جامعة فلســـــطين التقنية - خضـــوري
Palestine Technical University - Kadoorie

Digital Electronics and Logic Design

Number System

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Common Number Systems

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	No
Hexa-decimal	16	0, 1, … 9, A, B, … F	No	Yes

Quantities/Counting (1 of 3)

Decimal	Binary	Octal	Hexa-decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

Quantities/Counting (2 of 3)

Decimal	Binary	Octal	Hexa-decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

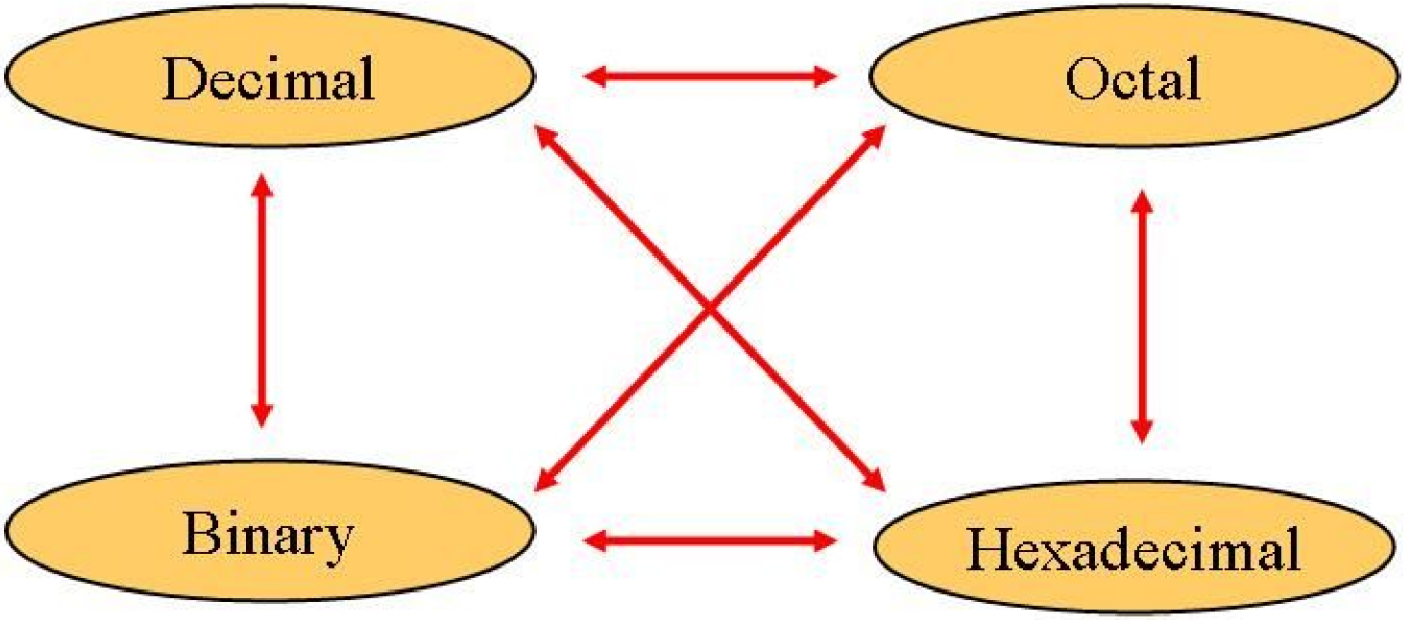
Quantities/Counting (3 of 3)

Decimal	Binary	Octal	Hexa-decimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17

Etc.

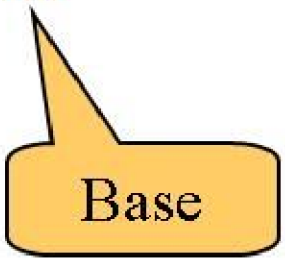
Conversion Among Bases

The possibilities:

