Supervision systems course

Chapter - 05: Frequency domain Signal Analysis

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Outline

Introduction

In this session focused on frequency domain analysis, our aim is to explore the transformation of a signal acquired in the time domain into the frequency domain. As emphasized in our previous discussion on time domain analysis, each component of machinery in condition-based maintenance reveals itself through characteristic frequencies within the signal. Consequently, a signal inherently carries these distinctive frequencies. In the realm of frequency domain analysis, our objective is to comprehend the various methods at our disposal for identifying the frequencies present in a signal. One fundamental approach involves initiating our analysis from the perspective of the signal's time period.

Need for Frequency Analysis

- Identifying Signal Characteristics: Frequency analysis helps in understanding the different frequency components present in a signal. This is valuable for characterizing and identifying various aspects of the signal, such as the presence of specific frequencies related to machine components in CBM.
- Fault Detection in Machinery: In fields like machinery condition monitoring, the characteristic frequencies of components can indicate potential faults. Frequency analysis allows us to detect abnormal patterns or deviations from expected frequencies, enabling early fault detection (Frequencies Signature).

- Communication Systems: In communication systems, frequency analysis is essential for modulating and demodulating signals. It helps allocate bandwidth efficiently and ensures reliable transmission of information. Audio and Music Processing:
- Frequency analysis is fundamental in audio and music processing to analyze and manipulate sound signals. It allows for tasks like equalization, filtering, and compression to enhance audio quality.
- For structural health monitoring and vibration analysis, frequency analysis is crucial. It helps identify resonant frequencies and provides insights into the dynamic behavior of structures.

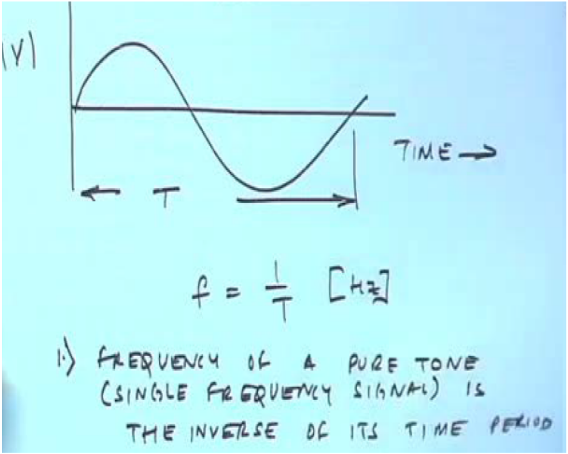
- Image Processing: In image processing, frequency analysis is employed for tasks such as image compression and enhancement. It helps identify patterns and features within images based on their frequency content. Medical Diagnostics:
- In medical diagnostics, frequency analysis is used in techniques like Fourier Transform in analyzing signals from medical instruments. It aids in identifying specific frequencies associated with physiological conditions. Vibration Analysis: Environmental Monitoring:
- Frequency analysis is used in environmental monitoring to analyze signals related to seismic activity, weather patterns, and other natural phenomena.

- Wireless Communication:
  In wireless communication, understanding frequency characteristics is vital for efficient spectrum utilization and interference avoidance. Frequency analysis ensures that different communication channels operate without significant interference.
- Quality Control in Manufacturing: In manufacturing processes, frequency analysis is applied to monitor and control the quality of products. It aids in identifying irregularities or deviations in the production process.

In essence, frequency analysis provides a powerful tool for understanding the content and characteristics of signals in diverse applications. Whether in diagnosing machinery issues, optimizing communication systems, or processing audio and visual information, the ability to analyze frequencies is fundamental to gaining insights and making informed decisions.

