جامعة فلســــطين التقنية - خضــــوري

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Digital Electronics and Logic Design

COMPLEMENTS

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Complements

- Complements are used in,
  - Digital computers for simplifying the subtraction operation.
  - There are two types of complements for each base-r system.
    - **r's complement**
    - **(r-1)'s complement**
  - When the value of the base is substituted the two types receive.
    - **2's and 1's complements for binary numbers, or**
    - **10's and 9's for decimal numbers.**

Complements

- ***The r'*s Complements**
  - Given a positive number N with an integer part of n digits, the r's complement of N is defined as,

$$(r^n - N)$$

  - For example,
    - **The 10's complement of $(52520)_{10}$ is 10^5-52520=47480**
      - **Where n is number of digits in the Number.**
    - **The 10's complement of $(0.3267)_{10}$ is 1- 0.3267=0.6733**
      - **No inter part so $10^n = 10^0 = 1$**
    - **The 10's complement of $(25.639)_{10}$ is 10^2-25.639=74.631**

Complements

- ***The r'*s Complements**
  - For example,
    - **The 2's complement of $(101110)_2$ is $(2^6)_{10} - (101100)_2$ $= (1000000 - 101100)_2 = 010100$**
    - **The 2's complement of $(0.0110)_2$ is $(1 - 0.0110)_2 = 0.1010$**
    - **The 10's complement of $(25.639)_{10}$ is $10^2 - 25.639 = 74.631$**
      - **Note: 2's complement = (1's complement) + 1**

Complements

Example: Find the 2's complement of 10110010.

$(10110010)_2$	Binary number
$(01001101)_2$	1's complement
$+1$	Add 1
$(01001110)_2$	2's complement

Complements

The 10's complement of 012398 is 987602.

The 10's complement of 246700 is 753300.

Change all bits to the left of the least significant 1 to get 2's complement.

The 2's complement of 1101100 is 0010100.

The 2's complement of 0110111 is 1001001.

Simple Observations

- 10's Complement of a number can be formed by leaving all least significant zeros unchanged, subtracting the first non zero least significant digit from 10, and then subtracting rest of the digits from 9.
- The 2's complement can be formed by leaving all least significant zeros and the first nonzero digit unchanged, and then replacing 1 by 0's and 0's by 1's in all other higher significant digits.

Etc.

Complements

- ***The (r-1)'s Complement***
  - Given a positive number N with an integer part of n digits and a fraction part of m digits,
  - the (r-1)'s complement of N is defined as,

$$(r^{n} - r^{-m} - N)$$

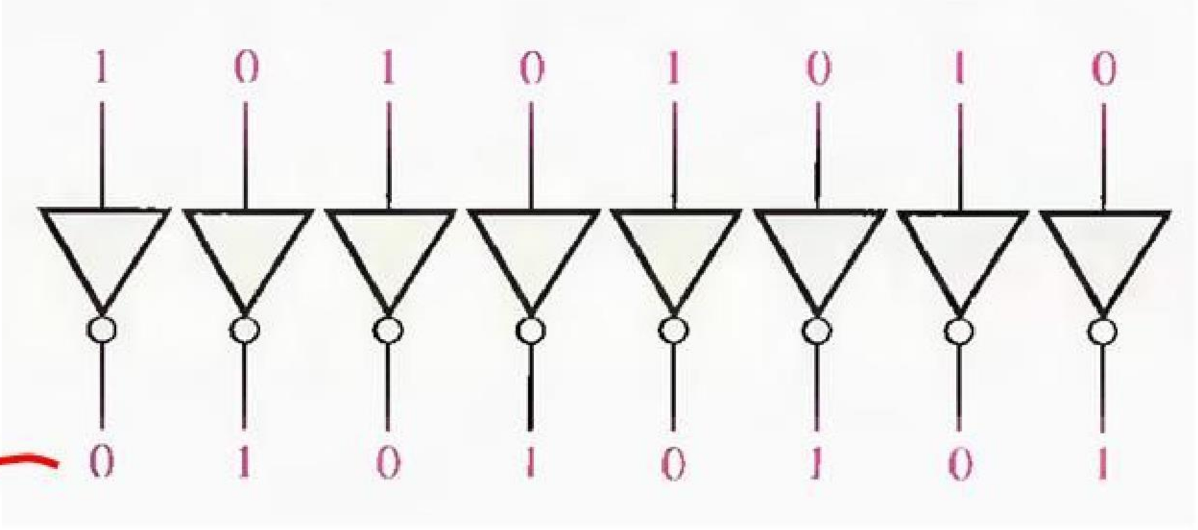
  - For example,
    - **The 9's complement of $(52520)_{10}$ is $(10^{5} - 1 - 52520) = 47479$**
      - **No fraction part, so $10^{-m} = 10^{0} = 1$**
    - **The 9's complement of $(0.3267)_{10}$ is $(1 - 10^{-4} - 0.3267)$**
      $= 0.9999 - 0.3267 = 0.6732$
      - **No inter part so $10^{n} = 10^{0} = 1$**
    - **The 9's complement of $(25.639)_{10}$ is $(10^{2} - 10^{-3} - 25.639)$**
      $= 99.999 - 25.639 = 74.360$

