Chapter 3

When this load is lifted at constant velocity, or is just suspended, then it is in a state of equilibrium. In this chapter we will study equilibrium for a particle and show how these ideas can be used to calculate the forces in cables used to hold suspended loads.

Equilibrium of a Particle

CHAPTER OBJECTIVES

- To introduce the concept of the free-body diagram for a particle.
- To show how to solve particle equilibrium problems using the equations of equilibrium.

3.1 Condition for the Equilibrium of a Particle

A particle is said to be in *equilibrium* if it remains at rest if originally at rest, or has a constant velocity if originally in motion. Most often, however, the term "equilibrium" or, more specifically, "static equilibrium" is used to describe an object at rest. To maintain equilibrium, it is *necessary* to satisfy Newton's first law of motion, which requires the *resultant force* acting on a particle to be equal to *zero*. This condition may be stated mathematically as

$$\Sigma \mathbf{F} = \mathbf{0} \tag{3–1}$$

where $\Sigma \mathbf{F}$ is the vector *sum of all the forces* acting on the particle.

Not only is Eq. 3–1 a necessary condition for equilibrium, it is also a *sufficient* condition. This follows from Newton's second law of motion, which can be written as $\Sigma \mathbf{F} = m\mathbf{a}$. Since the force system satisfies Eq. 3–1, then $m\mathbf{a} = \mathbf{0}$, and therefore the particle's acceleration $\mathbf{a} = \mathbf{0}$. Consequently, the particle indeed moves with constant velocity or remains at rest.

3.2 The Free-Body Diagram

To apply the equation of equilibrium, we must account for *all* the known and unknown forces ($\Sigma \mathbf{F}$) which act *on* the particle. The best way to do this is to think of the particle as isolated and "free" from its surroundings. A drawing that shows the particle with *all* the forces that act on it is called a *free-body diagram (FBD)*.

Before presenting a formal procedure as to how to draw a free-body diagram, we will first consider two types of connections often encountered in particle equilibrium problems.

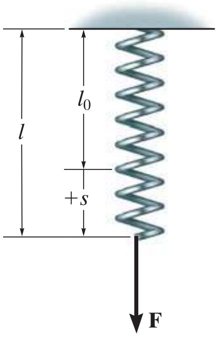


Fig. 3–1

Springs. If a *linearly elastic spring* (or cord) of undeformed length l_0 is used to support a particle, the length of the spring will change in direct proportion to the force $\mathbf{F}$ acting on it, Fig. 3–1. A characteristic that defines the "elasticity" of a spring is the *spring constant* or *stiffness* k.

The magnitude of force exerted on a linearly elastic spring which has a stiffness k and is deformed (elongated or compressed) a distance $s = l - l_0$, measured from its *unloaded* position, is

$$F = ks \tag{3–2}$$

If s is positive, causing an elongation, then $\mathbf{F}$ must pull on the spring; whereas if s is negative, causing a shortening, then $\mathbf{F}$ must push on it. For example, if the spring in Fig. 3–1 has an unstretched length of 0.8 m and a stiffness $k = 500\ \text{N/m}$ and it is stretched to a length of 1 m, so that $s = l - l_0 = 1\ \text{m} - 0.8\ \text{m} = 0.2\ \text{m}$, then a force $F = ks = 500\ \text{N/m}(0.2\ \text{m}) = 100\ \text{N}$ is needed.

Cables and Pulleys. Unless otherwise stated throughout this book, except in Sec. 7.4, all cables (or cords) will be assumed to have negligible weight and they cannot stretch. Also, a cable can support *only* a tension or "pulling" force, and this force always acts in the direction of the cable. In Chapter 5, it will be shown that the tension force developed in a continuous cable which passes over a frictionless pulley must have a *constant* magnitude to keep the cable in equilibrium. Hence, for any angle θ, shown in Fig. 3–2, the cable is subjected to a constant tension T throughout its length.

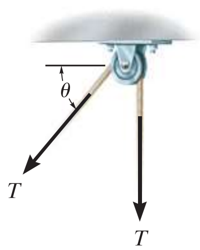


Fig. 3–2

Procedure for Drawing a Free-Body Diagram

Since we must account for *all the forces acting on the particle* when applying the equations of equilibrium, the importance of first drawing a free-body diagram cannot be overemphasized. To construct a free-body diagram, the following three steps are necessary.

Draw Outlined Shape.

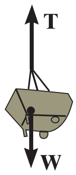
Imagine the particle to be *isolated* or cut "free" from its surroundings by drawing its outlined shape.

Show All Forces.

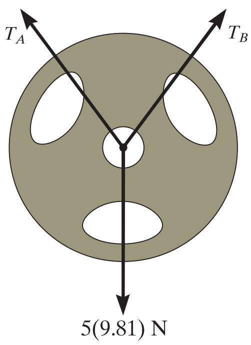
Indicate on this sketch *all* the forces that act *on the particle*. These forces can be *active forces*, which tend to set the particle in motion, or they can be *reactive forces* which are the result of the constraints or supports that tend to prevent motion. To account for all these forces, it may be helpful to trace around the particle's boundary, carefully noting each force acting on it.

Identify Each Force.

The forces that are *known* should be labeled with their proper magnitudes and directions. Letters are used to represent the magnitudes and directions of forces that are unknown.



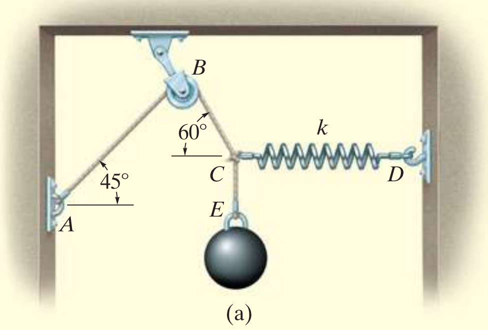
The bucket is held in equilibrium by the cable, and instinctively we know that the force in the cable must equal the weight of the bucket. By drawing a free-body diagram of the bucket we can understand why this is so. This diagram shows that there are only two forces *acting on the bucket*, namely, its weight **W** and the force **T** of the cable. For equilibrium, the resultant of these forces must be equal to zero, and so $T = W$.



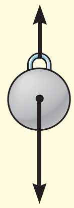
The 5-kg plate is suspended by two straps A and B. To find the force in each strap we should consider the free-body diagram of the plate. As noted, the three forces acting on it form a concurrent force system.

EXAMPLE 3.1

The sphere in Fig. 3–3*a* has a mass of 6 kg and is supported as shown. Draw a free-body diagram of the sphere, the cord *CE*, and the knot at *C*.



(a)



(b)

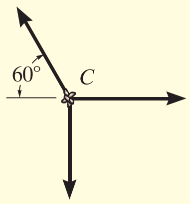
(c)

SOLUTION

Sphere. By inspection, there are only two forces acting on the sphere, namely, its weight, 6 kg $(9.81 \text{ m/s}^2) = 58.9$ N, and the force of cord *CE*. The free-body diagram is shown in Fig. 3–3*b*.

Cord *CE*. When the cord *CE* is isolated from its surroundings, its free-body diagram shows only two forces acting on it, namely, the force of the sphere and the force of the knot, Fig. 3–3*c*. Notice that $\mathbf{F}_{CE}$ shown here is equal but opposite to that shown in Fig. 3–3*b*, a consequence of Newton's third law of action–reaction. Also, $\mathbf{F}_{CE}$ and $\mathbf{F}_{EC}$ pull on the cord and keep it in tension so that it doesn't collapse. For equilibrium, $F_{CE} = F_{EC}$.

Knot. The knot at *C* is subjected to three forces, Fig. 3–3*d*. They are caused by the cords *CBA* and *CE* and the spring *CD*. As required, the free-body diagram shows all these forces labeled with their magnitudes and directions. It is important to recognize that the weight of the sphere does not directly act on the knot. Instead, the cord *CE* subjects the knot to this force.



(d)

Fig. 3–3

3.3 Coplanar Force Systems

If a particle is subjected to a system of coplanar forces that lie in the x–y plane, as in Fig. 3–4, then each force can be resolved into its **i** and **j** components. For equilibrium, these forces must sum to produce a zero force resultant, i.e.,

$$\Sigma \mathbf{F} = \mathbf{0}$$
$$\Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} = \mathbf{0}$$

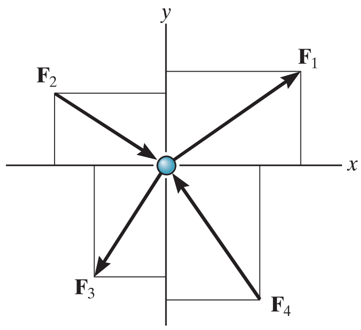


Fig. 3–4

For this vector equation to be satisfied, the resultant force's x and y components must both be equal to zero. Hence,

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0 \qquad (3\text{–}3)$$

These two equations can be solved for at most two unknowns, generally represented as angles and magnitudes of forces shown on the particle's free-body diagram.

When applying each of the two equations of equilibrium, we must account for the sense of direction of any component by using an *algebraic sign* which corresponds to the arrowhead direction of the component along the x or y axis. It is important to note that if a force has an *unknown magnitude*, then the arrowhead sense of the force on the free-body diagram can be *assumed*. Then if the *solution* yields a *negative scalar*, this indicates that the sense of the force is opposite to that which was assumed.

For example, consider the free-body diagram of the particle subjected to the two forces shown in Fig. 3–5. Here it is *assumed* that the *unknown force* **F** acts to the right to maintain equilibrium. Applying the equation of equilibrium along the x axis, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad +F + 10\text{ N} = 0$$

Both terms are "positive" since both forces act in the positive x direction. When this equation is solved, $F = -10$ N. Here the *negative sign* indicates that **F** must act to the left to hold the particle in equilibrium, Fig. 3–5. Notice that if the $+x$ axis in Fig. 3–5 were directed to the left, both terms in the above equation would be negative, but again, after solving, $F = -10$ N, indicating that **F** would have to be directed to the left.

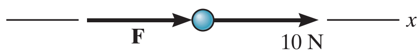


Fig. 3–5

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Procedure for Analysis

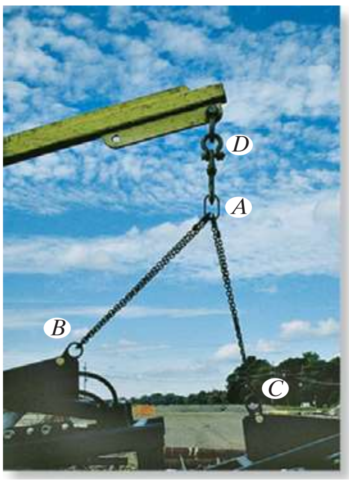
Coplanar force equilibrium problems for a particle can be solved using the following procedure.

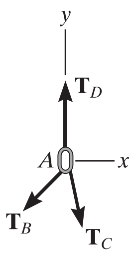
Free-Body Diagram.

- Establish the x, y axes in any suitable orientation.
- Label all the known and unknown force magnitudes and directions on the diagram.
- The sense of a force having an unknown magnitude can be assumed.

Equations of Equilibrium.

- Apply the equations of equilibrium, $\Sigma F_x = 0$ and $\Sigma F_y = 0$.
- Components are positive if they are directed along a positive axis, and negative if they are directed along a negative axis.
- If more than two unknowns exist and the problem involves a spring, apply $F = ks$ to relate the spring force to the deformation s of the spring.
- Since the magnitude of a force is always a positive quantity, then if the solution for a force yields a negative result, this indicates that its sense is the reverse of that shown on the free-body diagram.

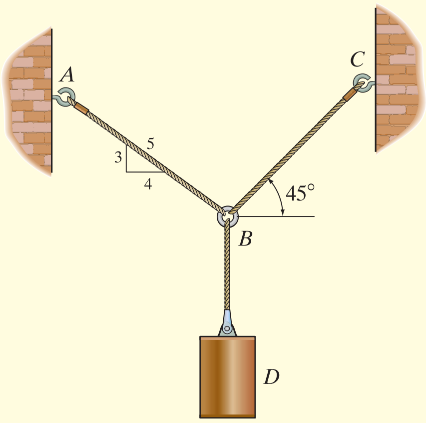




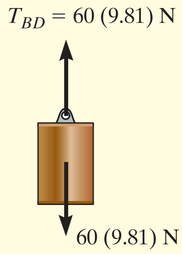
The chains exert three forces on the ring at A, as shown on its free-body diagram. The ring will not move, or will move with constant velocity, provided the summation of these forces along the x and along the y axis equals zero. If one of the three forces is known, the magnitudes of the other two forces can be obtained from the two equations of equilibrium.

