

# Digital Electronics and Logic Design Combinational Logic Analysis

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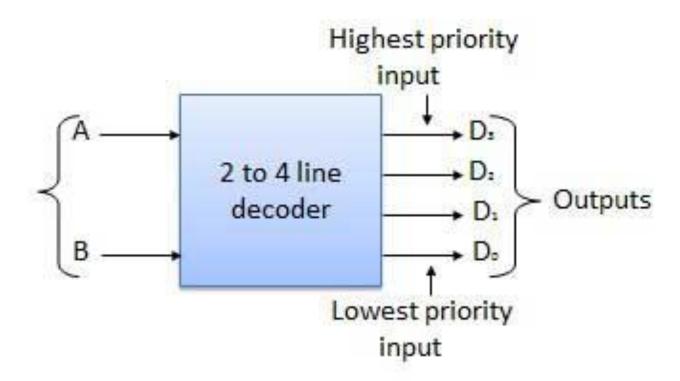
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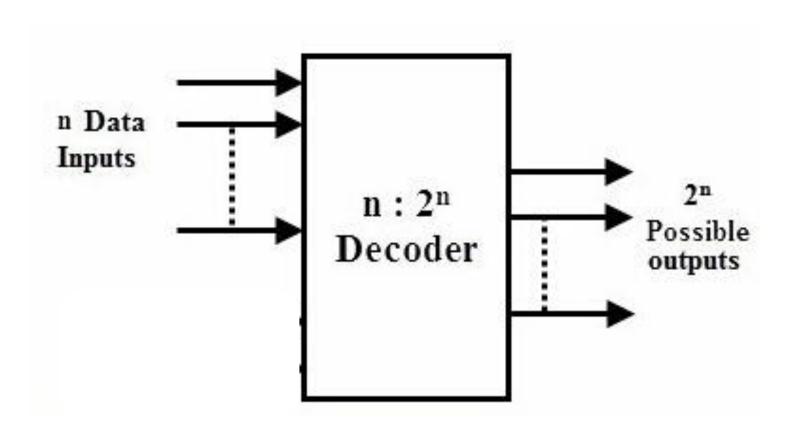
# Decoders

Discrete quantities of information are represented in digital systems by binary codes. A binary code of n bits is capable of representing up to  $2^n$  distinct elements of coded information.

A *decoder* is a combinational circuit that converts binary information from n input lines to a maximum of  $2^n$  unique output lines. If the n-bit coded information has unused combinations, the decoder may have fewer than  $2^n$  outputs.

The decoders presented here are called *n*-to-*m*-line decoders, where  $m \leq 2^n$ .

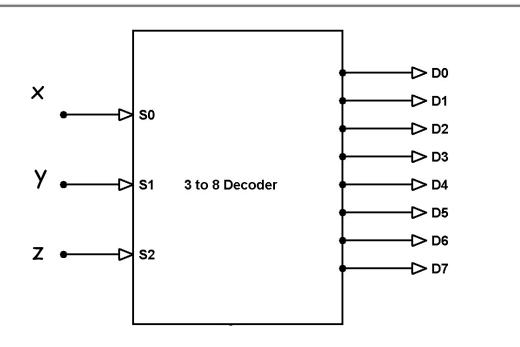


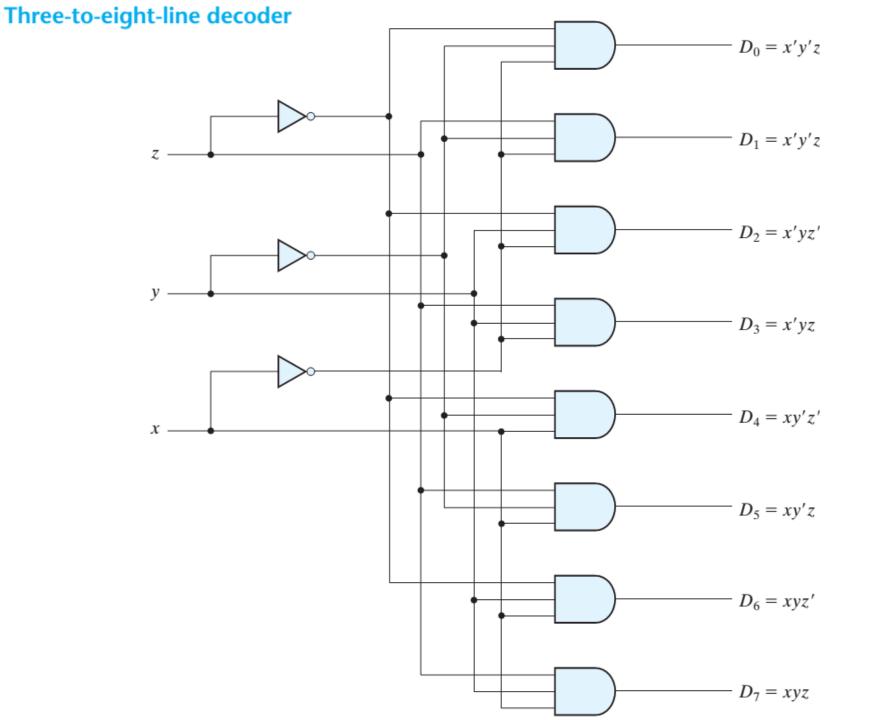


Truth Table of a Three-to-Eight-Line Decoder

	Inputs			Outputs								
X	y	Z	D <sub>0</sub>	<b>D</b> <sub>1</sub>	D <sub>2</sub>	$D_3$	$D_4$	$D_5$	D <sub>6</sub>	D <sub>7</sub>		
0	0	0	1	0	0	0	0	0	0	0		
0	0	1	0	1	0	0	0	0	0	0		
0	1	0	0	0	1	0	0	0	0	0		
0	1	1	0	0	0	1	0	0	0	0		
1	0	0	0	0	0	0	1	0	0	0		
1	0	1	0	0	0	0	0	1	0	0		
1	1	0	0	0	0	0	0	0	1	0		
1	1	1	0	0	0	0	0	0	0	1		



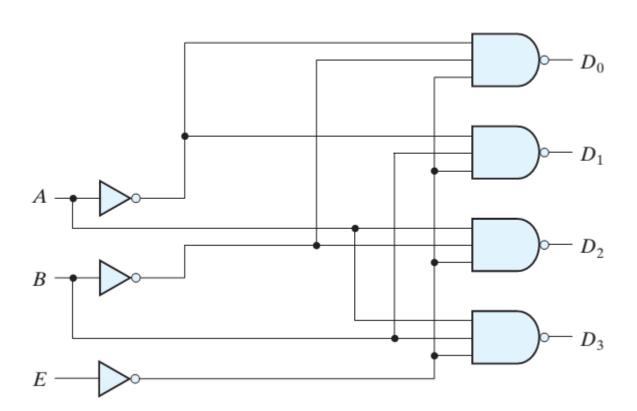




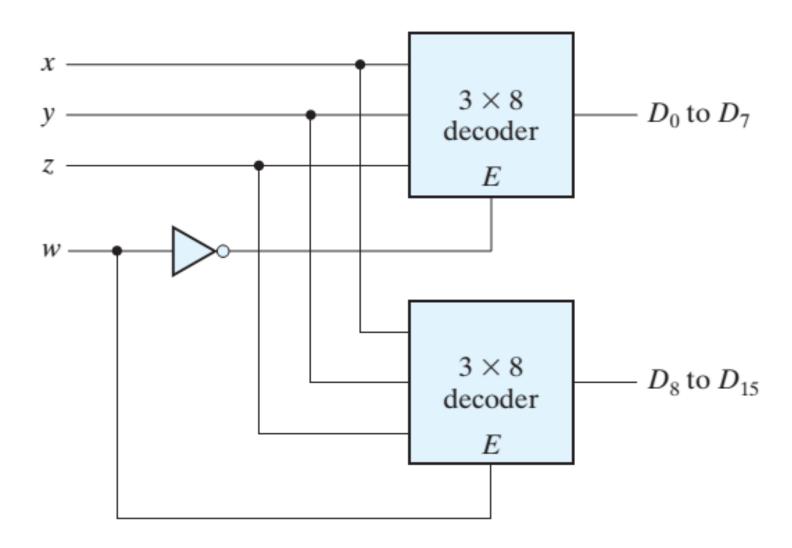
# Two-to-four-line decoder with enable input

