Noise in Analog communications

Introduction:

- Noise can broadly be defined as any unknown signal that affects the recovery of the desired signal.
- There may be many sources of noise in a communication system, but often the major sources are the communication devices themselves or interference encountered during the course of transmission.
- one of the most common ways is as an additive distortion

$$r(t) = s(t) + w(t)$$

Noise in communication systems:

- noise as a random process
- noise process is unpredictable, we use statistical parameters such as:
- The mean of the random process. noise have zero mean.
- The autocorrelation of the random process

$$R_w(\tau) = \frac{N_0}{2}\delta(\tau)$$

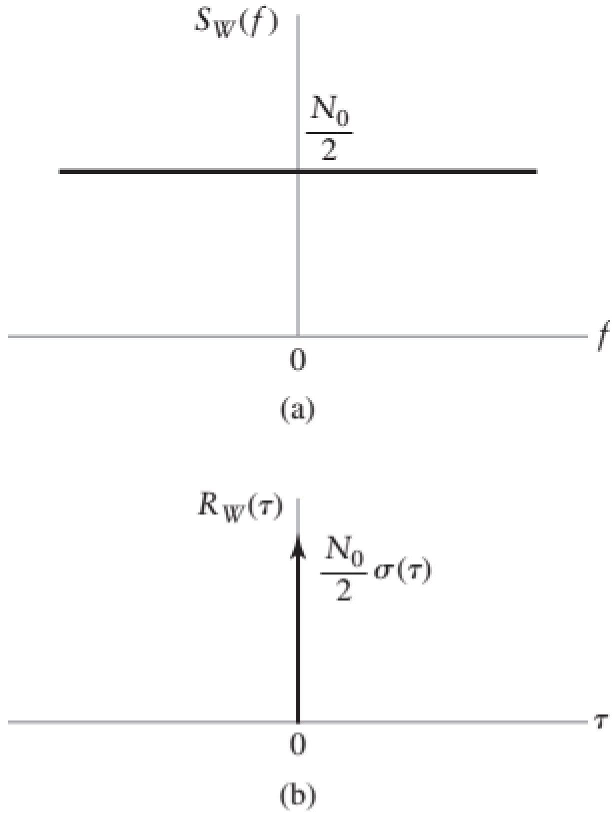
- The spectrum of the random process.

$$S_w(f) = \frac{N_0}{2}$$

White Gaussian noise:

- The noise analysis of communication systems is often based on an idealized noise process called white noise. The power spectral density of white noise is independent of frequency.
- Spectral density of white noise :

$$S_W(f) = \frac{N_0}{2}$$



- the autocorrelation of white noise is given by

$$R_W(\tau) = \frac{N_0}{2}\delta(\tau)$$

EXAMPLE 8.13 Ideal Low-Pass Filtered White Noise

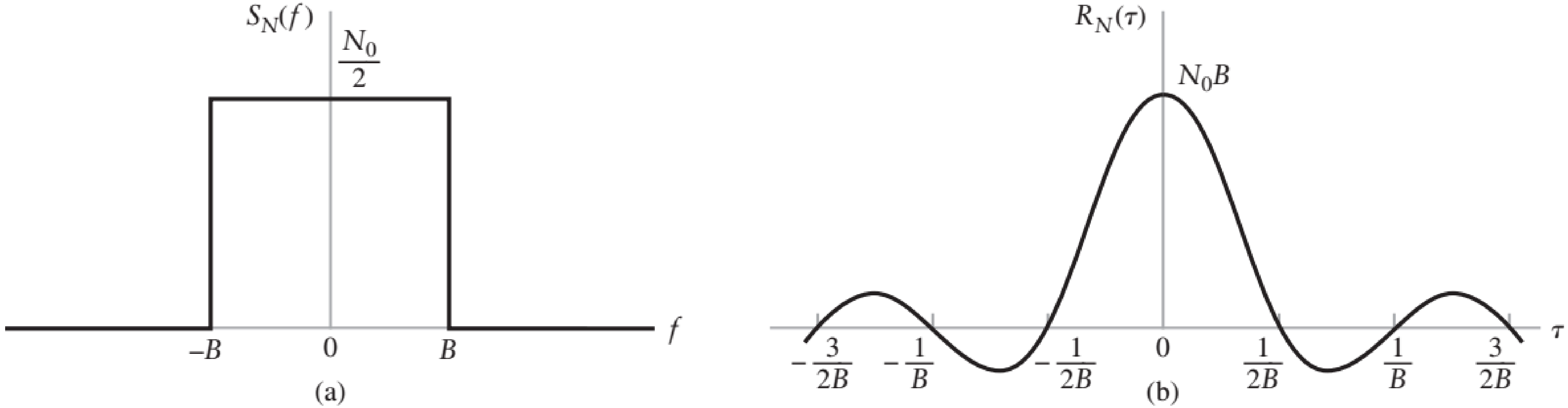
Suppose that a white noise process $W(t)$ of zero mean and power spectral density $N_0/2$ is applied to an ideal low-pass filter of bandwidth B with a unity gain passband amplitude response. The power spectral density of the noise process $N(t)$ appearing at the filter output is therefore (see Fig. 8.19(a))

$$S_N(f) = \begin{cases} \dfrac{N_0}{2}, & |f| < B \\ 0, & |f| > B \end{cases} \tag{8.94}$$