EXAMPLE 3.2

Determine the tension in cables BA and BC necessary to support the 60-kg cylinder in Fig. 3–6*a*.



(a)



(b)

SOLUTION

Free-Body Diagram. Due to equilibrium, the weight of the cylinder causes the tension in cable BD to be $T_{BD} = 60(9.81)$ N, Fig. 3–6*b*. The forces in cables BA and BC can be determined by investigating the equilibrium of ring B. Its free-body diagram is shown in Fig. 3–6*c*. The magnitudes of $\mathbf{T}_A$ and $\mathbf{T}_C$ are unknown, but their directions are known.

Equations of Equilibrium. Applying the equations of equilibrium along the x and y axes, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_C \cos 45^\circ - \left(\tfrac{4}{5}\right)T_A = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad T_C \sin 45^\circ + \left(\tfrac{3}{5}\right)T_A - 60(9.81)\text{ N} = 0 \qquad (2)$$

Equation (1) can be written as $T_A = 0.8839T_C$. Substituting this into Eq. (2) yields

$$T_C \sin 45^\circ + \left(\tfrac{3}{5}\right)(0.8839T_C) - 60(9.81)\text{ N} = 0$$

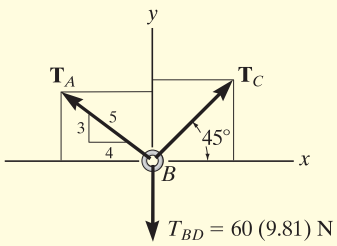
so that

$$T_C = 475.66\text{ N} = 476\text{ N} \qquad \textit{Ans.}$$

Substituting this result into either Eq. (1) or Eq. (2), we get

$$T_A = 420\text{ N} \qquad \textit{Ans.}$$

NOTE: The accuracy of these results, of course, depends on the accuracy of the data, i.e., measurements of geometry and loads. For most engineering work involving a problem such as this, the data as measured to three significant figures would be sufficient.



(c)

Fig. 3–6

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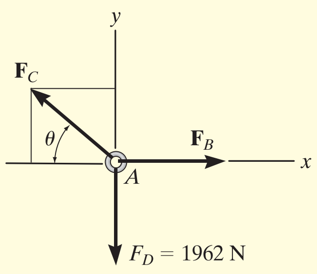
EXAMPLE 3.3

The 200-kg crate in Fig. 3–7*a* is suspended using the ropes *AB* and *AC*. Each rope can withstand a maximum force of 10 kN before it breaks. If *AB* always remains horizontal, determine the smallest angle θ to which the crate can be suspended before one of the ropes breaks.



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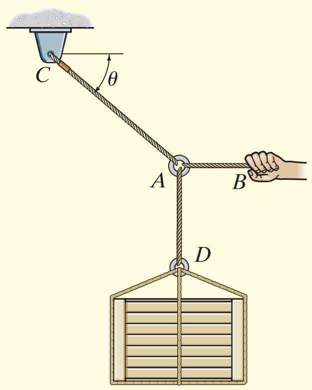


(a)

Fig. 3–7

SOLUTION

Free-Body Diagram. We will study the equilibrium of ring *A*. There are three forces acting on it, Fig. 3–7*b*. The magnitude of $\mathbf{F}_D$ is equal to the weight of the crate, i.e., $F_D = 200\,(9.81)\text{ N} = 1962\text{ N} < 10\text{ kN}$.

Equations of Equilibrium. Applying the equations of equilibrium along the *x* and *y* axes,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad -F_C \cos\theta + F_B = 0; \quad F_C = \frac{F_B}{\cos\theta} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad F_C \sin\theta - 1962\text{ N} = 0 \qquad (2)$$

From Eq. (1), F_C is always greater than F_B since $\cos\theta \le 1$. Therefore, rope *AC* will reach the maximum tensile force of 10 kN *before* rope *AB*. Substituting $F_C = 10$ kN into Eq. (2), we get

$$[10(10^3)\text{ N}]\sin\theta - 1962\text{ N} = 0$$

$$\theta = \sin^{-1}(0.1962) = 11.31^\circ = 11.3^\circ \qquad \textit{Ans.}$$

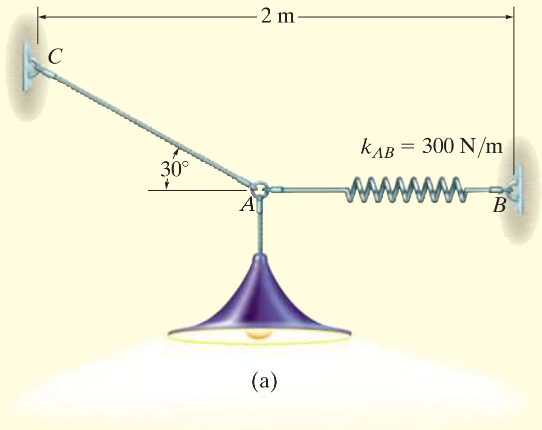
The force developed in rope *AB* can be obtained by substituting the values for θ and F_C into Eq. (1).

$$10(10^3)\text{ N} = \frac{F_B}{\cos 11.31^\circ}$$

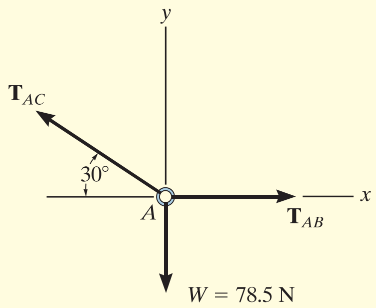
$$F_B = 9.81\text{ kN}$$

EXAMPLE 3.4

Determine the required length of cord AC in Fig. 3–8*a* so that the 8-kg lamp can be suspended in the position shown. The *undeformed* length of spring AB is $l'_{AB} = 0.4$ m, and the spring has a stiffness of $k_{AB} = 300$ N/m.



(a)



(b)

Fig. 3–8

SOLUTION

If the force in spring AB is known, the stretch of the spring can be found using $F = ks$. From the problem geometry, it is then possible to calculate the required length of AC.

Free-Body Diagram. The lamp has a weight $W = 8(9.81) = 78.5$ N and so the free-body diagram of the ring at A is shown in Fig. 3–8*b*.

Equations of Equilibrium. Using the x, y axes,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad T_{AB} - T_{AC}\cos 30^\circ = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad T_{AC}\sin 30^\circ - 78.5\text{ N} = 0$$

Solving, we obtain

$$T_{AC} = 157.0\text{ N}$$

$$T_{AB} = 135.9\text{ N}$$

The stretch of spring AB is therefore

$$T_{AB} = k_{AB}s_{AB}; \qquad 135.9\text{ N} = 300\text{ N/m}(s_{AB})$$

$$s_{AB} = 0.453\text{ m}$$

so the stretched length is

$$l_{AB} = l'_{AB} + s_{AB}$$

$$l_{AB} = 0.4\text{ m} + 0.453\text{ m} = 0.853\text{ m}$$

The horizontal distance from C to B, Fig. 3–8*a*, requires

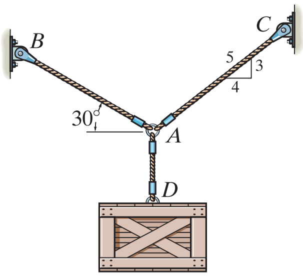
$$2\text{ m} = l_{AC}\cos 30^\circ + 0.853\text{ m}$$

$$l_{AC} = 1.32\text{ m} \qquad \textit{Ans.}$$

FUNDAMENTAL PROBLEMS

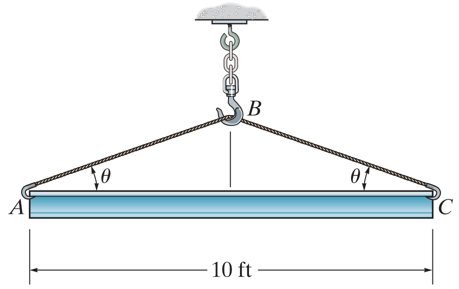
All problem solutions must include an FBD.

F3–1. The crate has a weight of 550 lb. Determine the force in each supporting cable.



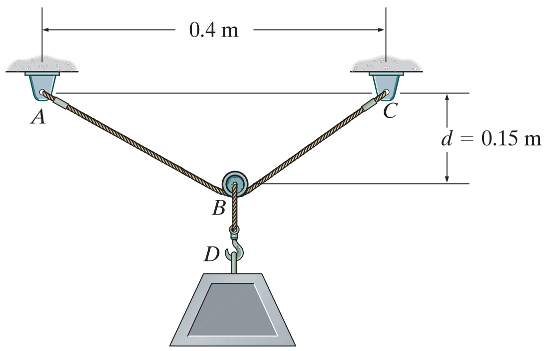
F3–1

F3–2. The beam has a weight of 700 lb. Determine the shortest cable ABC that can be used to lift it if the maximum force the cable can sustain is 1500 lb.



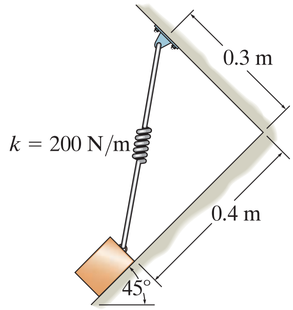
F3–2

F3–3. If the 5-kg block is suspended from the pulley B and the sag of the cord is $d = 0.15$ m, determine the force in cord ABC. Neglect the size of the pulley.



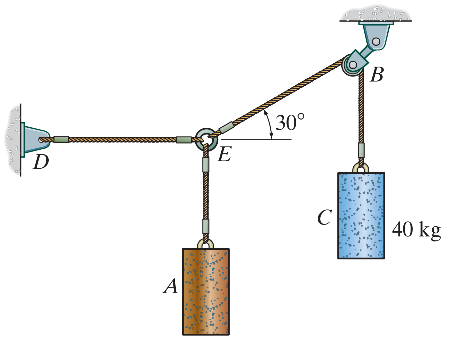
F3–3

F3–4. The block has a mass of 5 kg and rests on the smooth plane. Determine the unstretched length of the spring.



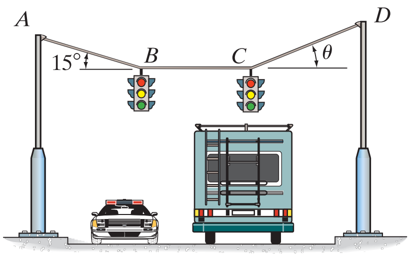
F3–4

F3–5. If the mass of cylinder C is 40 kg, determine the mass of cylinder A in order to hold the assembly in the position shown.



F3–5

F3–6. Determine the tension in cables AB, BC, and CD, necessary to support the 10-kg and 15-kg traffic lights at B and C, respectively. Also, find the angle θ.



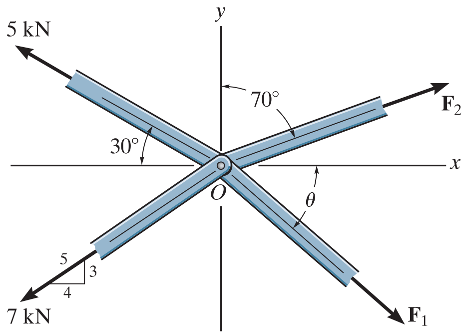
F3–6

PROBLEMS

All problem solutions must include an FBD.