Active Low
------------

E	A	B	$D_0$	$D_1$	$D_2$	$D_3$
1	X	X	-	-	1	-
0	0	0	0			1
0	0	1	1	0	1	1
0	1	0	1	1	0	1
0	1	1	1	1	1	1



## $4 \times 16$ decoder constructed with two $3 \times 8$ decoders

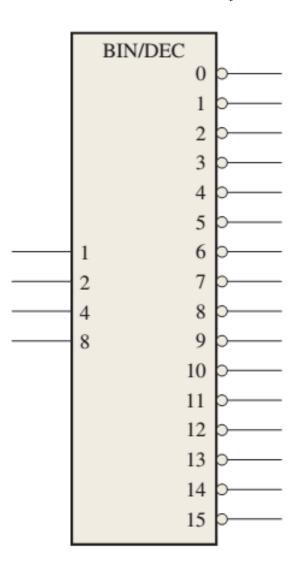


Decoding functions and truth table for a 4-line-to-16-line (1-of-16) decoder with active-LOW outputs.

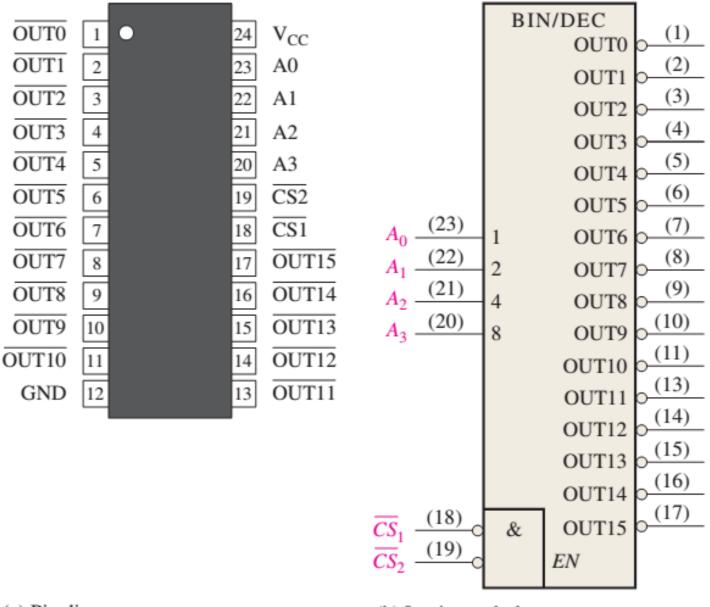
Decimal	I	Binar	y Inp	uts	Decoding								Out	puts							
Digit	$A_3$	$A_2$	$A_1$	$A_0$	Function	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	0	$\overline{A}_3\overline{A}_2\overline{A}_1\overline{A}_0$	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	1	$\overline{A}_3\overline{A}_2\overline{A}_1A_0$	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	1	0	$\overline{A}_3\overline{A}_2A_1\overline{A}_0$	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
3	0	0	1	1	$\overline{A}_3\overline{A}_2A_1A_0$	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
4	0	1	0	0	$\overline{A}_3 A_2 \overline{A}_1 \overline{A}_0$	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
5	0	1	0	1	$\overline{A}_3 A_2 \overline{A}_1 A_0$	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
6	0	1	1	0	$\overline{A}_3 A_2 A_1 \overline{A}_0$	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
7	0	1	1	1	$\overline{A}_3 A_2 A_1 A_0$	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
8	1	0	0	0	$A_3\overline{A}_2\overline{A}_1\overline{A}_0$	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
9	1	0	0	1	$A_3\overline{A}_2\overline{A}_1A_0$	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
10	1	0	1	0	$A_3\overline{A}_2A_1\overline{A}_0$	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
11	1	0	1	1	$A_3\overline{A}_2A_1A_0$	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
12	1	1	0	0	$A_3A_2\overline{A}_1\overline{A}_0$	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
13	1	1	0	1	$A_3A_2\overline{A}_1A_0$	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
14	1	1	1	0	$A_3A_2A_1\overline{A}_0$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
15	1	1	1	1	$A_3A_2A_1A_0$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

If an active-LOW output is required for each decoded number, the entire decoder can be implemented with NAND gates and inverters. In order to decode each of the sixteen binary codes, sixteen NAND gates are required (AND gates can be used to produce active-HIGH outputs).

Logic symbol for a 4-line-to-16-line (1-of-16) decoder.



#### The 74HC154 1-of-16 decoder.



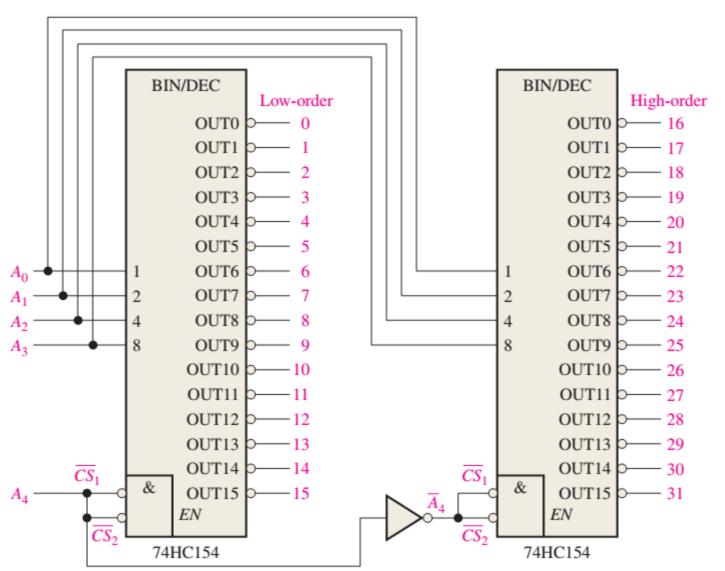
(a) Pin diagram

(b) Logic symbol

#### **EXAMPLE**

A certain application requires that a 5-bit number be decoded. Use 74HC154 decoders to implement the logic. The binary number is represented by the format  $A_4A_3A_2A_1A_0$ .

A 5-bit decoder using 74HC154s.



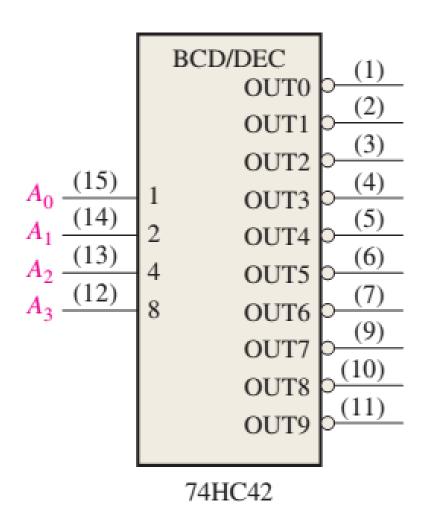
## The BCD-to-Decimal Decoder

4-line-to-10-line decoder

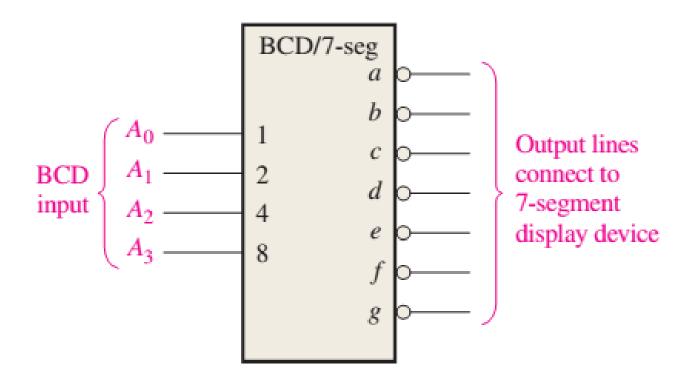
# BCD decoding functions.