The autocorrelation function of $N(t)$ is the inverse Fourier transform of the power spectral density shown in Fig. 8.19(a).

$$\begin{aligned} R_N(\tau) &= \int_{-B}^{B} \frac{N_0}{2} \exp(j2\pi f_c \tau)\, df \\ &= N_0 B \operatorname{sinc}(2B\tau) \end{aligned} \tag{8.95}$$

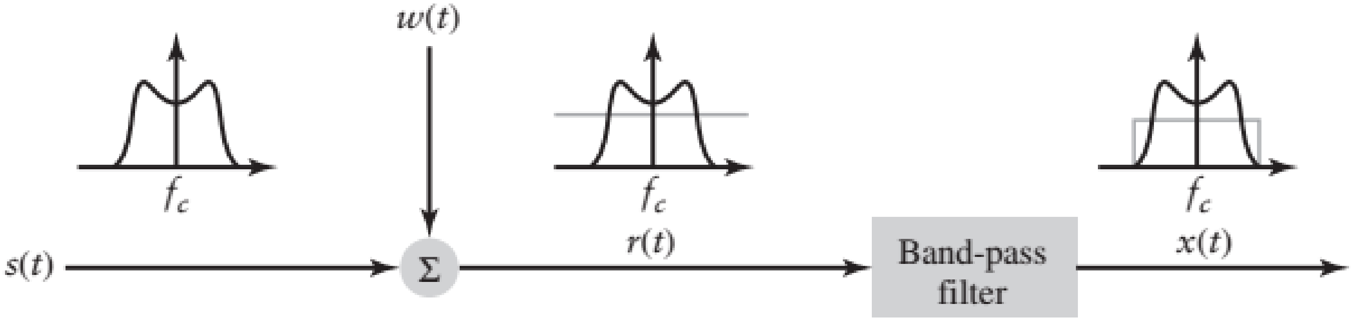
This autocorrelation function is plotted in Fig. 8.19(b). We see that $R_N(\tau)$ has its maximum value of N_0B at the origin, is symmetric in τ, and passes through zero at $\tau = \pm n/2B$ where $n = 1, 2, 3, \ldots$.


$S_N(f)$
$\frac{N_0}{2}$
f
$-B$
0
B
$R_N(\tau)$
N_0B
τ
$-\frac{3}{2B}$
$-\frac{1}{B}$
$-\frac{1}{2B}$
0
$\frac{1}{2B}$
$\frac{1}{B}$
$\frac{3}{2B}$
(a) (b)

How noise affects the signal :

- noise over a specified bandwidth:

$$N = N_0 B_T$$



$$x(t) = s(t) + n(t)$$

Signal-To-Noise Ratios

$$\text{SNR} = \frac{\mathrm{E}[s^2(t)]}{\mathrm{E}[n^2(t)]}$$

EXAMPLE 9.1 Sinusoidal Signal-to-Noise Ratio

Consider the case where the transmitted signal in Eq. (9.5) is

$$s(t) = A_c \cos(2\pi f_c t + \theta)$$

where the phase θ is unknown at the receiver. The signal is received in the presence of additive noise as shown in Fig. 9.2. The noise is white and Gaussian with power spectral density $N_0/2$.