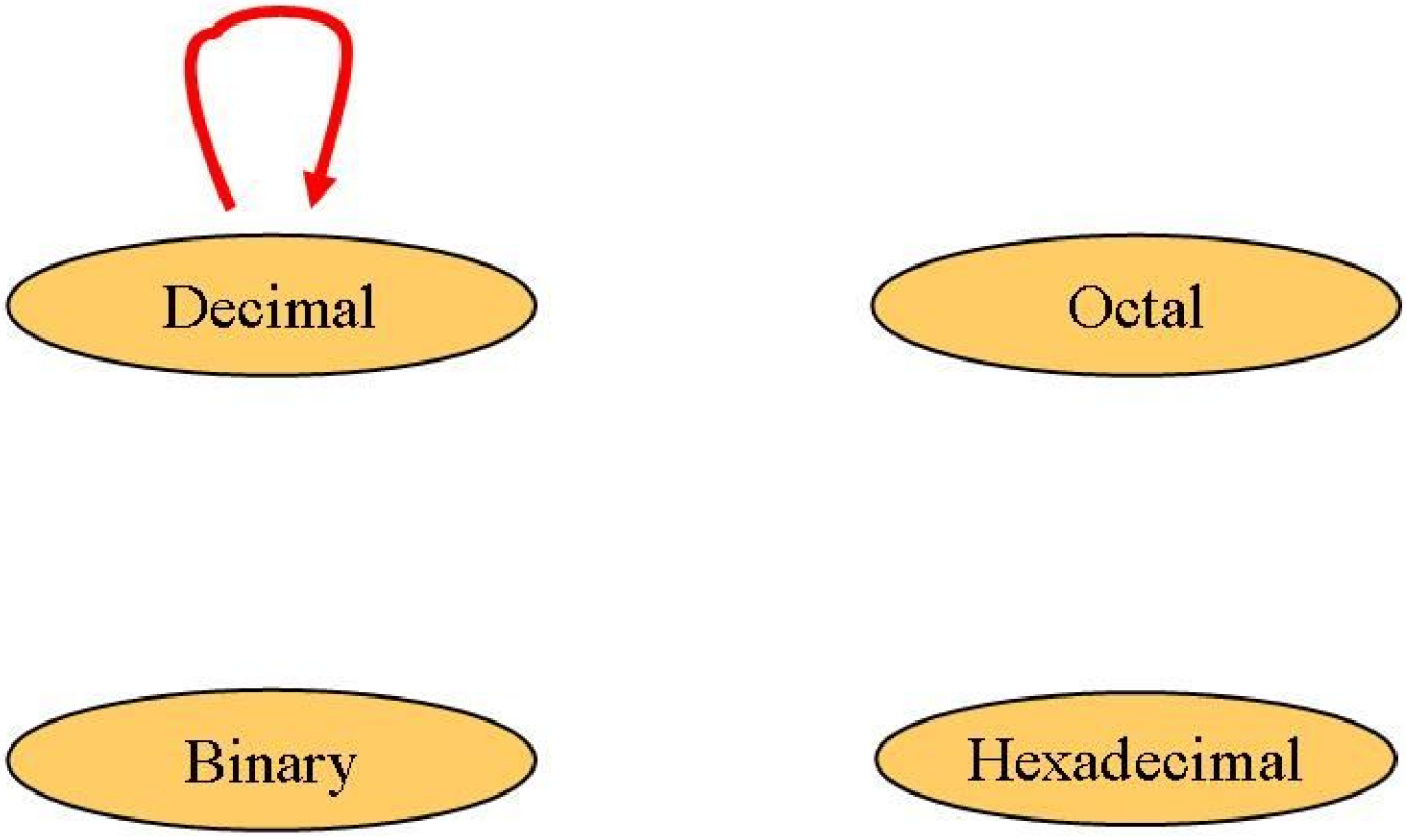


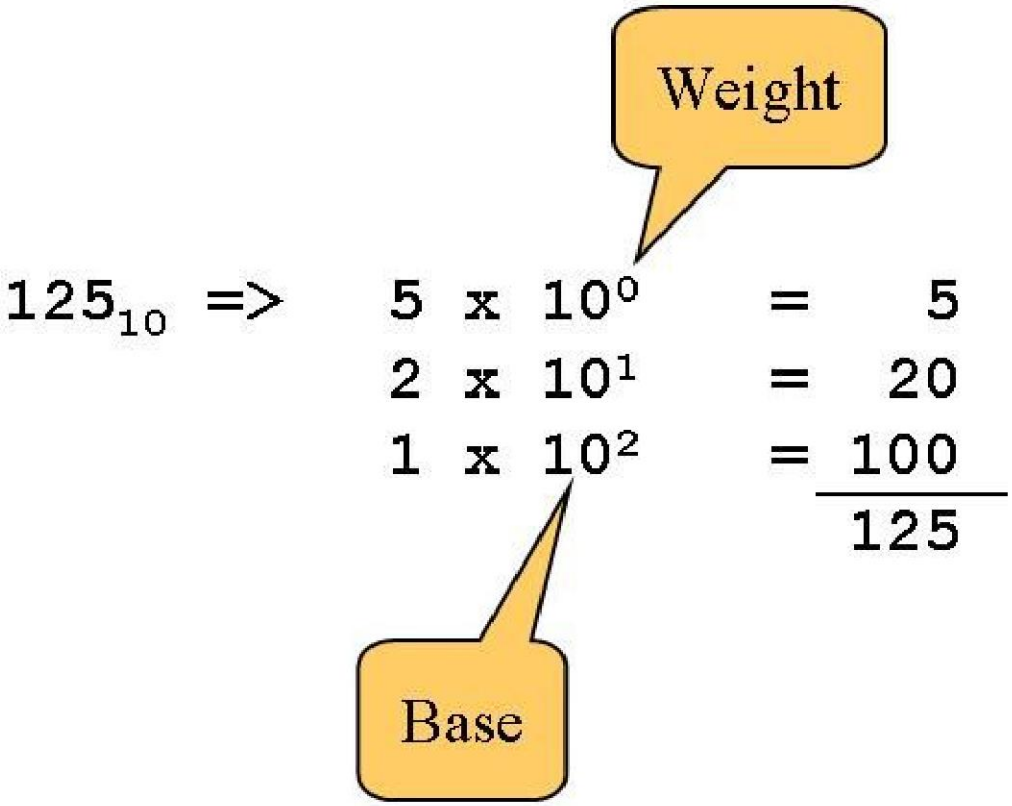
Quick Example

$$25_{10} = 11001_2 = 31_8 = 19_{16}$$

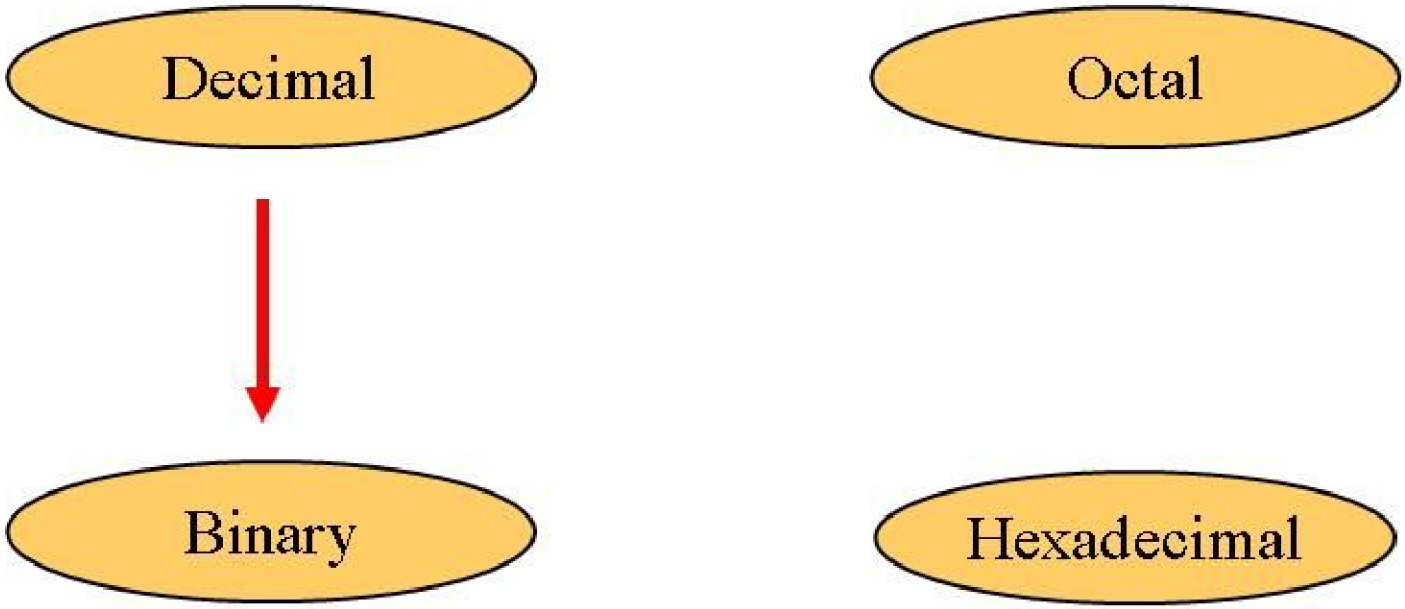


Decimal to Decimal (just for fun)




Weight
125_{10} =>
5 x 10^0 = 5
2 x 10^1 = 20
1 x 10^2 = 100
125
Base

Decimal to Binary



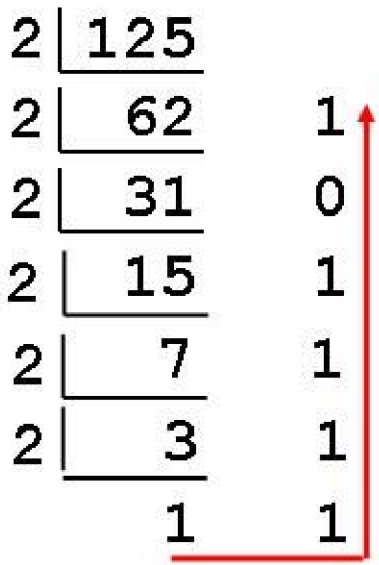
Decimal to Binary

Technique

- Divide by two, keep track of the remainder
- First remainder is bit 0 (LSB, least-significant bit)
- Second remainder is bit 1
- Etc.

Example

$125_{10} = ?_2$



$125_{10} = 1111101_2$

Decimal to Binary conversion

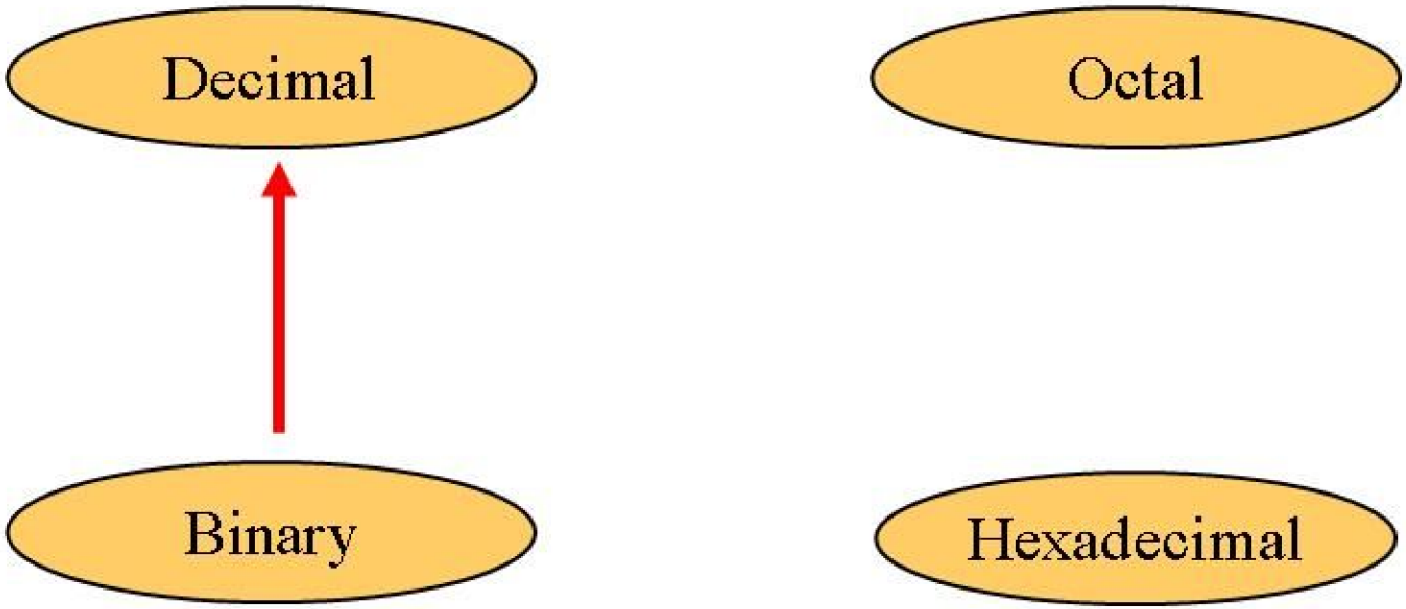
Convert $(0.6875)_{10}$ to binary.

