$y_n(t_0) \neq 0$, and suppose that

$$
c_1 y_1(t) + \dots + c_n y_n(t) = 0
$$
 (ii)

Chapter 4. Higher Order Linear Equations

(a) Suppose that $W(y_1, ..., y_n)(t_0) \neq 0$, and suppose that
 $c_1y_1(t) + \cdots + c_ny_n(t) = 0$ (ii)

for all t in I. By writing the equations corresponding to the first $n-1$ derivatives of **Chapter 4. Higher Order Linear Equations**
 $\binom{n}{0} \neq 0$, and suppose that
 $c_1 y_1(t) + \cdots + c_n y_n(t) = 0$ (ii)

ations corresponding to the first $n - 1$ derivatives of Eq. (ii)

ations corresponding to the first $n - 1$ deriv *l. Higher Order Linear Equations*
se that
 t) = 0 (ii)
to the first $n - 1$ derivatives of Eq. (ii)
 y_n are linearly independent.
pendent solutions of Eq. (i). If
a nonzero solution of Eq. (i) satisfying **Chapter 4. Higher Order Linear Equations**

(a) Suppose that $W(y_1, ..., y_n)(t_0) \neq 0$, and suppose that
 $c_1y_1(t) + \cdots + c_ny_n(t) = 0$ (ii)

for all t in I. By writing the equations corresponding to the first $n - 1$ derivatives of at t_0 , show that $c_1 = \cdots = c_n = 0$. Therefore, y_1, \ldots, y_n are linearly independent. **Chapter 4. Higher Order Linear Equations**

Suppose that $W(y_1, ..., y_n)(t_0) \neq 0$, and suppose that
 $c_1y_1(t) + \cdots + c_ny_n(t) = 0$ (ii)

Ill *t* in *I*. By writing the equations corresponding to the first $n - 1$ derivatives of Eq. **Chapter 4. Higher Order Linear Equations**
 $(t_0) \neq 0$, and suppose that
 $y_1(t) + \cdots + c_n y_n(t) = 0$ (ii)

ions corresponding to the first $n - 1$ derivatives of Eq. (ii)

0. Therefore, y_1, \ldots, y_n are linearly independent.
 chapter Order Linear Equations

at
 0 (ii)
 $\frac{1}{2}$ first $n - 1$ derivatives of Eq. (ii)
 $\frac{1}{2}$ are linearly independent.
 $\frac{1}{2}$ care solution of Eq. (i). If
 $\frac{1}{2}$ are solution of Eq. (i) satisfying
 $\$ **Chapter 4. Higher Order Linear Equations**

(a) Suppose that $W(y_1, ..., y_n)(t_0) \neq 0$, and suppose that
 $c_1y_1(t) + \cdots + c_ny_n(t) = 0$ (ii)

for all t in I. By writing the equations corresponding to the first $n - 1$ derivatives of y_n are linearly independent solutions of **Chapter 4. Higher Order Linear Equations**
 $(t_0) \neq 0$, and suppose that
 $y_1(t) + \cdots + c_n y_n(t) = 0$ (ii)

(iii)

(iii) on scorresponding to the first $n-1$ derivatives of Eq. (ii)

on Therefore, y_1, \ldots, y_n are linearly ind $W(y_1, \ldots, y_n)(t_0) = 0$ for some t_0 , show that there is a nonzero solution of I **Chapter 4. Higher Order Linear Equations**
 $v_1, \ldots, v_n(t_0) \neq 0$, and suppose that
 $\varepsilon_1 v_1(t) + \cdots + \varepsilon_n v_n(t) = 0$ (ii)
 $v_1 v_2 = 0$. Therefore, y_1, \ldots, y_n are linearly independent.
 $\cdots = \varepsilon_n = 0$. Therefore, $y_1, \ldots,$ **Chapter 4. Higher Order Linear Equations**
 t_0 \neq 0, and suppose that
 $t_1(t) + \cdots + c_n y_n(t) = 0$ (ii)

ons corresponding to the first $n - 1$ derivatives of Eq. (ii)

Therefore, y_1, \ldots, y_n are linearly independent.

or **Chapter 4. Higher Order Linear Equations**

(a) Suppose that $W(y_1, ..., y_n)(t_0) \neq 0$, and suppose that
 $c_1y_1(t) + \cdots + c_ny_n(t) = 0$ (ii)

for all t in I. By writing the equations corresponding to the first $n - 1$ derivatives of **Chapter 4. Higher Order Linear Equations**

(a) Suppose that $W(y_1, ..., y_n)(t_0) \neq 0$, and suppose that
 $c_1y_1(t) + \cdots + c_ny_n(t) = 0$ (ii)

for all *t* in *I*. By writing the equations corresponding to the first $n - 1$ derivative **Chapter 4. Higher Order Linear Equations**

(a) Suppose that $W(y_1, ..., y_n)(t_0) \neq 0$, and suppose that
 $c_1y_1(t) + \cdots + c_ny_n(t) = 0$ (ii)

for all t in *I*. By writing the equations corresponding to the first $n-1$ derivatives o **Chapter 4. Higher Order Linear Equations**

(a) Suppose that $W(y_1, ..., y_n)(t_0) \neq 0$, and suppose that
 $c_1y_1(t) + \cdots + c_ny_n(t) = 0$ (ii)

for all t in I . By writing the equations corresponding to the first $n - 1$ derivative **Chapter 4. Higher Order Linear Equations**

at $W(y_1, ..., y_n)(t_0) \neq 0$, and suppose that
 $c_1y_1(t) + \cdots + c_ny_n(t) = 0$ (ii)

writing the equations corresponding to the first $n - 1$ derivatives of Eq. (ii)

writing the equations **Chapter 4. Higher Order Linear Equations**

(a) Suppose that $W(y_1, ..., y_n)(t_0) \neq 0$, and suppose that

for all *t* in *I*. By writing then equations corresponding to the first $n-1$ derivatives of Eq. (ii)

for all *t* in **Chapter 4. Higher Order Linear Equations**
 $W(y_1, ..., y_n)(t_0) \neq 0$, and suppose that
 $c_1y_1(t) + \cdots + c_ny_n(t) = 0$ (ii)

writing the equations corresponding to the first $n - 1$ derivatives of Eq. (ii)
 $\frac{1}{1} = \cdots = c_n = 0$. Ther **Chapter 4. Higher Order Linear Equations**

(a) Suppose that $W(y_1,..., y_n)(t_0) \neq 0$, and suppose that
 $c_1y_1(t) + \cdots + c_ny_n(t) = 0$ (ii)

to rall t in t . By writing the equations corresponding to the first $n - 1$ derivative (a) Suppose that $W(y_1, ..., y_n)(t_0) \neq 0$, and suppose that
 $c_1y_1(t) + \cdots + c_ny_n(t) = 0$ (ii)

for all t in 1. By writing the cquations corresponding to the first $n - 1$ derivatives of Eq. (ii)

at t_0 , show that $c_1 = \cdots = c_n =$ (a) suppose that $W(y_1, ..., y_n)(x_0) \neq 0$, and suppose that
 $c_1y_1(t) + \cdots + c_ny_n(t) = 0$ (ii)

for all r in *I*. By writing the equations corresponding to the first $n - 1$ derivatives of Eq. (ii)

at t_0 show that $c_1 = \cdots = c_n =$

$$
y(t_0) = y'(t_0) = \dots = y^{(n-1)}(t_0) = 0.
$$

$$
y''' + p_1(t)y'' + p_2(t)y' + p_3(t)y = 0,
$$

$$
y_1v''' + (3y_1' + p_1y_1)v'' + (3y_1'' + 2p_1y_1' + p_2y_1)v' = 0.
$$

 $(t) = e^t$ 28. $t^2(t+3)y''' - 3t(t+2)y'' + 6(1+t)y' - 6y = 0$, $t > 0$; $y_1(t) = t^2$, $y_2(t) = t^3$

$$
L[y] = a_0 y^{(n)} + a_1 y^{(n-1)} + \dots + a_{n-1} y' + a_n y = 0,
$$
 (1)

Since $y = 0$ is a solution of this initial value problem, the uniqueness part of Theorem

4.1.1 yields a contradiction. Thus *W* is never zero.

