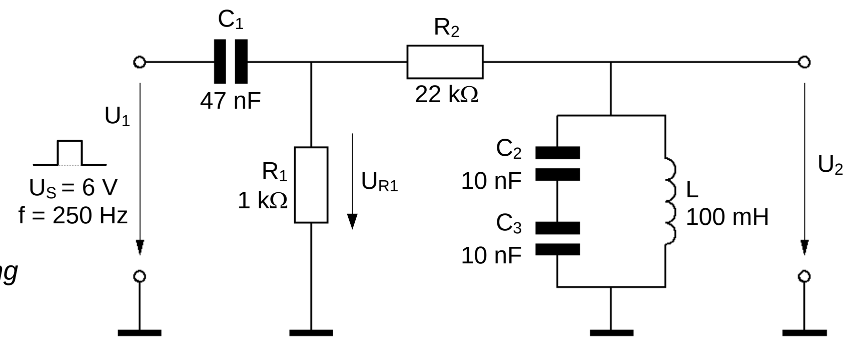


Fig. 16.3.9: Circuit for generating damped oscillations

- Assemble the circuit in Fig. 16.3.9 on the Electronic Circuits Board. Break the connection between the RC-element and oscillating circuit by removing the coupling resistor R_2.

Note: A suggested layout of the components on the Electronic Circuits Board will be found at the end of this section.

- Set the input voltage U_1 at the function generator to $U_p = 6$ V, f = 250 Hz.

First, a check must be made to ensure that needle pulses are formed at R_1 as required.

- Display U_1 and U_{R1} on the oscilloscope. Adjust the controls on the oscilloscope so that at least one period of the square-wave voltage is displayed.
- Draw the signal waveform in the chart (Fig. 16.3.10).

Oscilloscope settings:
X : 0,4 ms/ div.
Y_1 : 5 V/ div., DC
Y_2 : 2 V/ div., DC

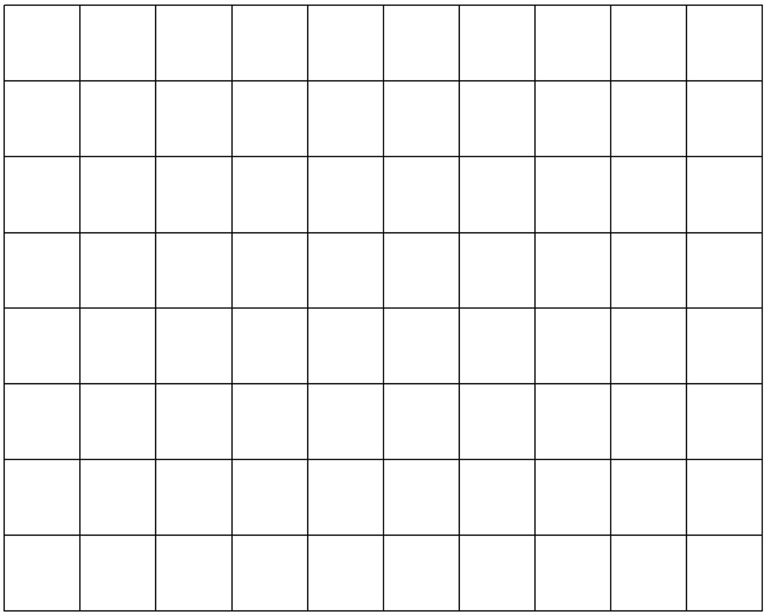


Fig. 16.3.10: Generation of needle pulses

- Switch off the function generator and complete the circuit with the coupling resistor R_2 = 22 kΩ.
- Calculate the resonant frequency f_o of the oscillating circuit.

- Display U_{R1} and U_2 on the oscilloscope. Adjust the controls on the oscilloscope so that at least one complete period of U_{R1} (needle pulses) is displayed.

Note: It should now be possible to display damped oscillations on the oscilloscope, at the output of the circuit (U_2).

- Draw the signal waveform in the chart (Fig. 16.3.11).
- Measure the periodic time of the damped oscillation and from this, calculate the actual resonant frequency f_o. Optimise the display of the oscillation on the oscilloscope. Compare the measured resonant frequency with that calculated.

T measured:

Oscilloscope settings:

X : 0,4 ms/ div.
Y_1 : 5 V/ div.
Y_2 : 0,5 V/ div.

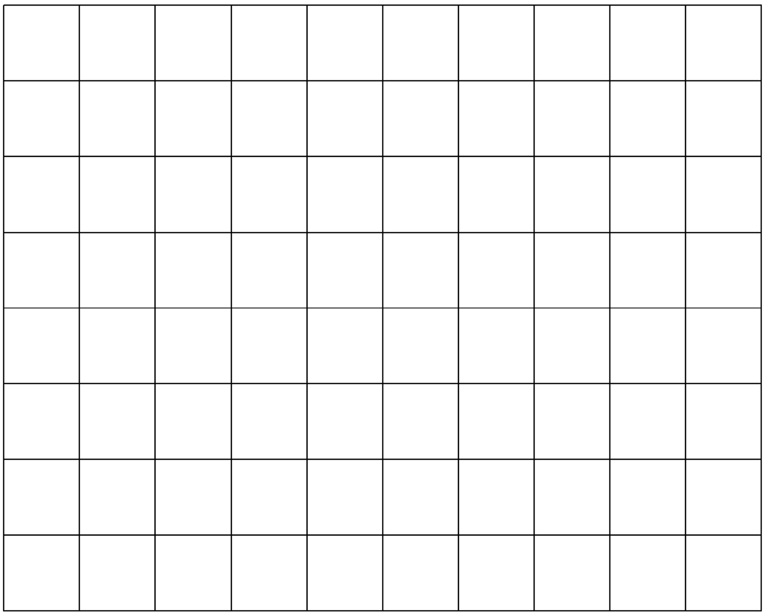


Fig. 16.3.11: Generation of damped oscillations

- What change do you expect when one of the capacitors C_2/C_3 is removed and replaced by a bridge? Check your answer by measurements.