	Integer		Fraction
$0.6875 \times 2 =$	1	+	0.3750
$0.3750 \times 2 =$	0	+	0.7500
$0.7500 \times 2 =$	1	+	0.5000
$0.5000 \times 2 =$	1	+	0.0000

Answer: $(0.6875)_{10} = (0.1011)_2$

$$(41.6875)_{10} = (101001.1011)_2$$

Binary to Decimal



Binary to Decimal

Technique

- Multiply each bit by 2^n, where n is the "weight" of the bit
- The weight is the position of the bit, starting from 0 on the right
- Add the results

Example

101011_2 =>

$1 \times 2^0 = 1$

$1 \times 2^1 = 2$

$0 \times 2^2 = 0$

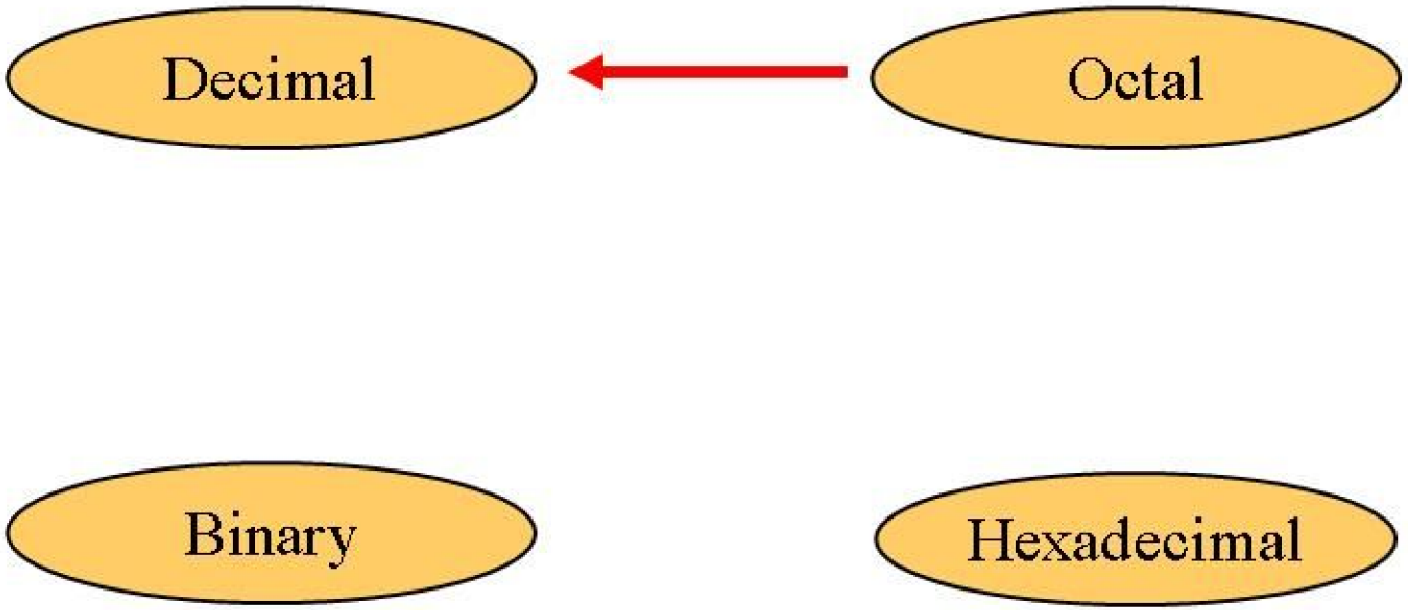
$1 \times 2^3 = 8$

$0 \times 2^4 = 0$

$1 \times 2^5 = 32$

43_{10}

Octal to Decimal



Octal to Decimal

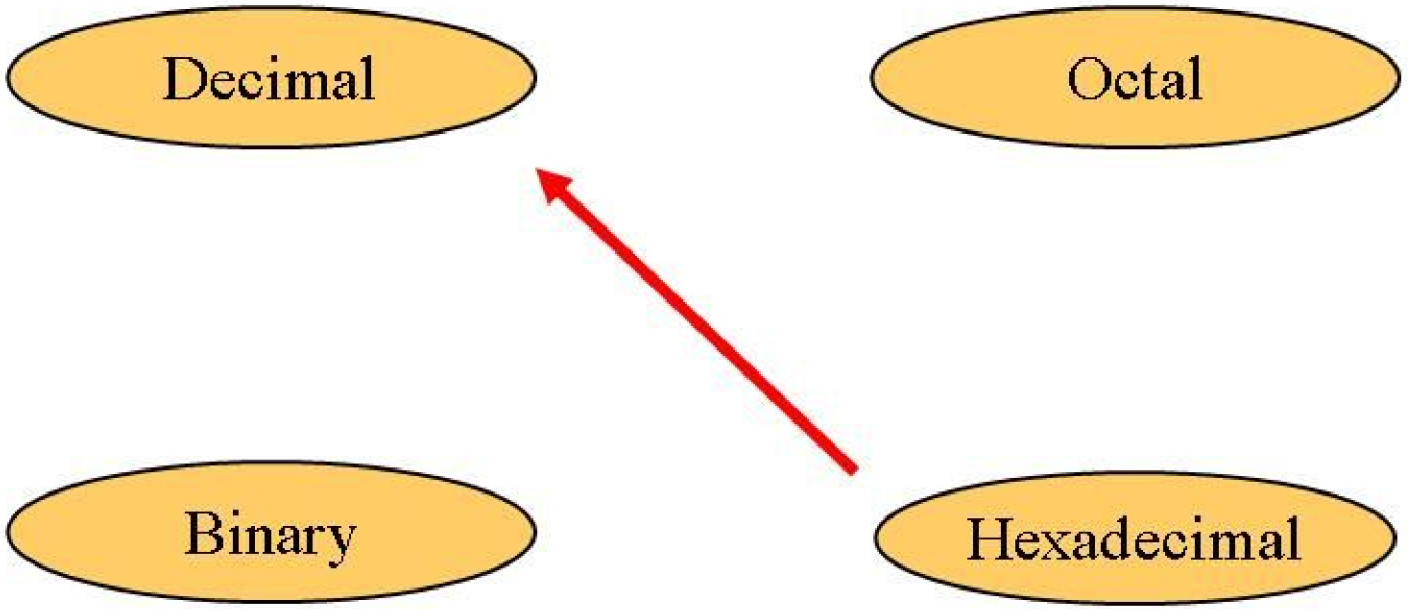
Technique

- Multiply each bit by 8^n, where n is the "weight" of the bit
- The weight is the position of the bit, starting from 0 on the right
- Add the results

Example

$$724_8 \Rightarrow 4 \times 8^0 = 4$$
$$2 \times 8^1 = 16$$
$$7 \times 8^2 = 448$$
$$\underline{\qquad\qquad}$$
$$468_{10}$$