Decimal		Decoding			
Digit	$A_3$	$A_2$	$A_1$	$A_0$	Function
0	0	0	0	0	$\overline{A}_3\overline{A}_2\overline{A}_1\overline{A}_0$
1	0	0	0	1	$\overline{A}_3\overline{A}_2\overline{A}_1A_0$
2	0	0	1	0	$\overline{A}_3\overline{A}_2A_1\overline{A}_0$
3	0	0	1	1	$\overline{A}_3\overline{A}_2A_1A_0$
4	0	1	0	0	$\overline{A}_3 A_2 \overline{A}_1 \overline{A}_0$
5	0	1	0	1	$\overline{A}_3 A_2 \overline{A}_1 A_0$
6	0	1	1	0	$\overline{A}_3 A_2 A_1 \overline{A}_0$
7	0	1	1	1	$\overline{A}_3A_2A_1A_0$
8	1	0	0	0	$A_3\overline{A}_2\overline{A}_1\overline{A}_0$
9	1	0	0	1	$A_3\overline{A}_2\overline{A}_1A_0$

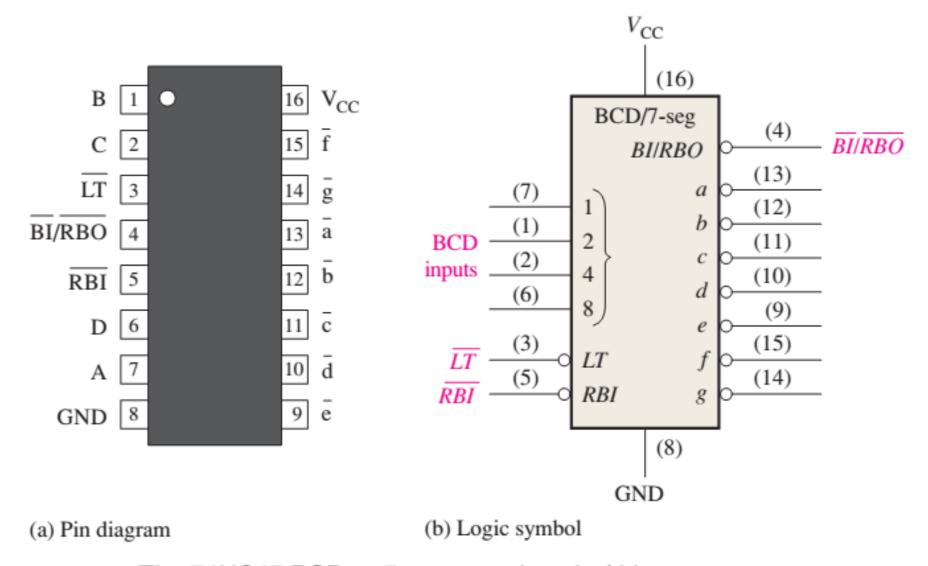
## The 74HC42 BCD-to-decimal decoder.



# The BCD-to-7-Segment Decoder



Logic symbol for a BCD-to-7-segment decoder/driver with active-LOW outputs.



The 74HC47 BCD-to-7-segment decoder/driver.

**Lamp Test** When a LOW is applied to the  $\overline{LT}$  input and the  $\overline{BI}/\overline{RBO}$  is HIGH, all of the seven segments in the display are turned on. Lamp test is used to verify that no segments are burned out.

## **Combinational Logic Implementation**

# Implementation of a full adder with a decoder

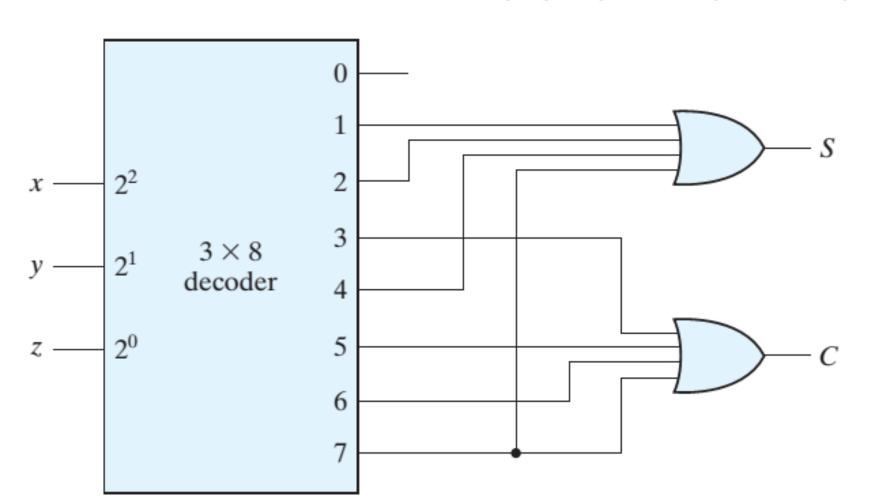
#### Full Adder

x	y	z	С	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1
	0 0 0	0 0 0 0 0 1	0 0 0 0 0 1 0 1 0 0 1 1	0 0 0 0 0 0 1 0 0 1 0 0 0 1 1 1

## Implementation of a full adder with a decoder

$$S(x, y, z) = \Sigma(1, 2, 4, 7)$$

$$C(x, y, z) = \Sigma(3, 5, 6, 7)$$



#### **ENCODERS**

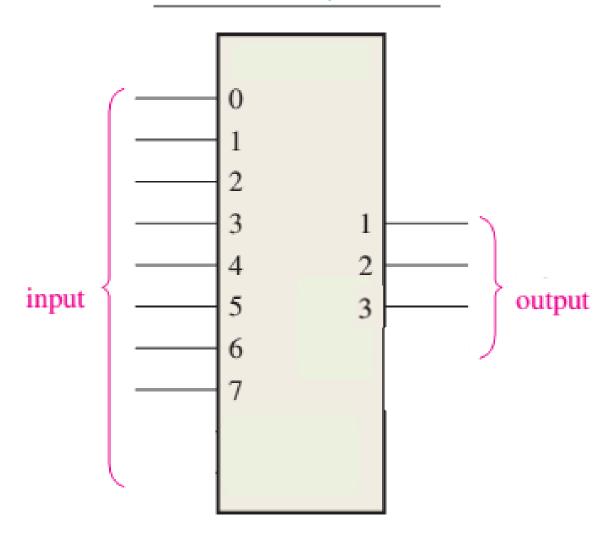
An encoder is a digital circuit that performs the inverse operation of a decoder. An encoder has  $2^n$  (or fewer) input lines and n output lines. The output lines, as an aggregate, generate the binary code corresponding to the input value. An example of an encoder is the octal-to-binary encoder

Truth Table of an Octal-to-Binary Encoder

				C	utput	s				
D <sub>0</sub>	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	$D_4$	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>	х	y	z
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1

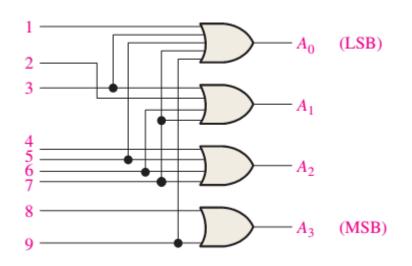
$$z = D_1 + D_3 + D_5 + D_7$$
  
 $y = D_2 + D_3 + D_6 + D_7$   
 $x = D_4 + D_5 + D_6 + D_7$ 

## an Octal-to-Binary Encoder



# The Decimal-to-BCD Encoder

	BCD Code						
<b>Decimal Digit</b>	$A_3$	$A_2$	$A_1$	$A_0$			
0	0	0	0	0			
1	0	0	0	1			
2	0	0	1	0			
3	0	0	1	1			
4	0	1	0	0			
5	0	1	0	1			
6	0	1	1	0			
7	0	1	1	1			
8	1	0	0	0			
9	1	0	0	1			