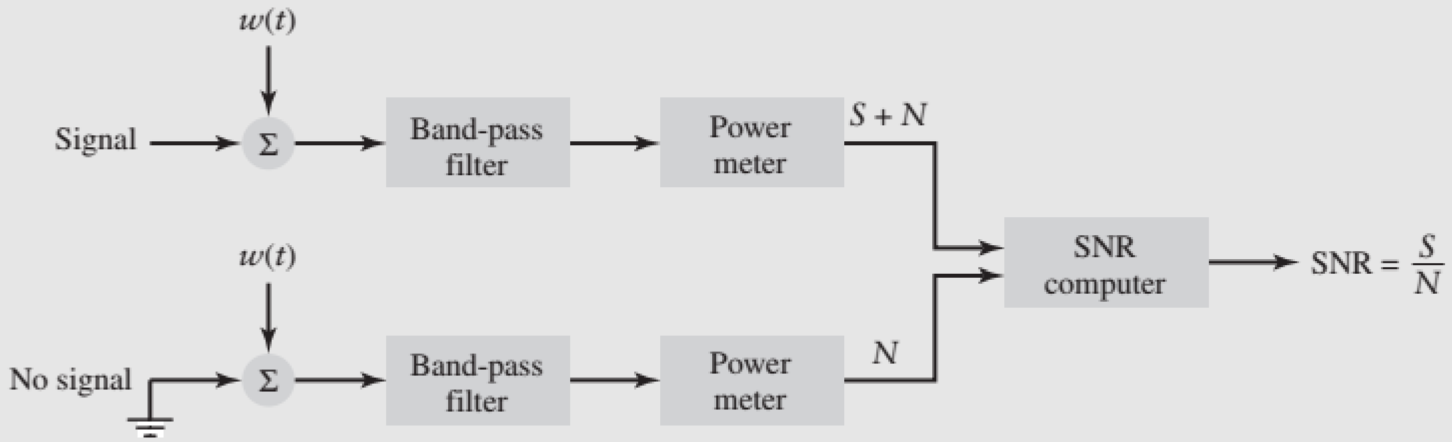


FIGURE 9.2 SNR measurement scheme for Example 9.1.

In this case, although the signal is random, it is also periodic. Consequently, we can estimate its average power by integrating over one cycle (i.e., equating an ensemble average with a time average).

$$\begin{aligned} \mathrm{E}[s^2(t)] &= \frac{1}{T}\int_0^T (A_c\cos(2\pi f_c t + \theta))^2\, dt \\ &= \frac{A_c^2}{2T}\int_0^T (1 + \cos(4\pi f_c t + 2\theta))\, dt \\ &= \frac{A_c^2}{2T}\left[t + \frac{\sin(4\pi f_c t + 2\theta)}{4\pi f_c}\right]_0^T \\ &= \frac{A_c^2}{2} \end{aligned} \tag{9.7}$$

Theoretically, the white noise extends to infinite frequency. The narrowband noise process, $n(t)$, is the result of passing the white noise process through a band-pass filter with noise-equivalent bandwidth B_T. Under this assumption, we compute the noise power

$$\begin{aligned}\mathbf{E}[n^2(t)] &= N \\ &= N_0 B_T\end{aligned} \tag{9.8}$$

Substituting the results of Eqs. (9.7) and (9.8) into (9.6), the signal-to-noise ratio becomes

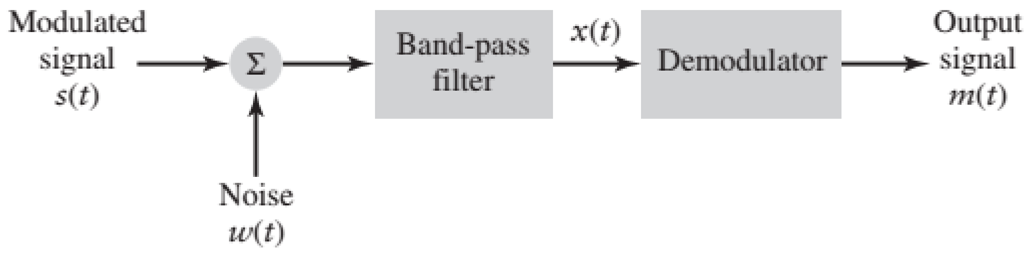
$$\mathrm{SNR} = \frac{A_c^2}{2N_0 B_T} \tag{9.9}$$

Since the bandwidth of the signal is arbitrarily narrow in this example, the choice of the bandwidth B_T is somewhat arbitrary. Consequently, in practice, the related *carrier-to-noise density ratio* is defined by

$$\frac{C}{N_0} = \frac{A_c^2}{2N_0} \tag{9.10}$$

which is not dimensionless like the SNR, but it is independent of the choice of bandwidth. The C/N_0 ratio has units of hertz where N_0 is measured in watts per hertz and the carrier power $C = A_c^2/2$ is measured in watts.

Signal to noise ratio is measured at the reciver.



We have two points of measurement

- If the signal-to-noise ratio is measured at the front-end of the receiver, then it is usually a measure of the quality of the transmission link and the receiver front-end.
- If the signal-to-noise ratio is measured at the output of the receiver, it is a measure of the quality of the recovered information-bearing signal whether it be audio, video, or otherwise.