Hexadecimal to Decimal



Hexadecimal to Decimal

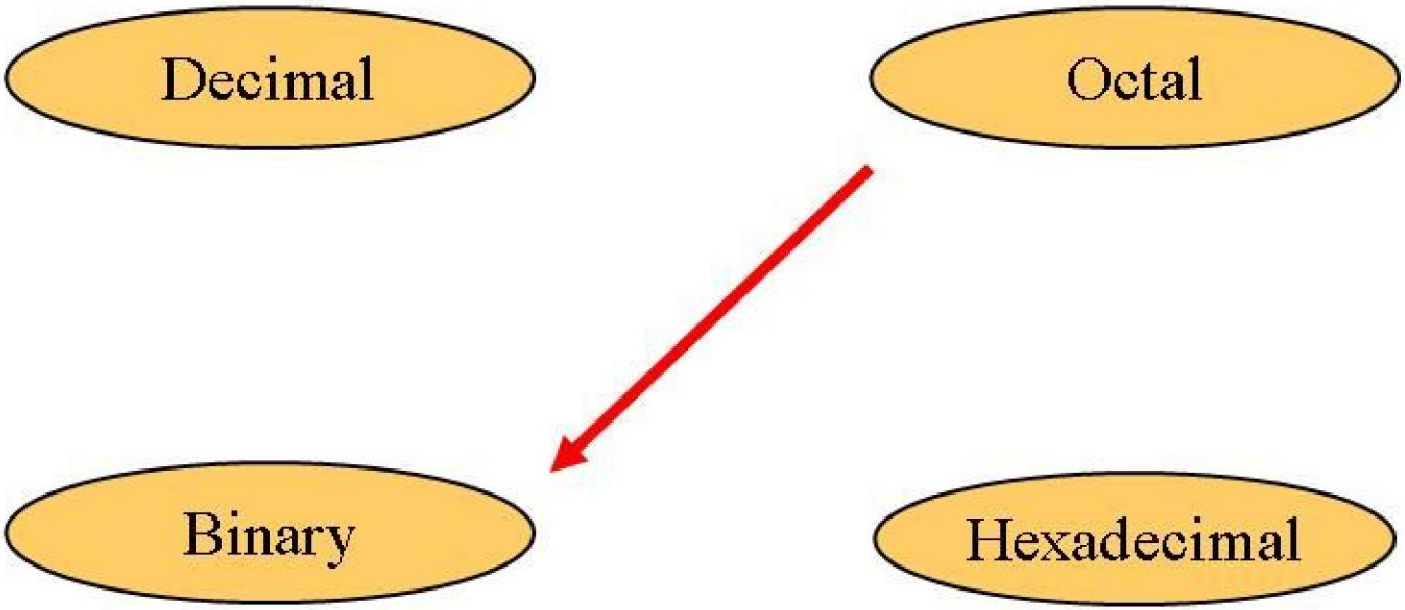
Technique

- Multiply each bit by 16^n, where n is the "weight" of the bit.
- The weight is the position of the bit, starting from 0 on the right.
- Add the results.

Example

$$ABC_{16} \Rightarrow C \times 16^0 = 12 \times 1 = 12$$
$$B \times 16^1 = 11 \times 16 = 176$$
$$A \times 16^2 = 10 \times 256 = 2560$$
$$2748_{10}$$

Octal to Binary



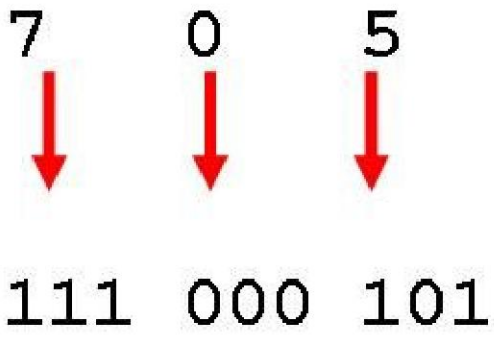
Octal to Binary

Technique

- Convert each octal digit to a 3-bit equivalent binary representation

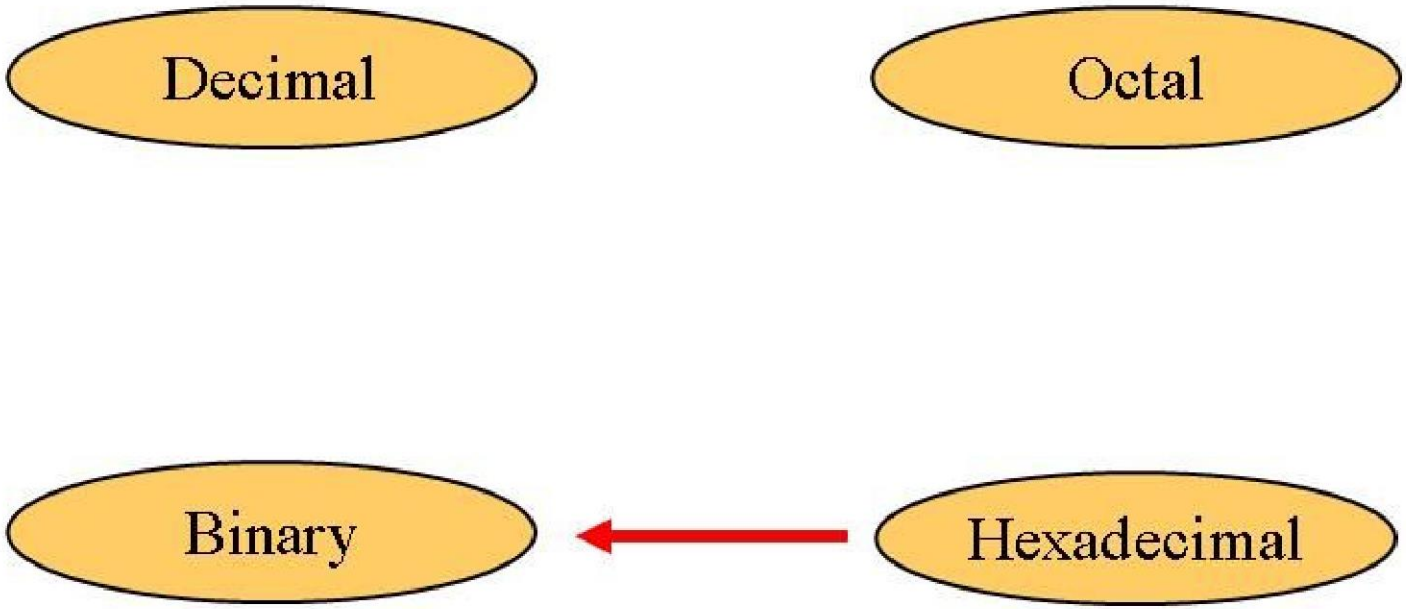
Example

$705_8 = ?_2$



$705_8 = 111000101_2$

Hexadecimal to Binary



Hexadecimal to Binary

Technique

- Convert each hexadecimal digit to a 4-bit equivalent binary representation.