Machine Signature Analysis

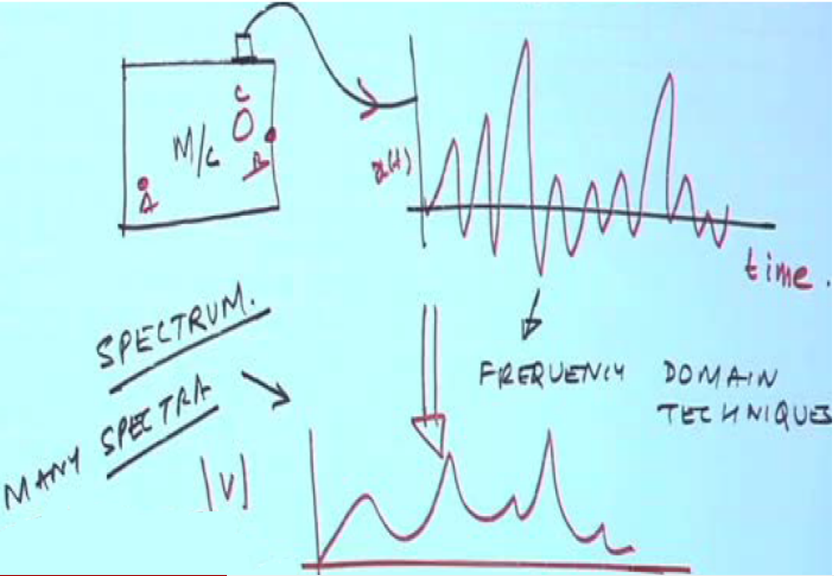
The frequency of a pure tone signal, often referred to as hertz, is essentially the reciprocal of its time period. In simpler terms, for a single-frequency signal or pure tone, its frequency is precisely the inverse of its time period.



Machine Signature Analysis

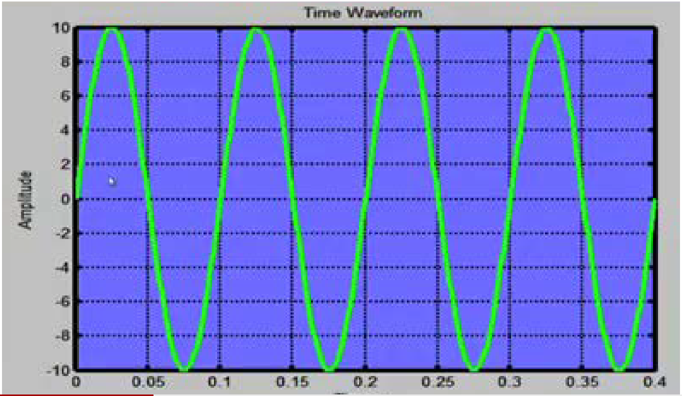
If a machine is equipped with a transducer placed at a specific location, considering the presence of various components labeled as A, B, C, etc., as depicted in the following slide, using the inverse time period technique becomes impractical for determining the frequency of this composite signal. Consequently, there arises a need to explore alternative frequency domain techniques to analyze and understand the signal's characteristics effectively.

Machine Signature Analysis

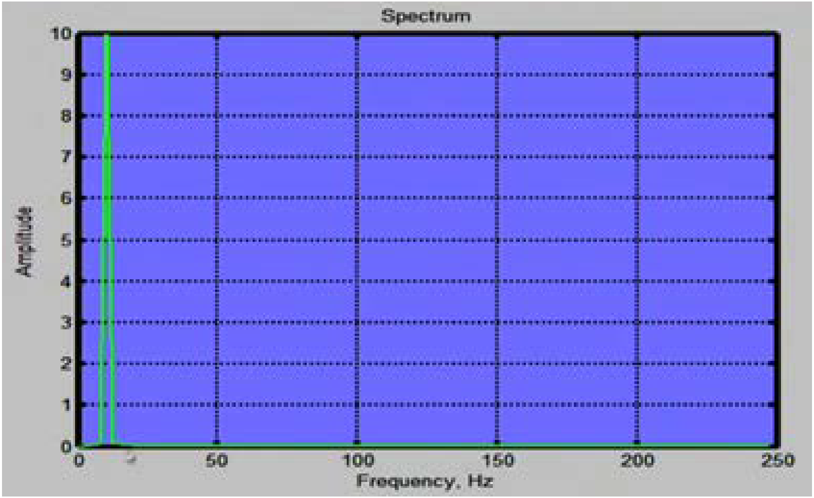


10 Hz Pure Tone

When dealing with signals, such as sinusoidal waves, examining the inverse of their time period provides us with a comprehensive spectrum, as illustrated in the accompanying figures. This approach allows us to gain valuable insights into the frequency composition of the signal and aids in understanding its spectral characteristics.