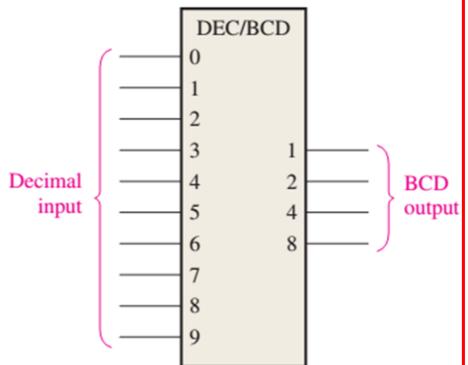
$$A_3 = 8 + 9$$

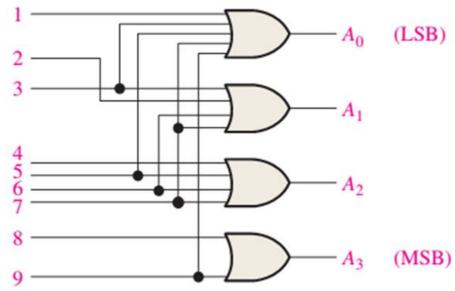
$$A_2 = 4 + 5 + 6 + 7$$

$$A_1 = 2 + 3 + 6 + 7$$

$$A_0 = 1 + 3 + 5 + 7 + 9$$

## The Decimal-to-BCD Encoder





$$A_3 = 8 + 9$$

$$A_2 = 4 + 5 + 6 + 7$$

$$A_1 = 2 + 3 + 6 + 7$$

$$A_0 = 1 + 3 + 5 + 7 + 9$$

# **Priority Encoder**

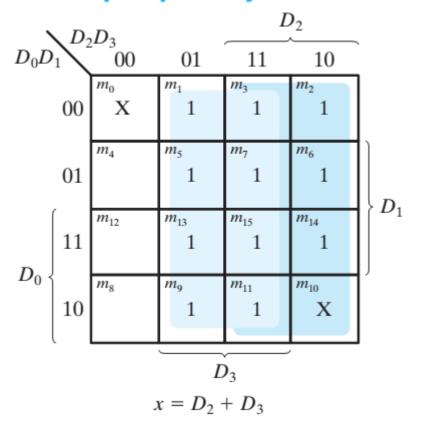
A priority encoder is an encoder circuit that includes the priority function. The operation of the priority encoder is such that if two or more inputs are equal to 1 at the same time, the input having the highest priority will take precedence. The truth table of a four-input priority encoder is given in Table A. In addition to the two outputs A and A, the circuit has a third output designated by A; this is a *valid* bit indicator that is set to 1 when one or more inputs are equal to 1. If all inputs are 0, there is no valid input and A is equal to 0.

Four-input priority encoder

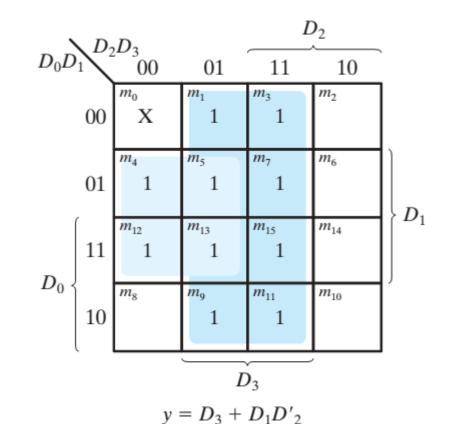
Truth Table of a Priority Encoder

	Inp	uts		Outputs				
$D_0$	<b>D</b> <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	х	y	V		
0	0	0	0	X	X	0		
1	0	0	0	0	0	1		
X	1	0	0	0	1	1		
X	X	1	0	1	0	1		
X	X	X	1	1	1	1		

## Four-input priority encoder



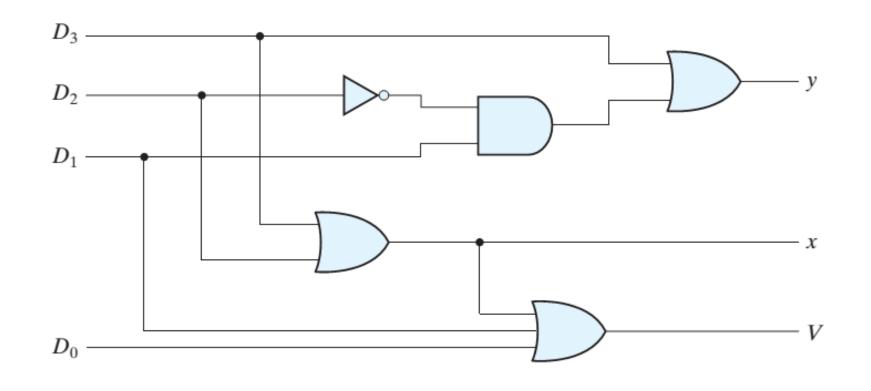
$$x = D_2 + D_3$$
  
 $y = D_3 + D_1 D_2'$   
 $V = D_0 + D_1 + D_2 + D_3$ 



#### Truth Table of a Priority Encoder

	Inp	uts	Outputs				
$D_0$	$D_1$	D <sub>2</sub>	$D_3$	x	y	V	
0	0	0	0	X	X	0	
1	0	0	0	0	0	1	
X	1	0	0	0	1	1	
X	X	1	0	1	0	1	
X	X	X	1	1	1	1	

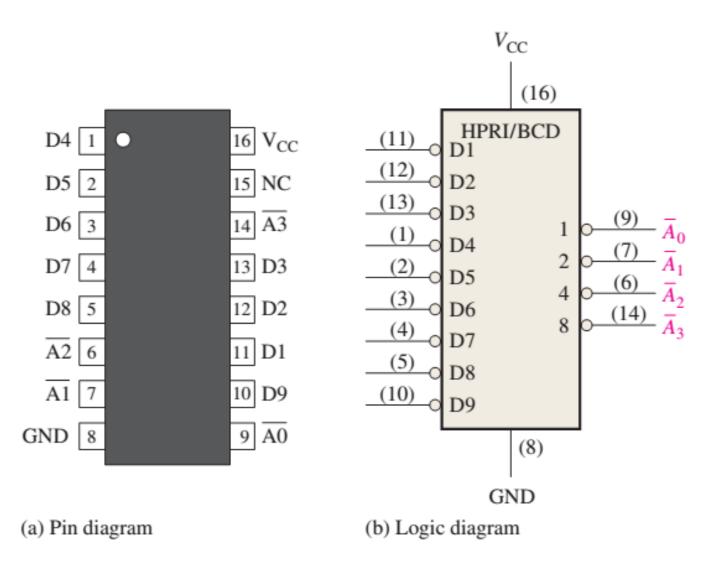
# Four-input priority encoder



$$x = D_2 + D_3$$
  
 $y = D_3 + D_1 D_2'$   
 $V = D_0 + D_1 + D_2 + D_3$ 

#### **DECIMAL-TO-BCD ENCODER**

The 74HC147 decimal-to-BCD encoder (HPRI means highest value input has priority.

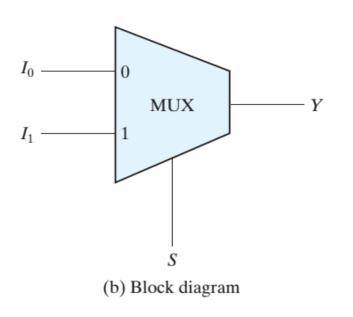


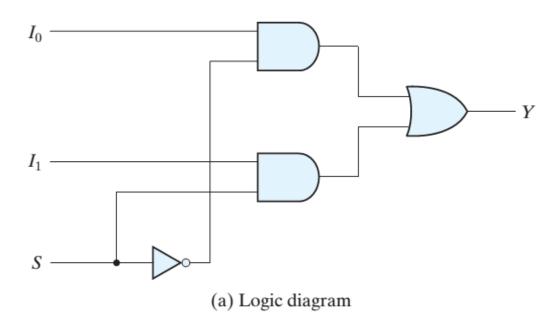
## Multiplexers (Data Selectors)

A multiplexer (MUX) is a device that allows digital information from several sources to be routed onto a single line for transmission over that line to a common destination.

many input lines and directs it to a single output line. The selection of a particular input line is controlled by a set of selection lines. Normally, there are  $2^n$  input lines and n selection lines whose bit combinations determine which input is selected.