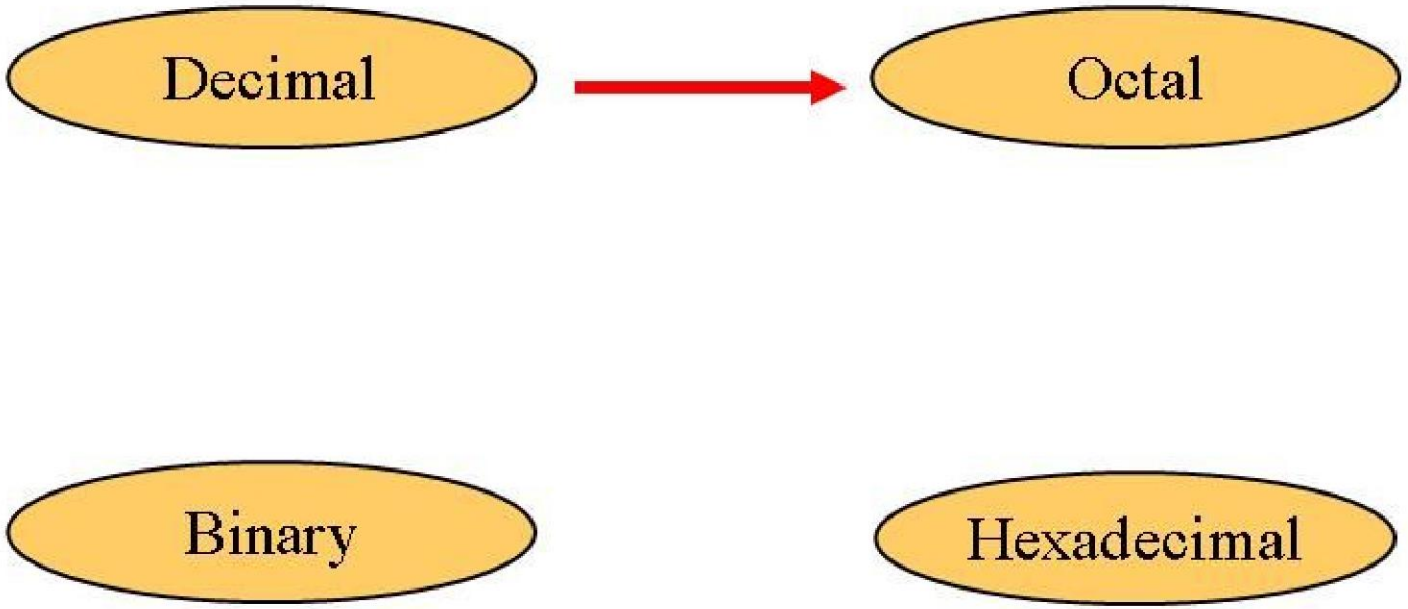
Example

$10AF_{16} = ?_2$

0001 0000 1010 1111

$10AF_{16} = 0001000010101111_2$

Decimal to Octal



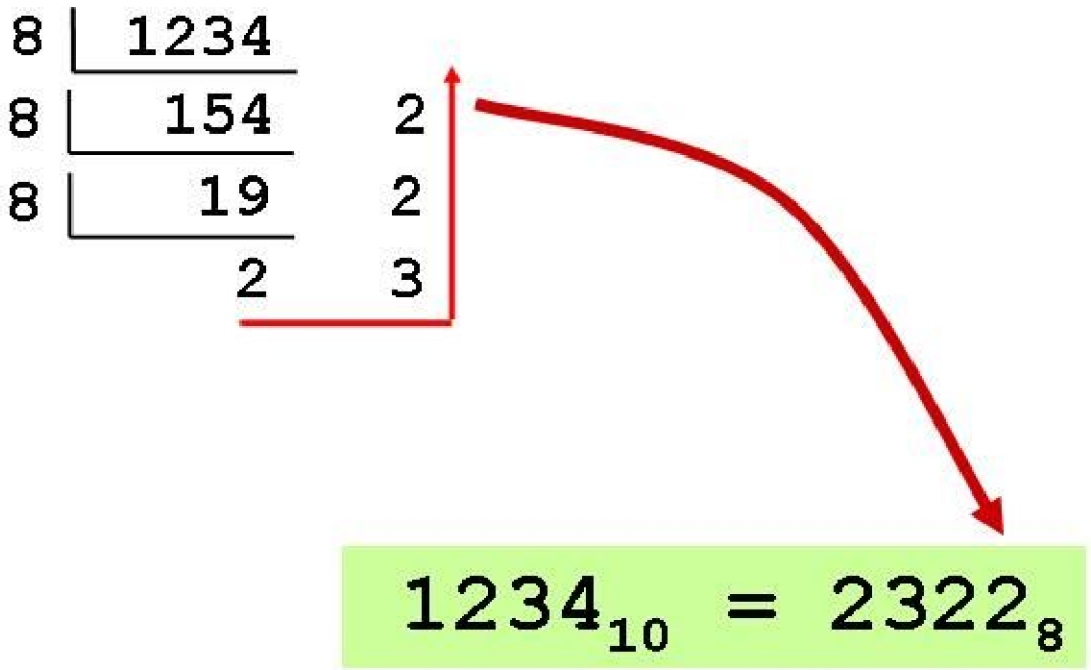
Decimal to Octal

Technique

- Divide by 8
- Keep track of the remainder

Example

$1234_{10} = ?_8$



Decimal to Octal Conversion

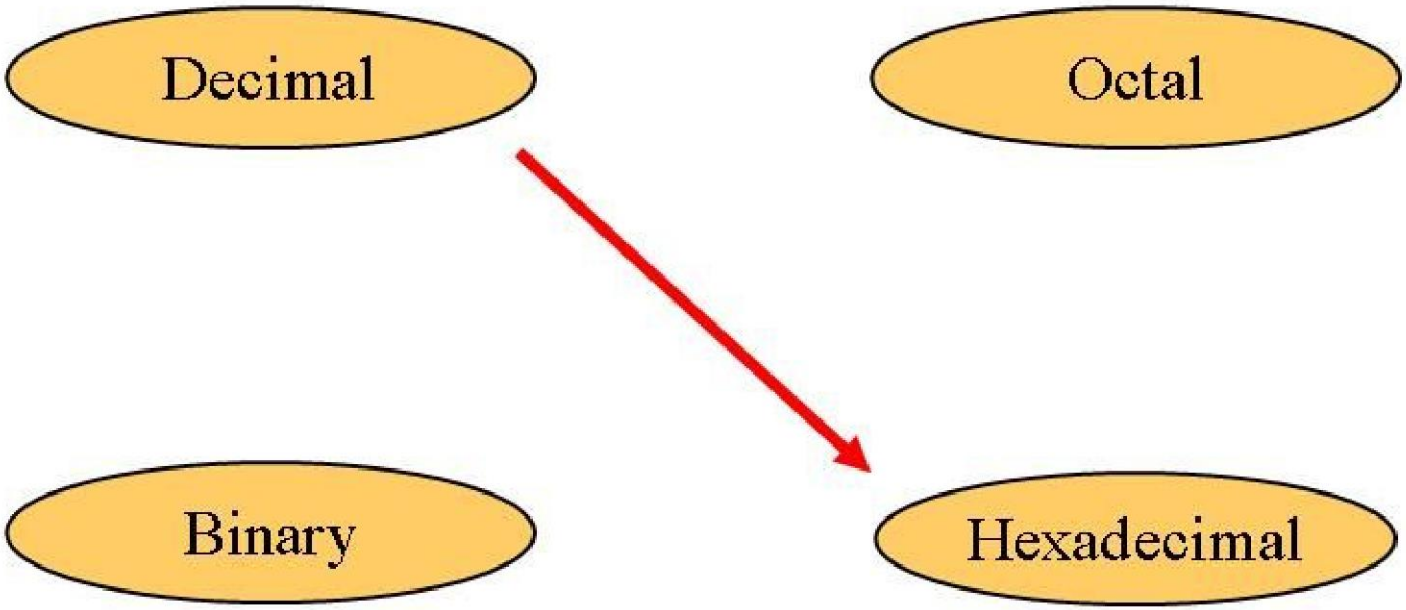
Convert $(0.513)_{10}$ to octal.

$$0.513 \times 8 = 4.104$$
$$0.104 \times 8 = 0.832$$
$$0.832 \times 8 = 6.656$$
$$0.656 \times 8 = 5.248$$
$$0.248 \times 8 = 1.984$$
$$0.984 \times 8 = 7.872$$