- What effect do you expect from a larger coupling resistor $R_2 = 100\ k\Omega$? Check your answer by measurement.

Exercise Layout, Damped Oscillation

Fig. 16.3.12 shows the exercise layout corresponding to the circuit of Fig. 16.3.9 for generating damped oscillations. The illustration shows the second measurement required on the oscilloscope. To reduce the amount of re-connection, a 2 mm connection lead is used.

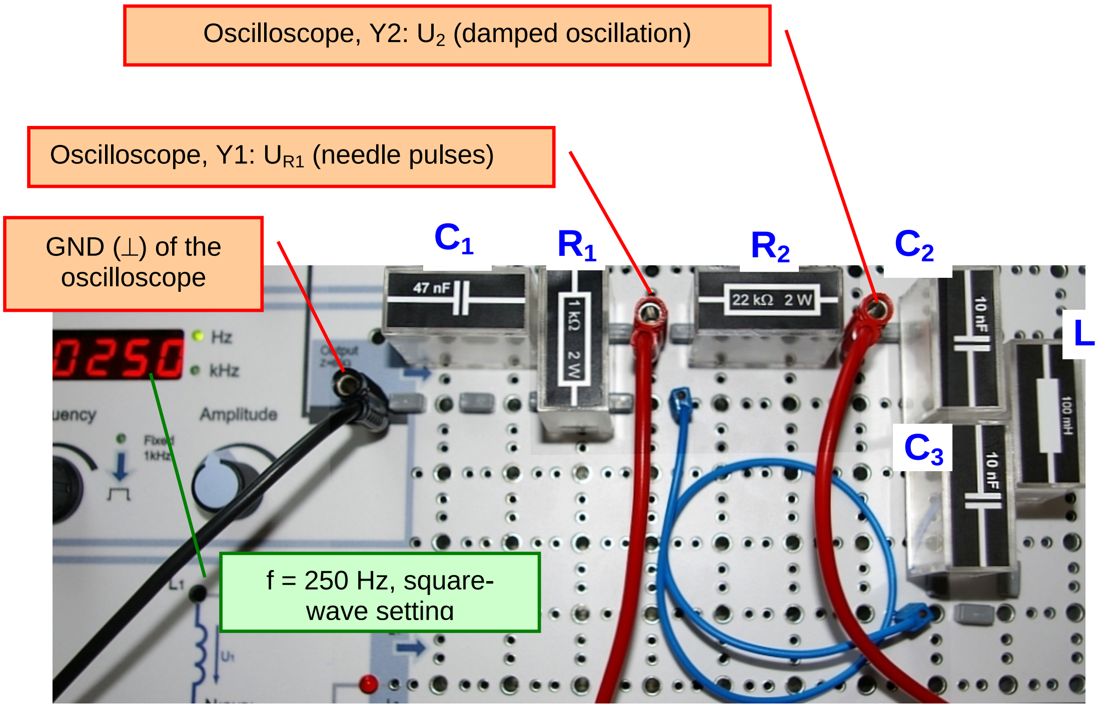


Fig. 16.3.12: Layout, Damped oscillation"

17. RLC Filter Circuit

A circuit with a frequency-dependent transfer response, is known as a **Filter**. The amplitude of a sinusoidal input voltage (U_1) is more, or less, reduced depending on the frequency of the input and is available at the output (U_2). This type of circuit is used to suppress unwanted components of a complex mixture of frequencies (frequency spectrum, signal voltages with different frequencies).

Fig. 17.1: Frequency response of a filter

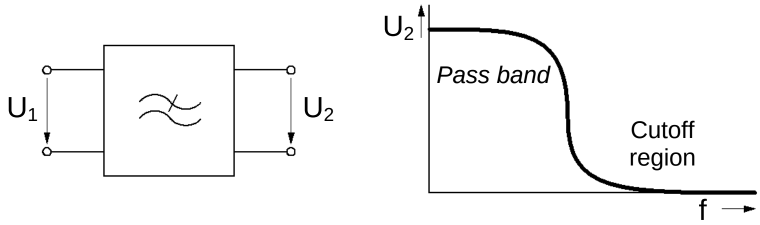


Fig. 17.1 shows the filtering principle of the frequency response of a so-called low-pass. Alternating voltages in the low-frequency range, are allowed to pass through the circuit with little or no, attenuation to the output of the circuit (U_2). This range of frequencies is known as the **pass band** of the filter. In contrast, the **cutoff region** specifies the range (in this case), of higher frequencies that are blocked (or heavily attenuated) and no signal (or very little) reaches the output.

17.1 Transfer Characteristics of Filters

In order to describe the frequency response of filters, the junction of pass band and cutoff region must be clearly defined. For this reason, the **cutoff frequency f_{cut}** of the filter (Fig. 17.1.1) is defined as the point at which the output voltage of the filter is 0,707-times the input voltage.

$$U_2 = \frac{1}{\sqrt{2}} \cdot U_1 = 0{,}707 \cdot U_1$$

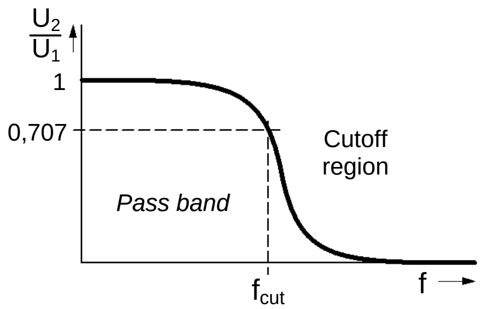


Fig. 17.1.1: Definition of the cutoff frequency f_{cut}

In many cases, a filter has the task of allowing a range of frequencies to pass through the filter (**bandpass**) or to suppress a range of frequencies (**bandstop**, Fig. 17.1.2). In the frequency response of such circuits, there are 2 limit frequencies, the upper (f_{hi}) and the lower cutoff frequency (f_{lo}). The range of frequencies between these 2 cutoff frequencies is known as the **bandwidth b** of the filter. **Bandpass** and **bandstop** filters are realised with oscillating circuits. This make use of the properties of