Complements

- ***The (r-1)'*s Complement**
  - For example,
    - **The 1's complement of $(101110)_2$ is $(2^6-1) - (101100)_2$**
      $= (111111 - 101100)_2 = 010011$
    - **The 1's complement of $(0.0110)_2$ is $(1 - 2^{-4})10 - 0.0110)_2$**
      $=(0.1111 - 0.0110) = 0.1001$
- We see from the above example that the 9's complement of a number is formed simply subtracting every digit from 9.
- 1's complement of a binary is formed by simply changing the bits from 1 to 0 and 0 to 1.

complements

The simplest way to obtain the l's complement of a binary number with a digital circuit is to use parallel inverters (NOT circuits),



1's complement

Subtraction with r's complement

- Subtraction of two +ive number (M-N) both of base r may be done as follows,
  - Add the Minuend M to the r's complement of the subtrahend N.
    - $M + (r^n - N) = M - N + r^n$
  - Inspect the result obtained in step 1 for an end carry:
    - **If an end carry occurs, discard it.**
    - **If an end carry does not occur, take the r's comp. of the number obtained in step 1 and place a –ive sign in front of it.**

Subtraction with r's complement

Example: Using 10's comp. subtract 72532-3250.

M = 72532

N = 03250

$$
\begin{aligned}
M &= \quad 72532 \\
\text{10's complement of } N &= + \underline{\ 96750} \\
\text{Sum} &= \quad 169282
\end{aligned}
$$

End Carry

Discard It

Answer = 69282

Subtraction with r's complement

Example: Using 10's comp. subtract (3250 – 72532).

M = 03250

N = 72532

$$M = 03250$$
$$10\text{'s complement of } N = +\ \underline{27468}$$
$$\text{Sum} = 30718$$

No End Carry

So Answer = - (10's complement of 30718)

= - 69282

Subtraction with r's complement

Example: Using 2's comp. subtract (1010100 – 1000100)

M = 1010100

N = 1000100

M = 1010100

2's complement of N =+0111100

End Carry → 1   **0010000**

So Answer = 10000

Subtraction with r's complement

Example: Using 2's comp. subtract (1010100 – 1000100)

M = 1000100

N = 1010100

M = 1000100

2's complement of N =+0101100

1110000

No Carry

So Answer = -(2's complement of 1110000)

=-(10000)

Examples

Given the two binary numbers $X = 1010100$ and $Y = 1000011$

Perform X – Y and Y-X using 2's complement

SOLUTION

$$X = 1010100$$

$$\text{2's complement of } Y = +\ 0111101$$

$$\text{Sum} = 10010001$$

Discard End Carry

$$\text{Answer: } X - Y = 0010001$$

Examples

Now Performing Y - X

SOLUTION

$$
\begin{aligned}
Y &= \quad 1000011 \\
\text{2's complement of } X &= +\ \underline{0101100} \\
\text{Sum} &= \quad 1101111
\end{aligned}
$$

There is no end carry.

Answer: $Y - X = -(\text{2's complement of } 1101111)$

$= -0010001$

Subtraction with (r-1)'s Complement

- Subtraction with (r-1)'s complement
- Subtraction of two +ive number (M-N) both of base r may be done as follows,
  - Add the Minuend M to the (r-1)'s complement of the subtrahend N.
  - Inspect the result obtained in step 1 for an end carry:
    - **If an end carry occurs, add 1 to least significant digit.**
    - **If an end carry does not occur, take the (r-1)'s comp. of the number obtained in step 1 and place a –ive sign in front of it.**

Subtraction with (r-1)'s complement

Example: Using 9's comp. subtract (72532 – 3250)

M = 72532

N = 03250

M = 72532

9's complement of N = +96749

1 69281

+1

69282

So Answer = 69282

Subtraction with (r-1)'s complement

Example: Using 9's comp. subtract (3250 - 72532)