26. Show that if y_1 is a solution of
 $y'' + p_1(t)y'(t) + p_2(t)y' + p_3(t)y = 0$,

the where a_0, a_1, \ldots, a_n are real constants. From our knowled $y''' + p_1(t)y'' + p_2(t)y' + p_3(t)y = 0,$
 $n y = y_1(t)v(t)$ leads to the following second order equation for v' :
 $v'' + (3y'_1 + p_1y_1)v'' + (3y''_1 + 2p_1y'_1 + p_2y_1)v' = 0.$

and 28 use the method of reduction of order (Problem 26) to solve the

o then the substitution $y = y_1(t)v(t)$ leads to the following second order equation for v' :
 $y_1v''' + (3y'_1 + p_1y_1)v'' + (3y''_1 + 2p_1y'_1 + p_2y_1)v' = 0$.

In each of Problems 27 and 28 use the method of reduction of order (Problem 26 quation for v':

26) to solve the
 $\frac{1}{2}$, $y_2(t) = t^3$

(1)

(1)

ad order linear
 r^r is a solution

(2) then the substitution $y = y_1(t)y(t)$ leads to the following second order equation for v :
 $y_1v''' + (3y'_1 + p_1y_1)v'' + (3y''_1 + 2p_1y'_1 + p_2y_1)v' = 0$.

In each of Problems 27 and 28 use the method of reduction of order (Problem 26) In each of Problems 2/ and 28 use the method of reduction of order (Problem 26) to solve the

neiven differential equation.

27. $(2-t)y'' + (2t-3)y'' - ty' + y = 0$, $t < 2$; $y_1(t) = e^t$

28. $t^2(t+3)y''' - 3t(t+2)y'' + 6(1+t)y' - 6y = 0$, $t > 0$

$$
L[e^{rt}] = e^{rt}(a_0r^n + a_1r^{n-1} + \dots + a_{n-1}r + a_n) = e^{rt}Z(r)
$$
 (2)

$$
Z(r) = a_0 r^n + a_1 r^{n-1} + \dots + a_{n-1} r + a_n.
$$
 (3)

Equalitions with Constant Coefficients

Consider the *n*th order linear homogenous differential equation

Where a_0 , a_1 , ..., a_n are real constants. From our knowledge of second order linear

equations with const cients

on
 ${}_{n}y = 0$, (1)

e of second order linear

that $y = e^{rt}$ is a solution
 $= e^{rt} Z(r)$ (2)
 a_{n} . (3)
 ${}_{n}r^{rt}$] = 0 and $y = e^{rt}$ is a

eristic polynomial, and

differential equation (1). (1)

linear

lution

(2)

(3)
 r^r is a
 il, and
 il, and
 in (1). **Consider the** *n***th order linear homogeneous differential equation**
 $L[y] = a_0 y^{(n)} + a_1 y^{(n-1)} + \cdots + a_{n-1} y' + a_n y = 0,$ (1)

where a_0, a_1, \ldots, a_n are real constants. From our knowledge of second order linear

equations with **neous Equations with Constant Coefficients**

Consider the *n*th order linear homogeneous differential equation
 $L[y] = a_0y^{(n)} + a_1y^{(n-1)} + \cdots + a_{n-1}y' + a_n y = 0$. (1)

where a_0, a_1, \ldots, a_n are real constants. From our know

Equations with Constant Coefficients

A polynomial of degree *n* has *n* zeros,¹ say $r_1, r_2, ..., r_n$, some of which may be equal;

hence we can write the characteristic polynomial in the form
 $Z(r) = a_0(r - r_1)(r - r_2) \cdots (r$ say r_1, r_2, \ldots, r_n , some of which may be equal; 215

, some of which may be equal;

form
 $-r_n$). (4)

sistic equation are real and no
 e^{rt} of Eq. (1) If these

$$
Z(r) = a_0(r - r_1)(r - r_2) \cdots (r - r_n).
$$
 (4)

Equations with Constant Coefficients

A polynomial of degree *n* has *n* zeros,¹ say $r_1, r_2, ..., r_n$, some of which may be equal;

hence we can write the characteristic polynomial in the form
 $Z(r) = a_0(r - r_1)(r - r_2) \cdots (r -$ **Coefficients**
 215

Is *n* zeros,¹ say $r_1, r_2, ..., r_n$, some of which may be equal;

acteristic polynomial in the form
 $= a_0(r - r_1)(r - r_2) \cdots (r - r_n)$. (4)

If the roots of the characteristic equation are real and no
 n **Equations with Constant Coefficients**
 A polynomial of degree *n* has *n* zeros,¹ say $r_1, r_2, ..., r_n$, some of which may be equal;

hence we can write the characteristic polynomial in the form
 $Z(r) = a_0(r - r_1)(r - r_2) \cdots ($ $e^{r_1 t}$, $e^{r_2 t}$, ..., $e^{r_n t}$ of Eq. (1). If these 215

Number of Eq. (1). If these
 $\begin{pmatrix} 4 \end{pmatrix}$

ation are real and no
 $\begin{pmatrix} n^t & \text{of Eq. (1). If these} \\ \text{q. (1) is} \end{pmatrix}$

(5) *Equations with Constant Coefficients*
 A polynomial of degree *n* has *n* zeros,¹ say r_1 , r_2 , ..., r_n , some of which may be equal;

hence we can write the characteristic polynomial in the form
 $Z(r) = a_0(r - r_1)($ **Equations with Constant Coefficients**
 A polynomial of degree *n* has *n* zeros,¹ say $r_1, r_2, ..., r_n$, some of which may be equal;

hence we can write the characteristic polynomial in the form
 $Z(r) = a_0(r - r_1)(r - r_2) \cdots ($ 215
hich may be equal;
(4)
on are real and no
of Eq. (1). If these
(1) is
(5)
is to evaluate their *Equations with Constant Coefficients*

A polynomial of degree *n* has *n* zeros,¹ say r_1 , r_2 , ..., r_n , some of which may be equal;

hence we can write the characteristic polynomial in the form
 $Z(r) = a_0(r - r_1)(r -$ Equations with Constant Coefficients

215

A polynomial of degree *n* has *n* zeros,¹ say r_1 , r_2 , ..., r_n , some of which may be equal;

hence we can write the characteristic polynomial in the form
 $Z(r) = a_0(r - r_1$ *Equations with Constant Coefficients*

215

A polynomial of degree *n* has *n* zeros,¹ say $r_1, r_2, ..., r_n$, some of which may be equal;

hence we can write the characteristic polynomial in the form
 $Z(r) = a_0(r - r_1)(r - r_2) \$ with Constant Coefficients

al of degree *n* has *n* zeros,¹ say $r_1, r_2, ..., r_n$, some of which may be equal;

no mitte the characteristic polynomial in the form
 $Z(r) = a_0(r - r_1)(r - r_2) \cdots (r - r_n)$. (4)

negual Roots. If the *Equations with Constant Coefficients*

A polynomial of degree *n* has *n* zeros,¹ say r_1 , r_2 , ..., r_n , some of which may be equal;

hence we can write the characteristic polynomial in the form
 $Z(r) = a_0(r - r_1)(r$ polynomial of degree *n* has *n* zeros,¹ say $r_1, r_2, ..., r_n$, some of which may be equal;
nee we can write the characteristic polynomial in the form
 $Z(r) = a_0(r - r_1)(r - r_2) \cdots (r - r_n)$. (4)
al and Unequal Roots. If the roots o *r* has *n* zeros,¹ say $r_1, r_2, ..., r_n$, some of which may be equal;

characteristic polynomial in the form
 $Z(r) = a_0(r - r_1)(r - r_2) \cdots (r - r_n)$. (4)
 ts. If the roots of the characteristic equation are real and no

have *n* $Z(r) = a_0(r - r_1)(r - r_2) \cdots (r - r_n)$. (4)