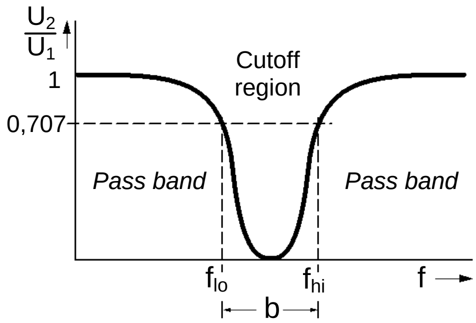


Fig. 17.1.2: Definition of upper and lower cutoff frequencies and bandwidth

the low and high impedances in the vicinity of the resonant frequency f_o.

There are 4 types of frequency filter, summarised in Table 17.1.3 together with their DIN-symbol, transfer characteristics and RLC example circuits.

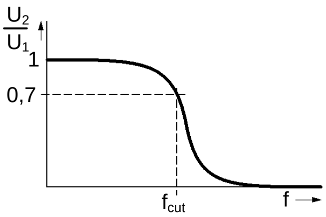
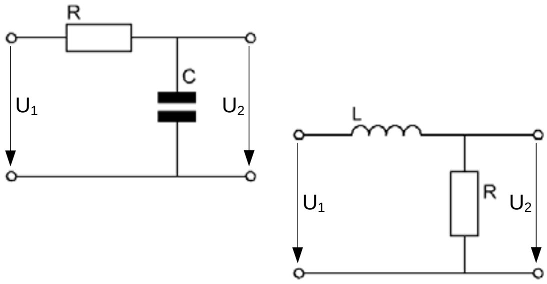
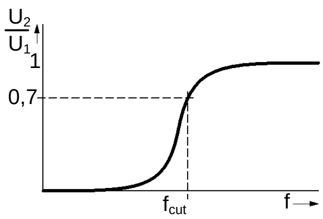
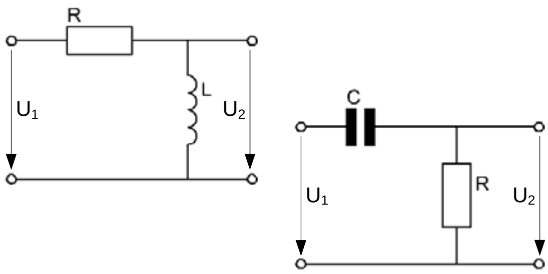
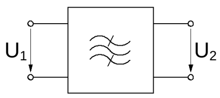
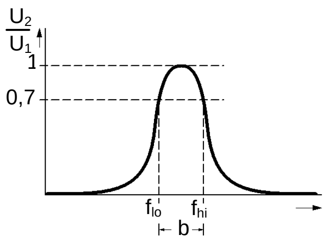
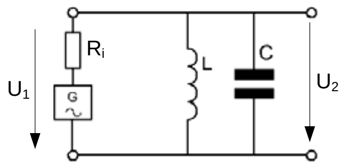
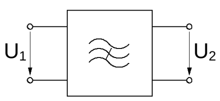
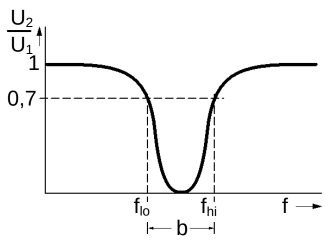
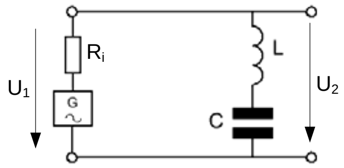
Filter type	Frequency response	RLC circuit example
Low-pass		
High-pass		
Bandpass 		
Bandstop 		

Table 17.1.3: Types of filter

For high- and low-pass filters, that use the RC or RL circuits shown in table 17.1.3, the following applies at the cutoff frequency, f_{cut}:

$$Reactance = Active\ resistance \quad => \quad X = R \quad => \quad U_X = U_R$$

Therefore at the cutoff frequency, the phase shift φ between input and output voltage is 45°. The vector diagrams in Fig. 17.1.4 show that this applies, irrespective of which component is used for measuring the output voltage.

The ratio of output voltage to input voltage is confirmed because sin φ = cos φ = 0,707:

$$U_2 = \frac{1}{\sqrt{2}} \cdot U_1 = 0{,}707 \cdot U_1$$

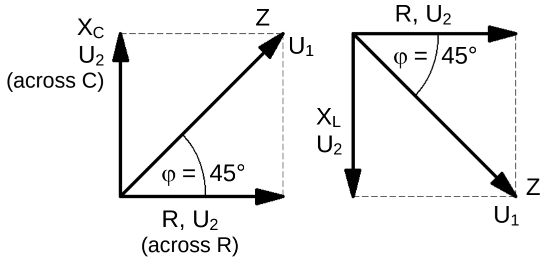


Fig. 17.1.4: Vector diagrams R, X_C and R, X_L at f_{cut}

For low- and high-pass filters, the cutoff frequency f_{cut} is given by:

RC-elements: $f_{cut} = \frac{1}{2\pi \cdot R \cdot C}$; RL-elements: $f_{cut} = \frac{R}{2\pi \cdot L}$

To improve the selectivity of filters, the edges of the pass band curve must be steeper. This is achieved in practice by modifying the basic circuits shown in table 17.1.3. More often, a completely different type of filter is used, for example quartz filters, ceramic filters or mechanical filters, to name a few.

17.2 Practical Proof of the Frequency Response of Filters

The transfer characteristics of filters will now be examined, using practical examples.