Answer $(0.513)_{10} = (0.406517\ldots)_8$

$$(153.513)_{10} = (231.406517)_8$$

Decimal to Hexadecimal



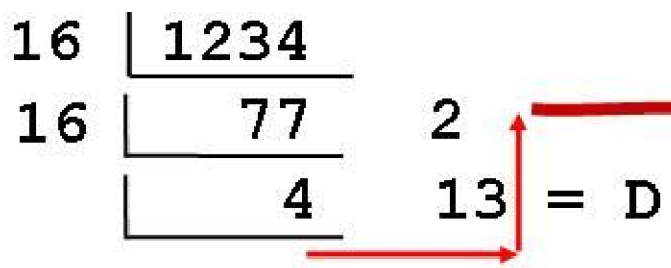
Decimal to Hexadecimal

Technique

- Divide by 16
- Keep track of the remainder

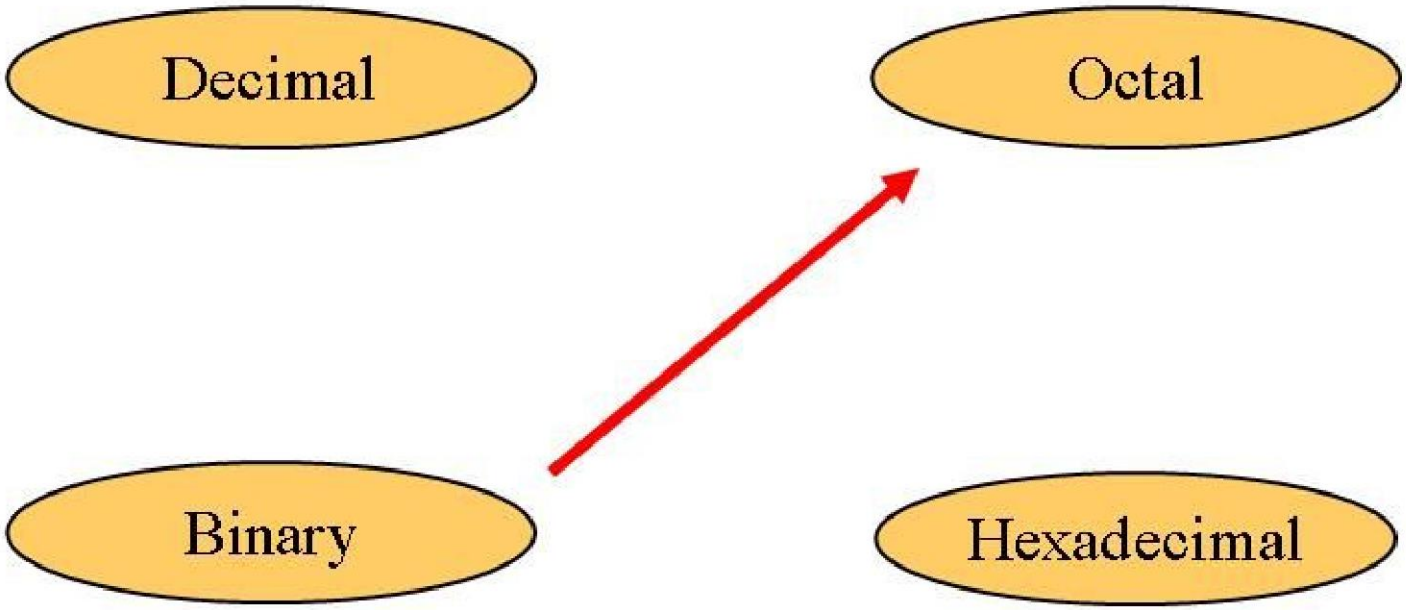
Example

$1234_{10} = ?_{16}$



$1234_{10} = 4D2_{16}$

Binary to Octal



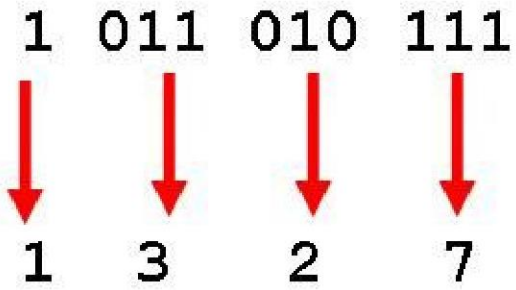
Binary to Octal

Technique

- Group bits in threes, starting on right
- Convert to octal digits

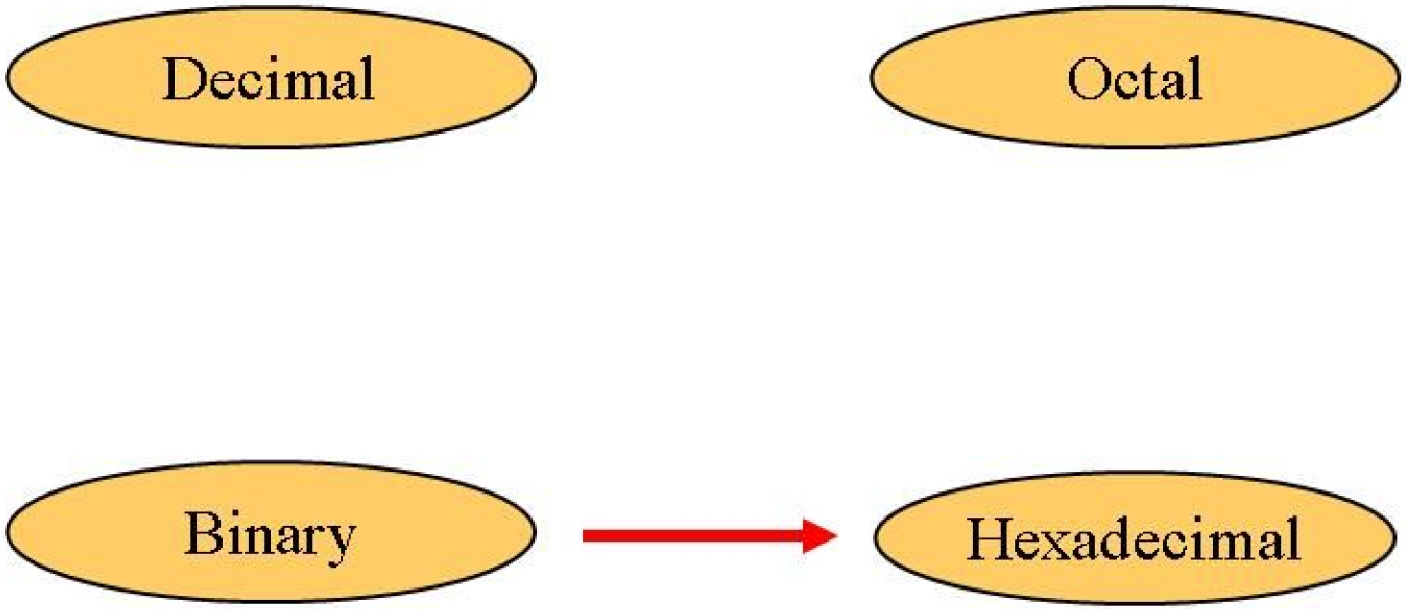
Example

$1011010111_2 = ?_8$



$1011010111_2 = 1327_8$

Binary to Hexadecimal



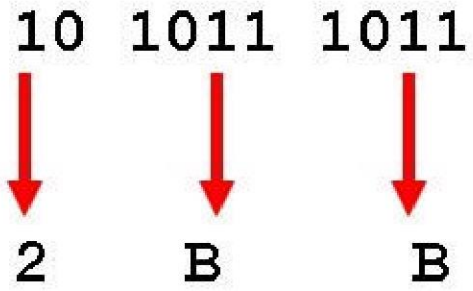
Binary to Hexadecimal

Technique

- Group bits in fours, starting on right
- Convert to hexadecimal digits

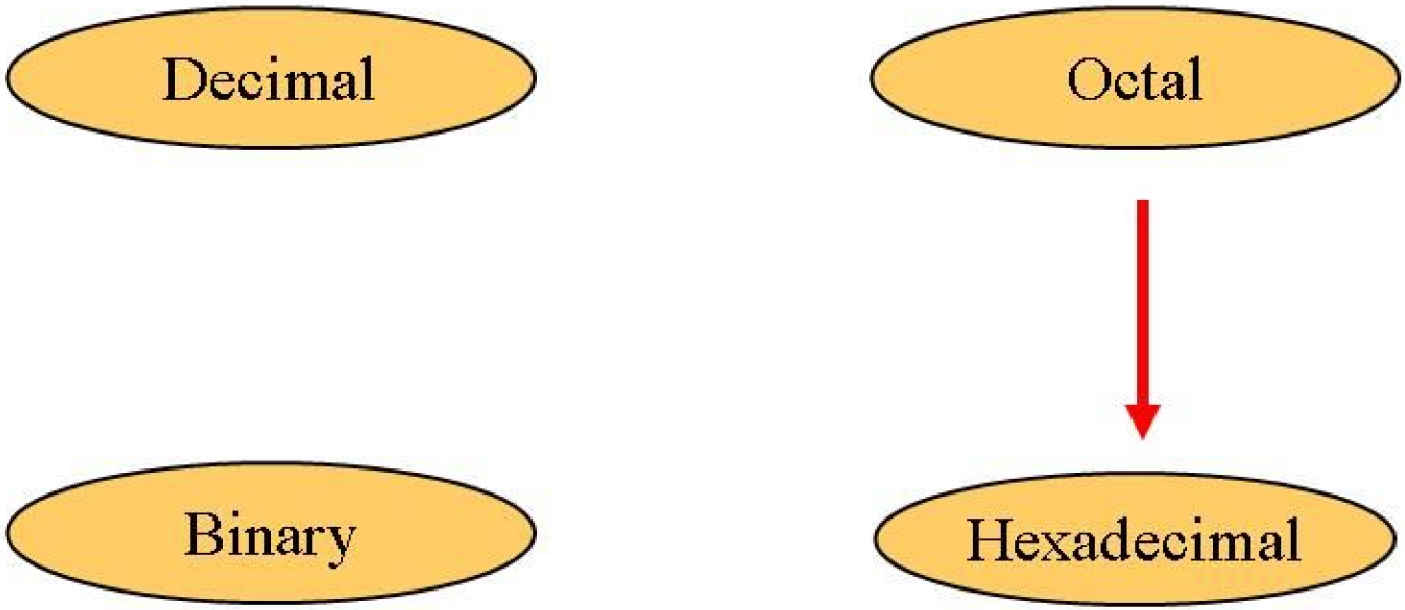
Example

$1010111011_2 = ?_{16}$



$1010111011_2 = 2BB_{16}$

Octal to Hexadecimal