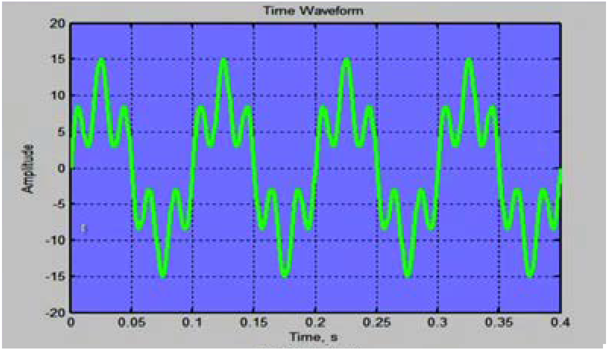


Spectrum of Pure Tone

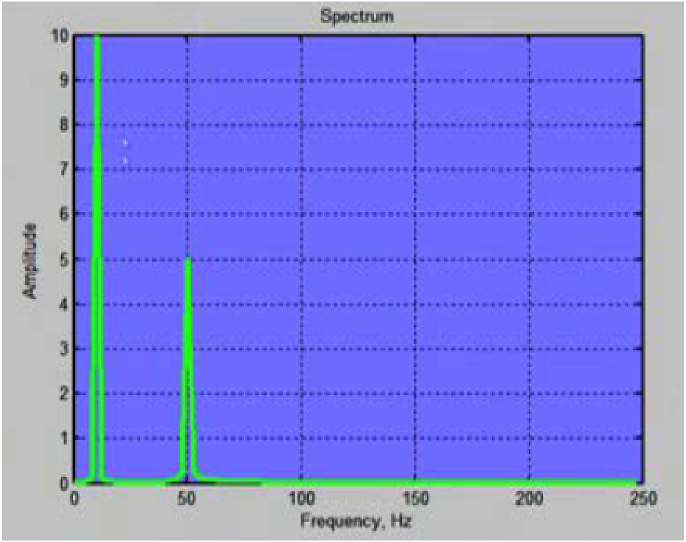


Summation of Two Sinusoids

In the scenario where multiple sinusoidal waves, let's say two, are present, the summation of these waves is visually represented in the figure. Furthermore, the spectrum of this resultant summation is depicted below, offering a clear illustration of the frequency components involved in the combined signal.



Spectrum of the summed sinusoids



What is the Fourier series?

The Fourier series serves as a method to express a non-sinusoidal periodic waveform as the sum of sinusoidal waveforms. This concept is valuable to us for two primary reasons:

- Frequency Distribution Insight: The Fourier series reveals that a periodic waveform can be deconstructed into sinusoidal components at various frequencies. This insight allows us to understand how the waveform is distributed across different frequency components.
- Steady-State Response Analysis: Utilizing the principle of superposition, we can employ the Fourier series to ascertain the steady-state response of a circuit. This is particularly useful when dealing with inputs represented by the Fourier series, enabling the determination of the circuit's steady-state response to periodic waveforms.

Fourier Series

$$\omega_0 = \frac{2\pi}{T}$$

The fundamental frequency has units of rad/s. Integer multiples of the fundamental frequency are called harmonic frequencies.

A periodic function f (t) can be represented by an infinite series of harmonically related sinusoids, called the (trigonometric) Fourier series, as follows:

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\,\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\,\omega_0 t$$

where ω_0 is the fundamental frequency and the (real) coefficients a_0, a_n, and b_n are called the Fourier trigonometric coefficients.