So we have two measurments :

- Pre detection SNR

- Post detection SNR

Calculation of post detection SNR:

- reference transmission model :This reference model is equivalent to transmitting the message at baseband. In this model, two assumptions are made:
- 1. The message power is the same as the modulated signal power of the modulation scheme under study.
  2. The baseband low-pass filter passes the message signal and rejects out-of-band noise. Accordingly, we may define the *reference signal-to-noise ratio*, SNR_{ref}, as

$$\text{SNR}_{\text{ref}} = \frac{\text{average power of the modulated message signal}}{\text{average power of noise measured in the message bandwidth}} \tag{9.11}$$

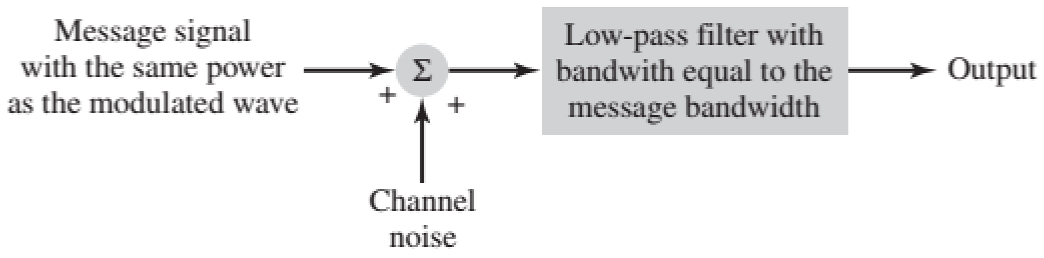


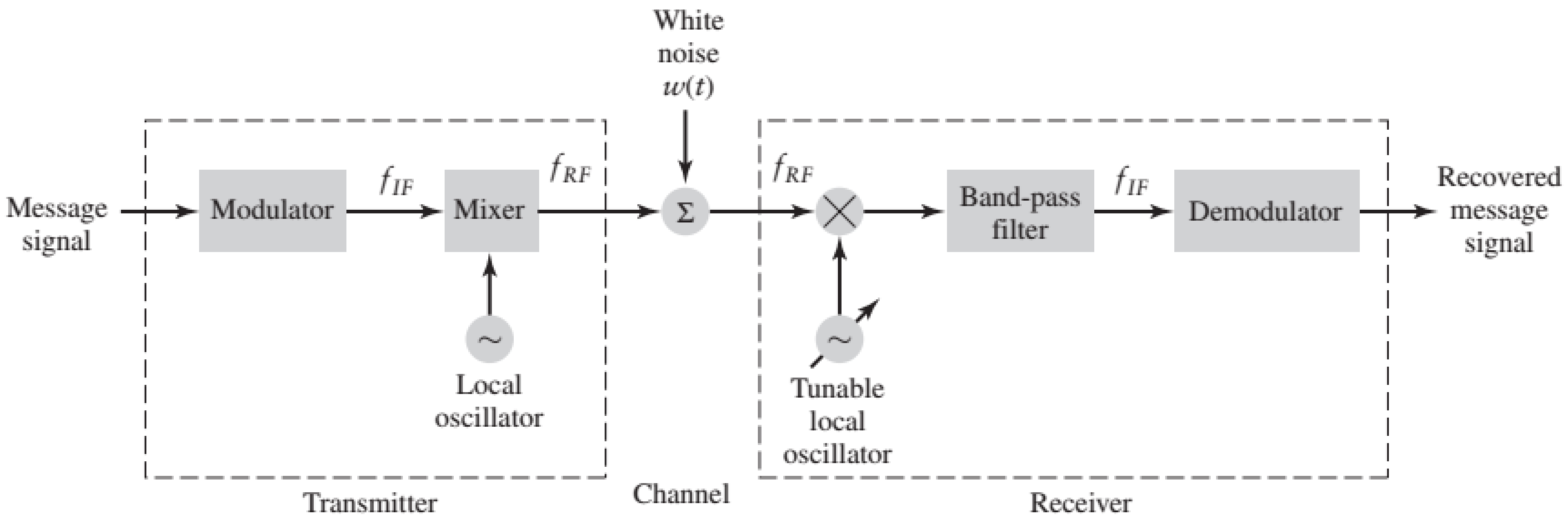
Figure of merit:

the higher the value that the figure of merit has, the better the noise performance of the receiver will be.

$$\text{Figure of merit} = \frac{\text{post}-\text{detection SNR}}{\text{reference SNR}}$$

- The pre-detection SNR is measured before the signal is demodulated.
- The post-detection SNR is measured after the signal is demodulated.
- The reference SNR is defined on the basis of a baseband transmission model.
- The figure of merit is a dimensionless metric for comparing different analog modulation–demodulation schemes and is defined as the ratio of the post-detection and reference SNRs.