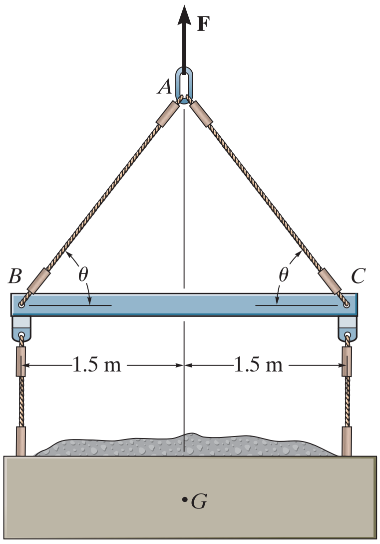
3–1. The members of a truss are pin connected at joint O. Determine the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ for equilibrium. Set $\theta = 60°$.

3–2. The members of a truss are pin connected at joint O. Determine the magnitude of $\mathbf{F}_1$ and its angle θ for equilibrium. Set $F_2 = 6$ kN.



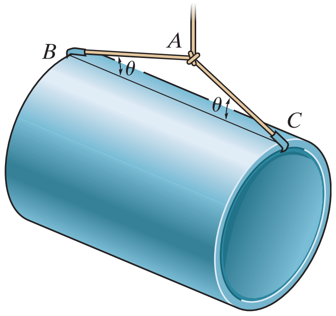
Probs. 3–1/2

3–3. The lift sling is used to hoist a container having a mass of 500 kg. Determine the force in each of the cables AB and AC as a function of θ. If the maximum tension allowed in each cable is 5 kN, determine the shortest lengths of cables AB and AC that can be used for the lift. The center of gravity of the container is located at G.



Prob. 3–3

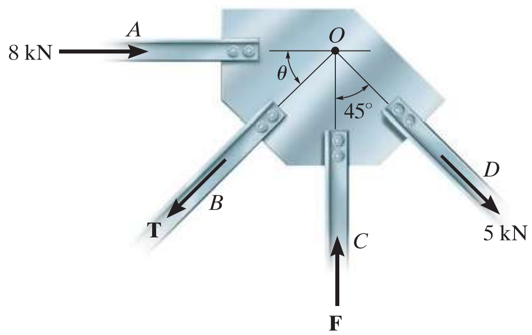
***3–4.** Cords AB and AC can each sustain a maximum tension of 800 lb. If the drum has a weight of 900 lb, determine the smallest angle θ at which they can be attached to the drum.



Prob. 3–4

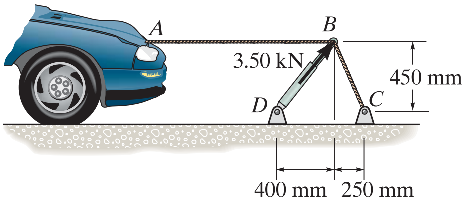
3–5. The members of a truss are connected to the gusset plate. If the forces are concurrent at point O, determine the magnitudes of $\mathbf{F}$ and $\mathbf{T}$ for equilibrium. Take $\theta = 30°$.

3–6. The gusset plate is subjected to the forces of four members. Determine the force in member B and its proper orientation θ for equilibrium. The forces are concurrent at point O. Take $F = 12$ kN.



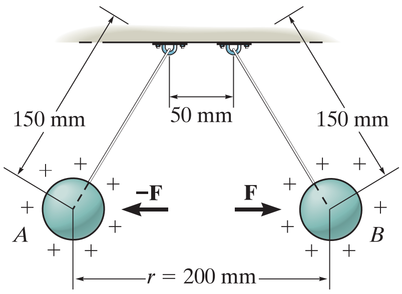
Probs. 3–5/6

3–7. The device shown is used to straighten the frames of wrecked autos. Determine the tension of each segment of the chain, i.e., AB and BC, if the force which the hydraulic cylinder DB exerts on point B is 3.50 kN, as shown.



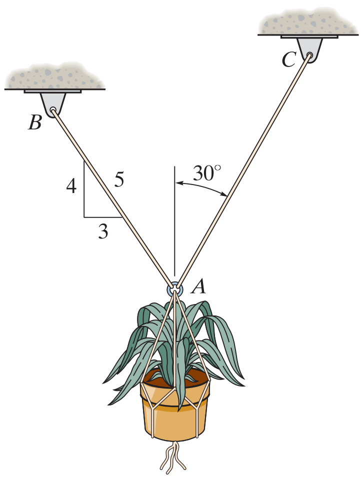
Prob. 3–7

***3–8.** Two electrically charged pith balls, each having a mass of 0.2 g, are suspended from light threads of equal length. Determine the resultant horizontal force of repulsion, F, acting on each ball if the measured distance between them is $r = 200$ mm.



Prob. 3–8

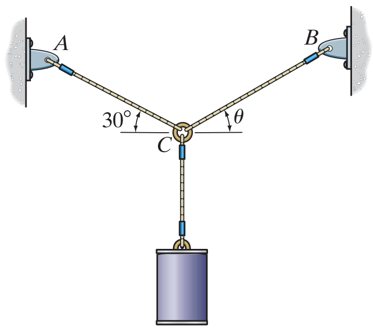
3–9. Determine the maximum weight of the flowerpot that can be supported without exceeding a cable tension of 50 lb in either cable AB or AC.



Prob. 3–9

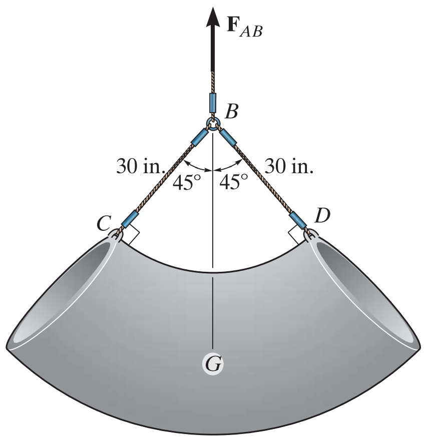
3–10. Determine the tension developed in wires CA and CB required for equilibrium of the 10-kg cylinder. Take $\theta = 40°$.

3–11. If cable CB is subjected to a tension that is twice that of cable CA, determine the angle θ for equilibrium of the 10-kg cylinder. Also, what are the tensions in wires CA and CB?



Probs. 3–10/11

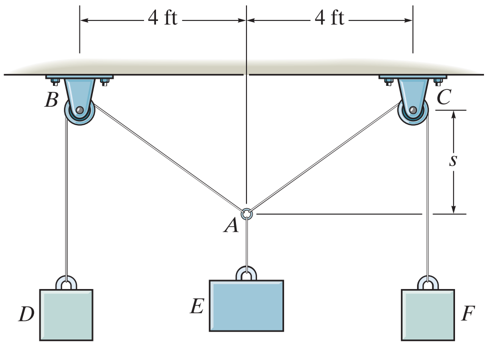
***3–12.** The concrete pipe elbow has a weight of 400 lb and the center of gravity is located at point G. Determine the force $\mathbf{F}_{AB}$ and the tension in cables BC and BD needed to support it.



Prob. 3–12

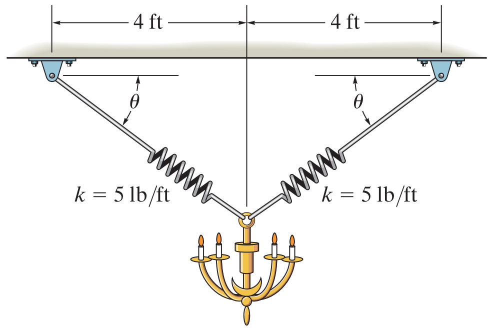
3–13. Blocks D and F weigh 5 lb each and block E weighs 8 lb. Determine the sag s for equilibrium. Neglect the size of the pulleys.

3–14. If blocks D and F weigh 5 lb each, determine the weight of block E if the sag $s = 3$ ft. Neglect the size of the pulleys.



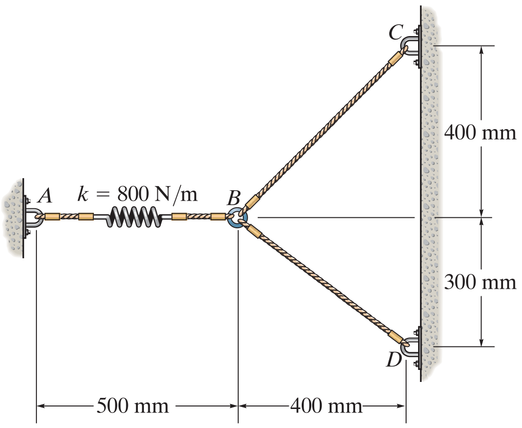
Probs. 3–13/14

*■**3–16.** The 10-lb lamp fixture is suspended from two springs, each having an unstretched length of 4 ft and stiffness of $k = 5$ lb/ft. Determine the angle θ for equilibrium.



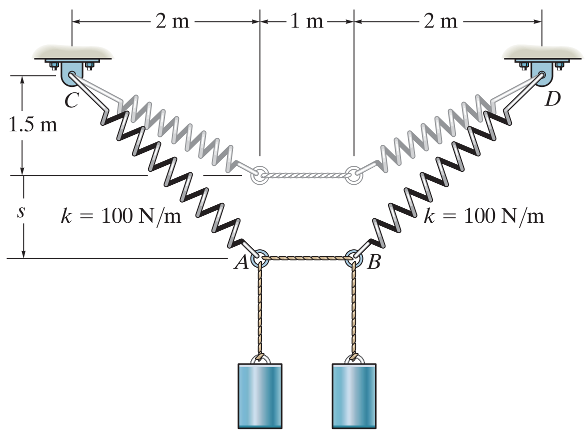
Prob. 3–16

3–15. The spring has a stiffness of $k = 800$ N/m and an unstretched length of 200 mm. Determine the force in cables BC and BD when the spring is held in the position shown.



Prob. 3–15

3–17. Determine the mass of each of the two cylinders if they cause a sag of $s = 0.5$ m when suspended from the rings at A and B. Note that $s = 0$ when the cylinders are removed.

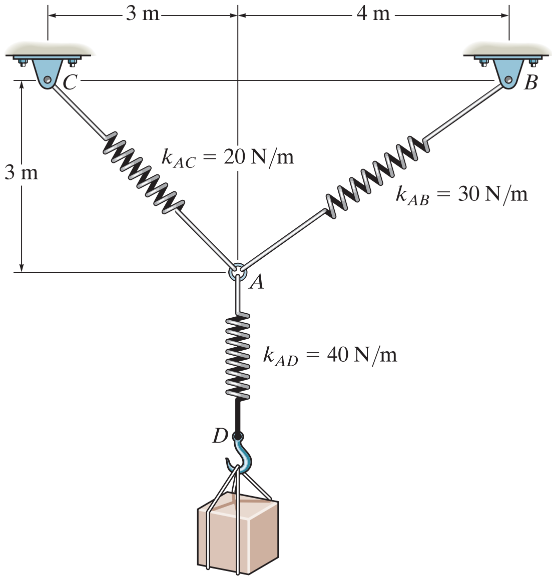


Prob. 3–17

3

3–18. Determine the stretch in each spring for equilibrium of the 2-kg block. The springs are shown in the equilibrium position.

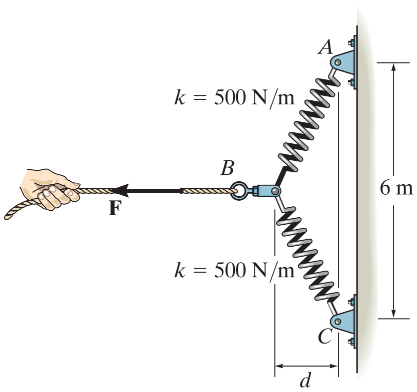
3–19. The unstretched length of spring AB is 3 m. If the block is held in the equilibrium position shown, determine the mass of the block at D.



Probs. 3–18/19

***3–20.** The springs BA and BC each have a stiffness of 500 N/m and an unstretched length of 3 m. Determine the horizontal force **F** applied to the cord which is attached to the *small* ring B so that the displacement of the ring from the wall is d = 1.5 m.