Fourier Series

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\,\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\,\omega_0 t$$

The Fourier trigonometric coefficients can be calculated using:

$$a_0 = \frac{1}{T}\int_{t_0}^{T+t_0} f(t)dt = \text{the average value of } f(t)$$

$$a_n = \frac{2}{T}\int_{t_0}^{T+t_0} f(t)\cos n\,\omega_0 t\,dt \quad n > 0$$

$$b_n = \frac{2}{T}\int_{t_0}^{T+t_0} f(t)\sin n\,\omega_0 t\,dt \quad n > 0$$

Fourier Series

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\,\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\,\omega_0 t$$

The conditions presented by Dirichlet are sufficient to guarantee the convergence of the trigonometric Fourier series. The Dirichlet conditions require that the periodic function f (t) satisfies the following mathematical properties:

1. f (t) is a single-valued function except at possibly a finite number of points.
2. f (t) is absolutely integral, that is,

$$\int_{t_0}^{t_0+T} |f(t)|dt < \infty \quad \text{for any } t_0$$

3. f (t) has a finite number of discontinuities within the period T.
4. f (t) has a finite number of maxima and minima within the period T.

Fourier Series

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\,\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\,\omega_0 t$$

❖ **Fourier series**

$$a_0 = \frac{1}{T}\int_{t_0}^{T+t_0} f(t)dt = \text{the average value of } f(t)$$

$$a_n = \frac{2}{T}\int_{t_0}^{T+t_0} f(t)\cos n\,\omega_0 t\,dt \quad n > 0$$

$$b_n = \frac{2}{T}\int_{t_0}^{T+t_0} f(t)\sin n\,\omega_0 t\,dt \quad n > 0$$

A Fourier series is an accurate representation of a periodic signal and consists of the sum of sinusoids at the fundamental and harmonic frequencies.

Given a periodic voltage or current waveform, we can obtain the Fourier representation of that voltage or current in four steps:

Step 1: Determine the period T and the fundamental frequency ω_0.

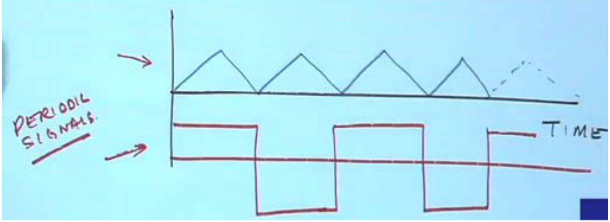
Step 2: Represent the voltage or current waveform as a function of t over one complete period.

Step 3: Determine the Fourier trigonometric coefficients a_0, a_n, and b_n.

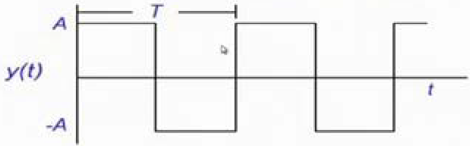
Step 4: Substitute the coefficients a_0, a_n, and b_n obtained in Step 3.

What is a periodic signal?

A continuous-time signal $x(t)$ is said to be periodic with period T if there is a positive nonzero value of T for which $x(t + T) = x(t)$ for all t it follows that $x(t + mT) = x(t)$ for all t and any integer m.



Example on Fourier Series

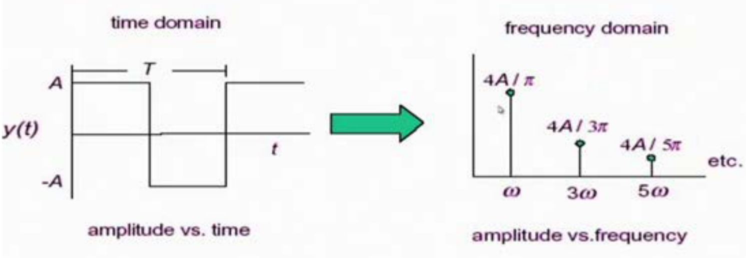


$$A_n = \frac{2}{T}\int_0^T y(t)\cos n\omega t dt = \frac{2}{T}\left[\int_0^{T/2} A\cos n\omega t dt + \int_{T/2}^{T} (-A)\cos n\omega t dt\right] = \underline{0}$$