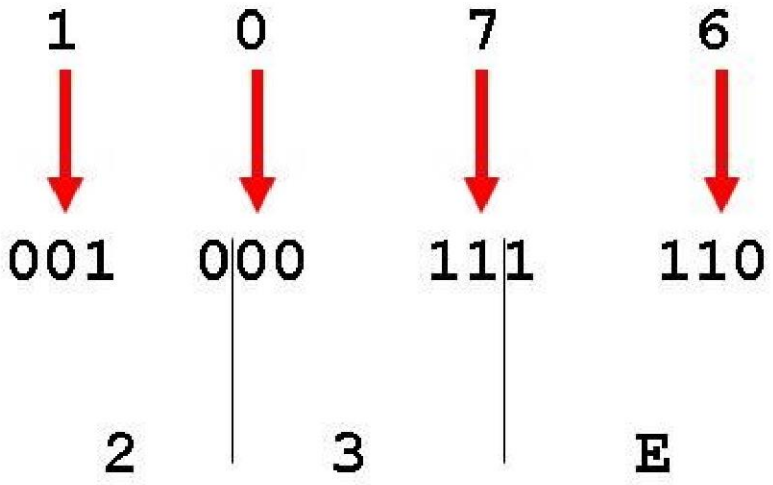
Octal to Hexadecimal

Technique

- Use binary as an intermediary

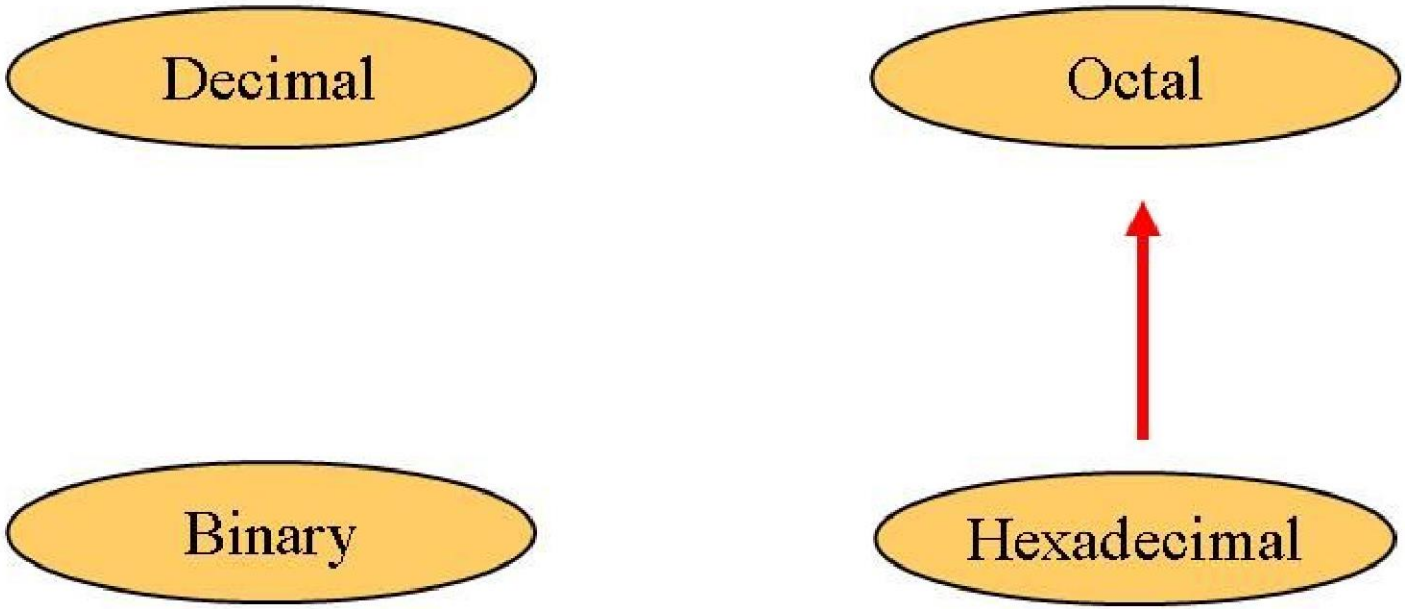
Example

$1076_8 = ?_{16}$



$1076_8 = 23E_{16}$

Hexadecimal to Octal



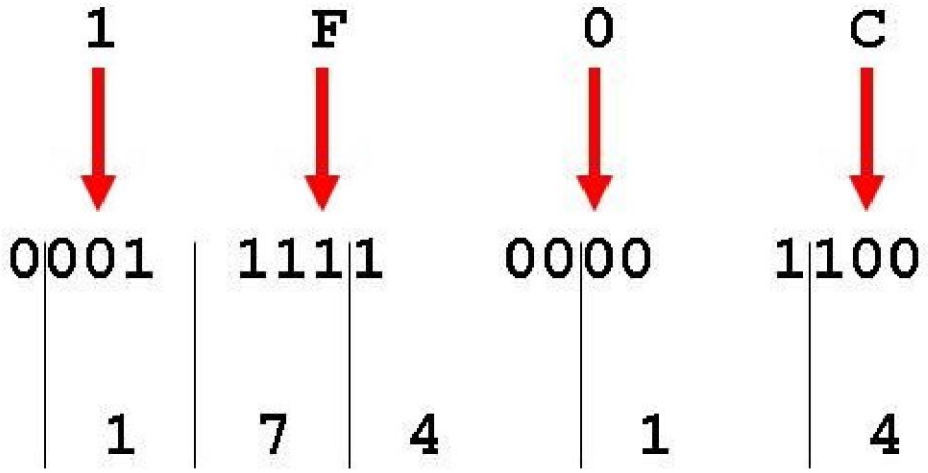
Hexadecimal to Octal

Technique

- Use binary as an intermediary

Example

$1F0C_{16} = ?_8$



$1F0C_{16} = 17414_8$

Exercise – Convert ...

Decimal	Binary	Octal	Hexa-decimal
33			
	1110101		
		703	
			1AF

Exercise – Convert …

Answer

Decimal	Binary	Octal	Hexa-decimal
33	100001	41	21
117	1110101	165	75
451	111000011	703	1C3
431	110101111	657	1AF

Common Powers (1 of 2)

Base 10

Power	Preface	Symbol	Value
10^{-12}	pico	p	.000000000001
10^{-9}	nano	n	.000000001
10^{-6}	micro	μ	.000001
10^{-3}	milli	m	.001
10^{3}	kilo	k	1000
10^{6}	mega	M	1000000
10^{9}	giga	G	1000000000
10^{12}	tera	T	1000000000000