Real and Unequal Roots. If the roots of the characteristic equation are real and no

two are equal, then we have *n* distinct solutions $e^{r_1}, e^{r_2}, \ldots, e^{r_n}$ of Eq. (1). If these

ti $r = r_n$). (4)

eristic equation are real and no
 $r_2t^2, ..., e^{r_n t}$ of Eq. (1). If these

ution of Eq. (1) is
 ${}_{n}e^{r_n t}$. (5)
 ${}_{n}e^{r_n t}$. (5)
 ${}_{n}e^{r_n t}$. (5)

bellem 40.
 $= 0$. (6)

s

2, $y'''(0) = -1$ (7)

ring the p (4)

are real and no

Eq. (1). If these

(5)

to evaluate their

(6)
 $= -1$ (7)

mial equation

(8)

3. Therefore the

(9)

7)

7) **Real and Unequal Roots.** If the roots of the characteristic equation are real and no

two are equal, then we have *n* distinct solutions $e^{r_1 t}, e^{r_2 t}, \ldots, e^{r_n t}$ of Eq. (1). If these

functions are linearly independent

$$
y = c_1 e^{r_1 t} + c_2 e^{r_2 t} + \dots + c_n e^{r_n t}.
$$
 (5)

 r_1 ^t, $e^{r_2 t}$, ..., $e^{r_n t}$ is to evaluate their

$$
y'''' + y''' - 7y'' - y' + 6y = 0.
$$
 (6)

$$
y(0) = 1,
$$
 $y'(0) = 0,$ $y''(0) = -2,$ $y'''(0) = -1$ (7)

EXAMPLE 1

$$
r^4 + r^3 - 7r^2 - r + 6 = 0.\tag{8}
$$

1, $r_2 = -1$, $r_3 = 2$, and $r_4 = -3$. Therefore the

$$
y = c_1 e^{t} + c_2 e^{-t} + c_3 e^{2t} + c_4 e^{-3t}.
$$
 (9)

The initial conditions (7) require that c_1, \ldots, c_4 satisfy the four equations

functions are linearly independent, then the general solution of Eq. (1) is
\n
$$
y = c_1e^{r_1t} + c_2e^{r_2t} + \cdots + c_ne^{r_nt}
$$
. (5)
\nOne way to establish the linear independence of e^{r_1t} , e^{r_2t} , ..., e^{r_nt} is to evaluate their
\nWronskian determinant. Another way is outlined in Problem 40.
\nFind the general solution of
\n $y^{(m)} + y^{(m} - 7y'' - y' + 6y = 0$. (6)
\nAlso find the solution that satisfies the initial conditions
\n $y(0) = 1$, $y'(0) = 0$, $y''(0) = -2$, $y'''(0) = -1$ (7)
\nand plot its graph.
\nAssuming that $y = e^{rt}$, we must determine r by solving the polynomial equation
\n $r^4 + r^3 - 7r^2 - r + 6 = 0$. (8)
\nThe roots of this equation are $r_1 = 1$, $r_2 = -1$, $r_3 = 2$, and $r_4 = -3$. Therefore the
\ngeneral solution of Eq. (6) is
\n $y = c_1e^t + c_2e^{-t} + c_3e^{2t} + c_4e^{-3t}$. (9)
\nThe initial conditions (7) require that $c_1, ..., c_4$ satisfy the four equations
\n $c_1 + c_2 + c_3 + c_4 = 1$,
\n $c_1 - c_2 + 2c_3 - 3c_4 = 0$,
\n $c_1 + c_2 + 4c_3 + 9c_4 = -2$,
\n $c_1 - c_2 + 8c_3 - 27c_4 = -1$.
\nBy solving this system of four linear algebraic equations, we find that
\n $c_1 = 11/8$, $c_2 = 5/12$, $c_3 = -2/3$, $c_4 = -1/8$.
\nTherefore the solution of the initial value problem is
\n $y = \frac{11}{8}e^t + \frac{5}{12}e^{-t} - \frac{2}{3}e^{2t} - \frac{1}{8}e^{-3t}$. (11)
\nThe graph of the solution is shown in Figure 4.2.1.
\n¹
\nAn important question in mathematics for more than 200 years was whether every polynomial equation

$$
c_1 = 11/8
$$
, $c_2 = 5/12$, $c_3 = -2/3$, $c_4 = -1/8$.

$$
y = \frac{11}{8}e^{t} + \frac{5}{12}e^{-t} - \frac{2}{3}e^{2t} - \frac{1}{8}e^{-3t}.
$$
 (11)

and plot its graph.

Assuming that $y = e^{rt}$, we must determine r by solving the polynomial equation

The roots of this equation are $r_1 = 1$, $r_2 = -1$, $r_3 = 2$, and $r_4 = -3$. Therefore the

general solution of Eq. (6) 1Alternative in the control of the control of the control of Eq. (6) is $r^4 + r^3 - 7r^2 - r + 6 = 0$. (8)

The roots of this equation are $r_1 = 1$, $r_2 = -1$, $r_3 = 2$, and $r_4 = -3$. Therefore the
 $y = c_1 e^t + c_2 e^{-t} + c_3 e^{2t} + c_$ $r^4 + r^3 - 7r^2 - r + 6 = 0.$ (8)

general solution of Eq. (6) is $r_1 = 1$, $r_2 = -1$, $r_3 = 2$, and $r_4 = -3$. Therefore the

general solution of Eq. (6) is $r = c_1 e^t + c_2 e^{-rt} + c_3 e^{2t} + c_4 e^{-3t}$. (9)

The initial conditions (7) The roots of this equation are $r_1 = 1$, $r_2 = -1$, $r_3 = 2$, and $r_4 = -3$. Therefore the general solution of Eq. (6) is
 $y = c_1 e^t + c_2 e^{-t} + c_3 e^{2t} + c_4 e^{-3t}$. (9)

The initial conditions (7) require that $c_1, ..., c_4$ satisf The roots of this equation is $r_1 = 1$, $r_2 = -1$, $r_3 = 2$, and $r_4 = -3$. Therefore the
general solution of Eq. (6) is
 $y = c_1 e' + c_2 e^{-t} + c_3 e^{2t} + c_4 e^{-3t}$. (9)
The initial conditions (7) require that $c_1, ..., c_4$ satisfy th general solution of Eq. (6) is
 $y = c_1 e^t + c_2 e^{-t} + c_3 e^{2t} + c_4 e^{-3t}$. (9)

The initial conditions (7) require that $c_1, ..., c_4$ satisfy the four equations
 $c_1 + c_2 + c_3 + c_4 = 1$,
 $c_1 - c_2 + 2c_3 - 3c_4 = 0$,
 $c_1 + c_2 + 4c_3 + 9$ guidarity of the properties of $c_1 + c_2 + c_3e^{-rt} + c_3e^{2t} + c_4e^{-3t}$. (9)

The initial conditions (7) require that $c_1, ..., c_4$ satisfy the four equations
 $c_1 + c_2 + c_3 + c_4 = 1$,
 $c_1 - c_2 + 2c_3 - 3c_4 = 0$.
 $c_1 + c_2 + 4c_3 + 9c_$

constants c_1, \ldots, c_n . While each of these tasks becomes mu

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calculator or computer.

analogous to the formula

an The state of the differential equation solver, so that the process of factors of the creation of the creation of the initial value problem of Example 1.

As Example 1 illustrates, the procedure for solving an *n*th order **EXECURE 4.2.1** Solution of the initial value problem of Example 1.

As Example 1 illustrates, the procedure for solving an *n*th order linear differential equation with constant coefficients depends on finding the roots **Produced and the matter of the matter of the matter is the produced automometries of the matter of the constant of the constant of the constant of the matter of the IF ACTIVE 4.2.1** Solution of the initial value problem of Example 1.

As Example 1 illustrates, the procedure for solving an *n*th order linear differential

ataion with constant coefficients depends on finding the roots ^{0.5}
 EXET ASSEE 1 EXET ASSEE ASSEED ASSEED AND A constrained equation with constant coefficients depends on finding the of Example 1.

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$$
a_0r^n + a_1r^{n-1} + \dots + a_{n-1}r + a_n = 0 \tag{12}
$$

Figure 1.1
 EXEREMALE 1.1 Solution of the initial value problem of Example 1.