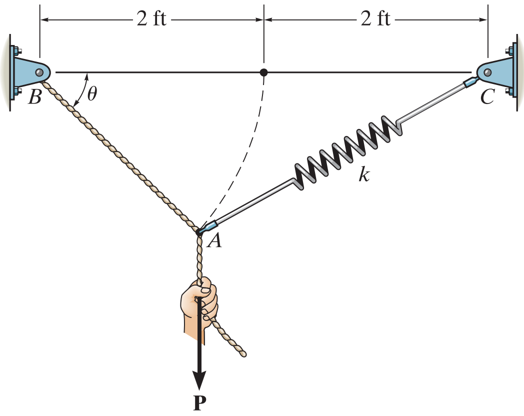
3–21. The springs BA and BC each have a stiffness of 500 N/m and an unstretched length of 3 m. Determine the displacement d of the cord from the wall when a force F = 175 N is applied to the cord.



Probs. 3–20/21

■**3–22.** A vertical force P = 10 lb is applied to the ends of the 2-ft cord AB and spring AC. If the spring has an unstretched length of 2 ft, determine the angle θ for equilibrium. Take k = 15 lb/ft.

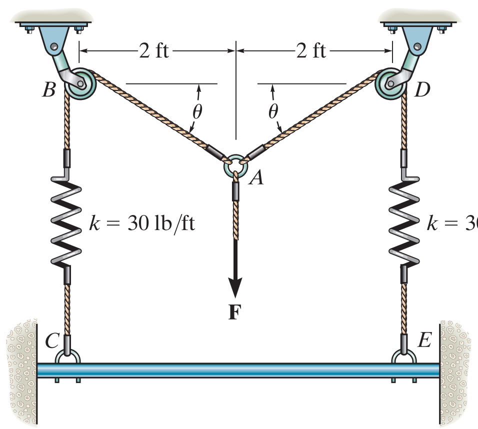
3–23. Determine the unstretched length of spring AC if a force P = 80 lb causes the angle $\theta = 60°$ for equilibrium. Cord AB is 2 ft long. Take k = 50 lb/ft.



Probs. 3–22/23

***3–24.** The springs on the rope assembly are originally unstretched when $\theta = 0°$. Determine the tension in each rope when F = 90 lb. Neglect the size of the pulleys at B and D.

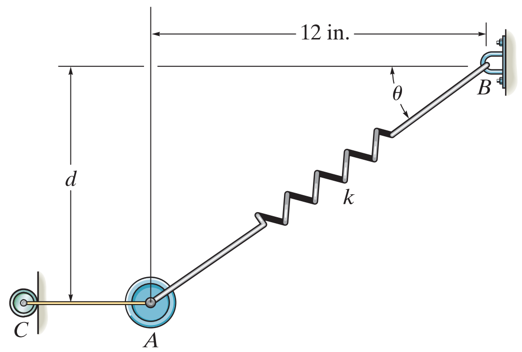
3–25. The springs on the rope assembly are originally stretched 1 ft when $\theta = 0°$. Determine the vertical force F that must be applied so that $\theta = 30°$.



Probs. 3–24/25

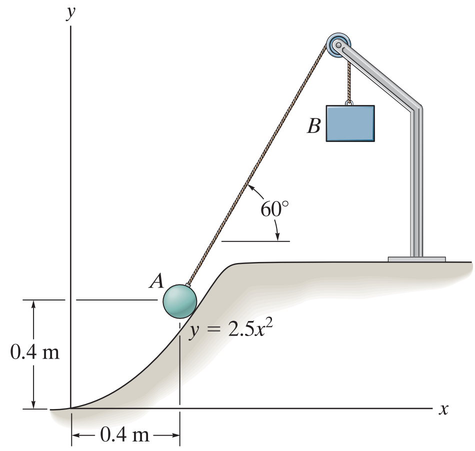
3–26. The 10-lb weight A is supported by the cord AC and roller C, and by the spring that has a stiffness of $k = 10\ \text{lb/in.}$ If the unstretched length of the spring is 12 in. determine the distance d to where the weight is located when it is in equilibrium.

3–27. The 10-lb weight A is supported by the cord AC and roller C, and by spring AB. If the spring has an unstretched length of 8 in. and the weight is in equilibrium when $d = 4$ in., determine the stiffness k of the spring.



Probs. 3–26/27

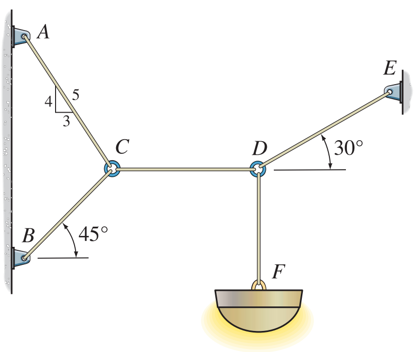
3–30. A 4-kg sphere rests on the smooth parabolic surface. Determine the normal force it exerts on the surface and the mass m_B of block B needed to hold it in the equilibrium position shown.



Prob. 3–30

***3–28.** Determine the tension developed in each cord required for equilibrium of the 20-kg lamp.

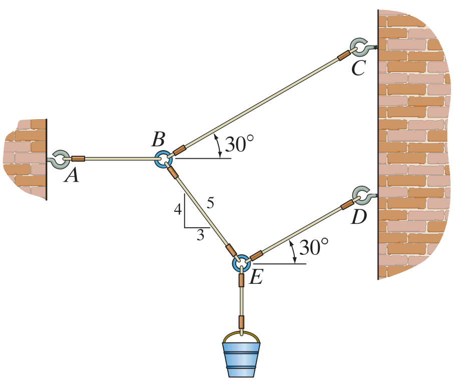
3–29. Determine the maximum mass of the lamp that the cord system can support so that no single cord develops a tension exceeding 400 N.



Probs. 3–28/29

3–31. If the bucket weighs 50 lb, determine the tension developed in each of the wires.

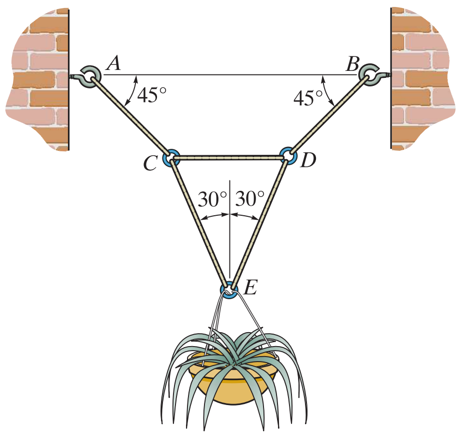
***3–32.** Determine the maximum weight of the bucket that the wire system can support so that no single wire develops a tension exceeding 100 lb.



Probs. 3–31/32

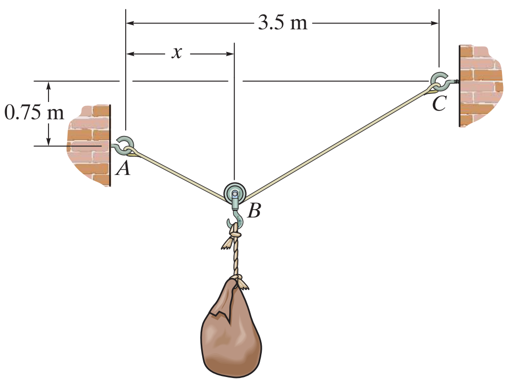
3–33. Determine the tension developed in each wire which is needed to support the 50-lb flowerpot.

3–34. If the tension developed in each of the wires is not allowed to exceed 40 lb, determine the maximum weight of the flowerpot that can be safely supported.



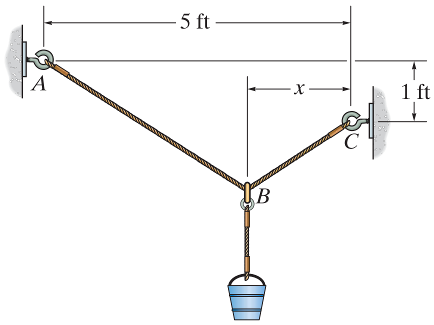
Probs. 3–33/34

3–35. Cable ABC has a length of 5 m. Determine the position x and the tension developed in ABC required for equilibrium of the 100-kg sack. Neglect the size of the pulley at B.



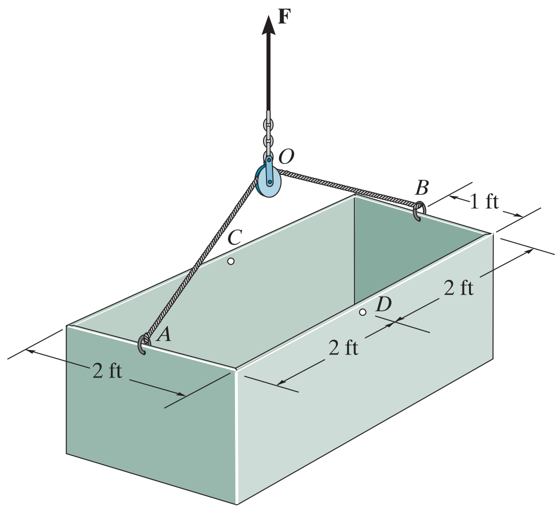
Prob. 3–35

*__3–36.__ The single elastic cord ABC is used to support the 40-lb load. Determine the position x and the tension in the cord that is required for equilibrium. The cord passes through the smooth ring at B and has an unstretched length of 6 ft and stiffness of $k = 50$ lb/ft.



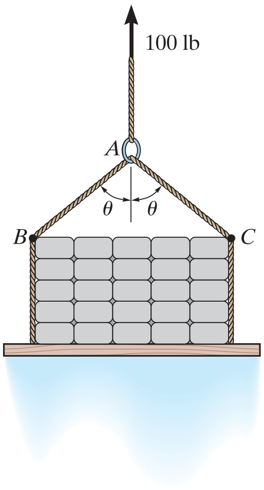
Prob. 3–36

3–37. The 200-lb uniform tank is suspended by means of a 6-ft-long cable, which is attached to the sides of the tank and passes over the small pulley located at O. If the cable can be attached at either points A and B, or C and D, determine which attachment produces the least amount of tension in the cable. What is this tension?



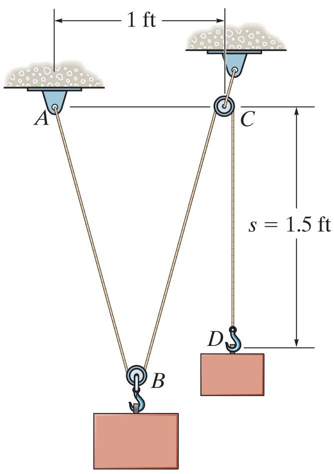
Prob. 3–37

3–38. The sling BAC is used to lift the 100-lb load with constant velocity. Determine the force in the sling and plot its value T (ordinate) as a function of its orientation θ where $0 \leq \theta \leq 90°$.



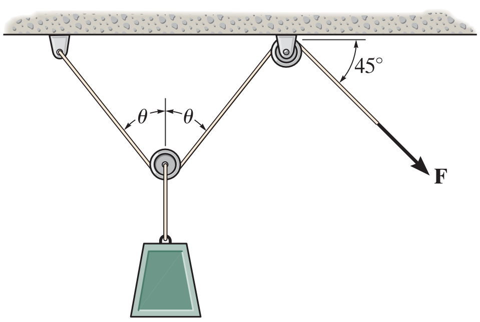
Prob. 3–38

■**3–39.** A "scale" is constructed with a 4-ft-long cord and the 10-lb block D. The cord is fixed to a pin at A and passes over two *small* pulleys. Determine the weight of the suspended block B if the system is in equilibrium when $s = 1.5$ ft.



Prob. 3–39

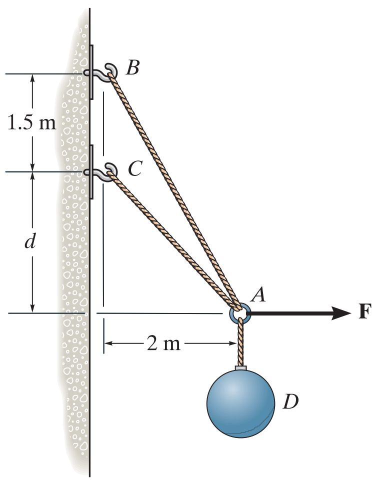
*__3–40.__ The load has a mass of 15 kg and is lifted by the pulley system shown. Determine the force **F** in the cord as a function of the angle θ. Plot the function of force F versus the angle θ for $0 \leq \theta \leq 90°$.