Exercise 1: High-pass Filter with RC-element

- Assemble the circuit in Fig. 17.2.1 on the Electronic Circuits Board.
- Calculate the cutoff frequency f_{cut} of the circuit, using nominal values. Round-up the result to a whole number.

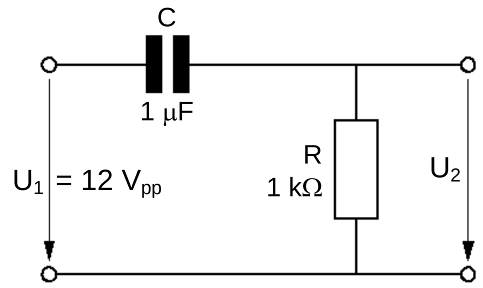
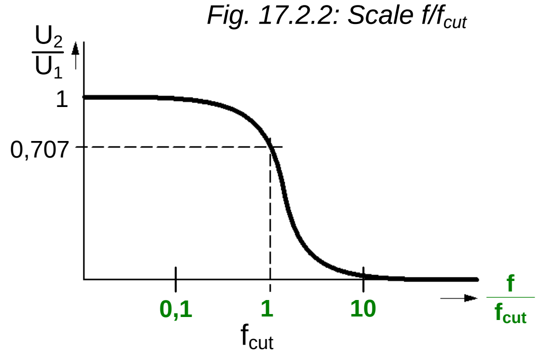


Fig. 17.2.1: RC-element as high-pass filter

Note: To be able to record the transfer characteristics for a frequency spectrum as wide as possible, values are measured in steps, at 10% of the cutoff frequency f_{cut}. In order to obtain a meaningful curve, it is necessary to plot the frequencies in the form f/f_{cut} referred to the cutoff frequency (Fig. 17.2.2).

Fig. 17.2.2: Scale f/f_{cut}



- Complete the line "f/f_{cut}" in table 17.2.3.
- Set the function generator for U_1 to a sine-wave voltage of $U_{pp} = 12$ V (Fig. 17.2.1). Check the amplitude on the oscilloscope.
- Measure the output voltage U_2 at the frequencies given in table 17.2.3 on the oscilloscope. Enter the values in the table.

Table 17.2.3: Measured values, RC high-pass filter

f	**1,6 Hz**	**16 Hz**	**f_{cut} = 160 Hz**	**1,6 kHz**	**16 kHz**
f / f_{cut}					
U_2 [V_{pp}]					
U_2 / U_1					

- Complete the line "U_2/U_1" in table 17.2.3.
- Draw the curve of the frequency response in the chart (Fig. 17.2.4).

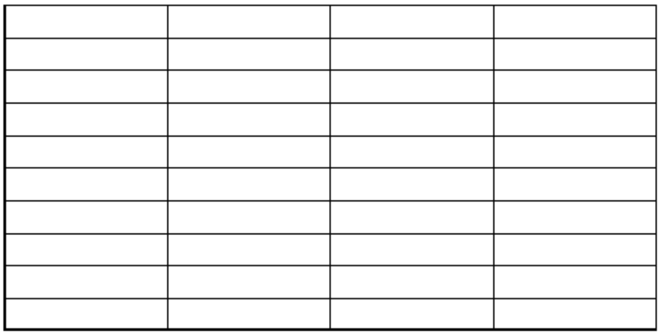


Fig. 17.2.4: Frequency response, RC high-pass

- What are the r.m.s. values of U_C and U_R when the high-pass filter transfers the cutoff frequency f_{cut}? Check your calculation by measurement with a multimeter.

Measurement:

U_C = ... ; U_R = ...

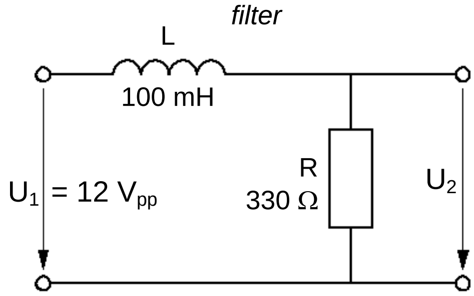
Exercise 2: Low-pass Filter with RL-element

- Assemble the circuit in Fig. 17.2.5 on the Electronic Circuits Board.

- Set the function generator for U_1 to a sine-wave voltage of $U_{pp} = 12$ V.

- Calculate the cutoff frequency f_{cut}, using nominal values.

Fig. 17.2.5: RL-element as low-pass filter



- How can the actual cutoff frequency f_{cut} be determined, using a voltmeter?

- Complete the measurements for determining the cutoff frequency f_{cut}.

- How can you explain the distinct difference between the calculated and measured values of f_{cut}?

- The frequency response of the low-pass filter will now be recorded. Complete the line "f" in table 17.2.6. Enter here, the value of cutoff frequency determined by measurement with the voltmeter.

- Measure the voltage $U_R = U_2$ at the frequencies entered in table 17.2.6, from the oscilloscope screen. Enter the values in the table.

Note: At each measurement, check the voltage of the function generator (U_1) and correct if necessary (more significant at very low and higher frequencies).

Table 17.2.6: Measured values, RL low-pass filter

f					
f/f_{cut}	**0,01**	**0,1**	**1**	**10**	**100**
U_2 [V_{pp}]					
U_2/U_1					

- Complete the line “U_2/U_1” in table 17.2.6.
- Draw the curve of the frequency response in the chart (Fig. 17.2.7).