As Fxample 1 illustrates, the procedure for solving an *n*th order linear differential

equation with constant coefficients depends on fin **FIGURE 4.2.1** Solution of the initial value problem of Example 1.
As Example 1 illustrates, the procedure for solving an *n*th order linear differential
equation with constant coefficients depends on inding the roots of **EXECTS 10** and the problem of Example 1.

ture for solving an *n*th order linear differential

eends on finding the roots of a corresponding

trial conditions are prescribed, then a system

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As Example 1 illustrates, the procedure for solving an *n*th order linear differential

equation with constant coefficients depends on finding the roots 2.1 Solution of the initial value problem of Example 1.

Actors, the procedure for solving an *n*th order linear differential

coefficients depends on finding the roots of a corresponding

equation. If initial conditions is an *n*th order linear differential
ing an *n*th order linear differential
ding the roots of a corresponding
nons are prescribed, then a system
betermine the proper values of the
formulas,² analoguas to the formula
iv **FIGURE 4.2.1** Solution of the initial value problem of Example 1.

As Example 1 illustrates, the procedure for solving an *n*th order linear differential

equation with constant coefficients depends on finding the roots As Example 1 illustrates, the procedure for solving an *m*th order linear differential
equation with constant coefficients depends on finding the roots of a corresponding
*m*th degree polynomial equation. If initial condi As Example 1 illustrates, the procedure for solving an *m*th order linear differential
equation with constant coefficients depends on finding the roots of a corresponding
the degree. Dolynomial equation. If initial condit As Example 1 illustrates, the procedure for solving an *n* th order linear differential
organito with constant coefficients depends on finding the roots of a corresponding
order or incerned polynomial equations. If initia equation with constant coefficients depends on finding the roots of a corresponding
or the factors of the orient algebraic equations must be solved to determine the proper values of the
onstants c_1, \ldots, c_n . While each o *nth* degree polynomial equation. If inital conditions are prescribed, then a system
of *n* linear algebraic equations must be solved to determine the proper values of the
constants $c_1, ..., c_n$. While each of these tasks be *n* linear algebraic equations must be solved to determine the proper values of the real and specific equations of the standard without differently with a calculator or computer.
For third and fourth degree polynomials th constants: c_1, \ldots, c_p . While each of these tasks becomes much more complicated as *n*
for thriad and fourth deprete polynomials there are formulas.
For thriad and fourth deprete polynomials there are formulas for anguat For thursd and fourth degree of homotomic sine there are formulas,² analogous to the formulas constrained galactoris and constructions and computers. Sometimes Roof-finding algorithms are easily available on calculators for quadratic equations but more complicated, that give exact expressions for the roots.
Roof-finding algorithms are readily available on calculators and computers. Sometimes
they are included in the differential equation Roof-finding algorithms are readily available on calculators and computers. Sometimes The characteristic polynomial is hidden and the solution of the differential equation is produced automatically.
In the characteristic they are included in the differential equation solver, so that the process of factoring
the characteristic polynomial is hidden and the solution of the differential equation is
produced automatically.
If you are fixed wit the mitorial in the unit of the context of the showed that no general solution of the differential equation is
produced automatically.
If you are fixed with the need to factor the characteristic polynomial by hand, here
i the consecteristic polynomial is finded by factor the characteristic polynomial by hand, here produced automatically.

If you are fixed with the need to factor the characteristic polynomial by hand, here is one result tha produced automateally.

If you are fraced with the need to factor the characteristic polynomial by hand, here

is one result that is sometimes helpful. Suppose that the polynomial
 $a_0r^n + a_1r^{n-1} + \cdots + a_{n-1}r + a_0 = 0$

ha

Equations with Constant Coefficients 217

solution will be dominated by the term corresponding to the algebraically largest root.

If this root is positive, then solutions will become exponentially unbounded, while if i *Equations with Constant Coefficients* 217

Solution will be dominated by the term corresponding to the algebraically largest root.

If this root is positive, then solutions will become exponentially unbounded, while if i **Equations with Constant Coefficients** 217
 Equations will be dominated by the term corresponding to the algebraically largest root.
 Solution will be dominated by the term corresponentially unbounded, while if it

is **Equations with Constant Coefficients**
 Equations will be dominated by the term corresponding to the algebraically largest root

if this root is positive, then solutions will tend exponentially to zero. Finally, if the **Equations with Constant Coefficients** 217

solution will be dominated by the term corresponding to the algebraically largest root.

If this root is positive, then solutions will become exponentially unbounded, while if i **Equations with Constant Coefficients** 217

Solution will be dominated by the term corresponding to the algebraically largest root.

If this root is positive, then solutions will lead exponentially to zero. Finally, if th **217**
 Constant Coefficients
 Coefficients
 Coefficients Equations with Constant Coefficients 217

Solution will be dominated by the term corresponding to the algebraically largest root.

If this root is positive, then solutions will become exponentially unbounded, while if 217

are algebraically largest root.

tially unbounded, while if it

Finally, if the largest root is

ecomes large. Of course, for

the next largest root.

the next largest root.

ex roots, they must occur in

are real nu *Equations with Constant Coefficients*
 **Equations will be dominated by the term corresponding to the algebraically largest root.

If this root is positive, then solutions will became exponentially unbounded, while if it
 Equations with Constant Coefficients** 217

solution will be dominated by the term corresponding to the algebraically largest root.

If this root is positive, then solutions will become exponentially unbounded, while if i *Equations with Constant Coefficients*
 217

solution will be dominated by the term corresponding to the algebraically largest root.

If this root is positive, then solutions will become exponentially unbounded, while i **217**
 217
 Example 10 the algebraically largest root.
 Example 10 the real-valued solution is the otherwise dominant term will be rero;

to constant as *t* becomes large. Of course, for

the otherwise dominant term *Equations with Constant Coefficients*
 217

Solution will be dominated by the term corresponding to the algebraically largest root.

If this root is positive, then solutions will become exponentially unbounded, while i 217

g to the algebraically largest root.

ponentially unbounded, while if it

zero. Finally, if the largest root is

as *t* becomes large. Of course, for

wise dominant term will be zero;

ed by the next largest root.

c *Equations with Constant Coefficients*

solution will be dominated by the term corresponding to the algebraically largest root.

If this root is positive, then solutions will become exponentially unbounded, while if it

i **Equations with Constant Coefficients**
 Equations will be dominated by the term corresponding to the algebraically largest root.

If this root is positive, then solutions will become exponentially unbounded, while if it **Equations with Constant Coefficients**
 Equations with Constant Coefficients

Solution will be dominated by the term corresponding to the algebraically largest root.

If this root is positive, then solutions will becaus If this root is positive, then solutions will become exponentially unbounded, while if it
is negative, then solutions will tend exponentially to zero. Finally, if the largest root is
zero, then solutions will approach a n

conjugate pairs, $\lambda \pm i\mu$, since the coefficients a_0, \ldots, a_n are real numbers. Provided that none of the roots is repeated, the general solution of Eq. (1) is still of the form (4). However, just as for the second order equation (Section 3.4), we can replace the complex-valued solutions $e^{(\lambda+i\mu)t}$ and $e^{(\lambda-i\mu)t}$ by the real-valued solutions e, then solutions will engine expondially to zero. Finally, the largest roto's considerated by solutions will approach a nonzero constant as *b* becomes large. Of course, for tital conditions the coefficient of the otherw certain initial conditions the coefficient of the otherwise dominant term will be zero;
then the nature of the solution for large *t* is determined by the next largest root.
 Complex Roots. If the characteristic equatio of the solution for large t is determined by the next largest root.