Prob. 3–40

3–41. Determine the forces in cables AC and AB needed to hold the 20-kg ball D in equlibrium. Take $F = 300$ N and $d = 1$ m.

3–42. The ball D has a mass of 20 kg. If a force of $F = 100$ N is applied horizontally to the ring at A, determine the dimension d so that the force in cable AC is zero.



Probs. 3–41/42

3

CONCEPTUAL PROBLEMS

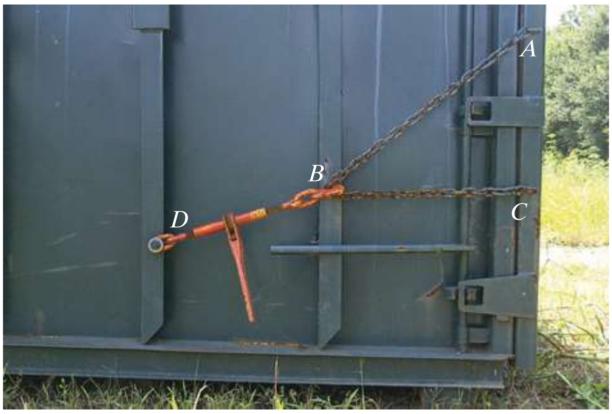
P3–1. The concrete wall panel is hoisted into position using the two cables *AB* and *AC* of equal length. Establish appropriate dimensions and use an equilibrium analysis to show that the longer the cables the less the force in each cable.

P3–1

P3–2. The hoisting cables *BA* and *BC* each have a length of 20 ft. If the maximum tension that can be supported by each cable is 900 lb, determine the maximum distance *AC* between them in order to lift the uniform 1200-lb truss with constant velocity.

P3–2

P3–3. The device *DB* is used to pull on the chain *ABC* to hold a door closed on the bin. If the angle between *AB* and *BC* is 30°, determine the angle between *DB* and *BC* for equilibrium.



P3–3

P3–4. Chain *AB* is 1-m long and chain *AC* is 1.2-m long. If the distance *BC* is 1.5 m, and *AB* can support a maximum force of 2 kN, whereas *AC* can support a maximum force of 0.8 kN, determine the largest vertical force *F* that can be applied to the link at *A*.

P3–4

3.4 Three-Dimensional Force Systems

In Section 3.1 we stated that the necessary and sufficient condition for particle equilibrium is

$$\Sigma \mathbf{F} = \mathbf{0} \qquad (3\text{–}4)$$

In the case of a three-dimensional force system, as in Fig. 3–9, we can resolve the forces into their respective **i**, **j**, **k** components, so that $\Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} + \Sigma F_z \mathbf{k} = \mathbf{0}$. To satisfy this equation we require

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_y = 0 \qquad (3\text{–}5)$$

These three equations state that the *algebraic sum* of the components of all the forces acting on the particle along each of the coordinate axes must be zero. Using them we can solve for at most three unknowns, generally represented as coordinate direction angles or magnitudes of forces shown on the particle's free-body diagram.

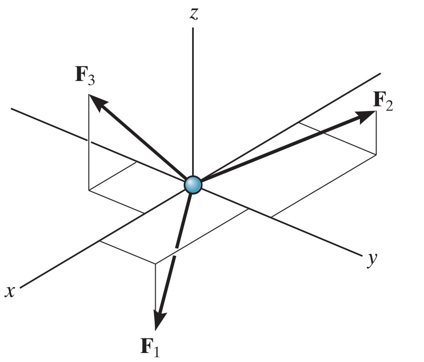


Fig. 3–9

Procedure for Analysis

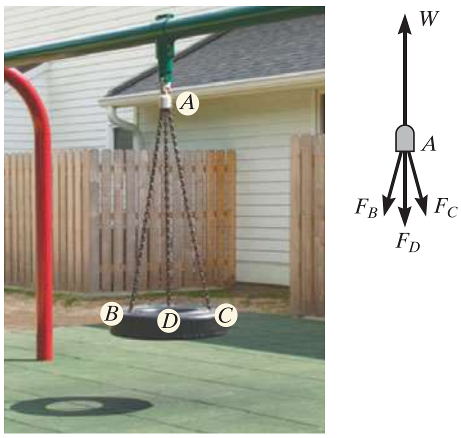
Three-dimensional force equilibrium problems for a particle can be solved using the following procedure.

Free-Body Diagram.

- Establish the *x, y, z* axes in any suitable orientation.
- Label all the known and unknown force magnitudes and directions on the diagram.
- The sense of a force having an unknown magnitude can be assumed.

Equations of Equilibrium.

- Use the scalar equations of equilibrium, $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$, in cases where it is easy to resolve each force into its *x, y, z* components.
- If the three-dimensional geometry appears difficult, then first express each force on the free-body diagram as a Cartesian vector, substitute these vectors into $\Sigma \mathbf{F} = \mathbf{0}$, and then set the **i**, **j**, **k** components equal to zero.
- If the solution for a force yields a negative result, this indicates that its sense is the reverse of that shown on the free-body diagram.



The joint at *A* is subjected to the force from the support as well as forces from each of the three chains. If the tire and any load on it have a weight *W*, then the force at the support will be **W**, and the three scalar equations of equilibrium can be applied to the free-body diagram of the joint in order to determine the chain forces, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$.

EXAMPLE 3.5

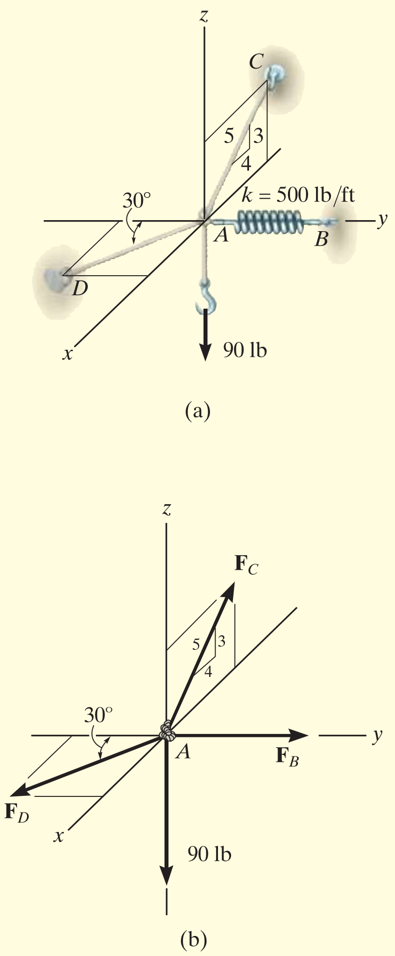


Fig. 3–10

A 90-lb load is suspended from the hook shown in Fig. 3–10*a*. If the load is supported by two cables and a spring having a stiffness $k = 500\ \text{lb/ft}$, determine the force in the cables and the stretch of the spring for equilibrium. Cable AD lies in the x–y plane and cable AC lies in the x–z plane.

SOLUTION

The stretch of the spring can be determined once the force in the spring is determined.

Free-Body Diagram. The connection at A is chosen for the equilibrium analysis since the cable forces are concurrent at this point. The free-body diagram is shown in Fig. 3–10*b*.

Equations of Equilibrium. By inspection, each force can easily be resolved into its x, y, z components, and therefore the three scalar equations of equilibrium can be used. Considering components directed along each positive axis as "positive," we have

$$\Sigma F_x = 0; \qquad F_D \sin 30^\circ - \left(\tfrac{4}{5}\right)F_C = 0 \qquad (1)$$

$$\Sigma F_y = 0; \qquad -F_D \cos 30^\circ + F_B = 0 \qquad (2)$$

$$\Sigma F_z = 0; \qquad \left(\tfrac{3}{5}\right)F_C - 90\ \text{lb} = 0 \qquad (3)$$

Solving Eq. (3) for F_C, then Eq. (1) for F_D, and finally Eq. (2) for F_B, yields

$$F_C = 150\ \text{lb} \qquad \textit{Ans.}$$

$$F_D = 240\ \text{lb} \qquad \textit{Ans.}$$

$$F_B = 207.8\ \text{lb} = 208\ \text{lb} \qquad \textit{Ans.}$$

The stretch of the spring is therefore

$$F_B = ks_{AB}$$

$$207.8\ \text{lb} = (500\ \text{lb/ft})(s_{AB})$$

$$s_{AB} = 0.416\ \text{ft} \qquad \textit{Ans.}$$

NOTE: Since the results for all the cable forces are positive, each cable is in tension; that is, it pulls on point A as expected, Fig. 3–10*b*.

EXAMPLE 3.6

The 10-kg lamp in Fig. 3–11*a* is suspended from the three equal-length cords. Determine its smallest vertical distance *s* from the ceiling if the force developed in any cord is not allowed to exceed 50 N.

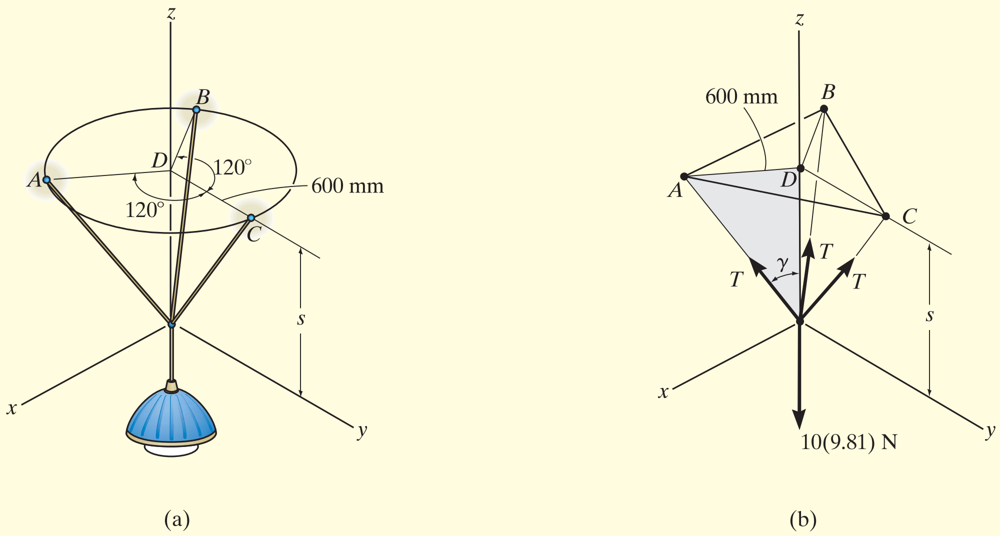


Fig. 3–11

SOLUTION

Free-Body Diagram. Due to symmetry, Fig. 3–11*b*, the distance $DA = DB = DC = 600$ mm. It follows that from $\Sigma F_x = 0$ and $\Sigma F_y = 0$, the tension *T* in each cord will be the same. Also, the angle between each cord and the *z* axis is γ.

Equation of Equilibrium. Applying the equilibrium equation along the *z* axis, with $T = 50$ N, we have

$$\Sigma F_z = 0; \qquad 3[(50\text{ N})\cos\gamma] - 10(9.81)\text{ N} = 0$$

$$\gamma = \cos^{-1}\frac{98.1}{150} = 49.16^\circ$$

From the shaded triangle shown in Fig. 3–11*b*,

$$\tan 49.16^\circ = \frac{600\text{ mm}}{s}$$

$$s = 519\text{ mm} \qquad \textit{Ans.}$$

EXAMPLE 3.7

Determine the force in each cable used to support the 40-lb crate shown in Fig. 3–12*a*.