Band-Pass Receiver Structures

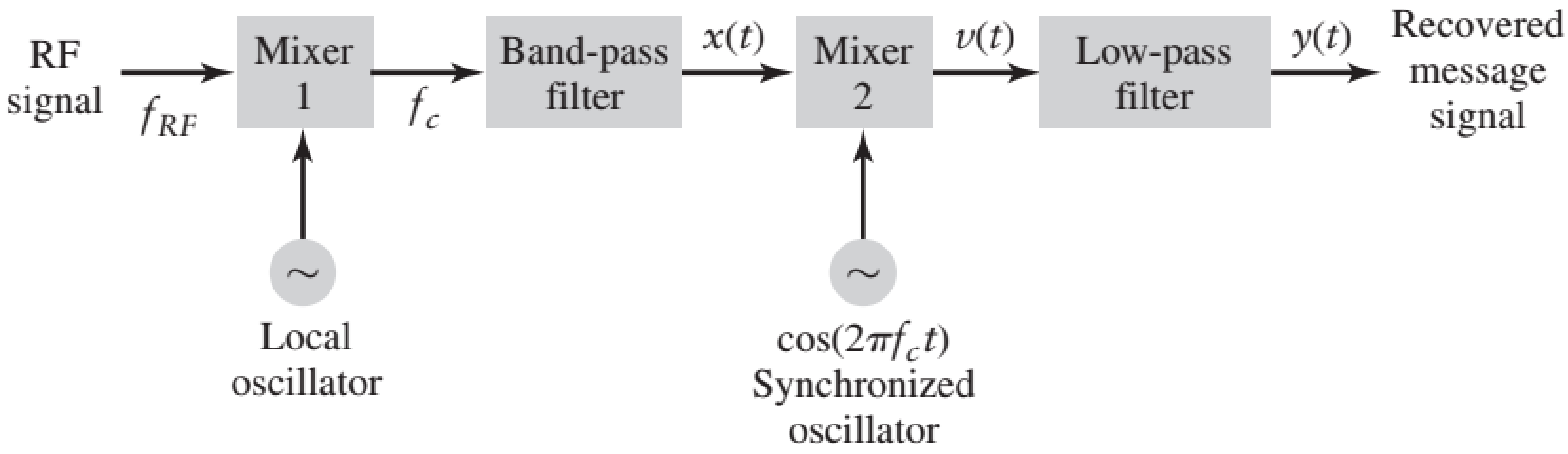


$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

Noise in Linear Receivers Using Coherent Detection

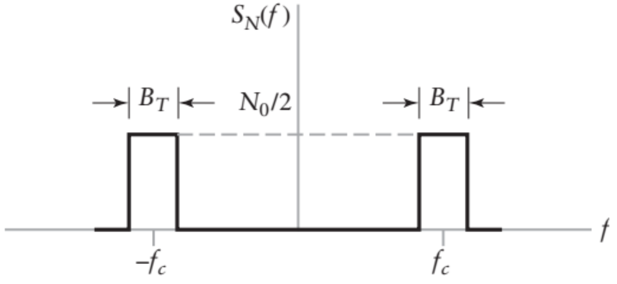
In the case of double sideband suppressed-carrier (DSB-SC) modulation, the modulated signal is represented as

$$s(t) = A_c m(t) \cos(2\pi f_c t + \theta)$$

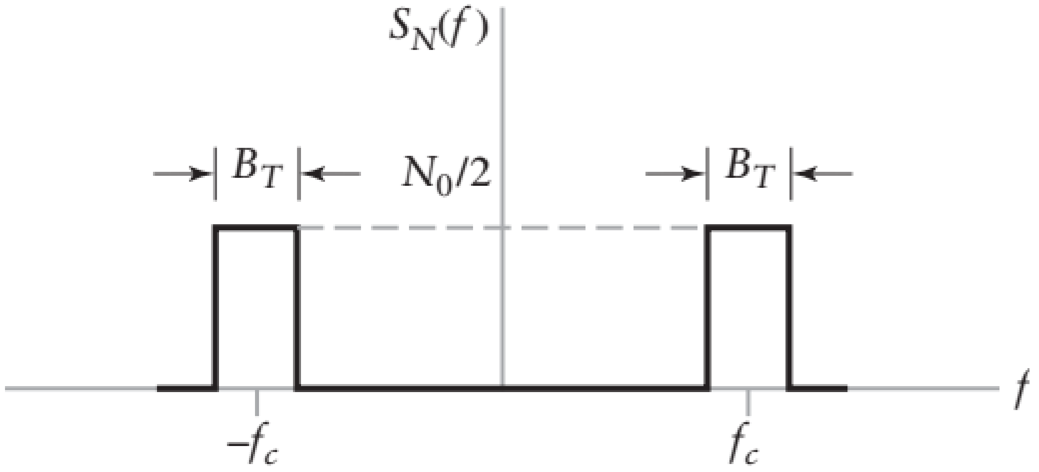


- the received RF signal is the sum of the modulated signal and white Gaussian noise .
- The received signal is down-converted to an IF by multiplication with a sinusoid of frequency $f_c - f_{RF}$
- This down-conversion is performed by the product modulator shown as mixer 1.
- After band-pass filtering, the resulting signal is