### Two-to-one-line multiplexer

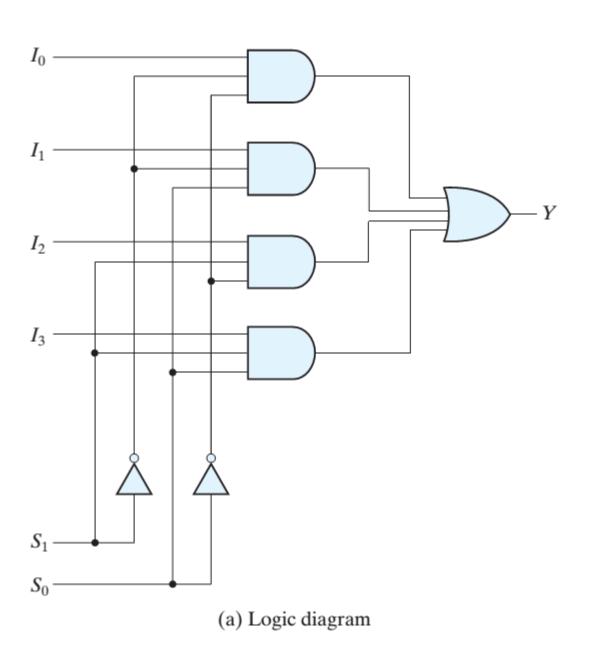


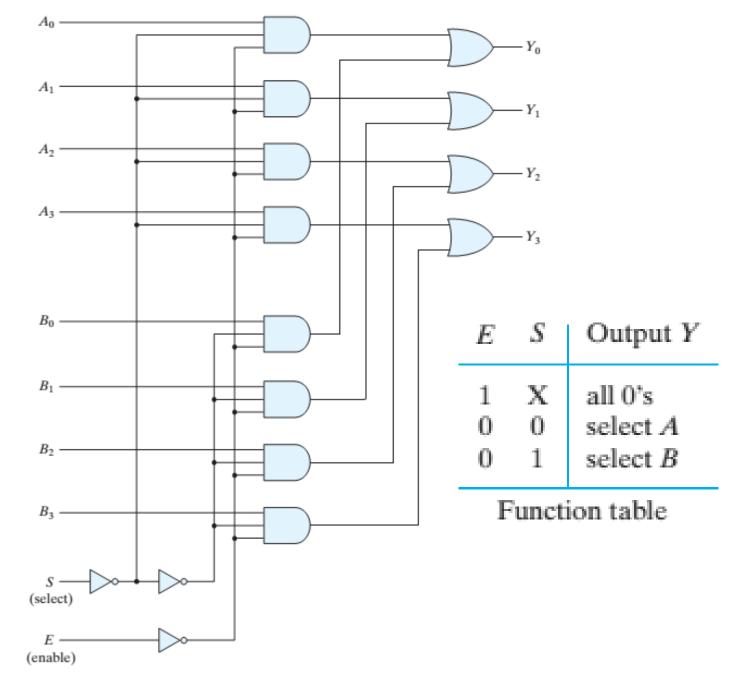


# Four-to-one-line multiplexer

$S_1$	$S_0$	Y
0 0 1 1	0 1 0 1	$I_0 \\ I_1 \\ I_2 \\ I_3$

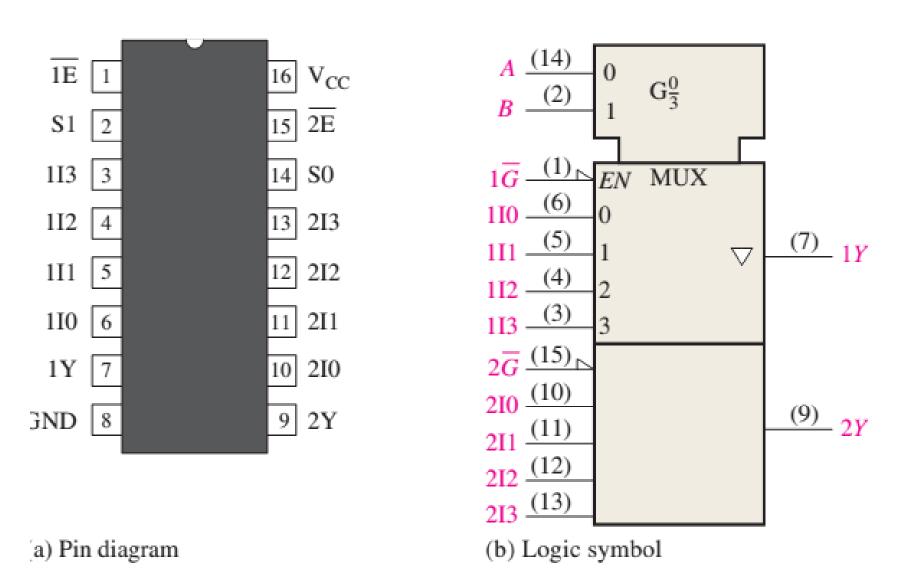
(b) Function table





Quadruple two-to-one-line multiplexer

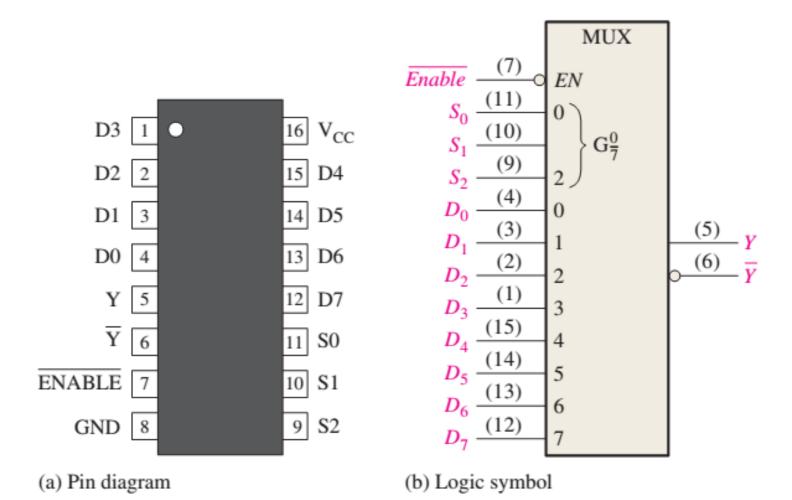
The 74HC153 is a dual four-input data selector/multiplexer.



The 74HC153 dual four-input data selector/multiplexer.

#### **EIGHT-INPUT DATA SELECTOR/MULTIPLEXER**

Fixed-Function Device The 74HC151 has eight data inputs  $(D_0-D_7)$  and, therefore, three data-select or address input lines  $(S_0-S_2)$ . Three bits are required to select any one of the eight data inputs  $(2^3 = 8)$ . A LOW on the Enable input allows the selected input data to pass through to the output. Notice that the data output and its complement are both available. The pin diagram is shown in Figure and the ANSI/IEEE logic symbol



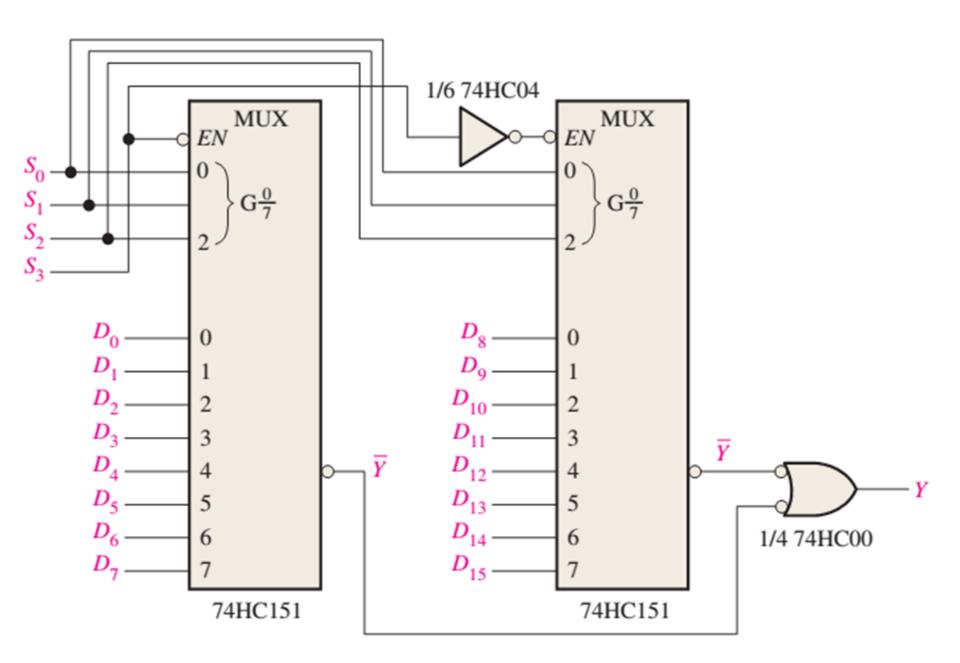
#### **EXAMPLE**

Use 74HC151s and any other logic necessary to multiplex 16 data lines onto a single data-output line.