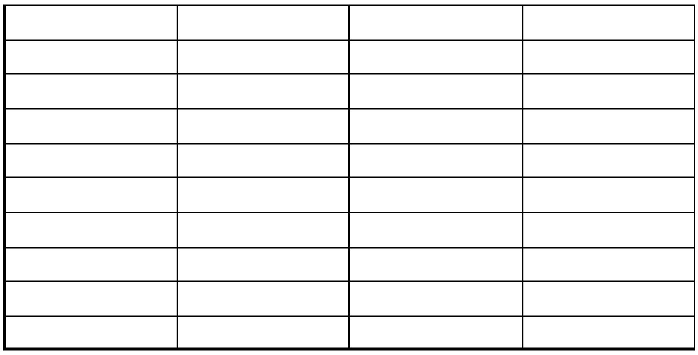


Fig. 17.2.7: Frequency response, RL low-pass

- How can you explain that at f_{cut}, less than 0,7-times the input voltage is available at the output?

Exercise 3: Bandstop with LC Series Oscillating Circuit

- Assemble the circuit in Fig. 17.2.8 on the Electronic Circuits Board.
- Set the function generator for U_1 to a sine-wave voltage of U_{rms} = 3 V.
- Calculate the resonant frequency f_o of the circuit from the nominal values. Round-up the result to a whole number.

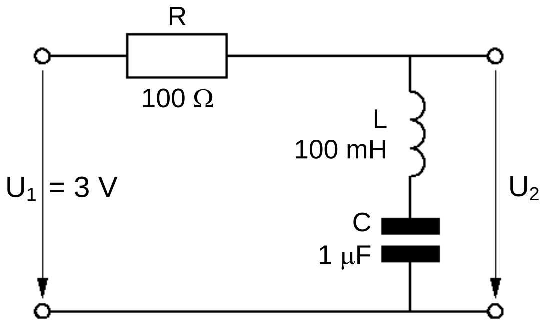


Fig. 17.2.8: Series oscillating circuit as bandstop

- Measure the voltage U_2 at the frequencies given in table 17.2.9. Enter the values in the table.

Table 17.2.9: Measured values, bandstop

f [kHz]	0,1	0,2	0,35	0,45	0,5	0,55	0,65	0,8	1
U_2 [V]									
U_2/U_1									

- Complete the line "U_2/U_1" in table 17.2.9.
- Draw the curve of the frequency response in the chart (Fig. 17.2.10).

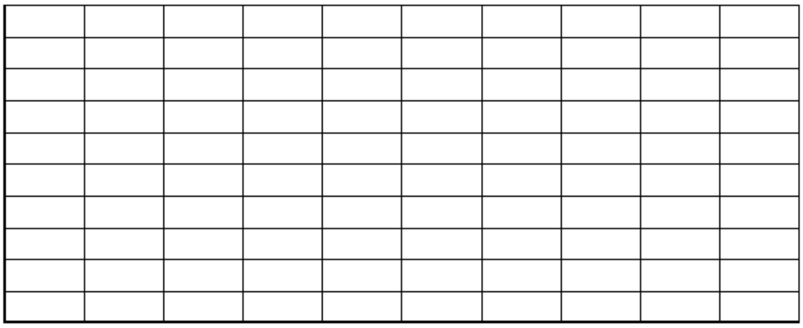


Fig. 17.2.10: Frequency response, bandstop

- Measure the upper and lower cutoff frequencies of the bandstop (f_{hi}, f_{lo}).

- What is the bandwidth b, of the bandstop filter?

- Mark the bandwidth and cutoff frequencies on the frequency response curve for the bandstop filter drawn in Fig. 17.2.10.

18. Transformers

Short form, sometimes used: **Transformer**

18.1 Tasks and Function of Transformers

Transformers consist of two windings or coils (**'primary'** and **'secondary'** windings), that are magnetically coupled together. The **primary winding** absorbs electrical energy from an AC voltage generator. The energy is converted to a changing magnetic field that cuts the other winding and also produces here, a changing magnetic field. This induces electrical energy in the **secondary winding** and is available at the output of the transformer. The transformation characteristics of a transformer depend mainly on the magnetic coupling between primary and secondary windings. The density of magnetic flux can be considerably increased by 'closing' the magnetic circuit using ferromagnetic material – in Fig. 18.1.1, this is in the form of an iron core.