$$x(t) = s(t) + n(t)$$



Power spectral density of the noise



PRE-DETECTION SNR

$$s(t) = A_c m(t) \cos(2\pi f_c t + \theta)$$

$$\mathbf{E}[s^2(t)] = \mathbf{E}[(A_c \cos(2\pi f_c t + \theta))^2]\mathbf{E}[m^2(t)]$$

$$P = \mathbf{E}[m^2(t)]$$

the average received signal power due to the modulated component is

$$\mathbf{E}[s^2(t)] = \frac{A_c^2 P}{2}$$

- then the noise power passed by the filter is $N_0 B_T$
- the signal-to-noise ratio of the signal is

$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{DSB}} = \frac{A_c^2 P}{2N_0 B_T}$$

POST-DETECTION SNR

- The post-detection signal-to-noise ratio is the ratio of the message signal power to the noise power after demodulation/detection.
- The post-detection SNR depends on both the modulation and demodulation techniques
- The signal at the input of the detector is

$$x(t) = s(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

The output of the detector is:

$$
\begin{aligned}
v(t) &= x(t)\cos(2\pi f_c t) \\
&= \tfrac{1}{2}(A_c m(t) + n_I(t)) \\
&\quad + \tfrac{1}{2}(A_c m(t) + n_I(t))\cos(4\pi f_c t) - \tfrac{1}{2} n_Q(t)\sin(4\pi f_c t)
\end{aligned}
$$

high-frequency components are removed with a low-pass filter and the result is:

$$
y(t) = \tfrac{1}{2}(A_c m(t) + n_I(t))
$$

Two observations can be made:

- The message signal and the in-phase component of the filtered noise appear additively in the output.
- The quadrature component of the noise is completely rejected by the demodulator

we may compute the output or post-detection signal to noise ratio by noting the following

- The message component is $\frac{1}{2}A_c m(t)$, so analogous to the computation of the pre-detection signal power, the post-detection signal power is $\frac{1}{4}A_c^2 P$ where P is the average message power as defined in Eq. (9.16).
- The noise component is $\frac{1}{2}n_I(t)$ after low-pass filtering. As described in Section 8.11, the in-phase component has a noise spectral density of N_0 over the bandwidth from $-B_T/2$ to $B_T/2$. If the low-pass filter has a noise bandwidth W, corresponding to the message bandwidth, which is less than or equal to $B_T/2$, then the output noise power is

$$\begin{aligned} \mathbf{E}[n_I^2(t)] &= \int_{-W}^{W} N_0 \, df \\ &= 2N_0 W \end{aligned} \tag{9.22}$$

Thus the power in $n_I(t)$ is $2N_0W$.

Combining these observations, we obtain the post-detection SNR of

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{DSB}} = \frac{\frac{1}{4}(A_c^2)P}{\frac{1}{4}(2N_0 W)}$$

$$= \frac{A_c^2 P}{2N_0 W}$$

if $W \approx B_T/2$, the post-detection SNR is twice the pre-detection SNR.

This is due to the fact that the quadrature component of the noise has been discarded by the synchronous demodulator.

FIGURE OF MERIT

- the average noise power for a message of bandwidth W is $N_0 W$.
- For DSB-SC modulation the average modulated message power is given by

$$\mathbf{E}[s^2(t)] = \frac{A_c^2 P}{2}$$

- the reference SNR for this transmission scheme is $\mathrm{SNR}_{\mathrm{ref}} = A_c^2 P/(2N_0 W)$