#### Solution

An expansion of two 74HC151s is shown in Figure 6–48. Four bits are required to select one of 16 data inputs ( $2^4 = 16$ ). In this application the  $\overline{Enable}$  input is used as the most significant data-select bit. When the MSB in the data-select code is LOW, the left 74HC151 is enabled, and one of the data inputs ( $D_0$  through  $D_7$ ) is selected by the other three data-select bits. When the data-select MSB is HIGH, the right 74HC151 is enabled, and one of the data inputs ( $D_8$  through  $D_{15}$ ) is selected. The selected input data are then passed through to the negative-OR gate and onto the single output line.

# A 16-input multiplexer.



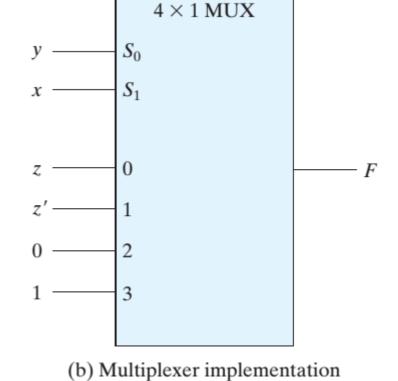
# Implementing a Boolean function with a multiplexer

consider the Boolean function

$$F(x, y, z) = \Sigma(1, 2, 6, 7)$$

This function of three variables can be implemented with a four-to-one-line multiplexer

x	y	z	F						
0	0	0 1	0 1	F = z					
0	1 1	0 1	1 0	F = z'					
1 1	0	0 1	0	F = 0					
1 1	1 1	0 1	1 1	<i>F</i> = 1					
	(a) Truth table								

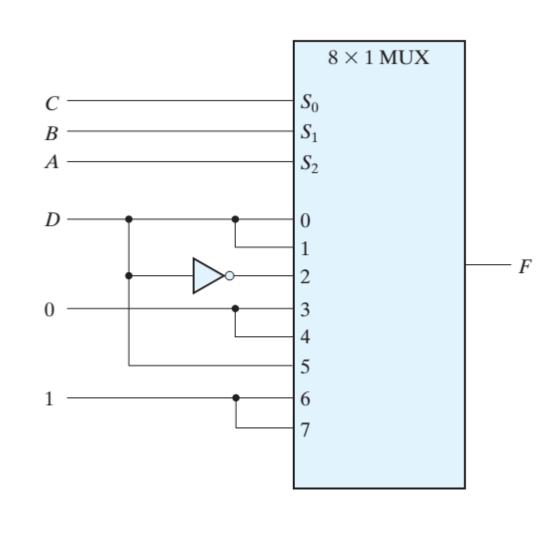


## Implementing a four-input function with a multiplexer

a second example, consider the implementation of the Boolean function

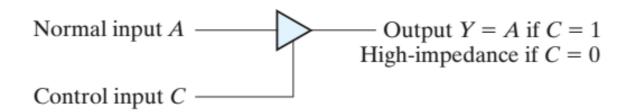
$$F(A, B, C, D) = \Sigma(1, 3, 4, 11, 12, 13, 14, 15)$$

A	B	C	D	F	
0	0	0	0 1	0 1	F = D
0	0	1 1	0 1	0 1	F = D
0	1 1	0	0 1	1 0	F = D'
0	1 1	1 1	0 1	0	F = 0
1 1	0	0	0 1	0	F = 0
1 1	0	1 1	0 1	0 1	F = D
1 1	1 1	0	0 1	1 1	F = 1
1 1	1 1	1 1	0 1	1 1	<i>F</i> = 1



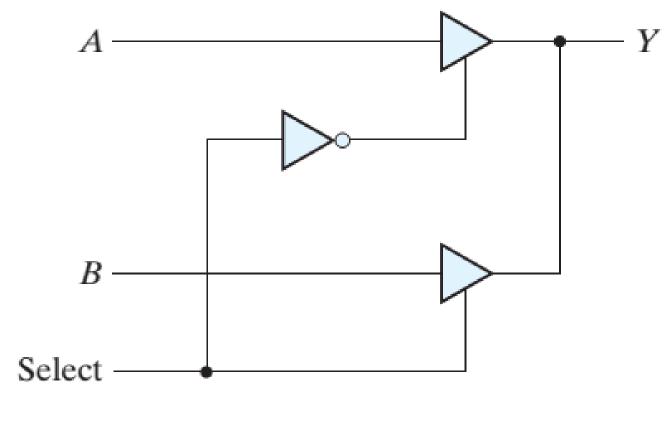
#### **Three-State Gates**

A multiplexer can be constructed with three-state gates—digital circuits that exhibit three states. Two of the states are signals equivalent to logic 1 and logic 0 as in a conventional gate. The third state is a *high-impedance* state in which (1) the logic behaves like an open circuit, which means that the output appears to be disconnected, (2) the circuit has no logic significance, and (3) the circuit connected to the output of the three-state gate is not affected by the inputs to the gate. Three-state gates may perform any conventional logic, such as AND or NAND. However, the one most commonly used is the buffer gate.



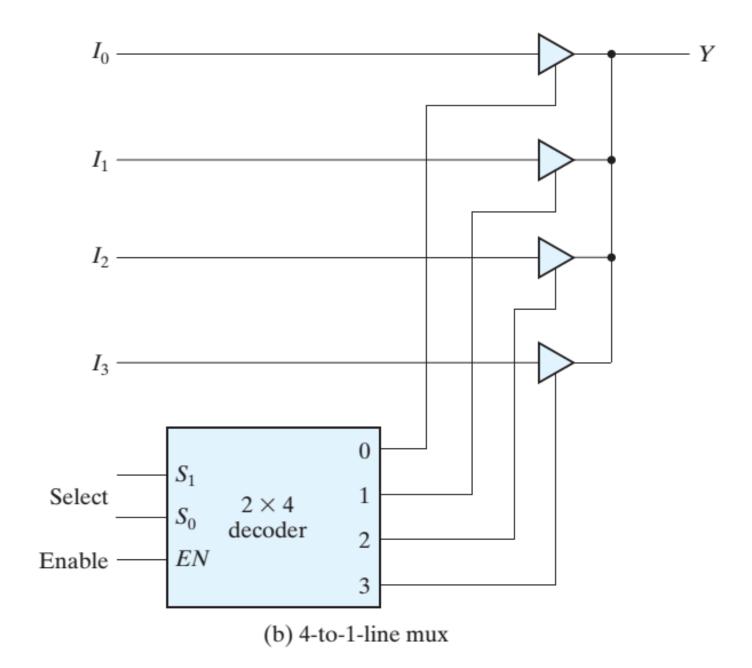
Graphic symbol for a three-state buffer

# Multiplexers with three-state gates



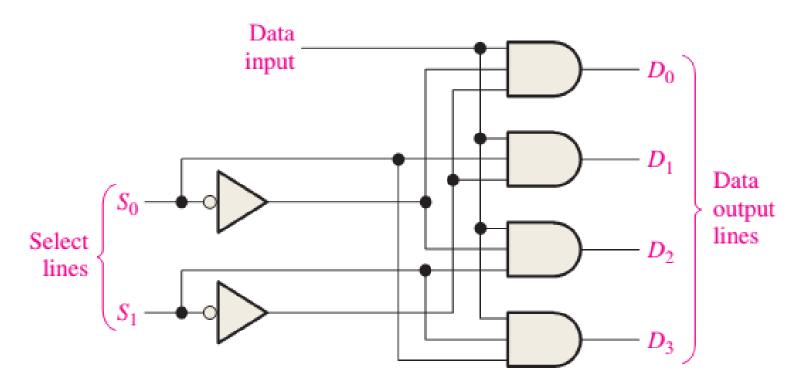
(a) 2-to-1-line mux

## Multiplexers with three-state gates



## Demultiplexers

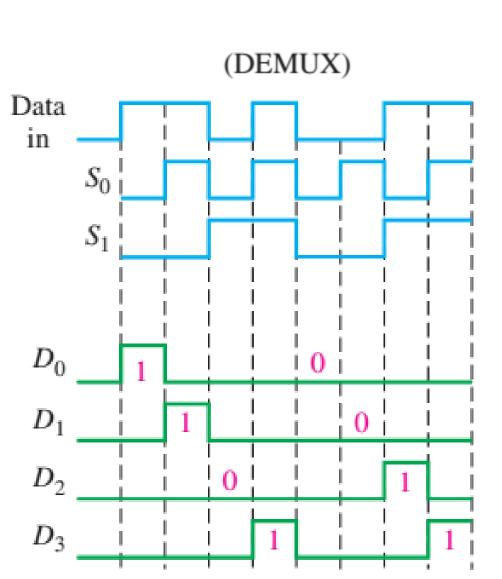
A demultiplexer (DEMUX) basically reverses the multiplexing function. It takes digital information from one line and distributes it to a given number of output lines. For this reason, the demultiplexer is also known as a data distributor. As you will learn, decoders can also be used as demultiplexers.

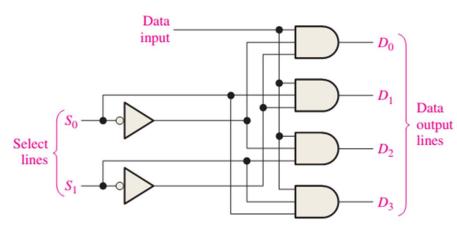


A 1-line-to-4-line demultiplexer.

In a demultiplexer, data are switched from one line to several lines.

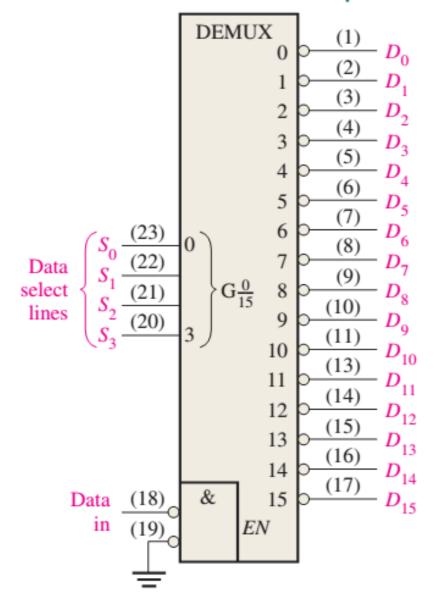
# Demultiplexers





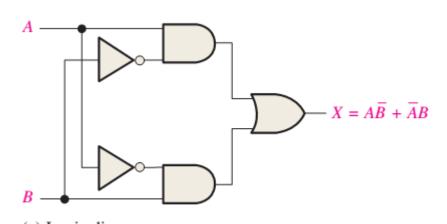
A 1-line-to-4-line demultiplexer.

## 4-Line-to-16-Line Decoder as a Demultiplexer



The decoder used as a demultiplexer.

### Exclusive-OR Logic



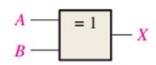
$$X = A \oplus B$$
$$X = A\overline{B} + \overline{A}B$$

#### Truth table for an exclusive-OR.

A	В	X
0	0	0
0	1	1
1	0	1
1	1	0

$$B \longrightarrow X$$

(b) ANSI distinctive shape symbol



(c) ANSI rectangular outline symbol

$$X \oplus 0 = X$$

$$X \oplus 1 = \overline{X}$$

$$X \oplus X = 0$$

$$X \oplus \overline{X} = 1$$
.

$$\overline{X} \oplus \overline{X} = 0$$
.

$$\overline{X} \oplus \overline{X} = X \oplus X$$

$$A \oplus AB = A\overline{B}_{\psi}$$

$$A \oplus A\overline{B} = AB$$

$$A \oplus \overline{AB} = A + B$$

$$(A \oplus B).(A \oplus C) = \overline{ABC} + A\overline{BC} + A$$

$$x \oplus v' = x' \oplus v = (x \oplus y)'$$

$$A \oplus B = B \oplus A$$

$$(A \oplus B) \oplus C = A \oplus (B \oplus C) = A \oplus B \oplus C$$

 $x \oplus 0 = x$ 

 $x \oplus 1 = x'$ 

 $x \oplus x = 0$ 

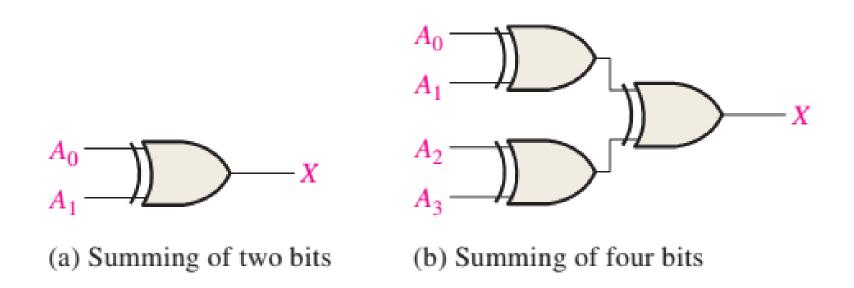
 $x \oplus x' = 1$ 

## Parity Generators/Checkers

A parity bit indicates if the number of 1s in a code is even or odd for the purpose of error detection.

The sum (disregarding carries) of an even number of 1s is always 0, and the sum of an odd number of 1s is always 1.

Therefore, to determine if a given code has **even parity** or **odd parity**, all the bits in that code are summed. As you know, the modulo-2 sum of two bits can be generated by an exclusive-OR gate, as shown in Figure 6-55(a); the modulo-2 sum of four bits can be formed by three exclusive-OR gates connected as shown in Figure 6-55(b); and so on. When the number of 1s on the inputs is even, the output X is 0 (LOW). When the number of 1s is odd, the output X is 1 (HIGH).



## Parity Method for Error Detection

A parity bit tells if the number of 1s is odd or even.

Many systems use a parity bit as a means for bit **error detection**. Any group of bits contain either an even or an odd number of 1s. A parity bit is attached to a group of bits to make the total number of 1s in a group always even or always odd. An even parity bit makes the total number of 1s even, and an odd parity bit makes the total odd.

A given system operates with even or odd **parity**, but not both. For instance, if a system operates with even parity, a check is made on each group of bits received to make sure the total number of 1s in that group is even. If there is an odd number of 1s, an error has occurred.

The BCD code with parity bits.

Even	Parity	Odd	Parity
P	BCD	P	BCD
0	0000	1	0000
1	0001	0	0001
1	0010	0	0010
0	0011	1	0011
1	0100	0	0100
0	0101	1	0101
0	0110	1	0110
1	0111	0	0111
1	1000	0	1000
0	1001	1	1001

Even Parity =0 ====> No. Of Ones is EVEN Even Parity =1 ====> No. Of Ones is Odd

Odd Parity =1 ====> No. Of Ones is EVEN Odd Parity =0 ====> No. Of Ones is Odd

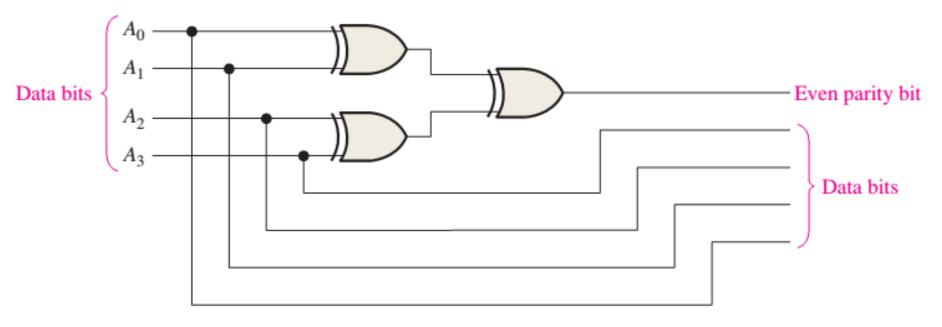
The BCD code with parity bits.

Even	Parity	Odd Parity		
P	BCD	P	BCD	
0	0000	1	0000	
1	0001	0	0001	
1	0010	0	0010	
0	0011	1	0011	
1	0100	0	0100	
0	0101	1	0101	
0	0110	1	0110	
1	0111	0	0111	
1	1000	0	1000	
0	1001	1	1001	

The parity bit can be attached to the code at either the beginning or the end, depending on system design. Notice that the total number of 1s, including the parity bit, is always even for even parity and always odd for odd parity.

#### **EXAMPLE**

Use exclusive-OR gates to implement an even-parity code generator for an original 4-bit code.



Even-parity generator.

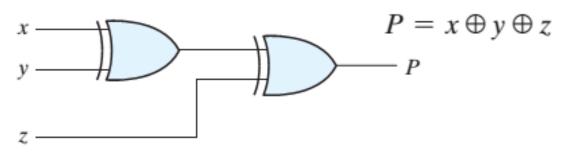
## **Parity Generation and Checking**

**Even-Parity-Generator Truth Table** 

Even Parity =0 ====> No. Of Ones is EVEN Even Parity =1 ====> No. Of Ones is Odd

Three-Bit Message		Parity Bit	
x	y	z	P
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

. For even parity, the bit P must be generated to make the total number of 1's (including P) even.



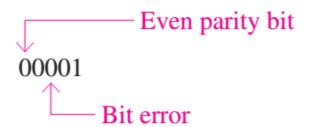
(a) 3-bit even parity generator

## **Detecting an Error**

A parity bit provides for the detection of a single bit error (or any odd number of errors, which is very unlikely) but cannot check for two errors in one group. For instance, let's assume that we wish to transmit the BCD code 0101. (Parity can be used with any number of bits; we are using four for illustration.) The total code transmitted, including the even parity bit, is



Now let's assume that an error occurs in the third bit from the left (the 1 becomes a 0).



When this code is received, the parity check circuitry determines that there is only a single 1 (odd number), when there should be an even number of 1s. Because an even number of 1s does not appear in the code when it is received, an error is indicated.

An odd parity bit also provides in a similar manner for the detection of a single error in a given group of bits.

## **Even-Parity-Checker Truth Table**

Even Parity =0 ====> No. Of Ones is EVEN Even Parity =1 ====> No. Of Ones is Odd

(b) 4-bit even parity checker

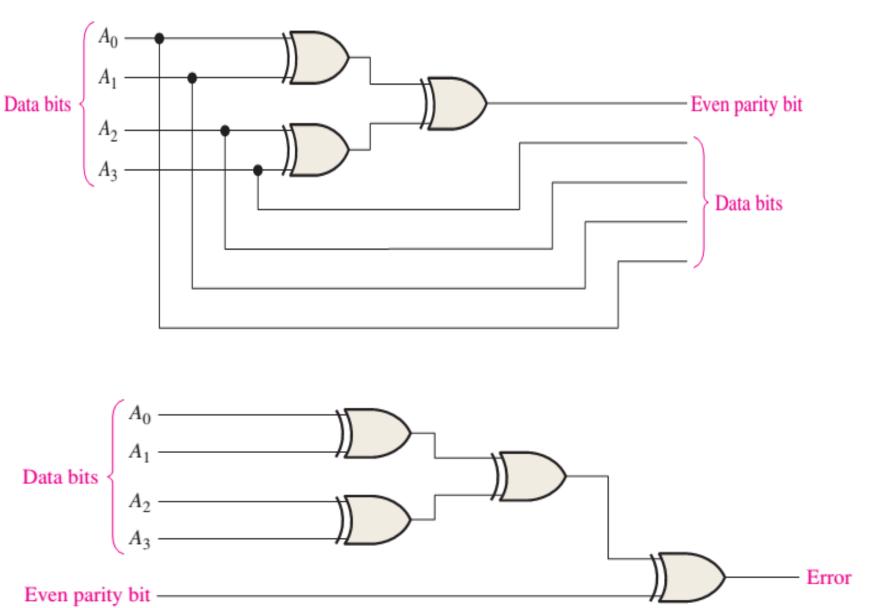
Four Bits Received				Parity Error Check C, will be equal to 1 if an error	
x	y	z	P	С	
0 0 0 0 0 0 0 1 1	0 0 0 0 1 1 1 1 0	0 0 1 1 0 0 1 1 0 0	0 1 0 1 0 1 0 1	0 1 1 0 1 0 0 1 1 1	$C = x \oplus y \oplus z \oplus P$ $x$ $y$ $z$ (a) 3-bit even parity generator
1 1 1 1 1	0 1 1 1 1	1 0 0 1 1	0 1 0 1 0 1	0 1 0 1 1 0	(a) 5-bit even parity generator $ \begin{array}{c c} x \\ y \\ - \\ P \end{array} $

#### **EXAMPLE**

Use exlusive-OR gates to implement an even-parity checker for the 5-bit code generated by the circuit in Example

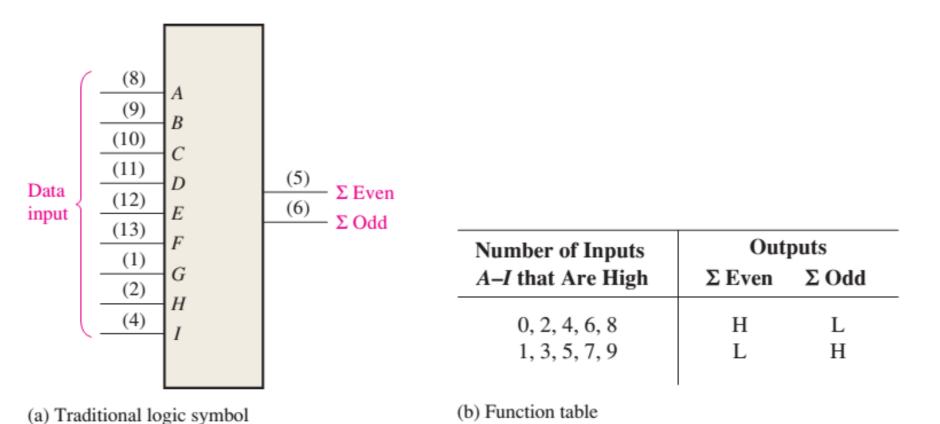
#### Solution

The circuit in Figure produces a 1 output when there is an error in the five-bit code and a 0 when there is no error.



#### 9-BIT PARITY GENERATOR/CHECKER

Fixed-Function Device The logic symbol and function table for a 74HC280 are shown in Figure 6–56. This particular device can be used to check for odd or even parity on a 9-bit code (eight data bits and one parity bit), or it can be used to generate a parity bit for a binary code with up to nine bits. The inputs are A through I; when there is an even number of 1s on the inputs, the  $\Sigma$  Even output is HIGH and the  $\Sigma$  Odd output is LOW.



The 74HC280 9-bit parity generator/checker.

**Parity Checker** When this device is used as an even parity checker, the number of input bits should always be even; and when a parity error occurs, the  $\Sigma$  Even output goes LOW and the  $\Sigma$  Odd output goes HIGH. When it is used as an odd parity checker, the number of input bits should always be odd; and when a parity error occurs, the  $\Sigma$  Odd output goes LOW and the  $\Sigma$  Even output goes HIGH.

**Parity Generator** If this device is used as an even parity generator, the parity bit is taken at the  $\Sigma$  Odd output because this output is a 0 if there is an even number of input bits and it is a 1 if there is an odd number. When used as an odd parity generator, the parity bit is taken at the  $\Sigma$  Even output because it is a 0 when the number of inputs bits is odd.

Thank you!