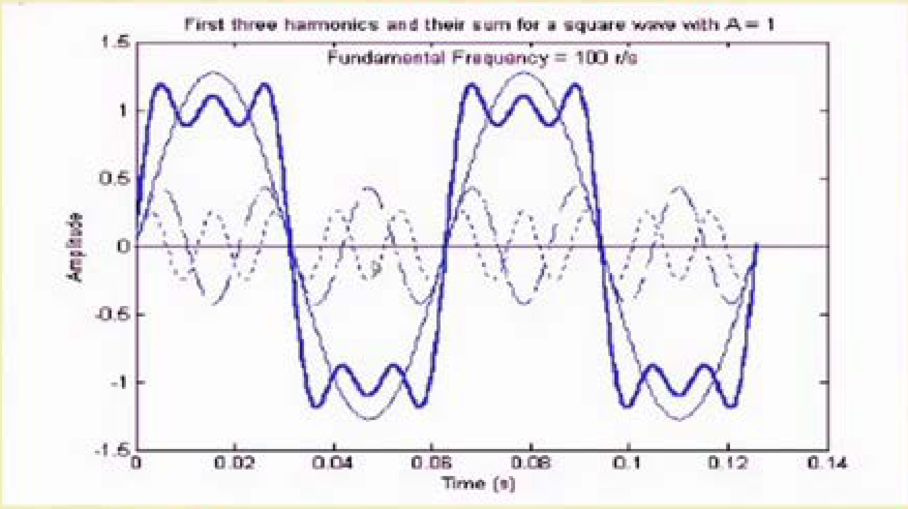
$$B_n = \frac{2}{T}\int_0^T y(t)\sin n\omega t dt = \begin{cases} 4A/n\pi & , n \text{ odd} \\ 0 , & n \text{ even} \end{cases}$$

$$C_n = \sqrt{A_n^2 + B_n^2} = B_n \qquad \phi_n = \tan^{-1}(B_n/A_n) = \pi/2$$

Example(cont'd)



Example(cont'd)



Example2

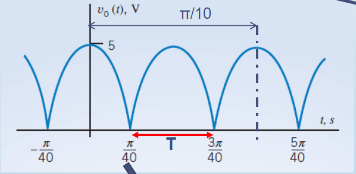
EXAMPLE:

Figure 1 shows a full-wave rectifier having a cosine input. The output of a full-wave input is the absolute value of its input, shown in Figure 2. A full-wave rectifier is an electronic circuit often used as a component of such diverse products as power supplies and AM radio receivers. Determine the Fourier series of the periodic waveform shown in Figure 2.

Solution

Step 1: From second figure, we see that the period of $v_o(t)$ is

input ω_0= 20 rad/s ➡ $\omega_0 = \frac{2\pi}{T}$ ➡ T= π/10 s



Note: Old 1 full period was 2pi→pi/10
Then, pi/2→pi/40

output

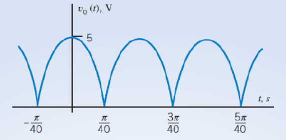
$$T = \frac{3\pi}{40} - \frac{\pi}{40} = \frac{\pi}{20}\ \text{s}$$

The fundamental radian frequency is

$$\omega_0 = \frac{2\pi}{T} = 40\ \text{rad/s}$$

Example2(cont'd)

Step 2: Equations require integration over one full period of $v_o(t)$. We are free to choose the starting point of that period, to make the integration as easy as possible. Often, we choose to integrate either from *0* to *T* or from *–T/2* to *+T/2*. In this example, the periodic waveform can be represented as



$$v_o(t) = \begin{cases} 5\cos(20t) & \text{when } -\frac{\pi}{40} \le t \le \frac{\pi}{40} \\ -5\cos(20t) & \text{when } \frac{\pi}{40} \le t \le \frac{3\pi}{40} \end{cases}$$

$$a_0 = \frac{20}{\pi}\int_0^{\pi/20} v_o(t)dt = \frac{20}{\pi}\int_0^{\pi/40} 5\cos(20t)dt + \frac{20}{\pi}\int_{\pi/40}^{\pi/20} -5\cos(20t)dt$$

On the other hand, if we choose to integrate from $-T/2$ to $T/2$, we have

$$a_0 = \frac{20}{\pi}\int_{-\pi/40}^{\pi/40} v_o(t)dt = \frac{20}{\pi}\int_{-\pi/40}^{\pi/40} 5\cos(20t)dt$$

The second equation is simpler, so we choose to integrate from $-T/2$ to $+T/2$ for convenience.

Example2(cont'd)

Step 3: Now we will determine the Fourier trigonometric coefficients a_0, a_n, and b_n. First,

$$a_0 = \frac{20}{\pi}\int_{-\pi/40}^{\pi/40} 5\cos(20t)dt = \frac{100}{\pi}\left(\frac{1}{20}\sin(20t)|_{-\pi/40}^{\pi/40}\right) = \frac{5}{\pi}\left(\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right) = \boxed{\frac{10}{\pi}}$$

Next,

$$a_n = \frac{40}{\pi}\int_{-\pi/40}^{\pi/40} 5\cos(20t)\cos(n\,\omega_0 t)dt = \frac{40}{\pi}\int_{-\pi/40}^{\pi/40} 5\cos\boxed{(20t)}\cos\boxed{(40nt)}dt$$

$\sin(A + B) = \sin A.\cos B + \cos A.\sin B$

$\sin(A - B) = \sin A.\cos B - \cos A.\sin B$

$\cos(A + B) = \cos A.\cos B - \sin A.\sin B$

$\cos(A - B) = \cos A.\cos B + \sin A.\sin B$

Using a trigonometric identity,

$$\cos(20t)\cos(40nt) = \frac{1}{2}(\cos(20t + 40nt) + \cos(20t - 40nt))$$
$$= \frac{1}{2}(\cos((1+2n)20t) + \cos((1-2n)20t))$$

Why different: Because, f(t) is [5cos 20t]. **The input curve is changing according to this expression**. Main ω_0 is the final signals's frequency in cos $n\omega_0 t$.

$$a_n = \frac{2}{T}\int_{t_0}^{T+t_0} f(t)\cos n\,\omega_0 t\,dt \quad n > 0$$

Example2(cont'd)

❖ Fourier series

Then,

$$a_n = \frac{100}{\pi}\int_{-\pi/40}^{\pi/40} (\cos((1+2n)20t) + \cos((1-2n)20t))dt$$

$$= \frac{100}{\pi}\left(\frac{\sin((1+2n)20t)}{(1+2n)20}\Big|_{-\pi/40}^{\pi/40} + \frac{\sin((1-2n)20t)}{(1-2n)20}\Big|_{-\pi/40}^{\pi/40}\right)$$

$$= \frac{5}{\pi}\left(\frac{\sin\left((1+2n)\frac{\pi}{2}\right) - \sin\left(-(1+2n)\frac{\pi}{2}\right)}{(1+2n)} + \frac{\sin\left((1-2n)\frac{\pi}{2}\right) - \sin\left(-(1-2n)\frac{\pi}{2}\right)}{(1-2n)}\right)$$

$$= \frac{5}{\pi}\left(\frac{2(-1)^n}{(1+2n)} + \frac{2(-1)^n}{(1-2n)}\right) = \frac{20(-1)^n}{\pi(1-4n^2)}$$

Similarly,

$$b_n = \frac{40}{\pi}\int_{-\pi/40}^{\pi/40} 5\cos(20t)\sin(40\,nt)dt$$

$$= \frac{100}{\pi}\int_{-\pi/40}^{\pi/40} (\sin((2n+1)20t) + \sin((2n-1)20t))dt$$

$$= \frac{100}{\pi}\left(\frac{-\cos((1+2n)20t)}{(1+2n)20}\Big|_{-\pi/40}^{\pi/40} + \frac{-\cos((1-2n)20t)}{(1-2n)20}\Big|_{-\pi/40}^{\pi/40}\right) = 0$$

$\sin(A + B) = \sin A.\cos B + \cos A.\sin B$

$\sin(A - B) = \sin A.\cos B - \cos A.\sin B$

$\cos(A + B) = \cos A.\cos B - \sin A.\sin B$

$\cos(A - B) = \cos A.\cos B + \sin A.\sin B$

$$= \sin\left((1+2n)\frac{\pi}{2}\right) - \sin\left(-(1+2n)\frac{\pi}{2}\right)$$

$$= \sin\left(\frac{\pi}{2}+n\pi\right) - \sin\left(-(\frac{\pi}{2}+n\pi)\right)$$

$$= Sin\,\frac{\pi}{2}.Cos\,n\pi + Cos\,\frac{\pi}{2}.Sin\,n\pi = Sin\,\frac{\pi}{2}.Cos\,n\pi$$

$$= Sin\,\frac{\pi}{2}.Cos\,n\pi = (-1)^n$$

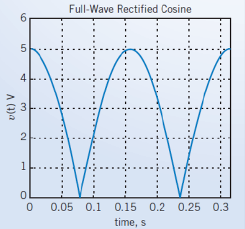
$$a_0 = \frac{10}{\pi},\ a_n = \frac{20(-1)^n}{\pi(1-4n^2)} \text{ and } b_n = 0$$

Example(cont'd)

Step 4: Substitute the coefficients a_0, a_n, and b_n

$$v_o(t) = \frac{10}{\pi} + \frac{20}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{1-4n^2}\cos(40\,nt)$$

Equation above represents the rectified cosine by its Fourier series, but this equation is complicated enough to make us wonder what we have accomplished.
How can we be sure that it actually represents a rectified cosine? If we plot it by MATLAB:



Limitation of Traditional Fourier Analysis

Difficult to implement numerically for a measured signal (e.g. one having no obvious mathematical form; I need to have expression y(t) known to me).

Limited to periodic signals (cannot handle transient wave forms or random signals)

What is the Fourier transform ?

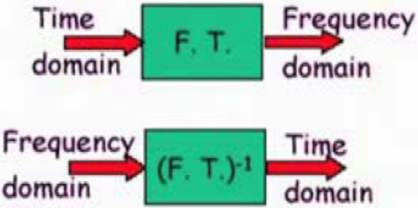
- We obtain the Fourier transform as a generalization of the Fourier series, taking the limit as the period of a periodic wave becomes infinite.
- The Fourier transform is useful to us in two ways:

1. The Fourier transform represents an aperiodic waveform in the frequency domain. That allows us to think about the way in which the waveform is distributed in frequency. For example, we can give meaning to such expressions as the high-frequency part of a pulse.
2. We can represent both the input to a circuit and the circuit itself in the frequency domain: the input represented by its Fourier transform and the circuit represented by its network function. The frequency-domain representation of circuit output is obtained as the product of the Fourier transform of the input and the network function of the circuit.

The Fourier Integral (Transform)

$$Y(f) = \int_{-\infty}^{+\infty} y(t) e^{-j2\pi ft} dt$$

$$y(t) = \int_{-\infty}^{+\infty} Y(f) e^{+j2\pi ft} df$$



- *y(t)* may be transient, random, or periodic
- *Y(f)* is in general complex
- *Y(f)* and *y(t)* form a *Fourier transform pair*
- *Y(f)* is related to the Laplace Transform

Thank You!