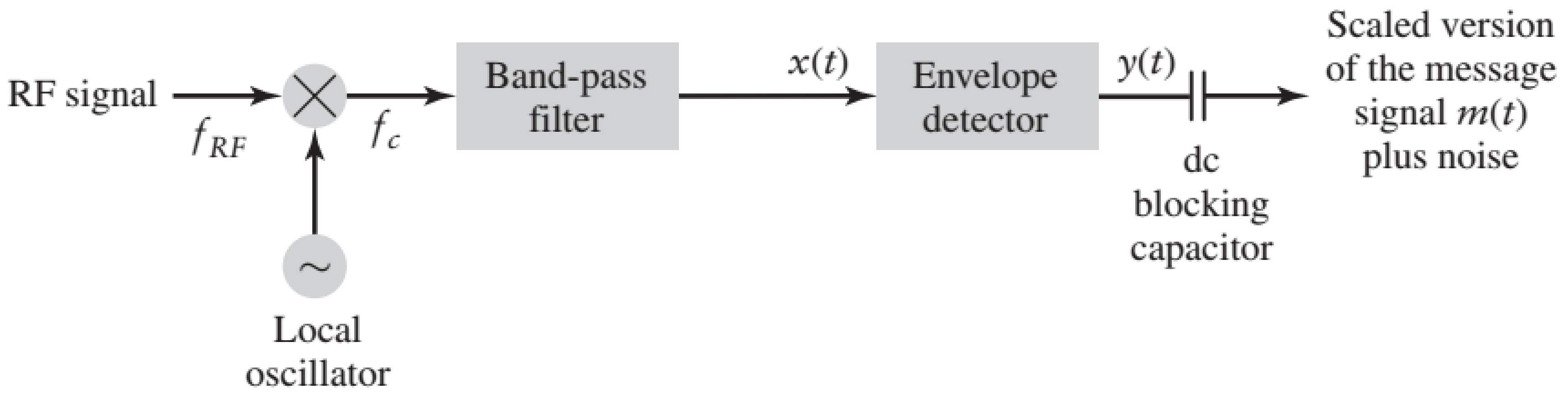
$$\text{Figure of merit} = \frac{\mathrm{SNR}_{\mathrm{post}}^{\mathrm{DSB}}}{\mathrm{SNR}_{\mathrm{ref}}} = 1$$

- we lose nothing in performance by using a band-pass modulation scheme compared to the baseband modulation scheme, even though the bandwidth of the former is twice as wide.
- Consequently, DSB-SC modulation provides a baseline against which we may compare other amplitude modulation detection schemes

Noise In AM Receivers Using Envelope Detection

- that the envelope-modulated signal is represented by

$$s(t) = A_c(1 + k_a m(t)) \cos(2\pi f_c t)$$



PRE-DETECTION SNR

- the average power of the carrier component is $A_c^2/2$

- The power in the modulated part of the signal is

$$
\begin{aligned}
\mathbf{E}[(1 + k_a m(t))^2] &= \mathbf{E}[1 + 2k_a m(t) + k_a^2 m^2(t)] \\
&= 1 + 2k_a \mathbf{E}[m(t)] + k_a^2 \mathbf{E}[m^2(t)] \\
&= 1 + k_a^2 P
\end{aligned}
$$

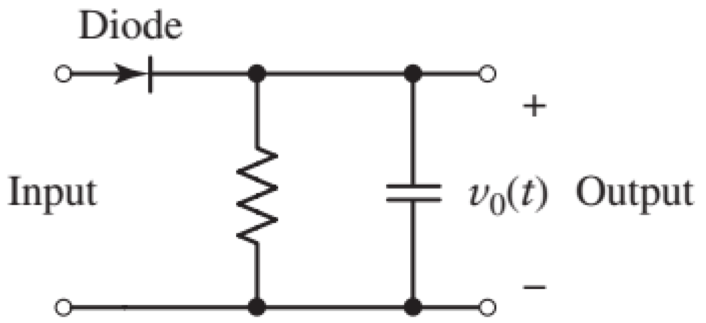
- The total received power is $A_c^2(1 + k_a^2 P)/2$

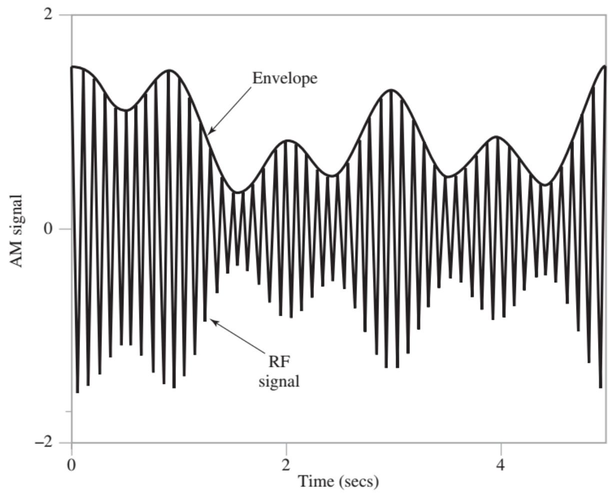
$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{AM}} = \frac{A_c^2(1 + k_a^2 P)}{2N_0 B_T}$$

POST-DETECTION SNR

- represent the noise in terms of its in-phase and quadrature components, and consequently model the input to the envelope detector as

$$
\begin{aligned}
x(t) &= s(t) + n(t) \\
&= [A_c + A_c k_a m(t) + n_I(t)] \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t)
\end{aligned}
$$