If the characteristic equation has complex roots, they must occur in
 $\lambda \pm i\mu$, since the coefficients a_0, \ldots, a_n are ereal numbers. Provided

roots conjugate pairs, $\lambda \pm i\mu$, since the coefficients a_0, \ldots, a_n are real numbers. Frowided
that none of the roots is repeated, the general solution of Eq. (1) is still of the form (4).
However, just as for the second orde

$$
e^{\lambda t}\cos\mu t, \qquad e^{\lambda t}\sin\mu t \tag{13}
$$

obtained as the real and imaginary parts of $e^{(\lambda+i\mu)t}$. Thus, even though some of the complex-valued solutions $e^{(x+ix)/t}$ and $e^{(x-ix)/t}$ by the real-valued solutions
 $e^{kt} \cos \mu t$, $e^{kt} \sin \mu t$. Thus, even though some of the

crosts of the characteristic equation are complex, it is still possible to expres obtained as the real and imaginary parts of $e^{kx+1/2}$. Thus, even though some of the choose of the characteristic equation are complex, it is still possible to express the general solution of Eq. (1) as a linear combina

$$
y^{\text{iv}} - y = 0. \tag{14}
$$

$$
y(0) = 7/2,
$$
 $y'(0) = -4,$ $y''(0) = 5/2,$ $y'''(0) = -2$ (15)

EXAMPLE $\bf{2}$

Substituting e^{rt} for y, we find that the characteristic equation is

$$
r^4 - 1 = (r^2 - 1)(r^2 + 1) = 0.
$$

$$
y = c_1 e^t + c_2 e^{-t} + c_3 \cos t + c_4 \sin t.
$$

$$
c_1 = 0
$$
, $c_2 = 3$, $c_3 = 1/2$, $c_4 = -1$;

$$
y = 3e^{-t} + \frac{1}{2}\cos t - \sin t.
$$
 (16)

Find the general solution of
 $y^{iv} - y = 0$. (14)

Also find the solution that satisfies the initial conditions
 $y(0) = 7/2$, $y'(0) = -4$, $y''(0) = 5/2$, $y'''(0) = -2$ (15)

and draw its sraph.

Substituting e^{t} for y , we fi observe that the initial conditions (15) cause the coefficient c_1 of the initial conditions or $y^2 - y = 0$.

(14) can be solution that statisfies the initial conditions $y'(0) = -2$ (15) d draw its graph.

Substituting $e^{$ (14)

"(0) = -2 (15)

of Eq. (14) is

(16)

.1;

(16)

of the exponentially

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socillation, as Figure

y, then c_1 is likely to

example, if the first

changed from -2 to Find the general solution of
 $y^w - y = 0$. (14)

Also find the solution that satisfies the initial conditions
 $y(0) = 7/2$, $y'(0) = -4$, $y''(0) = 5/2$, $y'''(0) = -2$ (15)

and draw its graph.

Substituting e^{rt} for y, we find $y^{iv} - y = 0.$ (14)

Also find the solution that satisfies the initial conditions
 $y(0) = 7/2$, $y'(0) = -4$, $y''(0) = 5/2$. $y'''(0) = -2$ (15)

and draw its graph.

Substituting e^{rt} for y, we find that the characteristic equat Also find the solution that suffice the initial conditions
 $y(0) = 7/2$, $y'(0) = -4$, $y''(0) = 5/2$, $y'''(0) = -2$ (15)

and draw its graph.

Substituting e^r for y, we find that the characteristic equation is
 $r^4 - 1 = (r^2 - 1$ (14)

2 (15)

(16)

(16)

0000 (16)

0000 (16)

0000 (16)

5 (16)

5 (17)

(17)

17) Also find the solution that satisfies the initial conditions
 $y(0) = 7/2$, $y'(0) = -4$, $y''(0) = 5/2$, $y'''(0) = -2$ (15)

and draw its graph.

Substituting e^{rt} for y , we find that the characteristic equation is
 $r^4 - 1 = ($ $y(0) = 7/2$, $y'(0) = -4$, $y''(0) = 5/2$, $y'''(0) = -2$ (15)
and draw its graph.
Substituting e^{rt} for y, we find that the characteristic equation is
Therefore the roots are $r = 1, -1, i, -i$, and the general solution of Eq. (14) $y(0) = 7/2$, $y'(0) = -4$, $y'(0) = 5/2$, $y''(0) = -2$ (15)

Substituting e^{t} for y , we find that the characteristic equation is

Substituting e^{t} for y , we find that the characteristic equation is
 $r^4 - 1 = (r^2 - 1)(r^2$

$$
y = \frac{1}{32}e^{t} + \frac{95}{32}e^{-t} + \frac{1}{2}\cos t - \frac{17}{16}\sin t.
$$
 (17)

Chapter 4. Higher Order Linear Equations
The coefficients in Eq. (17) differ only slightly from those in Eq. (16), but the expo-
nentially growing term, even with the relatively small coefficient of 1/32, completely
domina **Chapter 4. Higher Order Linear Equations**
The coefficients in Eq. (17) differ only slightly from those in Eq. (16), but the expo-
nentially growing term, even with the relatively small coefficient of 1/32, completely
domi **Chapter 4. Higher Order Linear Equations**
The coefficients in Eq. (17) differ only slightly from those in Eq. (16), but the exponentially growing term, even with the relatively small coefficient of 1/32, completely domin

 $a_0 y'' + a_1 y' + a_2 y = 0$, then the two linearly independent solutions are $e^{r_1 t}$ and $te^{r_1 t}$. **Example 12**
 i a fit is a reported to the solution (16). **FIGURE 4.2.3** Plots of the solutions (16) (light curve) and (17) (heavy curve).

he roots of the characteristic equation are not distinct, that is, if it is a r **FIGURE 4.2.2** A plot of the solution (16). **FIGURE 4.2.3** Plots of the solutions (16)

(light curve) and (17) (heavy curve).
 Repeated Roots. If the roots of the characteristic equation are not distinct, that is, if

s **FIGURE 4.2.** 2 A plot of the solution (16). **FIGURE 4.2.3** Plots of the solutions (16)

(light curve) and (17) (heavy curve).
 Repeated Roots. If the roots of the characteristic equation are not distinct, that is, if
 Repeated Roots. If the roots of the characteristic equation are not distinct, that is, if some of the roots are repeated, then the solution (5) is clearly not the general solution of $\sigma f_{\text{L}}(1)$. Recall that if r_1 Eq. (1). Recall that if r_1 is a repeated root for the second order linear equation
 $y'' + a_1y' + a_2y = 0$, then the two linearly independent solutions are $e^{r_1'}$ and $te^{r_1'}$,
 $r_1a_1e^{r_1}$, $t^2e^{r_1'}$, $t^2e^{r_1'}$,

$$
e^{r_1 t}, \quad t e^{r_1 t}, \quad t^2 e^{r_1 t}, \quad \ldots, \quad t^{s-1} e^{r_1 t} \tag{18}
$$

find 2s real-valued solutions by noting that the real and imaginary parts of $e^{(\lambda+i\mu)t}$, , $te^{(\lambda+i\mu)t}$, ..., $t^{s-1}e^{(\lambda+i\mu)t}$ are also linearly independent so $s \le n$), then
 $e^{r_1 t}$, $te^{r_1 t}$, $t^2 e^{r_1 t}$, $t^{2} e^{r_1 t}$, $t^{1-1} e^{r_1 t}$ (18)

are corresponding solutions of Eq. (1).

If a complex root $\lambda + i\mu$ is repeated s times, the complex-valued solutions by the find 2s

$$
e^{\lambda t} \cos \mu t
$$
, $e^{\lambda t} \sin \mu t$, $te^{\lambda t} \cos \mu t$, $te^{\lambda t} \sin \mu t$,
..., $t^{s-1}e^{\lambda t} \cos \mu t$, $t^{s-1}e^{\lambda t} \sin \mu t$.

EXAMPLE 3

$$
y^{\text{iv}} + 2y'' + y = 0. \tag{19}
$$

$$
r^4 + 2r^2 + 1 = (r^2 + 1)(r^2 + 1) = 0.
$$

 $y = c_1 \cos t + c_2 \sin t + c_3 t \cos t + c_4 t \sin t.$

The properties are all the contribution 219

In determining the roots of the characteristic equation it may be necessary to compute

c cube roots, or fourth roots, or even higher roots of a (possibly complex) number. **Equations with Constant Coefficients**

In determining the roots of the characteristic equation it may be necessary to compute

the cube roots, or fourth roots, or even higher roots of a (possibly complex) number. This

c **Equations with Constant Coefficients**

In determining the roots of the characteristic equation it may be necessary to compute

the cube roots, or fourth roots, or even higher roots of a (possibly complex) number. This

c the cube roots, or fourth roots, or even higher roots of a (possibly complex) number. This can usually be done most conveniently by using Euler's formula $e^{it} = \cos t + i \sin t$ **Equations with Constant Coefficients**

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the cube roots, or fourth roots, or even higher roots of a (possibly complex) number. This

c Equations with Constant Coefficients

In determining the roots of the characteristic equation it may be necessary to compute

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 Example 10

In determining the roots of the characteristic equation it may be necessary to compute

celucive cost, or fourth roots, or even higher roots of a (possibly complex) number. This

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c *Equations with Constant Coefficients*

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In determining the roots of the characteristic equation it may be necessary to compute

the cube roots or or fourth roots, or even higher roots of a foosithly complex) mumber. This
 the cube roots, or fourth roots, or even higher roots of a (possibly complex) number. This
can usually be done most conveniently by using Euler's formula $e^{it} = \cos t + i \sin t$
and the algebraic laws given in Section 3.4. This i

EXAMPLE 4

$$
y^{\text{iv}} + y = 0. \tag{20}
$$

 $r^4 + 1 = 0.$

 $1 = \cos \pi + i \sin \pi = e^{i\pi}$.