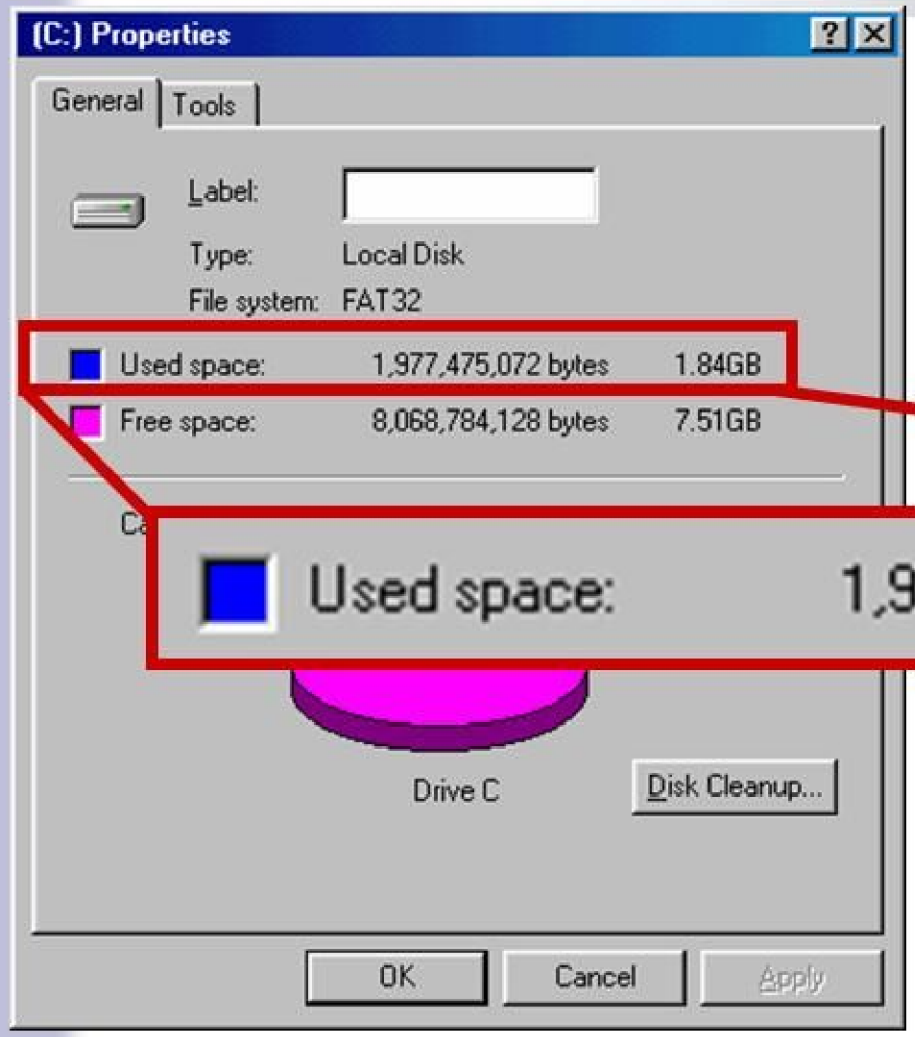
Common Powers (2 of 2)

Base 2

Power	Preface	Symbol	Value
2^{10}	kilo	k	1024
2^{20}	mega	M	1048576
2^{30}	Giga	G	1073741824

- What is the value of "k", "M", and "G"?
- In computing, particularly w.r.t. memory, the base-2 interpretation generally applies

Example



In the lab…

1. **Double click on My Computer**
2. **Right click on C:**
3. **Click on Properties**

Used space: 1,977,475,072 bytes 1.84GB

/ 2^{30} =

Review – multiplying powers

For common bases, add powers

$$a^b \times a^c = a^{b+c}$$

$$2^6 \times 2^{10} = 2^{16} = 65,536$$

or…

$$2^6 \times 2^{10} = 64 \times 2^{10} = 64k$$

Binary Addition (1 of 2)

Two 1-bit values

A	B	A + B
0	0	0
0	1	1
1	0	1
1	1	10

Binary Addition (2 of 2)

Two *n*-bit values

- Add individual bits
- Propagate carries
- E.g.,

```
  1     1          
  10101      21
+ 11001    + 25
-------    ----
 101110      46
```

Multiplication (1 of 3)

Decimal (just for fun)

```
   35
x 105
-----
  175
 000
 35
-----
 3675
```

Multiplication (2 of 3)

Binary, two 1-bit values

A	B	A × B
0	0	0
0	1	0
1	0	0
1	1	1

Multiplication (3 of 3)

Binary, two *n*-bit values

- As with decimal values
- E.g.,

```
    1110
  x 1011
------
    1110
   1110
  0000
 1110
------
10011010
```

Fractions

Decimal to decimal (just for fun)

$$3.14 \Rightarrow 4 \times 10^{-2} = 0.04$$
$$1 \times 10^{-1} = 0.1$$
$$3 \times 10^{0} = 3$$
$$3.14$$

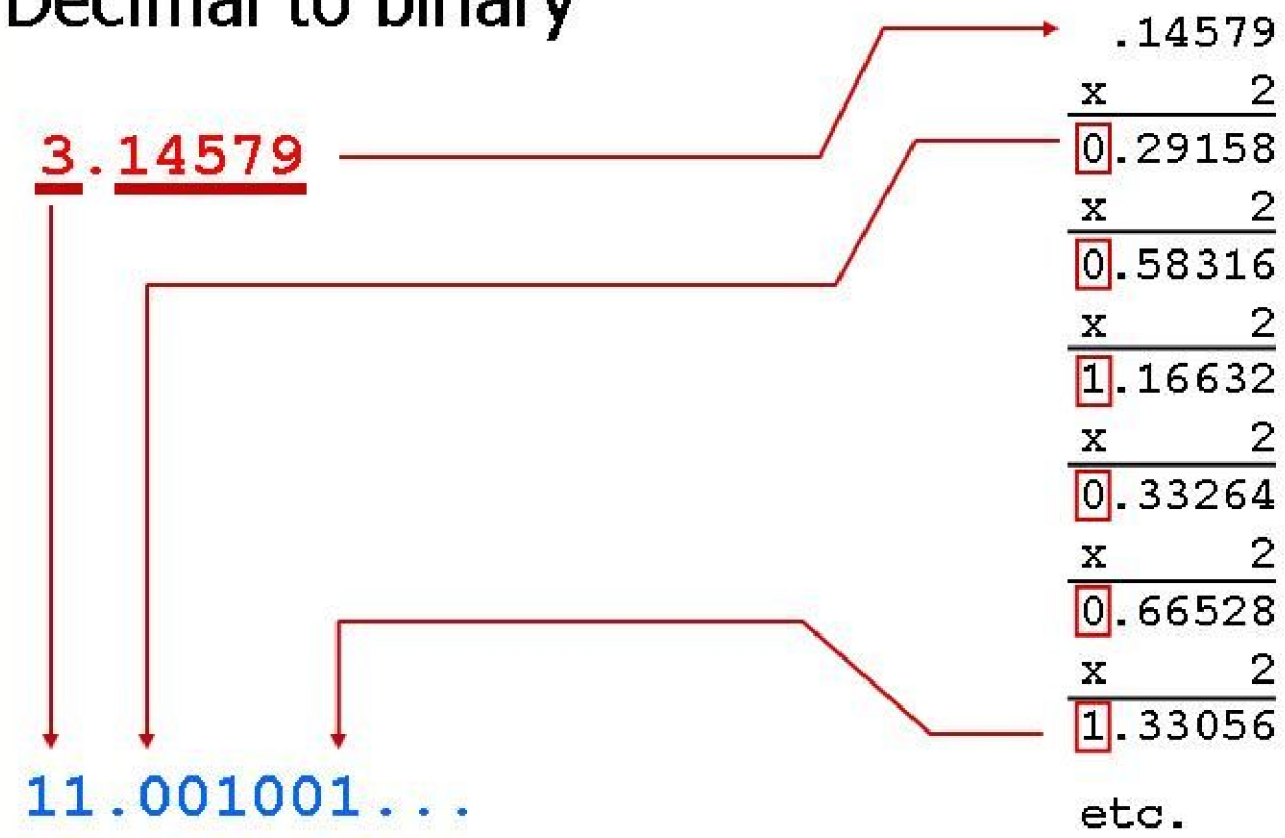
Fractions

Binary to decimal

$$
\begin{aligned}
10.1011 \Rightarrow\ & 1 \times 2^{-4} = 0.0625 \\
& 1 \times 2^{-3} = 0.125 \\
& 0 \times 2^{-2} = 0.0 \\
& 1 \times 2^{-1} = 0.5 \\
& 0 \times 2^{0} = 0.0 \\
& 1 \times 2^{1} = 2.0 \\
& \qquad\qquad\quad 2.6875
\end{aligned}
$$

Fractions

Decimal to binary



Exercise – Convert ...

Decimal	Binary	Octal	Hexa-decimal
29.8			
	101.1101		
		3.07	
			C.82

Exercise – Convert …

Answer

Decimal	Binary	Octal	Hexa-decimal
29.8	11101.110011…	35.63…	1D.CC…
5.8125	101.1101	5.64	5.D
3.109375	11.000111	3.07	3.1C
12.5078125	1100.10000010	14.404	C.82