M = 03250

N = 72532

M =	**03250**
9's complement of N =	**+27467**
No End Carry	**30717**

So Answer = -(9's complement of 30717)

=- 69282

Subtraction with (r-1)'s complement

Example: Using 1's comp. subtract (1010100 - 1000100)

M = 1010100

N = 1000100

M = 1010100

1's complement of N =+0111011

1 0001111

+1

0010000

So Answer = 10000

Subtraction with (r-1)'s complement

Example: Using 1's comp. subtract (1000100 - 1010100)

M = 1000100

N = 1010100

M =	1000100
1's complement of N =	+0101011
	1101111

No End Carry

So Answer = -(1's complement of 1101111)

=- 10000

Examples

(a) $X - Y = 1010100 - 1000011$

SOLUTION

$$
\begin{array}{rr}
X = & 1010100 \\
\text{1's complement of } Y = & +\ \underline{0111100} \\
\text{Sum} = & 10010000 \\
\text{End-around carry} & \underline{+\quad 1} \\
\text{Answer: } X - Y = & 0010001
\end{array}
$$

Example

(b) $Y - X = 1000011 - 1010100$

SOLUTION

$$
\begin{aligned}
Y &= \quad 1000011 \\
\text{1's complement of } X &= + \underline{0101011} \\
\text{Sum} &= \quad 1101110
\end{aligned}
$$

There is no end carry.

Answer: $Y - X = -(\text{1's complement of } 1101110)$

$= -0010001$

Signed Binary Numbers

- +ive integers including zero can be represented as a unsigned number.
- For representing –ive numbers we need a notation for –ive values.
- Due to H/W limitation computers must represent every thing in binary forms.
- To do this sign is represented by a bit which is at the left most position of the number.
- Sign bit 0 for +ive and 1 for –ive is used.

Signed binary numbers

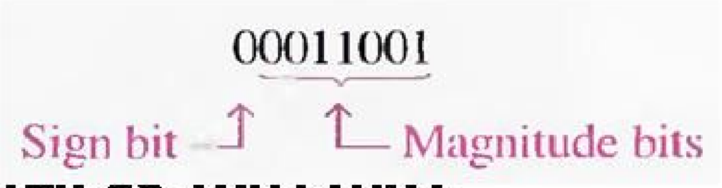
- Both signed and unsigned numbers consists of string of bits.
- The user determines whether the number is signed or unsigned, the left most bit is the signed bit but rest of the bits represent the number.
- If the number is assumed to be unsigned, then the left most bit of the number is most significant bit of the number.

Signed binary numbers

- For example 01001 is 9 in decimal (unsigned number) or +9 (signed binary) b/c the left most bit is 0.
- The string of bit 11001 is 25 in decimal when considered it as unsigned.
- -9 when considered as a signed b/c of the 1 in the left most position, which shows –ive. Other 4bits shows the binary number.
- This whole is called the ***singed magnitude convention.***

Signed binary number

- **In this convention a number has magnitude and a symbol (+ or -) or a bit (0, 1).**



- **-25 is represented as 10011001**
- **This is used in ordinary arithmetic.**
- **When representing it in computer, a different technique is used called ,**
- *signed complement system.*
  - *in this system a –ive is represented by its complement.*
- *2's complements is used to do the operation.*
- *Consider 9 in binary with 8bits,*
  - *+9 is represented with 0 bit in left most position followed by binary equalivant of 9 which is 00001001.*

Signed binary numbers

- There are three different ways to represent -9 with eight bits.

In signed-magnitude representation:	10001001
In signed-1's-complement representation:	11110110
In signed-2's-complement representation:	11110111

Signed binary numbers

- In signed magnitude -9 is obtained from +9 by changing the sign bit from 0 to 1.
- In signed compl. System -9 is obtained by taking the complement of all the bits of +9 including the sign bit.

+9 → 00001001

-9 → 11110111 (2,s Complement of +9)

Arithmetic addition

- The addition of two signed binary numbers with –ive numbers represented in signed 2,s complement form is obtained from the addition of the two numbers, including their sign bits.
- Carry is discarded of the sign bit position.

Arithmetic addition

- -ive numbers are initially in 2's complement and that the sum obtained after the addition if –ive is in 2's complement form.

+ 6	00000110
+13	00001101
+19	00010011

− 6	11111010
+13	00001101
+ 7	00000111

+ 6	00000110
−13	11110011
− 7	11111001

− 6	11111010
−13	11110011
−19	11101101