 $1 = \cos(\pi + 2m\pi) + i \sin(\pi + 2m\pi) = e^{i(\pi + 2m\pi)},$

$$
(-1)^{1/4} = e^{i(\pi/4 + m\pi/2)} = \cos\left(\frac{\pi}{4} + \frac{m\pi}{2}\right) + i\sin\left(\frac{\pi}{4} + \frac{m\pi}{2}\right).
$$

$$
\frac{1+i}{\sqrt{2}}, \frac{-1+i}{\sqrt{2}}, \frac{-1-i}{\sqrt{2}}, \frac{1-i}{\sqrt{2}}.
$$

Find the general solution of
 $y^{iv} + y = 0$. (20)

The characteristic equation is
 $r^4 + 1 = 0$.

To solve the equation we must compute the fourth roots of -1. Now -1, thought of as

a complex number, is $-1 + 0i$. It has mag Find the general solution of
 $y^N + y = 0$. (20)

The characteristic equation is
 $r^4 + 1 = 0$.

To solve the equation we must compute the fourth roots of -1. Now -1, thought of as

a complex number, is $-1 + 0i$. It has magn $y^{iv} + y = 0.$ (20)

The characteristic equation is
 $r^4 + 1 = 0.$

To solve the equation we must compute the fourth roots of -1. Now -1, thought of as

a complex number, is $-1 + 0i$. It has magnitude 1 and polar angle π .

$$
y = e^{t/\sqrt{2}} \left(c_1 \cos \frac{t}{\sqrt{2}} + c_2 \sin \frac{t}{\sqrt{2}} \right) + e^{-t/\sqrt{2}} \left(c_3 \cos \frac{t}{\sqrt{2}} + c_4 \sin \frac{t}{\sqrt{2}} \right). \tag{21}
$$

complex number, is $-1 + 0i$. It has magnitude 1 and polar angle π . Thus
 $-1 = \cos \pi + i \sin \pi = e^{i\pi}$.

orcover, the angle is determined only up to a multiple of 2π . Thus
 $-1 = \cos(\pi + 2m\pi) + i \sin(\pi + 2m\pi) = e^{i(\pi + 2m\pi)}$,

nete $-1 = \cos \pi + i \sin \pi = e^{i\pi}$

Moreover, the angle is determined only up to a multiple of 2π . Thus
 $-1 = \cos(\pi + 2m\pi) + i \sin(\pi + 2m\pi) = e^{i(\pi + 2m\pi)}$

where *m* is zero or any positive or negative integer. Thus
 $(-1)^{1/4} = e^{i(\pi/4 + m\$

 $-1 = \cos \pi + i \sin \pi = e^{\alpha}$.

Moreover, the angle is determined only up to a multiple of 2π . Thus
 $-1 = \cos(\pi + 2m\pi) + i \sin(\pi + 2m\pi) = e^{i(\pi + 2m\pi)}$,

where m is zero or any positive or negative integer. Thus
 $(-1)^{1/4} = e^{i(\pi/4 + m\pi$ Moreover, the angle is determined only up to a multiple of 2π . Thus
 $-1 = \cos(\pi + 2m\pi) + i \sin(\pi + 2m\pi) = e^{i(\pi + 2m\pi)}$,

where m is zero or any positive or negative integer. Thus
 $(-1)^{1/4} = e^{i(\pi/4 + m\pi/2)} = \cos(\frac{\pi}{4} + \frac{m\pi}{2}) +$ $-1 = \cos(\pi + 2m\pi) + i \sin(\pi + 2m\pi) = e^{i(\pi + 2m\pi)}$

here *m* is zero or any positive or negative integer. Thus
 $(-1)^{1/4} = e^{i(\pi/4 + m\pi/2)} = \cos\left(\frac{\pi}{4} + \frac{m\pi}{2}\right) + i \sin\left(\frac{\pi}{4} + \frac{m\pi}{2}\right)$.

Here four fourth roots of -1 are obtai a_1, \ldots, a_n in Eq. (1) are complex numbers, the s $m\pi$) + $i \sin(\pi + 2m\pi) = e^{i(\pi + 2m\pi)}$,

r negative integer. Thus
 $\frac{n\pi}{4} + \frac{m\pi}{2} + i \sin\left(\frac{\pi}{4} + \frac{m\pi}{2}\right)$.

btained by setting $m = 0, 1, 2,$ and 3; they are
 $\frac{1+i}{\sqrt{2}}, \frac{-1-i}{\sqrt{2}}, \frac{1-i}{\sqrt{2}}$.
 $\frac{-1+i}{\sqrt{2}}, \frac{-1-i}{\sqrt$ -1 = cos($\pi + 2m\pi$) + i still $\pi + 2m\pi$) = $e^{(x + 2m\pi)}$,

where *m* is zero or any positive or negative integer. Thus
 $(-1)^{1/4} = e^{i(n/4+m\pi/2)} = \cos(\frac{\pi}{4} + \frac{m\pi}{2}) + i \sin(\frac{\pi}{4} + \frac{m\pi}{2})$.

The four fourth roots of -1 are where *m* is zero or any positive or negative integer. Thus
 $(-1)^{1/4} = e^{i(\pi/4 + m\pi/2)} = \cos\left(\frac{\pi}{4} + \frac{m\pi}{2}\right) + i \sin\left(\frac{\pi}{4} + \frac{m\pi}{2}\right)$.

The four fourth roots of -1 are obtained by setting $m = 0, 1, 2,$ and 3; they are The four the same of $(-1)^{1/4} = e^{i(r/(4+mx)/2)} = \cos(\frac{\pi}{4} + \frac{m\pi}{2}) + i \sin(\frac{\pi}{4} + \frac{m\pi}{2})$.

The four fourth roots of -1 are obtained by setting $m = 0, 1, 2,$ and 3; they are
 $\frac{1+i}{\sqrt{2}}$, $\frac{-1+i}{\sqrt{2}}$, $\frac{-1-i}{\sqrt{2}}$, $\frac{1-i$ The four fourth roots of -1 are obtained by setting $m = 0, 1, 2,$ and 3; they are
 $\frac{1+i}{\sqrt{2}}$. $\frac{-1+i}{\sqrt{2}}$, $\frac{-1-i}{\sqrt{2}}$, $\frac{1-i}{\sqrt{2}}$.

It is easy to verify that for any other value of m we obtain $\cos \theta$ for t If $\frac{1+i}{\sqrt{2}}$, $\frac{-1+i}{\sqrt{2}}$, $\frac{-1+i}{\sqrt{2}}$, $\frac{1-i-i}{\sqrt{2}}$, $\frac{1-i-i}{\sqrt{2}}$.

It is easy to verify that for any other value of m we obtain one of these four roots.

For example, corresponding to $m = 4$, we obtain $(1+i)/$

PROBLEMS

1. $1 + i$

2. $-1 + \sqrt{3}i$

3. -3

4. $-i$

5. $\sqrt{3} - i$

6. $-1 - i$

Chapter 4. Higher Order Linear Equation

n each of Problems 7 through 10 follow the procedure illustrated in Example 4 to determine

in eindicated roots of the given complex number.

8. $(1-i)^{1/2}$

9. $1^{1/4}$

10. $[2$ **Chapter 4. Higher Order Linear Equation**

n each of Problems 7 through 10 follow the procedure illustrated in Example 4 to determine

indicated roots of the given complex number.