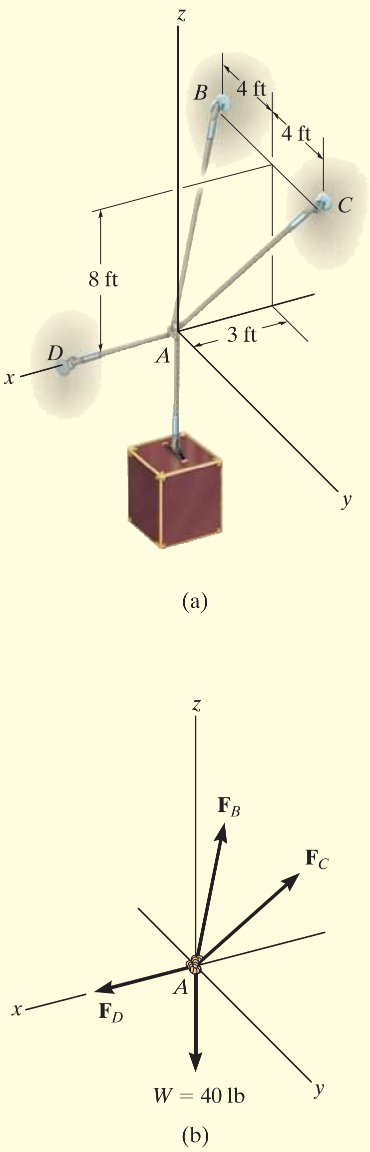


Fig. 3–12

SOLUTION

Free-Body Diagram. As shown in Fig. 3–12*b*, the free-body diagram of point *A* is considered in order to "expose" the three unknown forces in the cables.

Equations of Equilibrium. First we will express each force in Cartesian vector form. Since the coordinates of points *B* and *C* are *B*(−3 ft, −4 ft, 8 ft) and *C*(−3 ft, 4 ft, 8 ft), we have

$$\mathbf{F}_B = F_B\left[\frac{-3\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}}{\sqrt{(-3)^2 + (-4)^2 + (8)^2}}\right]$$

$$= -0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}$$

$$\mathbf{F}_C = F_C\left[\frac{-3\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}}{\sqrt{(-3)^2 + (4)^2 + (8)^2}}\right]$$

$$= -0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k}$$

$$\mathbf{F}_D = F_D\mathbf{i}$$

$$\mathbf{W} = \{-40\mathbf{k}\}\text{ lb}$$

Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = \mathbf{0}$$

$$-0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}$$

$$-0.318F_C\mathbf{i} + 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k} + F_D\mathbf{i} - 40\mathbf{k} = \mathbf{0}$$

Equating the respective **i**, **j**, **k** components to zero yields

$$\Sigma F_x = 0; \qquad -0.318F_B - 0.318F_C + F_D = 0 \qquad (1)$$

$$\Sigma F_y = 0; \qquad -0.424F_B + 0.424F_C = 0 \qquad (2)$$

$$\Sigma F_z = 0; \qquad 0.848F_B + 0.848F_C - 40 = 0 \qquad (3)$$

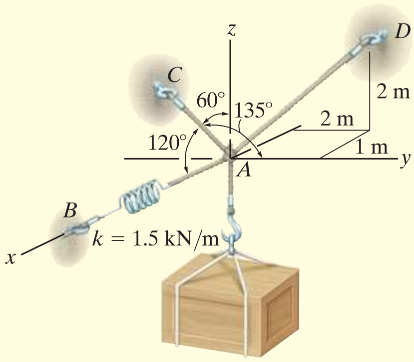
Equation (2) states that $F_B = F_C$. Thus, solving Eq. (3) for F_B and F_C and substituting the result into Eq. (1) to obtain F_D, we have

$$F_B = F_C = 23.6\text{ lb} \qquad \textit{Ans.}$$

$$F_D = 15.0\text{ lb} \qquad \textit{Ans.}$$

EXAMPLE 3.8

Determine the tension in each cord used to support the 100-kg crate shown in Fig. 3–13*a*.



(a)

SOLUTION

Free-Body Diagram. The force in each of the cords can be determined by investigating the equilibrium of point A. The free-body diagram is shown in Fig. 3–13*b*. The weight of the crate is $W = 100(9.81) = 981$ N.

Equations of Equilibrium. Each force on the free-body diagram is first expressed in Cartesian vector form. Using Eq. 2–9 for $\mathbf{F}_C$ and noting point $D(-1\text{ m}, 2\text{ m}, 2\text{ m})$ for $\mathbf{F}_D$, we have

$$\mathbf{F}_B = F_B\mathbf{i}$$

$$\mathbf{F}_C = F_C\cos 120^\circ\mathbf{i} + F_C\cos 135^\circ\mathbf{j} + F_C\cos 60^\circ\mathbf{k}$$

$$= -0.5F_C\mathbf{i} - 0.707F_C\mathbf{j} + 0.5F_C\mathbf{k}$$

$$\mathbf{F}_D = F_D\left[\frac{-1\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}}{\sqrt{(-1)^2 + (2)^2 + (2)^2}}\right]$$

$$= -0.333F_D\mathbf{i} + 0.667F_D\mathbf{j} + 0.667F_D\mathbf{k}$$

$$\mathbf{W} = \{-981\mathbf{k}\}\text{ N}$$

Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = \mathbf{0}$$

$$F_B\mathbf{i} - 0.5F_C\mathbf{i} - 0.707F_C\mathbf{j} + 0.5F_C\mathbf{k}$$

$$-0.333F_D\mathbf{i} + 0.667F_D\mathbf{j} + 0.667F_D\mathbf{k} - 981\mathbf{k} = \mathbf{0}$$

Equating the respective **i**, **j**, **k** components to zero,

$$\Sigma F_x = 0; \qquad F_B - 0.5F_C - 0.333F_D = 0 \qquad (1)$$

$$\Sigma F_y = 0; \qquad -0.707F_C + 0.667F_D = 0 \qquad (2)$$

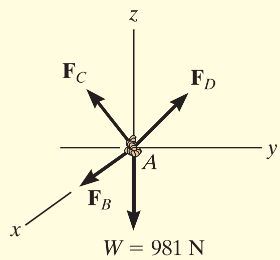
$$\Sigma F_z = 0; \qquad 0.5F_C + 0.667F_D - 981 = 0 \qquad (3)$$

Solving Eq. (2) for F_D in terms of F_C and substituting this into Eq. (3) yields F_C. F_D is then determined from Eq. (2). Finally, substituting the results into Eq. (1) gives F_B. Hence,

$$F_C = 813\text{ N} \qquad \textit{Ans.}$$

$$F_D = 862\text{ N} \qquad \textit{Ans.}$$

$$F_B = 694\text{ N} \qquad \textit{Ans.}$$



(b)

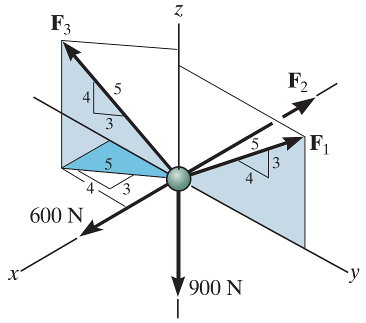
Fig. 3–13

3

FUNDAMENTAL PROBLEMS

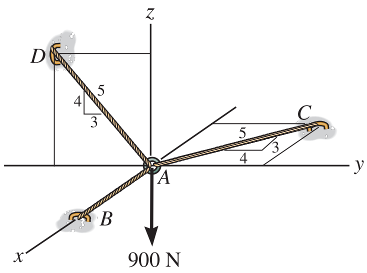
All problem solutions must include an FBD.

F3–7. Determine the magnitude of forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, so that the particle is held in equilibrium.



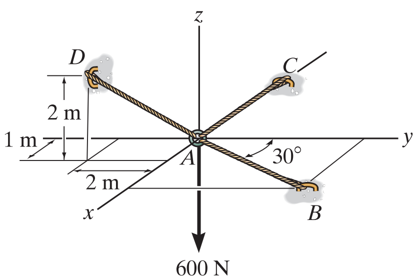
F3–7

F3–8. Determine the tension developed in cables AB, AC, and AD.



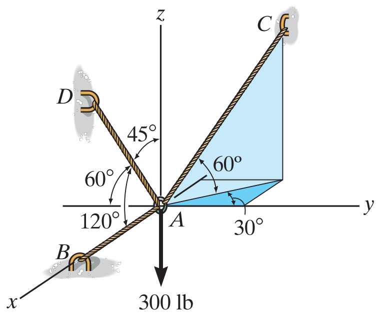
F3–8

F3–9. Determine the tension developed in cables AB, AC, and AD.



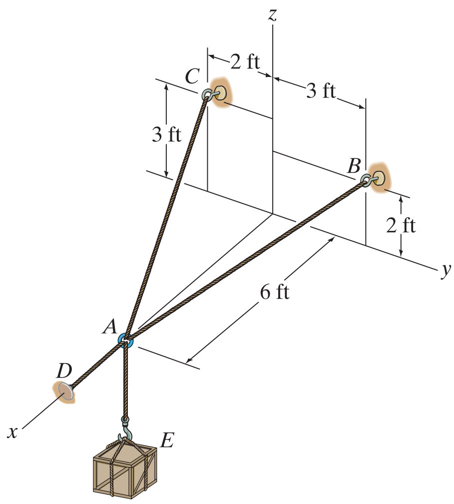
F3–9

F3–10. Determine the tension developed in cables AB, AC, and AD.



F3–10

F3–11. The 150-lb crate is supported by cables AB, AC, and AD. Determine the tension in these wires.

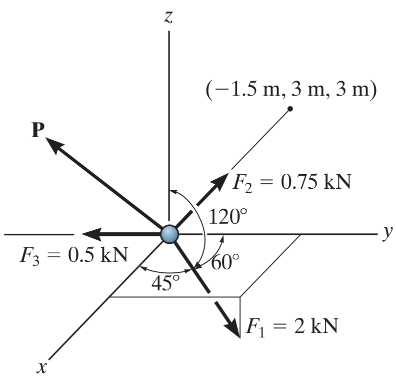


F3–11

PROBLEMS

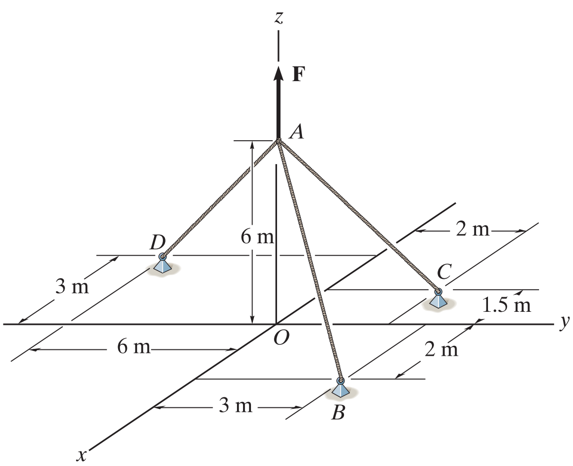
All problem solutions must include an FBD.

3–43. Determine the magnitude and direction of the force **P** required to keep the concurrent force system in equilibrium.



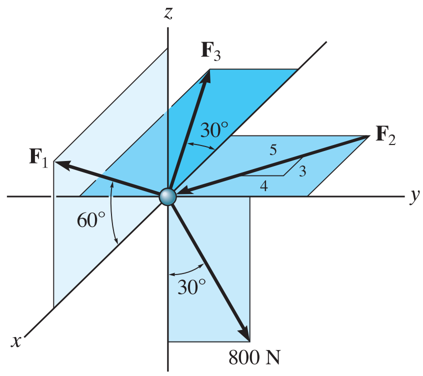
Prob. 3–43

*__3–44.__ If cable AB is subjected to a tension of 700 N, determine the tension in cables AC and AD and the magnitude of the vertical force **F**.



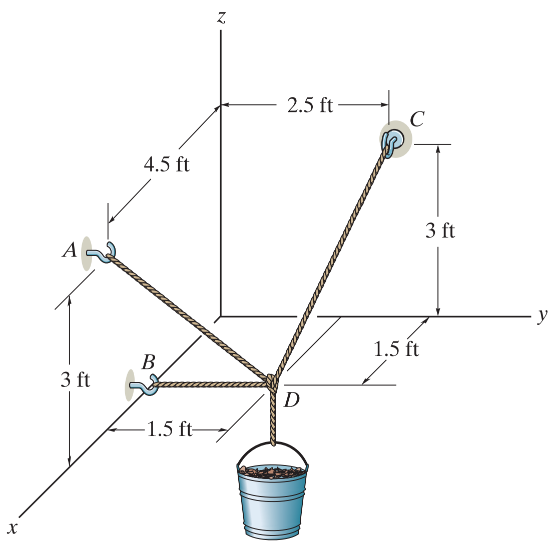
Prob. 3–44

3–45. Determine the magnitudes of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ for equilibrium of the particle.