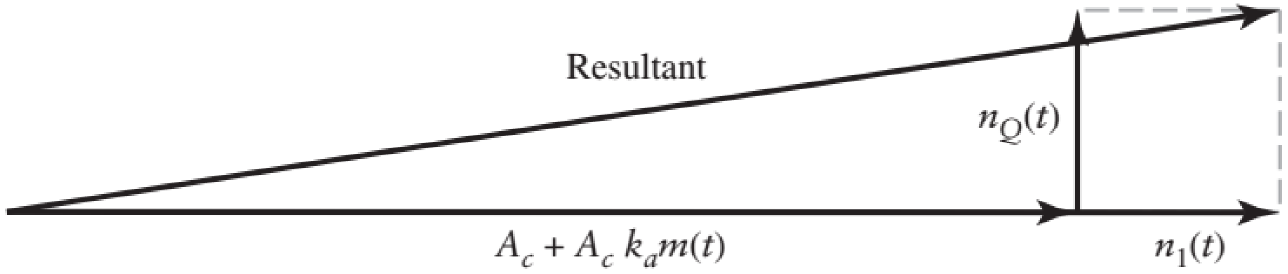
- the output of the envelope detector is the amplitude of the phasor x(t) representing and it is given by

$$\begin{aligned} y(t) &= \text{envelope of } x(t) \\ &= \{[A_c(1 + k_a m(t)) + n_I(t)]^2 + n_Q^2(t)\}^{1/2} \end{aligned}$$

- When the signal is larger than the noise :

$$y(t) \approx A_c + A_c k_a m(t) + n_I(t)$$

- under high SNR conditions. This new expression for the demodulated signal has three components:
- dc component,
- signal component,
- and noise component.



- the post-detection SNR for the envelope detection of AM, using a message bandwidth W, is given by

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{AM}} = \frac{A_c^2 k_a^2 P}{2N_0 W}$$

This evaluation of the output SNR is only valid under two conditions:

- The SNR is high.
- k_a is adjusted for 100% modulation or less, so there is no distortion of the signal envelope.

FIGURE OF MERIT

- the reference SNR

$$A_c^2(1 + k_a^2 P)/(2N_0 W).$$

$$\text{Figure of merit} = \frac{\text{SNR}_{\text{post}}^{\text{AM}}}{\text{SNR}_{\text{ref}}} = \frac{k_a^2 P}{1 + k_a^2 P}$$

- Always less than one. in general for envelop detection its less than 0.5
- Hence, the noise performance of an envelope-detector receiver is always inferior to a DSB-SC receiver, the reason is that at least half of the power is wasted transmitting the carrier as a component of the modulated (transmitted) signal

Noise in SSB

$$s(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) + \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t)$$

PRE-DETECTION SNR

$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{SSB}} = \frac{A_c^2 P}{4 N_0 W}$$

POST-DETECTION SNR

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{SSB}} = \frac{A_c^2 P}{4 N_o W}$$

FIGURE OF MERIT

$$\text{Figure of merit} = \frac{\mathrm{SNR}_{\mathrm{post}}^{\mathrm{SSB}}}{\mathrm{SNR}_{\mathrm{ref}}} = 1$$

Detection of Frequency Modulation (FM)

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$$

PRE-DETECTION SNR

$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{FM}} = \frac{A_c^2}{2N_0 B_T}$$

Post detection SNR:

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{FM}} = \frac{3A_c^2 k_f^2 P}{2N_0 W^3}$$

FIGURE OF MERIT

$$\begin{aligned}\text{Figure of merit} &= \frac{\mathrm{SNR}_{\mathrm{post}}^{\mathrm{FM}}}{\mathrm{SNR}_{\mathrm{ref}}} = \frac{\dfrac{3A_c^2 k_f^2 P}{2N_0 W^3}}{\dfrac{A_c^2}{2N_0 W}} \\ &= 3\left(\frac{k_f^2 P}{W^2}\right) \\ &= 3D^2\end{aligned}$$

$$\text{Figure of merit} \approx \frac{3}{4}\left(\frac{B_T}{W}\right)^2$$

THRESHOLD EFFECT

- The formula the post-detection SNR ratio of an FM receiver, is valid only if the pre-detection SNR, measured at the discriminator input, is high compared to unity.
- If the pre-detection SNR is lowered, the FM receiver breaks down. At first, individual clicks are heard in the receiver output, and as the pre-detection SNR decreases further, the clicks merge to a crackling or sputtering sound.
- At and below this breakdown point, the equation fails to accurately predict the post-detection SNR.
- This phenomenon is known as the threshold effect;