8. $(1 - i)^{1/2}$

9. $(1 - i)^{1/2}$

10. $[$ mapter 4. Higher Order Linear Equations

seedure illustrated in Example 4 to determine

8. $(1-i)^{1/2}$

10. $[2(\cos \pi/3 + i \sin \pi/3)]^{1/2}$

al solution of the given differential equation.

12. $y''' - 3y'' + 3y' - y = 0$

14. $y^{iv} - 4y$ **Chapter 4. Higher Order Linear Equations**

In each of Problems 7 through 10 follow the procedure illustrated in Example 4 to determine
 $\frac{7}{7}$. $\frac{1^{1/3}}{9}$, $\frac{1^{1/4}}{1}$
 $\frac{8}{10}$. $\frac{(1-i)^{1/2}}{2(1 \cos \pi/3 + i \sin \pi/$ **Chapter 4. Higher Order Linear Equations**

In each of Problems 7 through 10 follow the procedure illustrated in Example 4 to determine

the indicated roots of the given complex number.

1. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$ **Chapter 4. Higher Order Linear Equations**

In each of Problems 7 through 10 follow the procedure illustrated in Example 4 to determine

the indical cross of the given complex number.

3. $(1 - i)^{1/2}$

9. $1^{1/4}$

10. $[$ $i v - 4y''' + 4y'' = 0$ **Chapter 4. Higher Order Linear Equations**

In each of Problems 7 through 10 follow the procedure illustrated in Example 4 to determine

the indicated roots of the given complex number.

7. $1^{1/3}$

9. 1^{1/3}

9. 1^{1/3}
 $i v - 5y'' + 4y = 0$ 17. $y^{vi} - 3y^{iv} + 3y'' - y = 0$ 18. **Chapter 4. Higher Order Linear Equations**

bolems 7 through 10 follow the procedure illustrated in Example 4 to determine

roots of the given complex number.

8. $(1-i)^{1/2}$

10. $[2(\cos \pi/3 + i \sin \pi/3)]^{1/2}$

bolems 11 throu 19. $y^{\rm v} - 3y^{\rm iv} + 3y''' - 3y'' + 2y' = 0$ 20 **Chapter 4. Higher Order Linear Equations**

coblems 7 through 10 follow the procedure illustrated in Example 4 to determine

roots of the given complex number.

8. $(1-i)^{1/2}$

8. $(1-i)^{1/2}$

oblems 11 through 28 find th 18. $y'' - y'' = 0$

20. $y'' - 8y' = 0$ 21. $y^{viii} + 8y^{iv} + 16y = 0$
23. $y''' - 5y'' + 3y' + y = 0$ **Chapter 4. Higher Order Linear Equations**

blems 7 through 10 follow the procedure illustrated in Example 4 to determine

oots of the given complex number.

8. $(1 - i)^{1/2}$

blems 11 through 28 find the general solution 20. $y'' + 2y'' + y = 0$

24. $y''' + 5y'' + 6y' + 2y = 0$ **Chapter 4. Higher Order Linear Equations**

In each of Problems 7 through 10 follow the procedure illustrated in Example 4 to determine

the indicated roots of the given complex number.

7. $1^{1/4}$

9. $(1-i)^{1/2}$

1. y **Chapter 4. Higher Order Linear Equation**

In each of Problems 7 through 10 follow the precedure illustrated in Example 4 to determine

1. $1^{1/3}$

9. $1^{1/4}$

9. $1^{1/4}$

10. $[2(\cos \pi/3 + i \sin \pi/3)]^{1/2}$

11. $y''' - y'' - y' + y =$ \triangleright 26. $v^{\text{iv}} - 7v''' + 6v'' + 30v' - 36v = 0$ **Chapter 4. Higher Order Linear Equation**

In each of Problems 7 through 10 follow the procedure illustrated in Example 4 to determine
 $\frac{7}{11^{1/3}}$
 $\frac{1}{21^{1/4}}$
 $\frac{1}{21^{1/4}}$
 $\frac{1}{21^{1/4}}$
 $\frac{1}{21^{1/4}}$
 $\frac{1$ i^{iv} + 31y''' + 75y'' + 37y' + 5y = 0 > 28. y^{iv} + 6y''' + 17y'' + 22y' + 14y = 0 **Chapter 4. Higher Order Linear Equations**

In each of Problems 7 through 10 follow the procedure illustrated in Example 4 to determine

1. $y^n - y^n + y = 0$

8. $(1 - i)^{1/2}$

In each of Problems 11 through 28 find the general **Chapter 4. Higher Order Linear Equations**

In each of Problems 7 through 10 follow the procedure illustrated in Example 4 to determine

1. 1^{1/4}

9. 1^{1/4}

1. 12/² → *y*² → *y*² → *y*² + 3*i* = 10 solution of In each of Problems 7 through 10 follow the procedure illustrated in Example 4 to determinine

1. $\int_{-1}^{1/3}$

9. $1^{1/4}$

9. $1^{1/4}$

9. $\int_{-1}^{1/3}$

9. $\int_{-1}^{1/3}$

10. $[2(\cos \pi/3 + i \sin \pi/3)]^{1/2}$

11. \int_{-1}^{∞} 7. $1^{1/4}$

8. $(1 - i)^{1/2}$

9. $(1 - i)^{1/2}$

10. $2(y^2 - y^2 - y' + y = 0$

11. $y''' - y'' - y' + y = 0$

12. $y''' - 3y'' + 3y' - y = 0$

13. $2y''' - 4y' - 2y' + 4y = 0$

14. $y^2 - 4y'' - 2y' + 4y = 0$

14. $y^3 - 3y'' - 3y = 0$

14. $y^4 - 4y = 0$

14. $y^$ 9. $1^{1/4}$ 10. $[2(\cos \pi/3 + i \sin \pi/3)]^{1/2}$

10. $[2(\cos \pi/3 + i \sin \pi/3)]^{1/2}$

In each of Problems 11 through 28 find the general solution of the given differential equation.

11. $y'' - y' - y' + y = 0$

12. $y''' + 3y'' - y = 0$

14. $y^k - 4$ 13. $2y'' - y - y + y = 0$

13. $2y'' - 4y'' - 2y' + 4y = 0$

14. $y'' - 4y'' - 4y' + 4y' = 0$

15. $y^{24} + y - 0$

17. $y^{34} - 3y^{34} + 3y'' - y = 0$

16. $y'' - 3y'' - 3y'' - 4y = 0$

19. $y'' - 3y'' + 3y'' - y = 0$

19. $y'' - 3y'' + 3y'' - y = 0$

22. $y''' + 2y'' + y = 0$
 14. $y'' - 3y'' + 3y'' - y = 0$

14. $y''' - 4y''' + 4y'' = 0$

16. $y''' - 5y'' + 4y = 0$

18. $y'' + y'' = 0$

20. $y'' - 8y' = 0$

22. $y'' + 2y'' + y = 0$

24. $y'' - 5y'' + 6y'' + 2y = 0$

24. $y''' - 7y''' + 6y'' + 2y = 0$

b 26. $y''' - 7y''' + 6y'' + 30y' - 36y = 0$

f to so $y'' - 3y'' + 3y'' - 3y'' + 2y' = 0$
 $y'' = 4y'' + 8y' + 16y = 0$
 $y''' - 4y'' + 8y'' + 6y' + y = 0$
 $y''' + 2y'' + 2y' + y = 0$
 $y''' + 2y'' + 14y' + 4y = 0$
 $y''' + 2y'' + 14y' + 4y = 0$
 $y''' + 2y''' + 14y' + 4y = 0$
 $y''' + 2y''' + 14y' + 4y = 0$
 $y''' + 2y''' + 14y' + 4y = 0$
 y^{wii 2}y $y'' + 16y = 0$
 $y'' + 3y' + y = 0$
 $y'' + 3y' + y = 0$
 $y'' + 3y'' + 3y'' + 5y = 0$
 $y'' + 2y'' + 3y'' + 5y = 0$
 $y''' + 2y''' + 3y''' + 5y''' + 5y''' + 6y''' + 17y'' + 22y' + 14y = 0$
 $y''' + 21y''' + 75y'' + 37y'' + 5y = 0$
 $y''' + 2y''' + 3y'' + 5y = 0$
 $y''' + 2y''' + 3y''' +$ 25. $18y''' + 21y'' + 14y' + 4y = 0$

27. $2y''' + 21y'' + 14y' + 4y = 0$

27. $2y''' + 21y''' + 75y'' + 37y' + 5y = 0$

28. $y''' + 7y''' + 6y'' + 17y'' + 22y' + 14y = 0$

In each of Problems 29 through 36 find the solution of the given initial value probl

- 29. $y''' + y' = 0$; $y(0) = 0$, $y'(0) = 1$, $y''(0) = 2$
- 30. $y^{iv} + y = 0$; $y(0) = 0$, $y'(0) = 0$, $y''(0) = -1$, $y'''(0) = 0$
- 31. $y^{iv} 4y''' + 4y'' = 0$; $y(1) = -1$, $y'(1) = 2$, $y''(1) = 0$, $y'''(1) = 0$
- 32. $y''' y'' + y' y = 0$; $y(0) = 2$, $y'(0) = -1$, $y''(0) = -2$
- 33. $2y^{iv} y''' 9y'' + 4y' + 4y = 0$; $y(0) = -2$, $y'(0) = 0$, $y''(0) = -2$,
-
- y^{*w*}(0) = 0

34. $4y''' + y' + 5y = 0$; $y(0) = 2$, $y'(0) = 1$, $y''(0) = -1$

35. $6y''' + 5y'' + y' = 0$; $y(0) = -2$, $y'(0) = 2$, $y''(0) = 0$
- 36. $y^{iv} + 6y''' + 17y'' + 22y' + 14y = 0$; $y(0) = 1$, $y'(0) = -2$, $y''(0) = 0$, $y'''(0) = 3$
	-

$$
y = c_1 \cos t + c_2 \sin t + c_3 \cosh t + c_4 \sinh t.
$$

 ι and ι e^{-t} ? t ? (b) $x^2 + 2(y^2 + 44y^2 = 0$ **b** 26. $y'' - 4y' + 6y'' + 3(y' - 3y - 4y' + 2y' + 4y' - 3y - 4y' + 2y' + 2y' + 2y' + 2y' + 2y' + 2y' +$

- 38. Consider the equation $y^{\text{iv}} y = 0$.
	- t , e^{-t} , cos t, and sin t.
	-
	-
- 12y^{jn} + 31yⁿ + 75yⁿ + 37yⁿ + 5y = 0 ▶ 28. yⁿ + 6yⁿ + 17yⁿ + 22y² + 14y = 0

ch of Problems 29 through 36 find the solution of the given initial value problem and plot

of Problems 29 through 36 find the s ch of Problems 29 through 36 find the solution of the given initial value problem and plot

spin- How does to solution behave as $t \rightarrow \infty$?
 $y'' + y = 0$: $y(0) = 0$, $y'(0) = 1$, $y''(0) = 2$
 $y'' + y' = 4y'' + 4y'' = 0$; $y(0) = 0$, $y'($ In each of Problems 29 through 36 find the solution of the given initial value problem and plot

its graph. How does the solution behave $\mathbf{s}t \to \infty$?
 $20 \cdot y'' + y = 0;$ $y(0) = 0,$ $y'(0) = 0,$ $y''(0) = -1,$ $y''(0) = 0$
 $31 \cdot y''$ $y'' + y' = 0;$ $y(0) = 0,$ $y'(0) = 1,$ $y''(0) = 2$
 $y^2 + y = 0;$ $y(0) = 0,$ $y'(0) = 0,$ $y''(0) = 0$
 $y'' - y'' + y' - y - 0;$ $y(0) = 2,$ $y'(0) = -1,$ $y''(0) = -2$
 $2y^2 - y'' + y' - y - 0;$ $y(0) = 2,$ $y'(0) = -2,$ $y'(0) = 0,$
 $y'''(0) = 0$
 $y'''(0) + y' + y' =$ $y^w + y = 0$; $y(0) = 0$, $y'(0) = 0$, $y''(0) = -1$, $y''(0) = 0$
 $y^w - 4y^w + 4y^w = 0$; $y(1) = -1$, $y'(1) = 0$
 $y^w - 4y^w + 4y^w = 0$; $y(1) = 2$, $y'(1) = 0$, $y''(1) = 0$, $y'''(1) = 0$
 $y^w - y^w + y^w = 0$; $y(0) = 2$, $y'(0) = -1$, $y'(0) = -2$,
 $y'' - y'' + 4y'' = 0$; $y(0) = -1$, $y'(1) = 2$, $y''(1) = 0$, $y'''(1) = 0$
 $y''' - y'' + 4y'' - y = 0$; $y(0) = -2$, $y'(0) = -1$, $y'(0) = -2$
 $y'''(0) = -2$, $y'(0) = 0$, $y''(0) = -2$, $y'(0) = 0$, $y''(0) = -2$,
 $y'''(0) = 0$, $y'''(0) = 0$, $y'''(0) = -2$, $y'(0) = 1$, b $y'(1) = 2$, $y''(1) = 0$, $y'''(1) = 0$
 $'(0) = -1$, $y''(0) = -2$
 $(0) = -2$, $y'(0) = 0$, $y''(0) = -2$,
 $y(0) = 2$, $y''(0) = 0$
 $y(0) = 1$, $y'(0) = 0$, $y''(0) = 0$,
 $y(0) = 1$, $y'(0) = -2$, $y''(0) = 0$,
 $y(0) = 1$, $y'(0) = -2$, $y''(0) = 0$,
 $y = -ay + yy - y = 0;$ $y(1) = -1;$ $y'(1) = 0;$ $y'(1) = 0;$ $y'(0) = -2;$
 $2y'' - y'' + y' = 0;$ $y(0) = 2;$ $y'(0) = -1;$ $y''(0) = -2;$
 $y'''(0) = 0;$ $y'''(0) = 9y'' + 4y' + 4y = 0;$ $y(0) = -2;$ $y'(0) = 1,$ $y''(0) = -1;$
 $y'''(0) = 0;$ $y''' + y' + 5y = 0;$ $y(0) = 2$ $y''(0) = 0$
 $(y''(0) = 0)$
 $(y''' + y' + 5y = 0;$ $y(0) = 2, y'(0) = 1, y''(0) = -1$
 $(y''' + 5y'' + y' = 0; y(0) = -2, y'(0) = 2, y''(0) = 0$.
 $y''' + 6y'' + 17y'' + 22y' + 14y = 0; y(0) = 1, y'(0) = -2, y''(0) = 0$.
 $y''(0) = 3$
 $y'''(0) = 3$
 $y'''(0) = 3$

Show that the ge $v'(0) = 1$, $y''(0) = -1$
 $y'(0) = 2$, $y''(0) = 0$
 $y(0) = 1$, $y'(0) = -2$, $y''(0) = 0$,
 $y = 0$ can be written as
 $\sin t + c_3 \cosh t + c_4 \sinh t$.

mital conditions $y(0) = 0$, $y'(0) = 0$, $y''(0) = 1$,

the solutions $\cosh t$ and $\sinh t$ rather $4y'' + y' + 5y = 0$; $y(0) = 1$, $y''(0) = -1$, $y''(0) = -1$
 $6y'''' + y'' + 7y'' + 22y' + 14y = 0$; $y(0) = 1$, $y'(0) = -2$, $y''(0) = 0$,
 $y''' + 6y''' + 17y'' + 22y' + 14y = 0$; $y(0) = 1$, $y'(0) = -2$, $y''(0) = 0$,

Show that the general solution of y^k $y''(0) = 3$

Show that the general solution of $y'' - y = 0$ can be written as
 $y = c_1 \cos t + c_2 \sin t + c_3 \cosh t + c_4 \sinh t$.

Determine the solution satisfying the initial conditions $y(0) = 0$, $y'(0) = 0$, $y''(0) = 1$,
 $y''(0) = 1$. Why is

(a) Show that the displacements u_1 and u_2 of the masses from their respective equilibrium

$$
u_1'' + 5u_1 = 2u_2, \qquad u_2'' + 2u_2 = 2u_1. \tag{i}
$$

:

$$
u_1^{\text{iv}} + 7u_1'' + 6u_1 = 0. \tag{ii}
$$