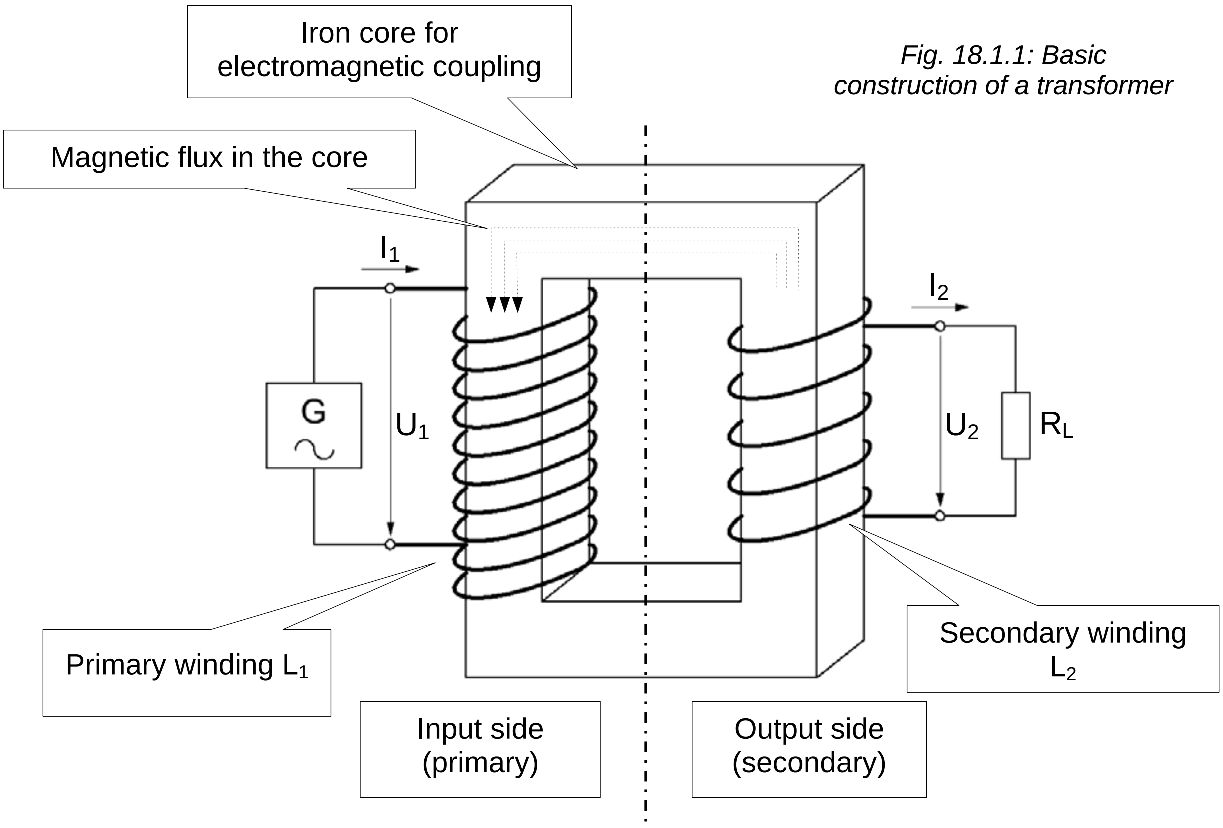


Fig. 18.1.1: Basic construction of a transformer

Transformers have the task of matching the electrical properties of an alternating voltage (primary side), to the requirements of a consumer (secondary side). A common example of such a task, is the mains transformer in domestic equipment. Mains outlet sockets, connected to the public mains network, provide a voltage of 220 V. The consumer's equipment however, usually requires a much lower voltage.

Apart from the transformation of energy, transformers can also be used for transforming (matching) currents and resistances (impedances), i.e. matching the output of one circuit to the input of a subsequent circuit. There are many types of matching transformer used in almost all frequeny ranges, including high-frequency techniques. In addition to their use for matching electrical properties, transformers are also used for electrical isolation between circuits where DC voltages must be 'blocked'. This is known as **galvanic isolation**.

There are 3 circuit symbols in common use as shown in Fig. 18.1.2. Dots at the side of the windings, indicate the ends of the winding with the same phase relationship.

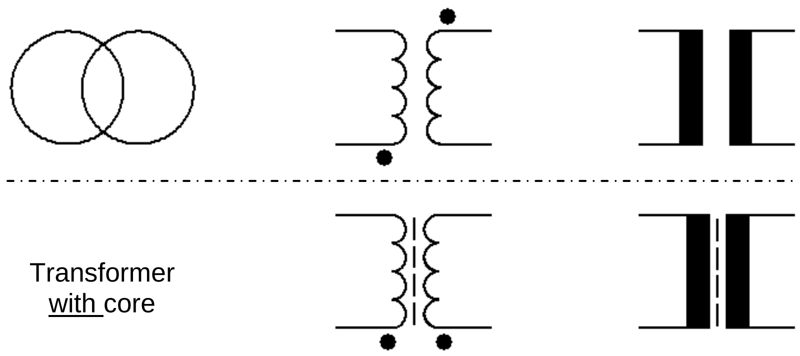


Fig. 18.1.2: Transformer circuit symbols

18.2 Differences between Real and Ideal Transformers

To describe the properties of a transformer, reference is usually made to an ideal transformer:

- In an ideal transformer, there are no losses. Thus, its efficiency is $\eta = 1$; i.e. input and output power are equal.
- When off-load (i.e. no load resistance), an ideal transformer does not consume any active energy.
- All the magnetic flux in the primary coil cuts (or 'threads') the secondary coil; i.e. outside of the core, there is no component present of the magnetic field (no lines of force exist in the free space surrouding the coils); thus, the magnetic coupling between the windings is 100%.
- The waveforms of the input and output alternating voltages are identical; i.e. the signal waveform at the output of the transformer is not distorted.

Its a different picture for real transformers: For example, there are losses of energy that result in an efficiency of $\eta < 1$. They are mainly due to resistance losses in the windings and the eddy currents that are produced in the coils. To keeep these eddy current losses as small as possible, the cores are constructed from thin laminated strips of soft iron. To reduce distortion of the output signal, the laminations of the core are separated by a very small air gap. However, this leads to a restriction of the magnetic flux that cuts the secondary coil and thus, there is a poorer degree of coupling between primary and secondary sides. The voltage output from the secondary circuit is then less than expected. Power losses or restrictions in the functioning of transformers must be accepted. The formulae described in the following section do not include any considerations of the losses and therefore, apply only to an ideal transformer.

18.3 Voltage, Current and Resistance Transformation (transformation ratio)

The ratio of the number of windings in the primary coil (N_1) to the number of windings in the secondary coil (N_2), is known as the **turns** or **transformation ratio** and is often indicated by the letter 'T':

$$T = \frac{N_1}{N_2}$$

In an off-load transformer and ignoring losses, this ratio T, also corresponds to the ratio between input and output voltages. To ensure the same power is available in the output circuit as in the primary circuit, the currents are in anti-phase to the voltages:

$$T = \frac{U_1}{U_2} = \frac{I_2}{I_1}$$

The load resistance in the secondary circuit (R_L or R_2, Fig. 18.3.1), can be calculated with Ohm's law from the current and voltage. The same applies for R_1, which is the load of the transformer effective at the AC voltage source:

$$R_2 = R_L = \frac{U_2}{I_2} \quad \text{and} \quad R_1 = \frac{U_1}{I_1}$$

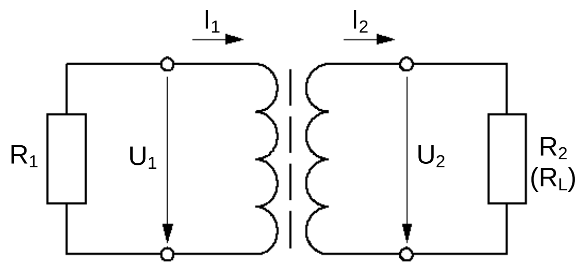


Fig. 18.3.1: Transformer characteristic quantities

When the resistances are formed as a ratio, then :

$$\frac{R_1}{R_2} = \frac{\frac{U_1}{I_1}}{\frac{U_2}{I_2}} = \frac{U_1 \cdot I_2}{U_2 \cdot I_1} = \frac{U_1}{U_2} \cdot \frac{I_2}{I_1} = T \cdot T = T^2 \quad \Rightarrow \quad \boldsymbol{R_1 = R_2 \cdot T^2} \quad \text{and} \quad \boldsymbol{T = \sqrt{\frac{R_1}{R_2}}}$$

Thus, the load resistance R_L (R_2) is transformed to the primary side as the square of the transformation ratio, T.

18.4 Practical Proof of Transformer Effect

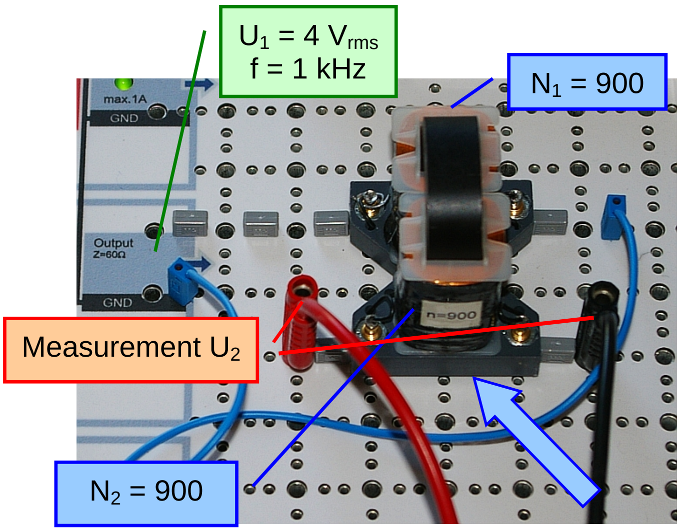
The transformation of voltage, current and resistance and their dependence on the transformation ratio, T as well as the effects of various degrees of coupling, will now be examined with practical exercises.

Exercise 1: Various Degrees of Coupling

The dependence of output voltage U_2 and transformation ratio, T on the magnetic coupling between the windings, will now be examined in 4 different configurations:

- Transformer with core
- Transformer with core and air gap
- Transformer with half-core
- Transformer without core

Fig. 18.4.1: Layout, transformer N_1=N_2=900 with core



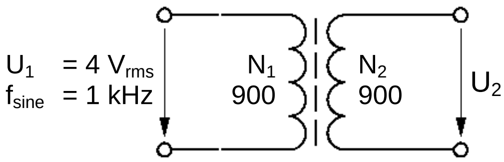


Fig. 18.4.2: Transformer: T = 1

- Assemble the circuit as shown in Fig. 18.4.2 on the Electronic Circuits Board.

Note: Ensure that the transformer and coil holders are in contact with the surface of the Board (i.e. not mounted on short circuit plugs, see arrow in Fig. 18.4.1). This will ensure that the core sections are in tight contact with each other.

- Set the function generator for U_1 to sine-wave voltage U_{rms} = 4 V, f = 1 kHz.
- Measure the secondary voltage of the transformer when off-load. Enter the value in table 18.4.3.
- Remove the top half of the core and repeat the measurement (enter the value in the table).

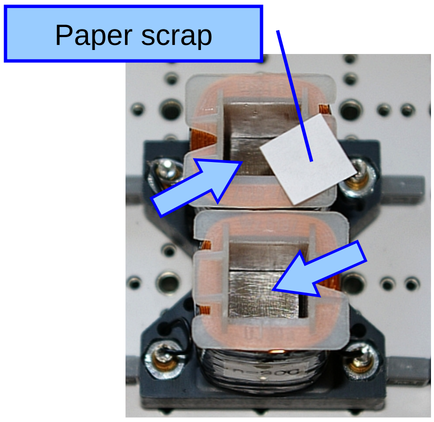
Configuration	U_2 [V_{rms}]
with core	
with core and air gap	
with ½ core	
without core	

Table 18.4.3: Measured

- To simulate an air gap between the core sections, cut 2 pieces of paper (Fig. 18.4.4 on the next page). Position the paper in the body of the core and replace the upper half of the core.
- Measure U_2 with core and air gap (enter the value in the table).

- Remove the transformer coils, short circuit plugs and connecting leads from the Board. Using 4 mm connecting leads, connect the primary winding to the input voltage U_1. couple the voltmeter in the same way, to the output of the secondary coil (Fig. 18.4.5).
- Measure the output voltage U_2 without core (enter the value in table 18.4.3). During measurement, make sure that the upper opening of both coil formers align with each other, so that as much of the magnetic lines of force from the primary winding, cut the secondary coil (Fig. 18.4.5).

Fig. 18.4.4: U_2 measurement, core with air gap



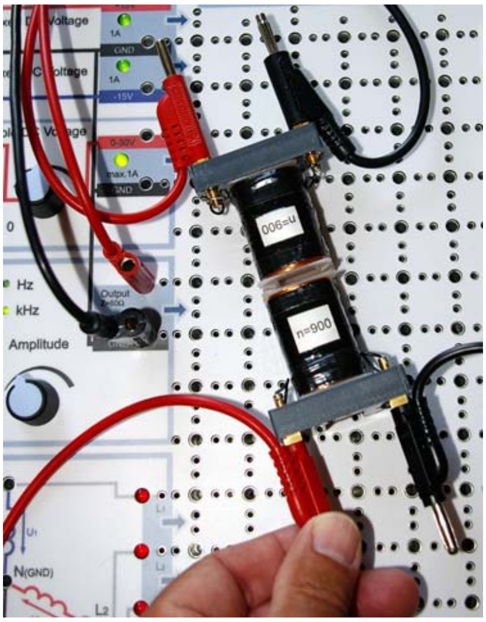


Fig. 18.4.5: U_2 measurement, without core

- Explain the different values measured for the secondary voltage.

