Equivalent Systems

انظمة متكافئة :-

Two systems of equations involving the same variables, and they have the same solution set.

نفس مجموعة الحل :

ex

the following are two equivalent systems :-

(A)

$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= -2 \\ x_2 &= 3 \\ 2x_3 &= 4 \end{aligned}$$

(B)

$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= -2 \\ -3x_1 - x_2 + x_3 &= 5 \\ 3x_1 + 2x_2 + x_3 &= 2 \end{aligned}$$

حله سهل .

use backsubstitution

systems

$n \times n$

overdetermined system

underdetermined system

$\boxed{\text{I}}$ $n \times n$ system

number of equations = number of unknowns = n.

strict triangular form مثلثي

If the system has in the k^{th} equation the coefficients of the first $(k-1)$ variables are all zeros and the coefficient of x_k is non zero $(k = 1, 2, \ldots, n)$.

ex on strict triangular form.

$$\begin{aligned} 3x_1 + 2x_2 + x_3 &= 1 \\ x_2 - x_3 &= 2 \\ 2x_3 &= 4 \end{aligned}$$

3×3 system.

$k = 2 \Rightarrow$ eq: 2 , x_1 معامله $= 0$

x_2, x_3, x_4, x_5 معاملاته $\neq 0$

to solve the previous system use Back substitution

$$\frac{2x_3}{2} = \frac{4}{2} \rightarrow \boxed{x_3 = 2}$$

$$x_2 - x_3 = 2$$

$$x_2 - 2 = 2 \rightarrow \boxed{x_2 = 4}$$

$$3x_1 + 2x_2 + x_3 = 1$$

$$3x_1 + 2(4) + (2) = 1$$

$$3x_1 + 10 = 1 \rightarrow 3x_1 = -9 \Rightarrow \boxed{x_1 = -3}$$

solution: $(-3, 4, 2)$.

H.W

$$2x_1 - x_2 + 3x_3 - 2x_4 = 1$$

$$x_2 - 2x_3 + 3x_4 = 2$$

$$4x_3 + 3x_4 = 3$$

$$4x_4 = 4$$

ex solve the system

$$
\begin{aligned}
x_1 + 2x_2 + x_3 &= 3\\
3x_1 - x_2 - 3x_3 &= -1\\
2x_1 + 3x_2 + x_3 &= 4
\end{aligned}
$$

sol:

write the augmented matrix

$$
\left[\begin{array}{ccc|c}
1 & 2 & 1 & 3\\
3 & -1 & -3 & -1\\
2 & 3 & 1 & 4
\end{array}\right] \rightarrow
$$

$$\cancel{3x_1 + 2x_2 = 5}$$

$$-3 \rightarrow \cancel{x_1 - 7x_2 = 3}$$

Elementary Row Operations :-

1. Interchange two rows.
2. Multiply a row by a non-zero real number.
3. Replace a row by its sum with a multiple of another row.

note:

Elementary row operations dont change the solution of the system

Pivot

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 3 & -1 & -3 & -1 \\ 2 & 3 & 1 & 4 \end{array}\right] \longrightarrow \text{pivotal row}$$

مطلوب تحويلهم لأصفار

$$\xrightarrow[-2R_1 + R_3]{-3R_1 + R_2}$$

Pivot جديد

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -7 & -6 & -10 \\ 0 & \boxed{-1} & -1 & -2 \end{array}\right] \xrightarrow{\frac{-1}{7} R_2 + R_3}$$

$$-7 * \frac{1}{7} = 1$$

مطلوب تحويله إلى صفر

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -7 & -6 & -10 \\ 0 & 0 & \frac{-1}{7} & \frac{-4}{7} \end{array}\right]$$

$$\frac{6}{7} + {-1} = \frac{6}{7} - \frac{7}{7} = \frac{-1}{7}$$

$$\frac{10}{7} + {-2} = \frac{10}{7} - \frac{14}{7} = \frac{-4}{7}$$

system :

$$x_1 + 2x_2 + x_3 = 3$$

$$-7x_2 - 6x_3 = -10$$

$$-\frac{1}{7} x_3 = \frac{-4}{7}$$

Now solve this system using <u>backsubstitution</u>.

$-\frac{1}{7}x_3 = \frac{-4}{7} \rightarrow -x_3 = -4 \rightarrow \boxed{x_3 = 4}$

$-7x_2 - 6x_3 = -10$

$-7x_2 - 6(4) = -10 \Rightarrow -7x_2 - 24 = -10$

$\frac{-7x_2}{-7} = \frac{14}{-7} \rightarrow \boxed{x_2 = -2}$

$x_1 + 2x_2 + x_3 = 3$

$x_1 + 2(-2) + (4) = 3 \longrightarrow x_1 - 4 + 4 = 3 \longrightarrow \boxed{x_1 = 3}$

Sol: $(3, -2, 4)$ unique solution.