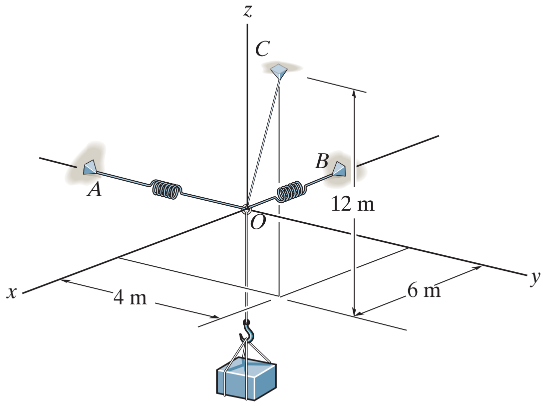
Prob. 3–45

3–46. If the bucket and its contents have a total weight of 20 lb, determine the force in the supporting cables DA, DB, and DC.



Prob. 3–46

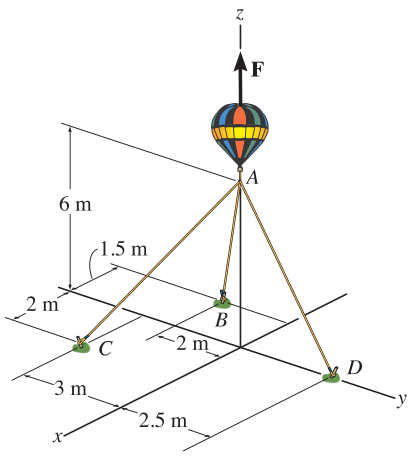
3–47. Determine the stretch in each of the two springs required to hold the 20-kg crate in the equilibrium position shown. Each spring has an unstretched length of 2 m and a stiffness of $k = 300$ N/m.



Prob. 3–47

***3–48.** If the balloon is subjected to a net uplift force of $F = 800$ N, determine the tension developed in ropes AB, AC, AD.

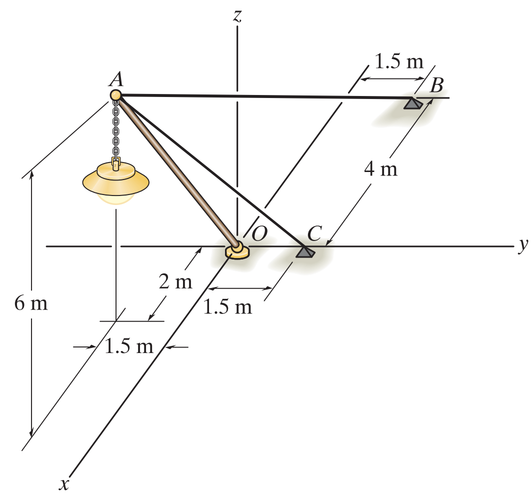
3–49. If each one of the ropes will break when it is subjected to a tensile force of 450 N, determine the maximum uplift force **F** the balloon can have before one of the ropes breaks.



Probs. 3–48/49

■3–50. The lamp has a mass of 15 kg and is supported by a pole AO and cables AB and AC. If the force in the pole acts along its axis, determine the forces in AO, AB, and AC for equilibrium.

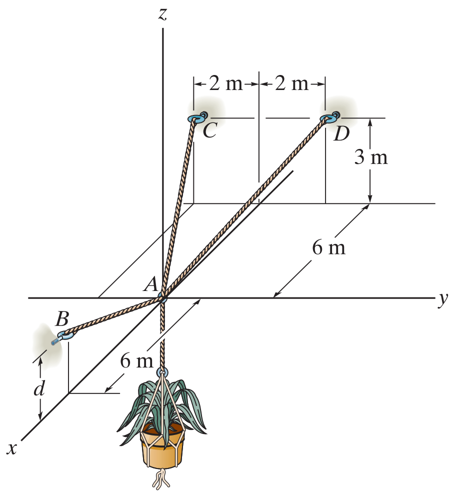
3–51. Cables AB and AC can sustain a maximum tension of 500 N, and the pole can support a maximum compression of 300 N. Determine the maximum weight of the lamp that can be supported in the position shown. The force in the pole acts along the axis of the pole.



Probs. 3–50/51

***3–52.** The 50-kg pot is supported from A by the three cables. Determine the force acting in each cable for equilibrium. Take $d = 2.5$ m.

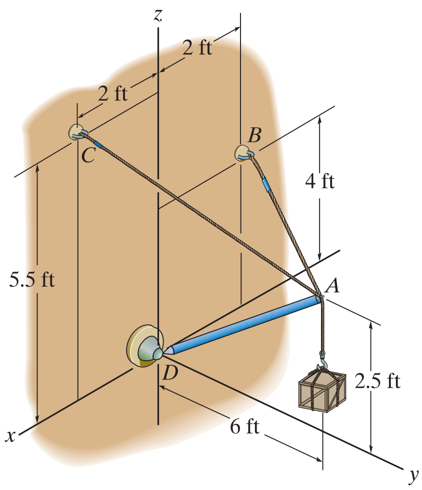
3–53. Determine the height d of cable AB so that the force in cables AD and AC is one-half as great as the force in cable AB. What is the force in each cable for this case? The flower pot has a mass of 50 kg.



Probs. 3–52/53

3–54. Determine the tension developed in cables AB and AC and the force developed along strut AD for equilibrium of the 400-lb crate.

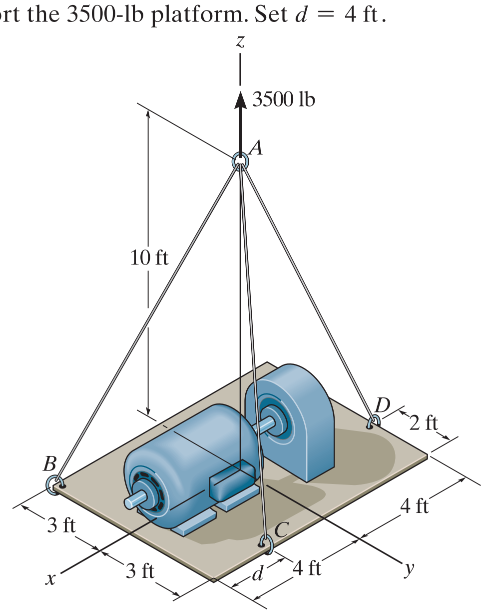
3–55. If the tension developed in each of the cables cannot exceed 300 lb, determine the largest weight of the crate that can be supported. Also, what is the force developed along strut AD?



Probs. 3–54/55

*3–56. Determine the force in each cable needed to support the 3500-lb platform. Set $d = 2$ ft.

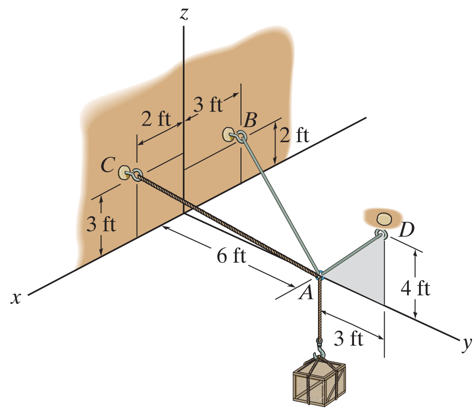
3–57. Determine the force in each cable needed to support the 3500-lb platform. Set $d = 4$ ft.



Probs. 3–56/57

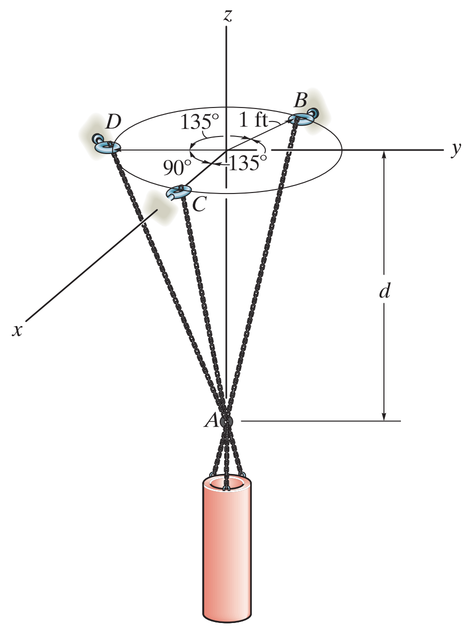
3–58. Determine the tension developed in each cable for equilibrium of the 300-lb crate.

3–59. Determine the maximum weight of the crate that can be suspended from cables AB, AC, and AD so that the tension developed in any one of the cables does not exceed 250 lb.



Probs. 3–58/59

*3–60. The 800-lb cylinder is supported by three chains as shown. Determine the force in each chain for equilibrium. Take $d = 1$ ft.

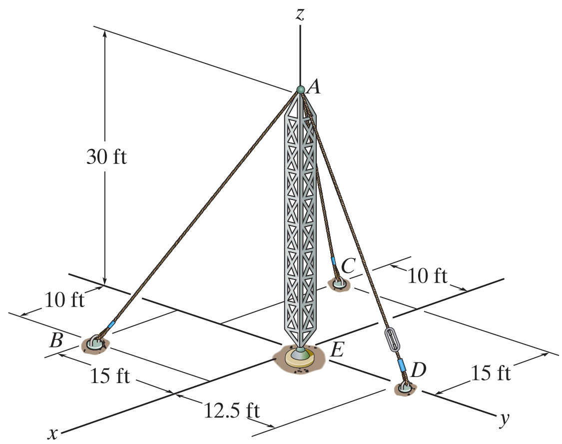


Prob. 3–60

3

3–61. If cable AD is tightened by a turnbuckle and develops a tension of 1300 lb, determine the tension developed in cables AB and AC and the force developed along the antenna tower AE at point A.

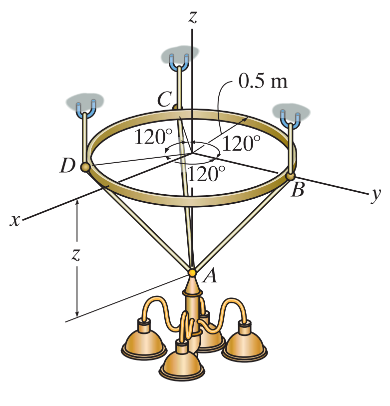
3–62. If the tension developed in either cable AB or AC can not exceeded 1000 lb, determine the maximum tension that can be developed in cable AD when it is tightened by the turnbuckle. Also, what is the force developed along the antenna tower at point A?



Probs. 3–61/62

3–63. The thin ring can be adjusted vertically between three equally long cables from which the 100-kg chandelier is suspended. If the ring remains in the horizontal plane and $z = 600$ mm, determine the tension in each cable.

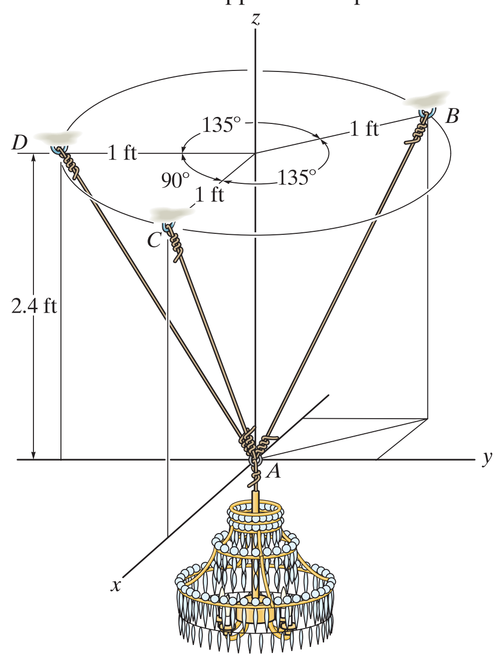
***3–64.** The thin ring can be adjusted vertically between three equally long cables from which the 100-kg chandelier is suspended. If the ring remains in the horizontal plane and the tension in each cable is not allowed to exceed 1 kN, determine the smallest allowable distance z required for equilibrium.



Probs. 3–63/64

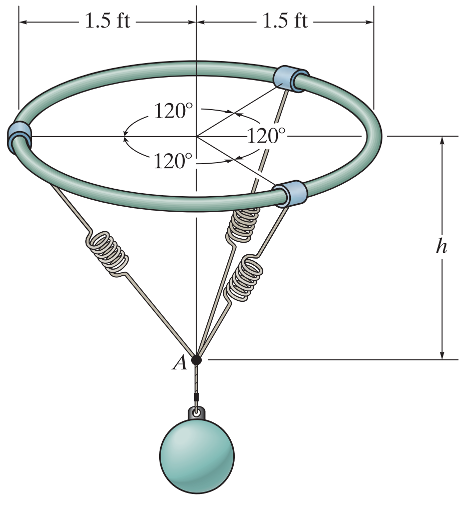
3–65. The 80-lb chandelier is supported by three wires as shown. Determine the force in each wire for equilibrium.

3–66. If each wire can sustain a maximum tension of 120 lb before it fails, determine the greatest weight of the chandelier the wires will support in the position shown.



Probs. 3–65/66

■**3–67.** The 80-lb ball is suspended from the horizontal ring using three springs each having an unstretched length of 1.5 ft and stiffness of 50 lb/ft. Determine the vertical distance h from the ring to point A for equilibrium.



Prob. 3–67

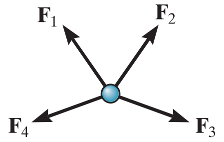
CHAPTER REVIEW

Particle Equilibrium

When a particle is at rest or moves with constant velocity, it is in equilibrium. This requires that all the forces acting on the particle form a zero resultant force.

In order to account for all the forces that act on a particle, it is necessary to draw its free-body diagram. This diagram is an outlined shape of the particle that shows all the forces listed with their known or unknown magnitudes and directions.

$$\mathbf{F}_R = \Sigma\mathbf{F} = \mathbf{0}$$



Two Dimensions

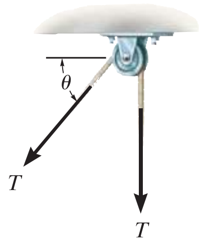
The two scalar equations of force equilibrium can be applied with reference to an established *x, y* coordinate system.

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

The tensile force developed in a *continuous cable* that passes over a frictionless pulley must have a *constant* magnitude throughout the cable to keep the cable in equilibrium.

If the problem involves a linearly elastic spring, then the stretch or compression *s* of the spring can be related to the force applied to it.

$$F = ks$$



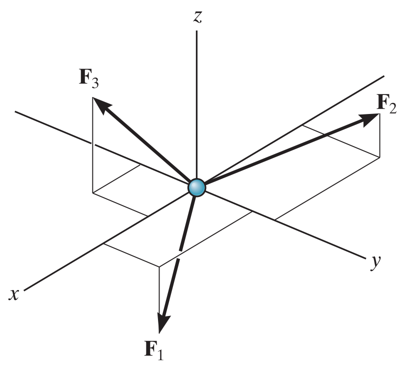
Cable is in tension

Three Dimensions

If the three-dimensional geometry is difficult to visualize, then the equilibrium equation should be applied using a Cartesian vector analysis. This requires first expressing each force on the free-body diagram as a Cartesian vector. When the forces are summed and set equal to zero, then the **i**, **j**, and **k** components are also zero.

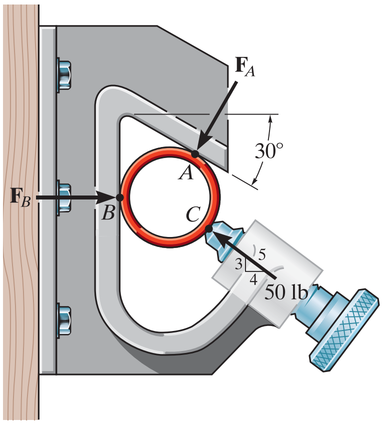
$$\Sigma\mathbf{F} = \mathbf{0}$$

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$



REVIEW PROBLEMS

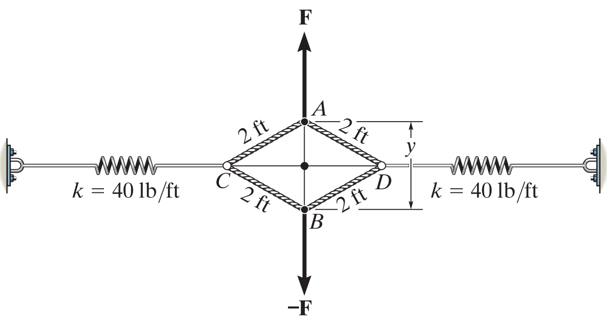
***3–68.** The pipe is held in place by the vise. If the bolt exerts a force of 50 lb on the pipe in the direction shown, determine the forces F_A and F_B that the smooth contacts at A and B exert on the pipe.



Prob. 3–68

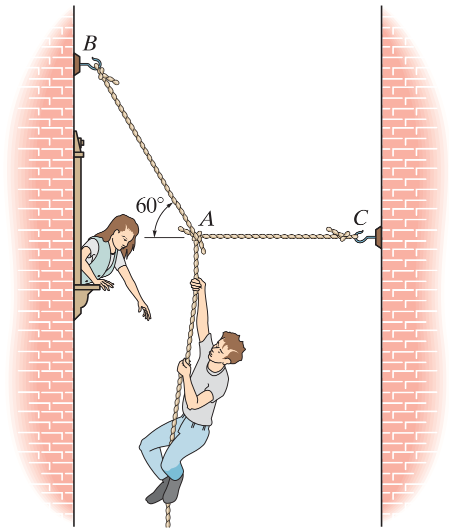
3–69. When y is zero, the springs sustain a force of 60 lb. Determine the magnitude of the applied vertical forces $\mathbf{F}$ and $-\mathbf{F}$ required to pull point A away from point B a distance of $y = 2$ ft. The ends of cords CAD and CBD are attached to rings at C and D.

3–70. When y is zero, the springs are each stretched 1.5 ft. Determine the distance y if a force of $F = 60$ lb is applied to points A and B as shown. The ends of cords CAD and CBD are attached to rings at C and D.



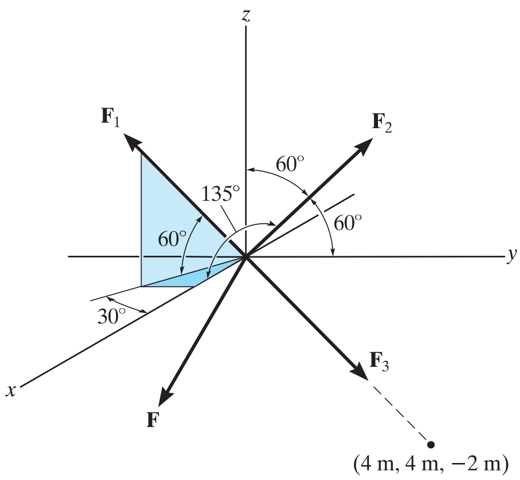
Probs. 3–69/70

3–71. Romeo tries to reach Juliet by climbing with constant velocity up a rope which is knotted at point A. Any of the three segments of the rope can sustain a maximum force of 2 kN before it breaks. Determine if Romeo, who has a mass of 65 kg, can climb the rope, and if so, can he along with Juliet, who has a mass of 60 kg, climb down with constant velocity?



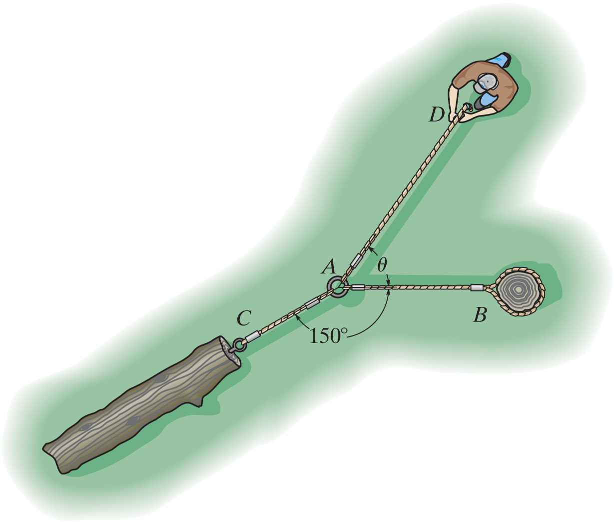
Prob. 3–71

***■3–72.** Determine the magnitudes of forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ necessary to hold the force $\mathbf{F} = \{-9\mathbf{i} - 8\mathbf{j} - 5\mathbf{k}\}$ kN in equilibrium.



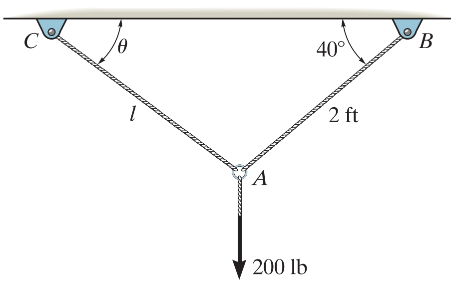
Prob. 3–72

3–73. The man attempts to pull the log at C by using the three ropes. Determine the direction θ in which he should pull on his rope with a force of 80 lb, so that he exerts a maximum force on the log. What is the force on the log for this case? Also, determine the direction in which he should pull in order to maximize the force in the rope attached to B. What is this maximum force?



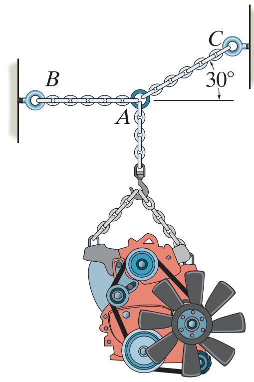
Prob. 3–73

■3–74. The ring of negligible size is subjected to a vertical force of 200 lb. Determine the required length l of cord AC such that the tension acting in AC is 160 lb. Also, what is the force acting in cord AB? *Hint:* Use the equilibrium condition to determine the required angle θ for attachment, then determine l using trigonometry applied to ΔABC.



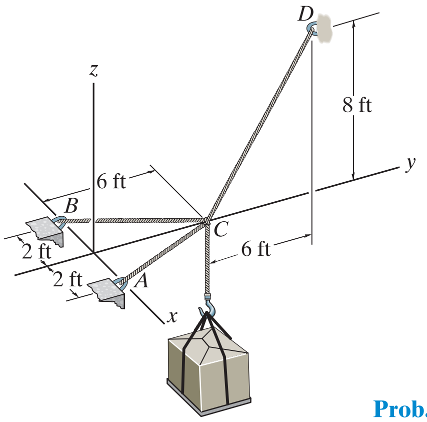
Prob. 3–74

3–75. Determine the maximum weight of the engine that can be supported without exceeding a tension of 450 lb in chain AB and 480 lb in chain AC.



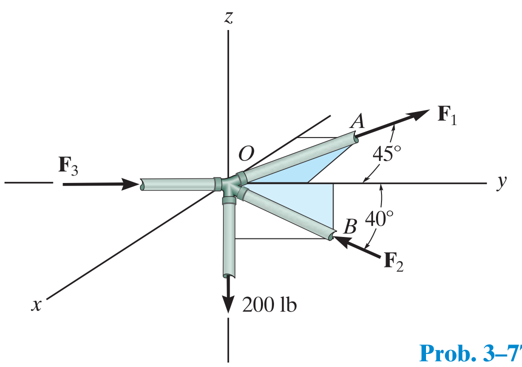
Prob. 3–75

***3–76.** Determine the force in each cable needed to support the 500-lb load.



Prob. 3–76

3–77. The joint of a space frame is subjected to four member forces. Member OA lies in the $x-y$ plane and member OB lies in the $y-z$ plane. Determine the forces acting in each of the members required for equilibrium of the joint.



Prob. 3–77