- To what extent does the formula for the transformation ratio (or turns ratio) T, apply the the 4 configurations?

Exercise 2: Proving the Transformation Ratio (or turns ratio), T

By taking measurements of voltage and current on 2 transformers with a different secondary winding, the relationship between the transformation ratio T and the number of turns in the winding N, will be examined.

- Assemble the circuit as shown in Fig. 18.4.6 on the Electronic Circuits Board. First, use the secondary coil with $N_2 = 900$ windings.

- Set the function generator for U_1 to sine-wave voltage $U_{rms} = 5$ V, f = 1 kHz.

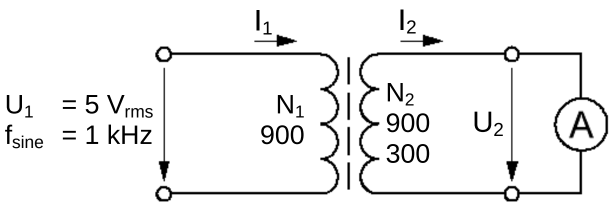


Fig. 18.4.6: Exercise circuit, transformer N_2=900 or 300 with core

- Remove the ammeter, and measure the secondary voltage (with $N_2 = 900$). Replace the secondary coil with the $N_2 = 300$ windings and repeat the voltage measurement. Enter both values measured in table 18.4.7.

- What is the transformation ratio T, when the number of windings in the coils are taken for the calculation?

  With $N_2 = 900$: With $N_2 = 300$:

- Calculate the transformation ratio from the measured values of secondary voltage (enter the values in table 18.4.7).

  With $N_2 = 900$: With N = 300:

N_1	N_2	U_1 [V]	U_2 [V]	T
900	900	5		
	300			

Table 18.4.7: Voltage measurement

N_1	N_2	I_1 [mA]	I_2 [mA]	T
900	900			
	300			

Table 18.4.8: Current measurement

- Now, measure the primary and secondary current for both versions of the transformer. Here, the very low internal resistance of the ammeter present a load to the secondary ciercuit – as shown in Fig. 18.4.6. Enter the measured values in table 18.4.7.

- Calculate the transformation ratio from the measured values of secondary current (enter the values in table 18.4.8).

  With $N_2 = 900$: With $N_2 = 300$:

- The coils are now changed over (Fig. 18.4.9). Now, the primary coil has $N_1 = 300$ windings and the secondary, $N_2 = 900$ windings. Calculate the value that must be set for the input voltage U_1 to give an off-load voltage at the secondary of the transformer of $U_2 = 6{,}5$ V, f = 100 Hz?

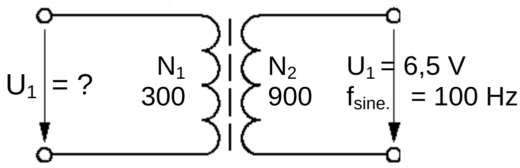


Fig. 18.4.9: Exercise circuit, transformer $N_1 = 300$, $N_2 = 900$ with core

- Turn the voltage control on the function generators fully counter-clockwise (output voltage = 0 V) and switch off the voltage supply to the Electronic Circuits Board.
- Assemble the transformer on the Electronic Circuits Board as shown in Fig. 18.4.9 and prove your calculations by measurement (Slowly and carefully, increase the voltage at the function generator).

  At $U_2 = 6{,}5$ V (secondary), measured at the primary side: U_1 = ...

- During the measurements, sounds can be heard at the transformer, that apparently depend on the frequency of the transformed alternating voltage. At f = 1 kHz this is an unmistakable whistle, then at f = 100 Hz, a humming tone. How are these sounds produced?

Exercise 3: Resistance (Impedance) Transformation

By voltage and current measurements on a transformer with different transformation ratio's and 2 different load resistors, the relationship between the resistance of primary and secondary circuits will be examined.

- Assemble the circuit shown in Fig. 18.4.10 on the Electronic Circuits Board. First, use the secondary coil with $N_2 = 900$ windings.
- Set the function generator for U_1 to sine-wave voltage $U_{rms} = 6$ V, f = 1 kHz.

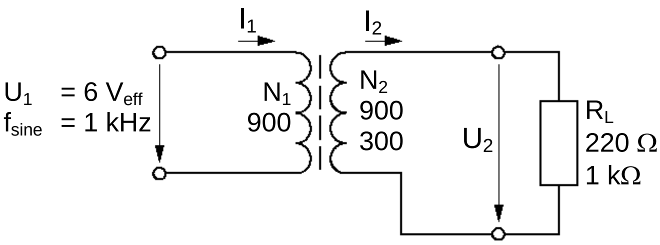


Fig. 18.4.10: Exercise circuit, transformer N_2=900 or 300 with core

- Enter the transformation ratio T for both transformer versions in table 18.4.11.
- Measure the voltage and current for both transformer versions and both load resistors. Enter the values in table 18.4.11.

N_1	N_2	T	R_L [kΩ]	U_1 [V]	U_2 [V]	I_1 [mA]	I_2 [mA]	R_1 [kΩ]	R_2 (R_L) [kΩ]
900	900		1	6					
	300		0,22						

Table 18.4.11: Measured values, Resistance transformation

- Calculate the transformation ratio T from the measured values of resistance.

For N_2 = 900 and R_L = 1 kΩ:

For N_2 = 300 and R_L = 220 Ω: