Signals and systems

Chapter 1

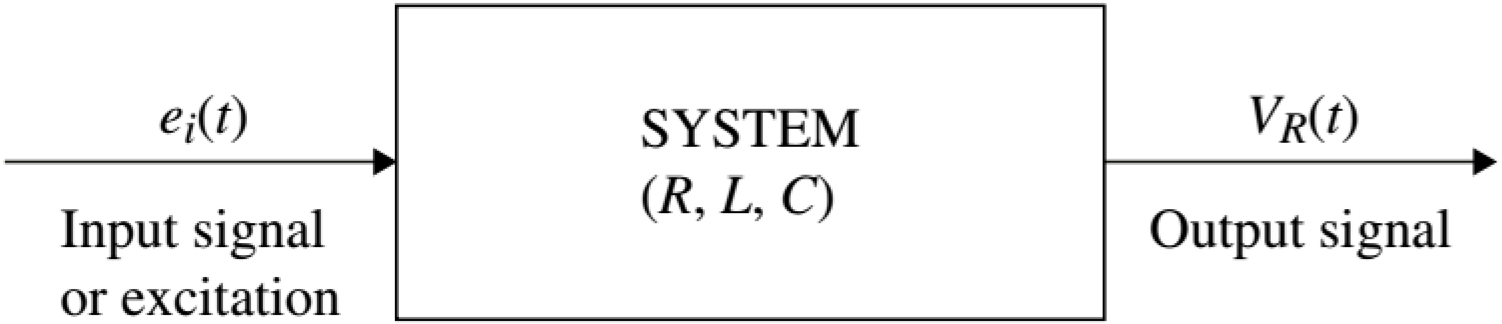
Introduction

- Signals: represent some independent variables that contain some information about the behavior of some natural phenomenon.
- When these signals are operated on some objects, they give out signals in the same or modified form. These objects are called systems.

Definitions :

- Signal: A signal is defined as a physical phenomenon that carries some information or data.
- The signals are usually functions of independent variable time.
- There are some cases where the signals are not functions of time.
- The electrical charge distributed in a body is a signal which is a function of space and not time.

- System: A system is defined as the set of interconnected objects with a definite relationship between objects and attributes.
- The inter-connected components provide desired function.

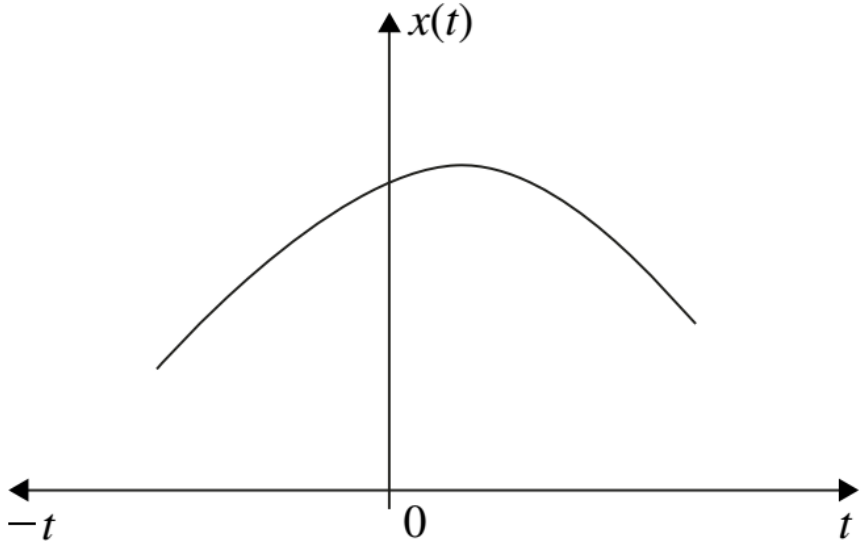


Signals are broadly classified as follows:

- 1. Continuous Time signal (CT signal).
- 2. Discrete Time signal (DT signal).

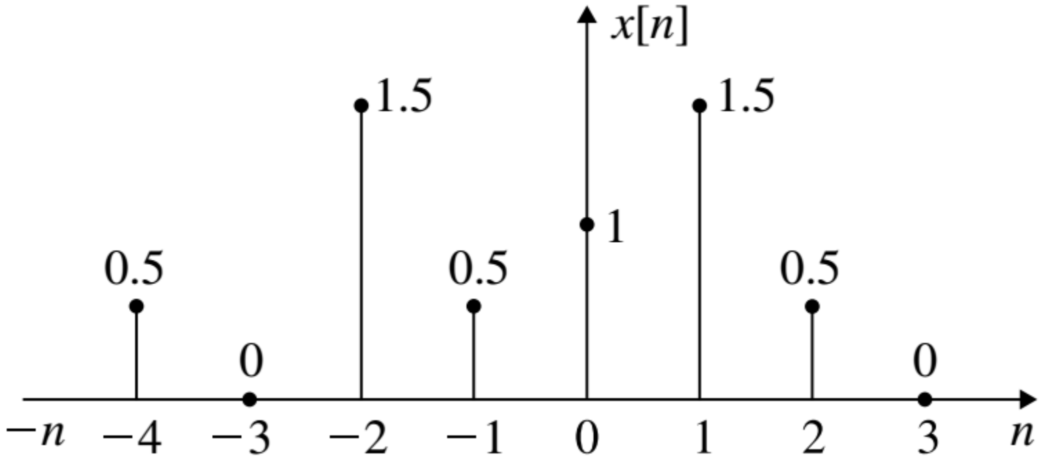
Continuous time signal

- The signal that is specified for every value of time t is called continuous time signal
- denoted by x(t)



Discreet time signal

- The discrete time signal is represented as
- a sequence of numbers and is denoted by x[n] where n is an integer. Here time t is divided into n discrete time intervals.



A discrete time signal $x[n]$ is represented by the following two methods:

1.

$$x[n] = \begin{cases} \left(\frac{1}{a}\right)^n & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{1.1}$$

Substituting various values of n where $n \geq 0$ in Eq. (1.1) the sequence for $x[n]$ which is denoted by $x\{n\}$ is written as follows:

$$x[n] = \left\{1, \frac{1}{a}, \frac{1}{a^2}, \ldots, \frac{1}{a^n}\right\}$$

2. The sequence is also represented as given below.

$$x[n] = \{3,\ 2,\ \underset{\uparrow}{5},\ 4,\ 6,\ 8,\ 2\}$$

The arrow indicates the value of $x[n]$ at $n = 0$ which is 5 in this case. The numbers to the left of the arrow indicate to the negative sequence $n = -1, -2$, *etc.* The numbers to the right of the arrow correspond to $n = 1, 2, 3, 4$, *etc.* Thus, for the above sequence, $x[-1] = 2$; $x[-2] = 3$; $x[0] = 5$; $x[1] = 4$; $x[2] = 6$; $x[3] = 8$ and $x[4] = 2$. If no arrow is marked for a sequence, the sequence starts from the first term in the extreme left. Consider the sequence

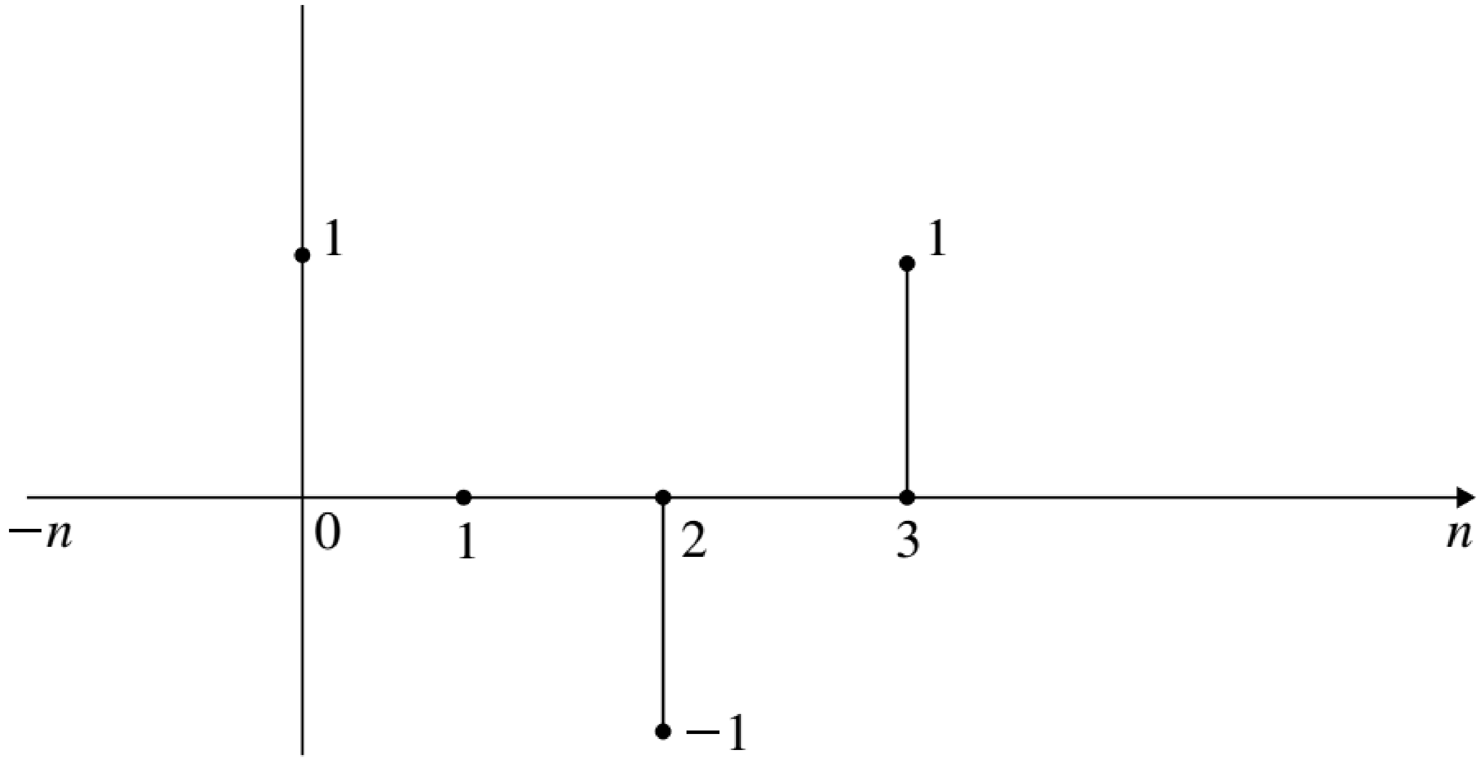
$$x[n] = \{5,\ 3,\ 4,\ 2\}.$$

Here, $x[0] = 5$; $x[1] = 3$; $x[2] = 4$ and $x[3] = 2$. There is no negative sequence here.

■ Example 1.1

Graphically represent the following sequence:

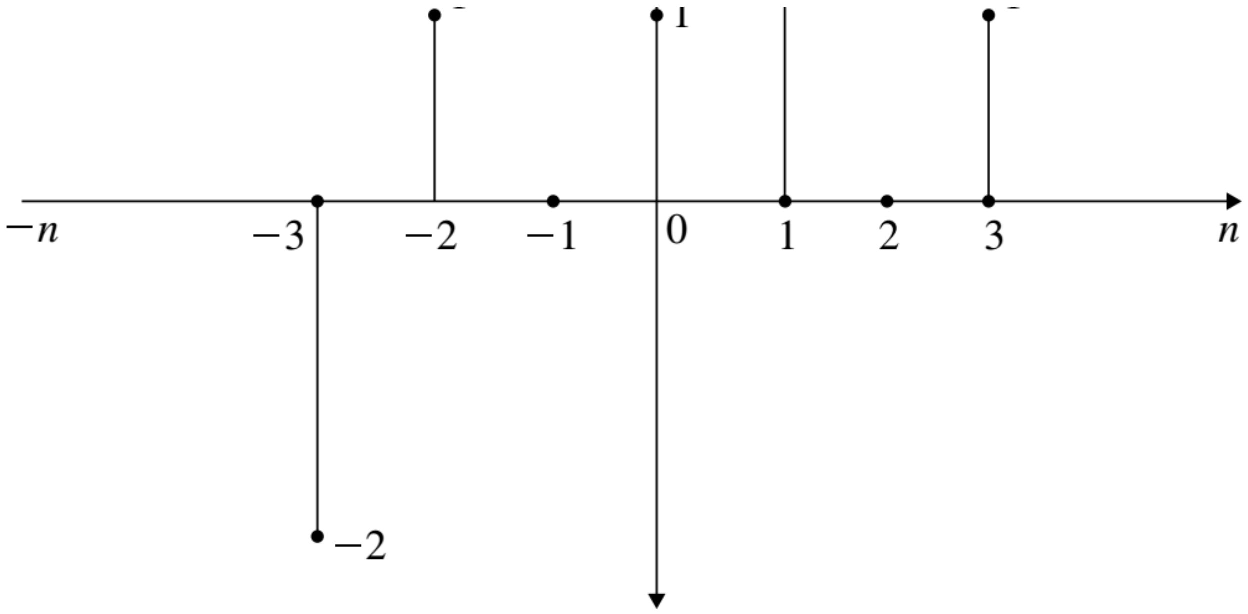
$$x[n] = \{1,\ 0,\ -1,\ 1\}$$



■ Example 1.2

Graphically represent the following sequence:

$$x[n] = \{-2,\ 1,\ 0,\ \underset{\uparrow}{1},\ 2,\ 0,\ 1\}$$

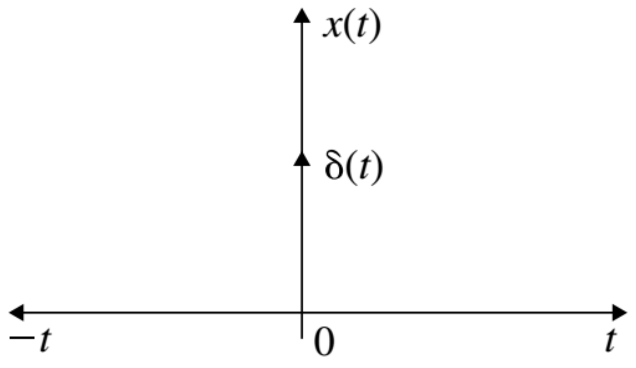


Basic Continuous Time Signals

- Unit Impulse Function

The unit impulse function is also known as **Dirac delta** function which is represented in Fig. 1.6. The unit impulse function is denoted as $\delta(t)$ and its mathematical description is given below.

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ 1 & t = 0 \end{cases} \tag{1.2}$$



Importance of Impulse Function

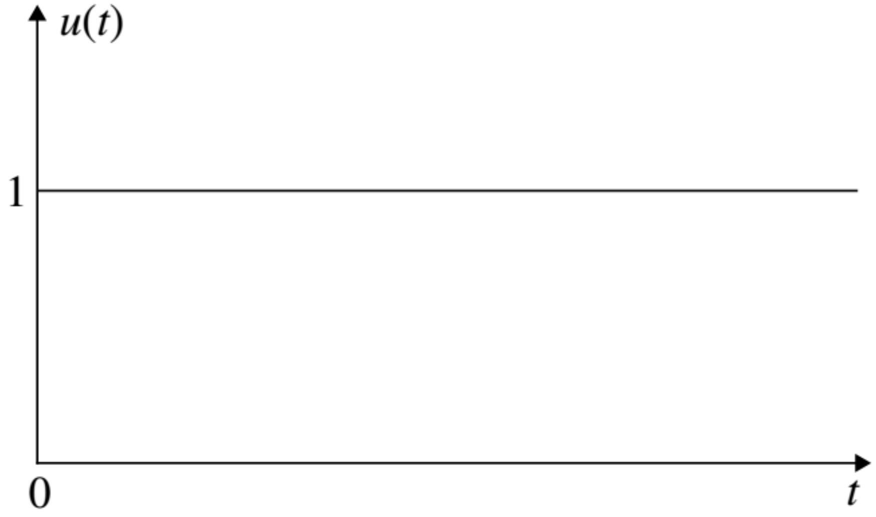
- 1. By applying impulse signal to a system, one can get the impulse response of the system. From impulse response, it is possible to get the transfer function of the system.
- 2. For a linear time invariant system, if the area under the impulse response curve is finite, then the system is said to be stable.
- 3. From the impulse response of the system, one can easily get the step response and ramp response by integrating it once and twice, respectively.
- 4. Impulse signal is easy to generate and apply to any system.

Properties of Impulse Function

1. $\delta(at) = \frac{1}{a}\delta(t)$
2. $\delta(-t) = \delta(t)$
3. $x(t)\delta(t) = x(0)\delta(t)$
4. $x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$
5. $\int_{-\infty}^{\infty} \delta(t)dt = 1$
6. $t\delta(t) = 0$
7. $t\frac{d\delta(t)}{dt} = -\delta(t)$
8. $x(t) * \delta(t - t_0) = x(t - t_0)$

Unit Step Function

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



The step function is denoted by $u(t)$. Any causal signal which begins at $t = 0$ (which has a value of zero for $t < 0$) is multiplied by the signal by $u(t)$. For example, **a causal exponentially decaying signal e^{-at} $(t \geq 0)$ is represented as $x(t) = e^{-at}u(t)$. Similarly e^{-at} $(t < 0)$ is represented as $x(t) = e^{-at}u(-t)$.**

Importance of Step Function

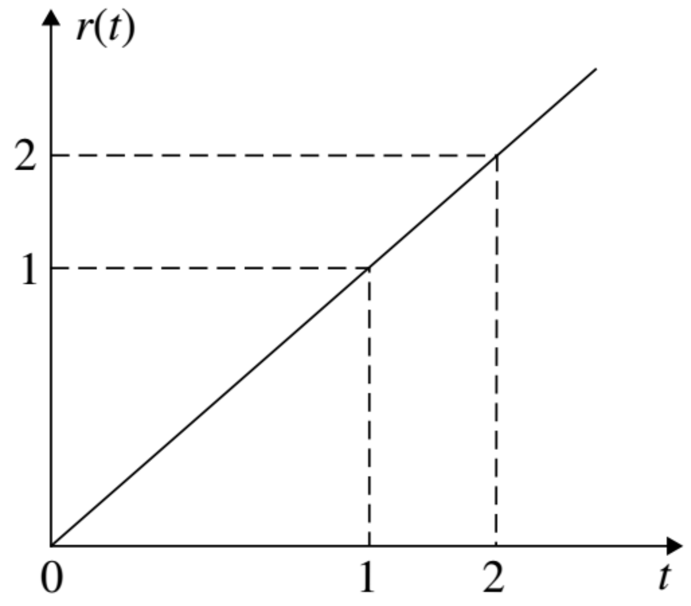
- 1-Step function is easy to generate and apply to the system.
- 2. By differentiating the step response, the impulse response can be obtained. By integrating the step response, the ramp response can be obtained.
- 3. Step signal is considered as a white noise which is drastic. If the system response is satisfactory for a step signal, it is likely to give a satisfactory response to other types of signals.
- 4. Application of step signal is equivalent to the application of numerous sinusoidal signals with a wide range of frequencies

Unit Ramp Function

$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

For a causal signal ($t \geq 0$), the ramp function can also be expressed as

$$r(t) = t\,u(t)$$



Relationships between Impulse, Step and Ramp Signals

$$\int u(t)dt = \int dt = t$$

$$\frac{du(t)}{dt} = \delta(t)$$

$$\frac{d^2 r(t)}{dt^2} = \frac{du(t)}{dt} = \delta(t)$$

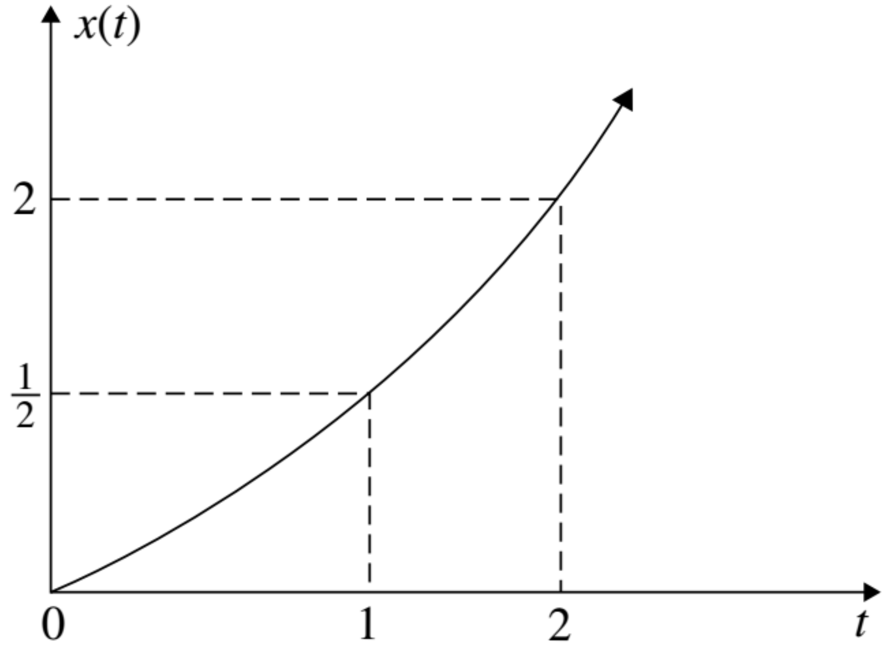
$$r(t) = \int\int \delta(t)\, dt$$

$$\delta(t) \xrightarrow{\text{integrate}} u(t) \xrightarrow{\text{integrate}} r(t)$$

$$r(t) \xrightarrow{\text{differentiate}} u(t) \xrightarrow{\text{differentiate}} \delta(t)$$

Unit Parabolic Function

$$\boxed{x(t) = \frac{1}{2}t^2} \qquad t \geq 0$$



$$\frac{dx(t)}{dt} = t \qquad t \geq 0.$$

Step, ramp and parabolic functions are called singularity functions.

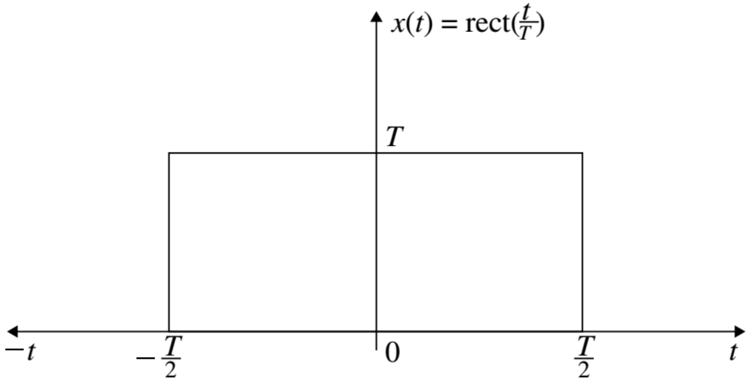
Unit Rectangular Pulse (or Gate) Function

$$x(t) = \begin{cases} 1 & \text{for } |t| \leq \frac{T}{2} \\ 0 & \text{otherwise} \end{cases}$$

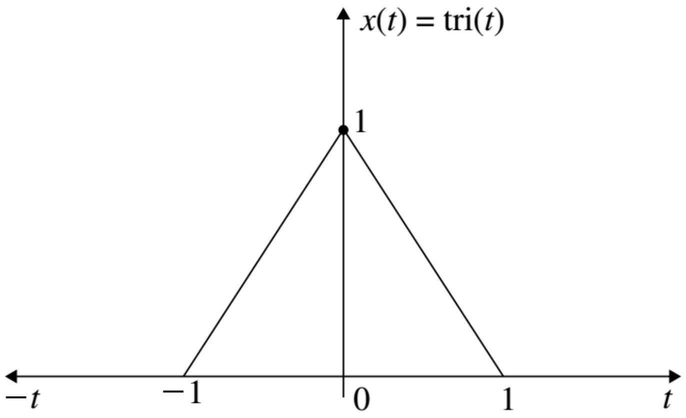
The above equation is also written in the following form:

$$x(t) = 1 \qquad -\frac{T}{2} \leq t \leq \frac{T}{2}$$

The function is written as $x(t) = \text{rect}(\frac{t}{T})$.



Unit Area Triangular Function



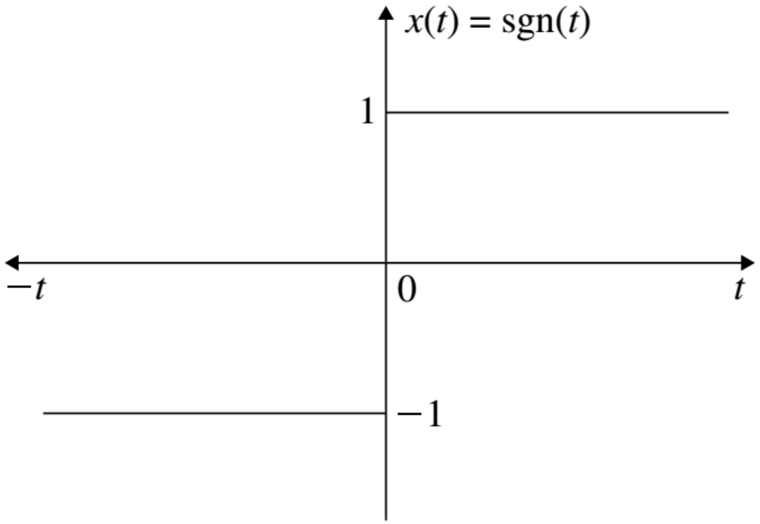
The unit area triangular function is represented in Fig. 1.11. It is symbolically written as $x(t) = \text{tri}(t)$. It is defined as

$$\text{tri}(t) = \begin{cases} [1 - |t|] & |t| \leq 1 \\ 0 & |t| > 1 \end{cases} \tag{1.13}$$

The above equation can be written in the following form also:

$$\begin{aligned} \text{tri}(t) &= [1 + t] \qquad -1 \leq t \leq 0 \\ &= [1 - t] \qquad 0 \leq t \leq 1 \end{aligned}$$

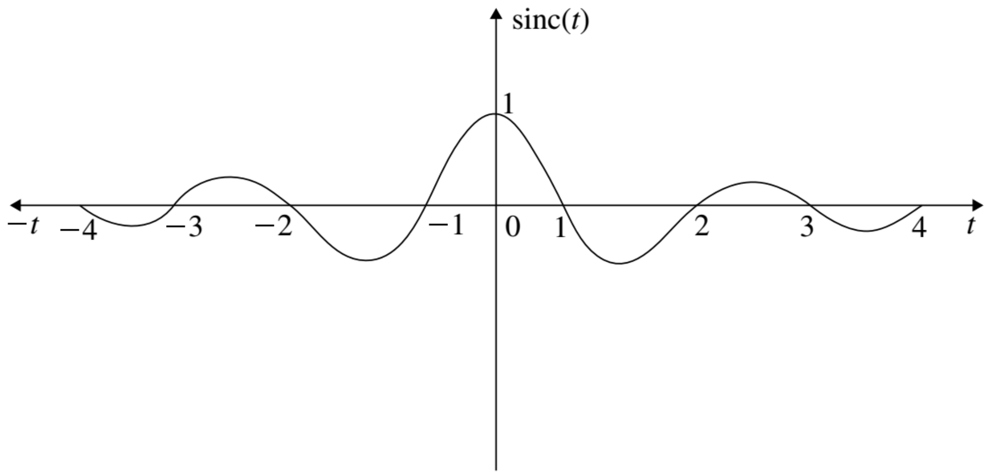
Unit Signum Function



The signum function is written in the abbreviated form as sgn(t). It represents the characteristics of an ideal relay. This is shown in Fig. 1.12. It is defined by the following equations:

$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases} \tag{1.14}$$

Unit Sinc Function



The unit sinc function is represented in Fig. 1.13. It is defined as

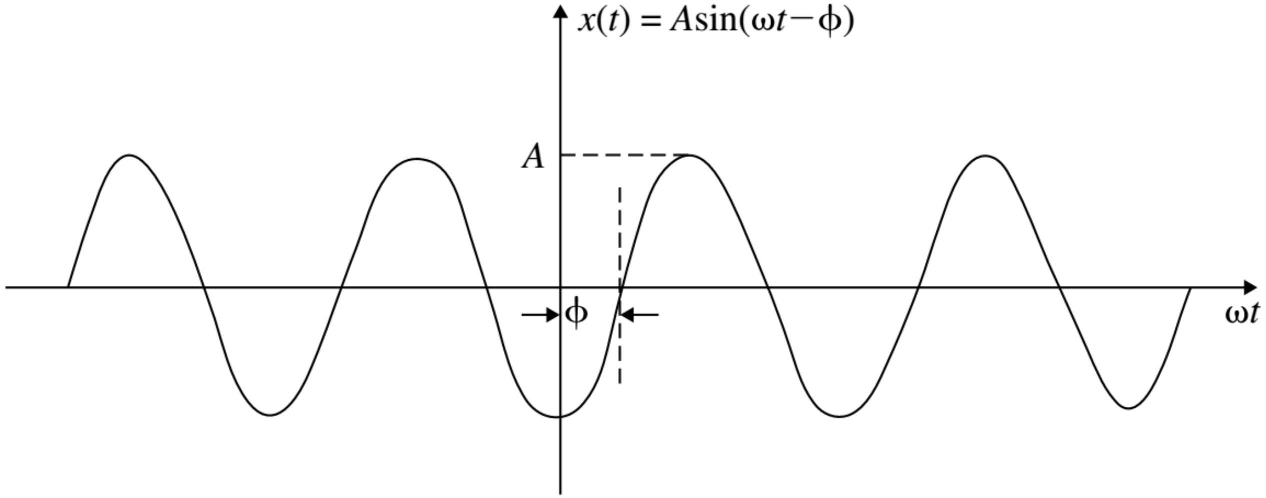
$$\text{sinc}(t) = \frac{\sin \pi t}{\pi t} \qquad -\infty < t < \infty.$$

Sinusoidal Signal

The sinusoidal signal is represented in Fig. 1.14. It is defined as

$$x(t) = A\sin(\omega t - \phi)$$

where $A =$ Peak amplitude, $\omega =$ radian frequency and $\phi =$ phase shift.



Real Exponential Signal

$$x(t) = e^{st}$$

where $s = \sigma + j\omega$ is a complex number. The signal $x(t)$ in Eq. (1.17) is called general complex exponential. Equation (1.17) is written in the following form:

$$\begin{aligned} x(t) &= e^{(\sigma + j\omega)t} \\ &= e^{\sigma t} e^{j\omega t} \\ &= e^{\sigma t}(\cos \omega t + j \sin \omega t) \end{aligned} \tag{1.18}$$

If $\omega = 0$,

$$x(t) = e^{\sigma t}$$

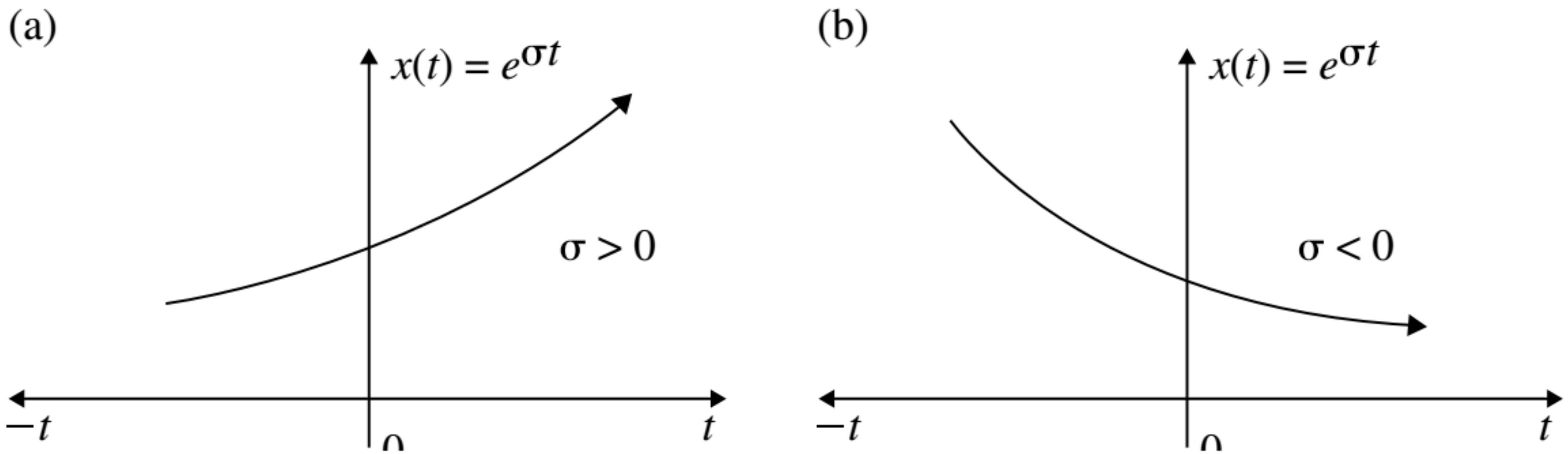


Fig. 1.15 Representation of real exponential signals. **a** Growing exponential; **b** Decaying exponential

Equation (1.19) is real exponential. The plot of $x(t)$ with respect to t for $\sigma > 0$ and $\sigma < 0$ is shown in Fig. 1.15a and b, respectively. For $\sigma > 0$, the signal is exponentially growing and for $\sigma < 0$, it is exponentially decaying.

Complex Exponential Signal

$$x(t) = e^{-\sigma t}(\cos \omega t + j \sin \omega t)$$

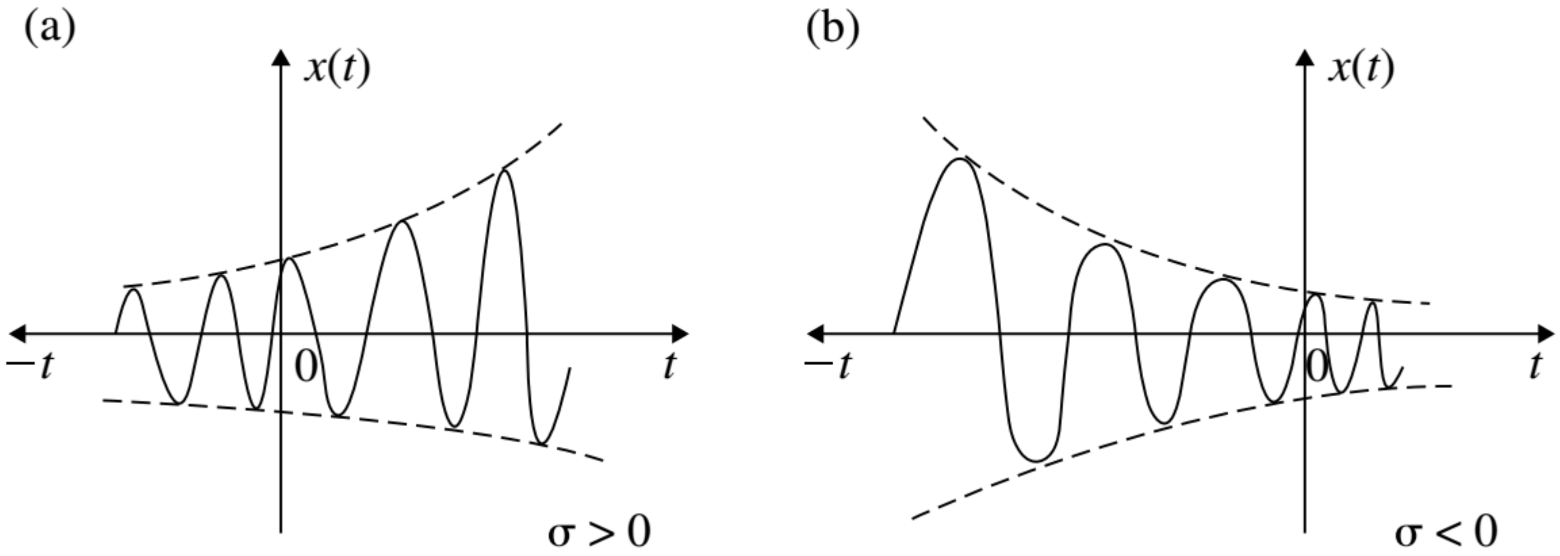
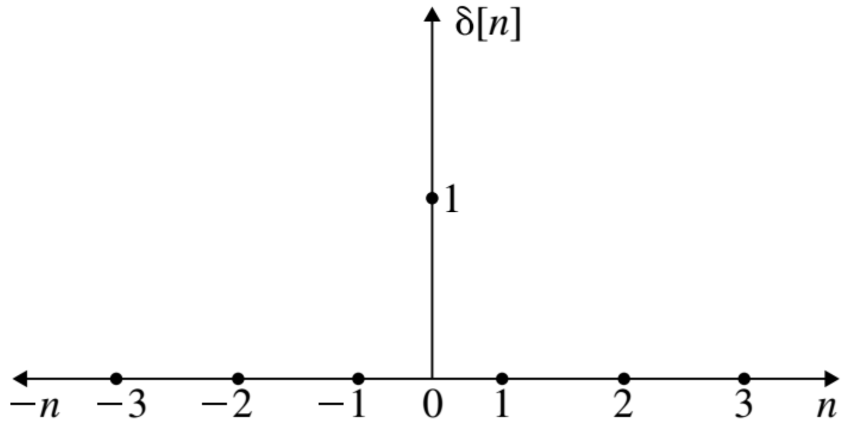


Fig. 1.16 Complex exponential signals. **a** Exponentially growing ($\sigma > 0$); **b** Exponentially decaying ($\sigma < 0$)

Basic Discrete Time Signals

The Unit Impulse Sequence



The basic impulse sequence is shown in Fig. 1.17. The unit impulse sequence also called sample is defined as

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \tag{1.20}$$

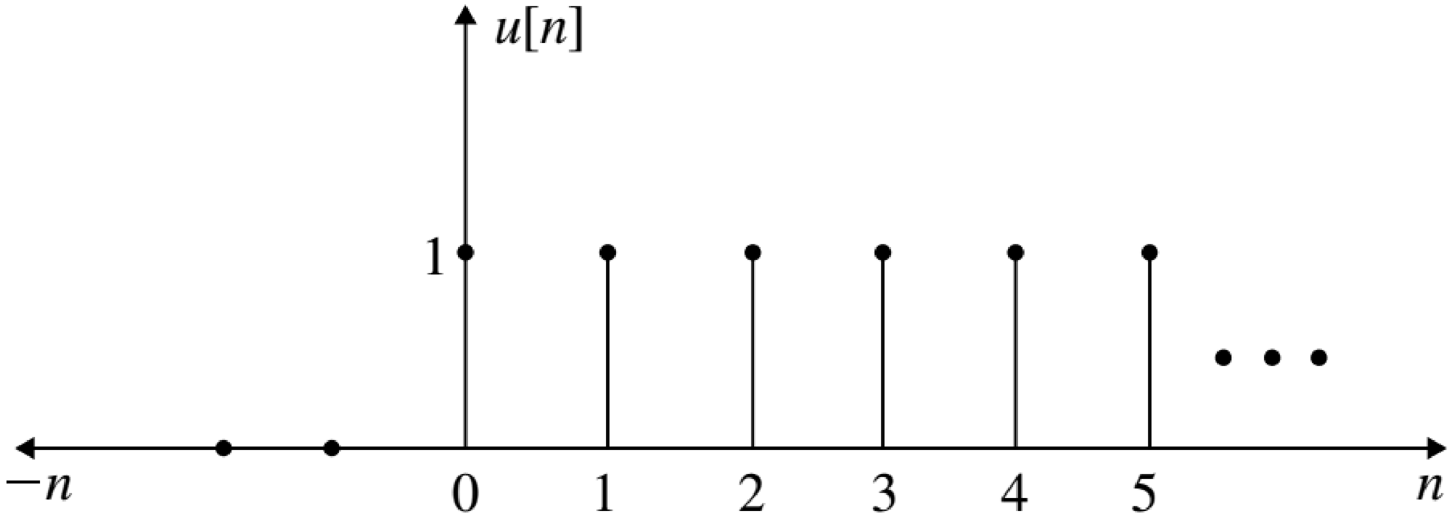
$\delta[n]$ **is also called Kronicker delta function.**

The Basic Unit Step Sequence

The basic unit step sequence is represented in Fig. 1.18. It is denoted by $u(n)$. It is defined as

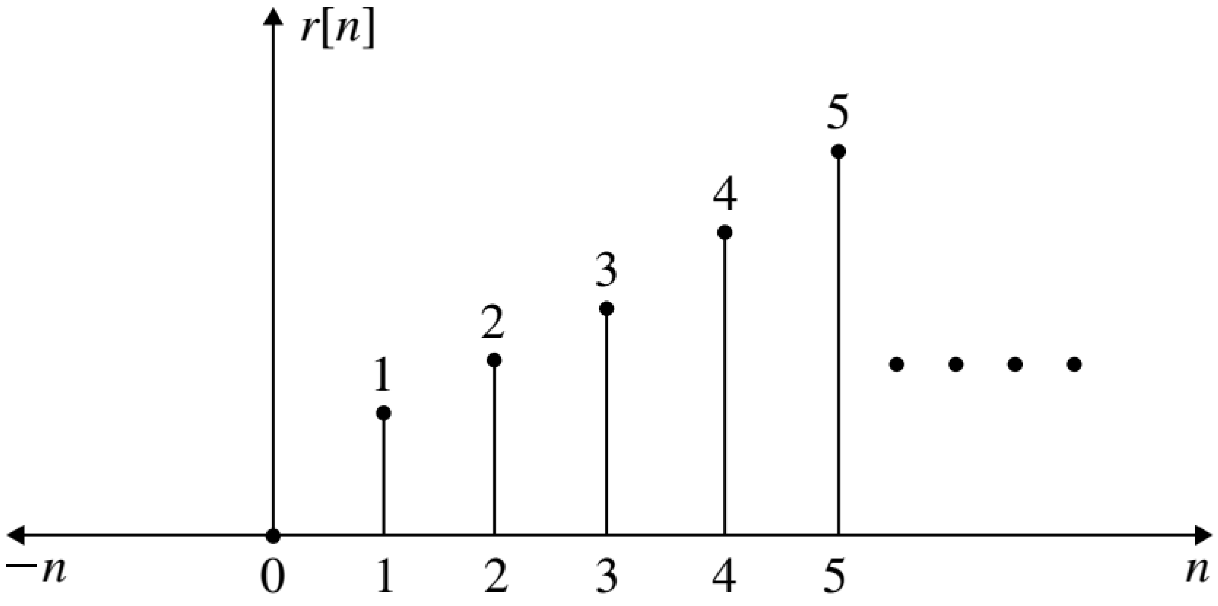
$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{1.21}$$

Any discrete sequences $x[n]$ for $n \geq 0$ is expressed as $x[n]u[n]$. For $n < 0$, it is expressed as $x[n]u[-n]$. It is be noted that at $n = 0$, the value of $u[n] = 1$.



The Basic Unit Ramp Sequence

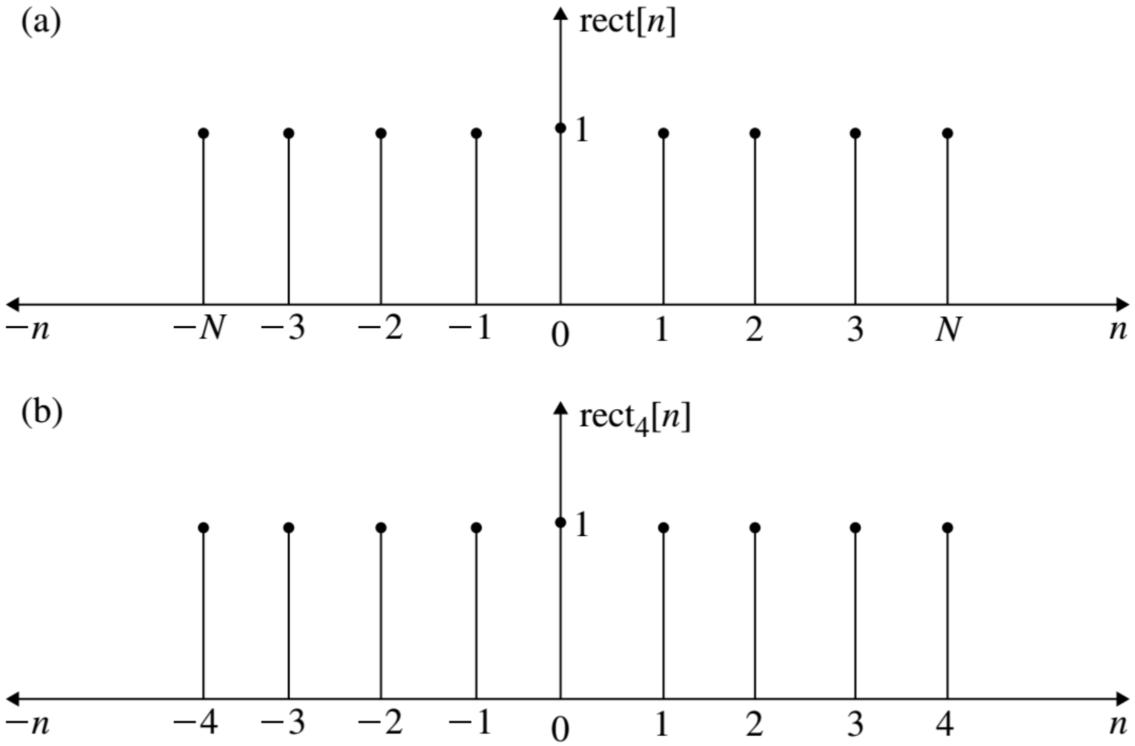
$$r[n] = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$



Unit Rectangular Sequence

$$\text{rect}[n] = \begin{cases} 1 & |n| \le N \\ 0 & |n| > N \end{cases}$$

$$\text{rect}[n] = \begin{cases} 1 & -N \le n \le N \\ 0 & \text{otherwise} \end{cases}$$



Sinusoidal Sequence

$$x[n] = Ae^{-\alpha n} \sin(\omega_0 n + \phi)$$

- A purely sinusoidal sequence ($\alpha = 0$).
- Decaying sinusoidal sequence ($\alpha > 0$).
- Growing sinusoidal sequence ($\alpha < 0$).

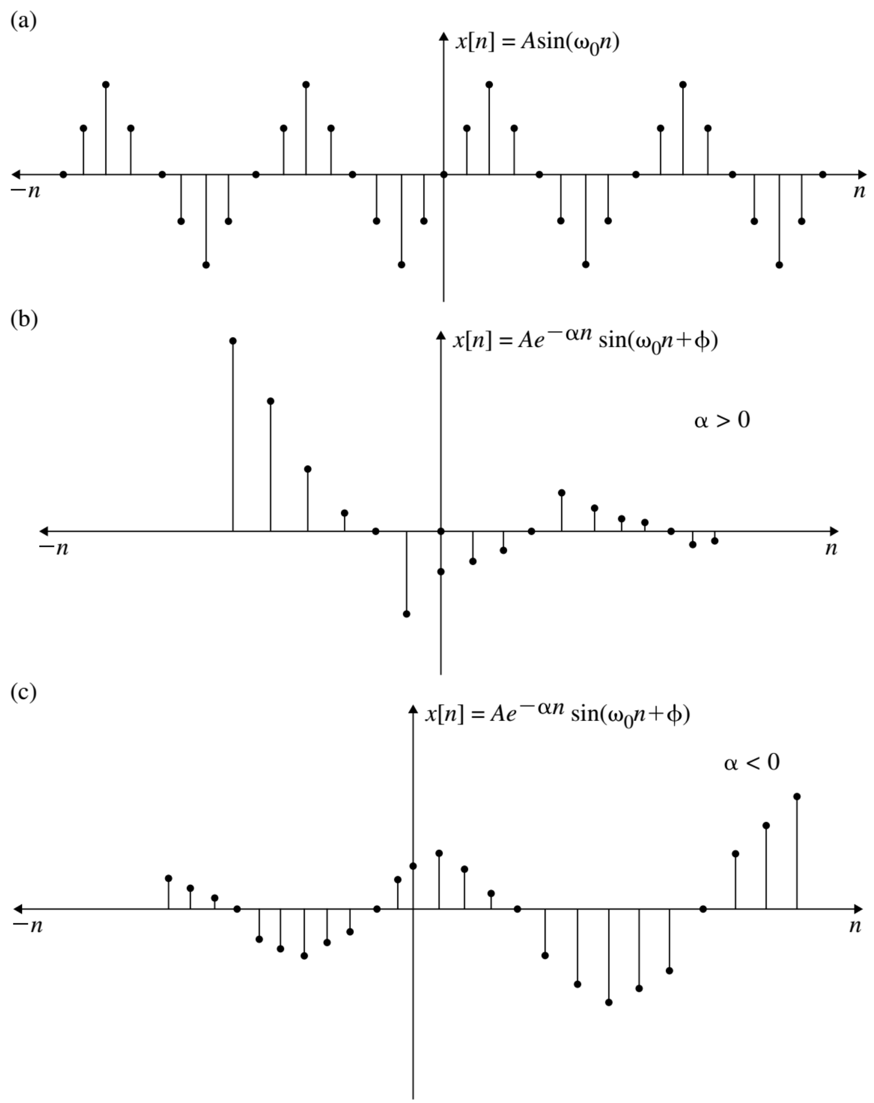


Fig. 1.21 Discrete time sinusoidal signal. **a** Purely sinusoidal; **b** Decaying sinusoidal; **c** Growing sinusoidal

Discrete Time Real Exponential Sequence

The general complex exponential sequence is defined as

$$x[n] = A\alpha^n$$

where A and α are in general complex numbers.

if A and α are real, the sequence is called real exponential.

1. Exponentially growing signal ($\alpha > 1$, Fig. 1.22a).
2. Exponentially decaying signal ($0 < \alpha < 1$, Fig. 1.22b).
3. Exponentially growing for alternate value of n ($\alpha < -1$, Fig. 1.22c).
4. Exponentially decaying for alternate value of n ($-1 < \alpha < 0$, Fig. 1.22d).

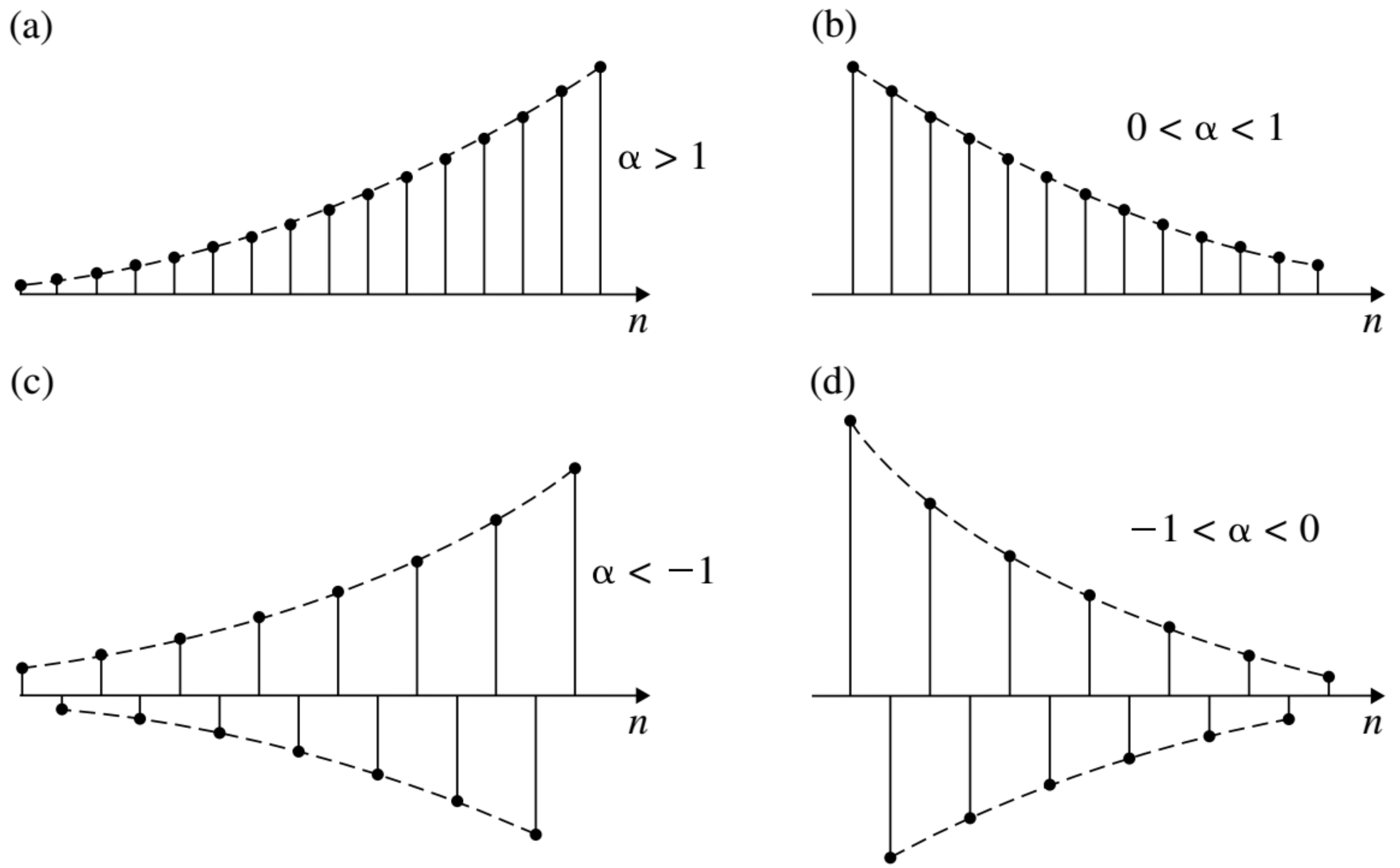
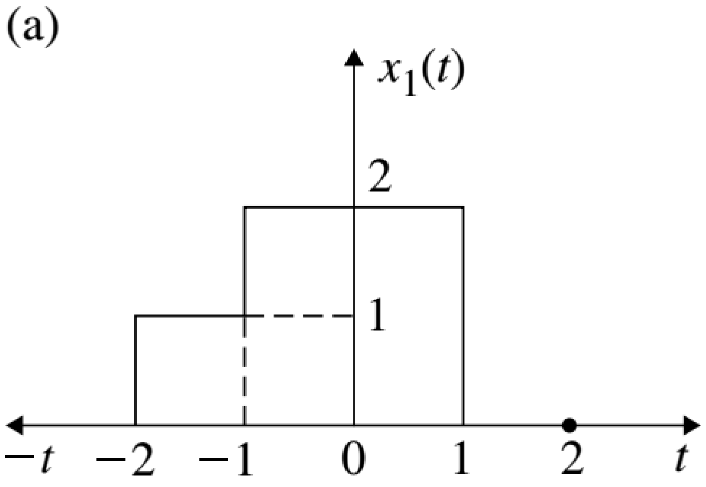
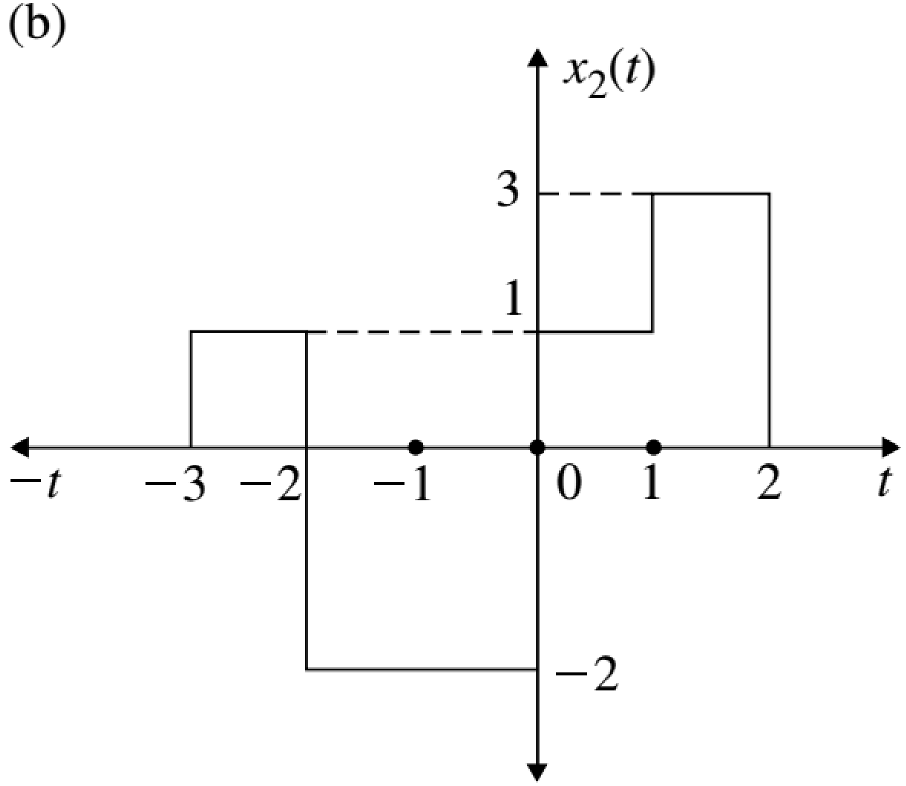


Fig. 1.22 Discrete time real exponential sequences. **a** $\alpha > 1$; **b** $0 < \alpha < 1$; **c** $\alpha < -1$; **d** $-1 < \alpha < 0$

Basic Operations on Continuous Time Signals

Addition of CT Signals

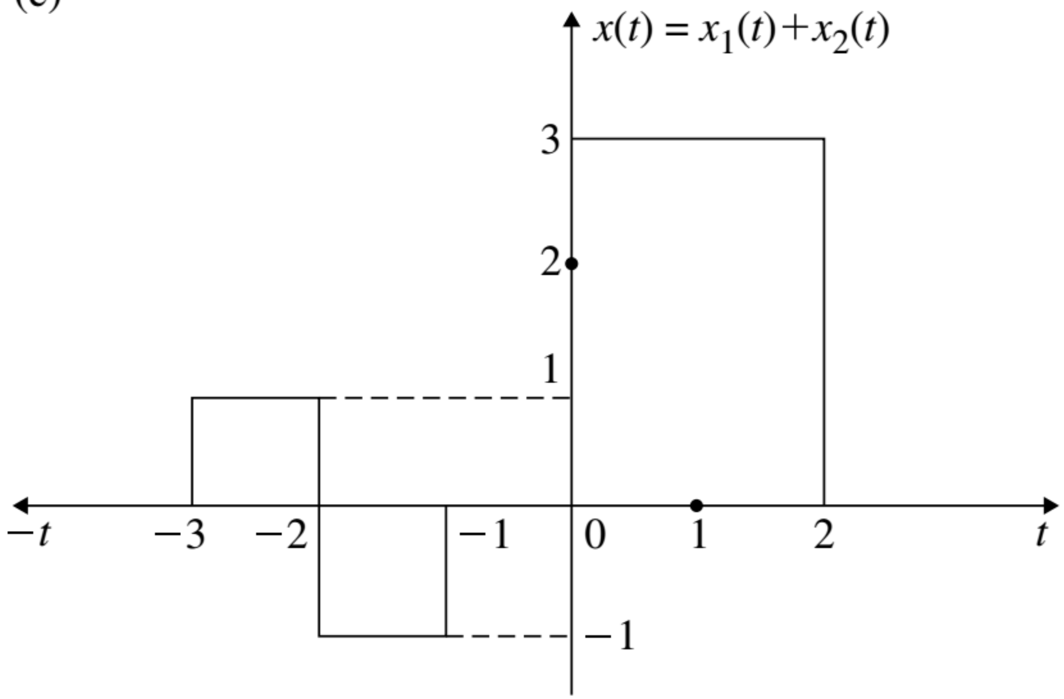




$$x(t) = x_1(t) + x_2(t)$$

t	-3	-2	-1	0	1	2
$x_1(t)$	0	1	2	2	0	0
$x_2(t)$	1	-2	-2	1	3	0
$x(t) = x_1(t) + x_2(t)$	1	-1	0	3	3	0

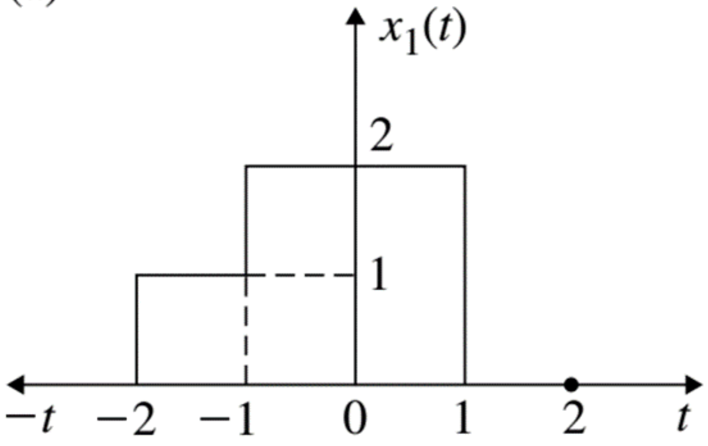
(c)



Multiplications of CT Signals

$$x(t) = x_1(t) \times x_2(t)$$

(a)



(b)

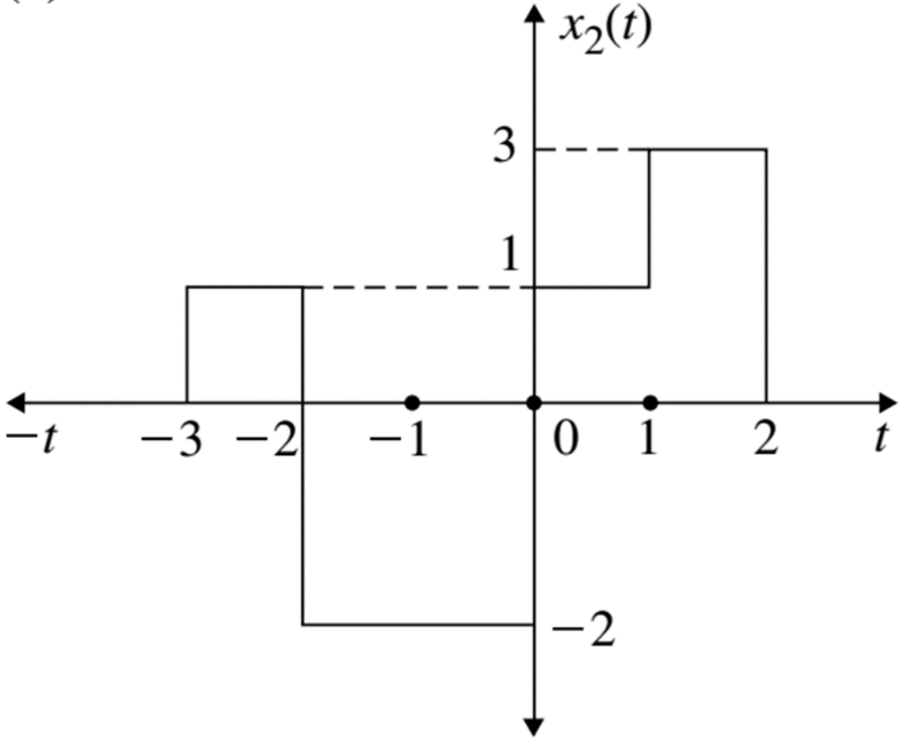
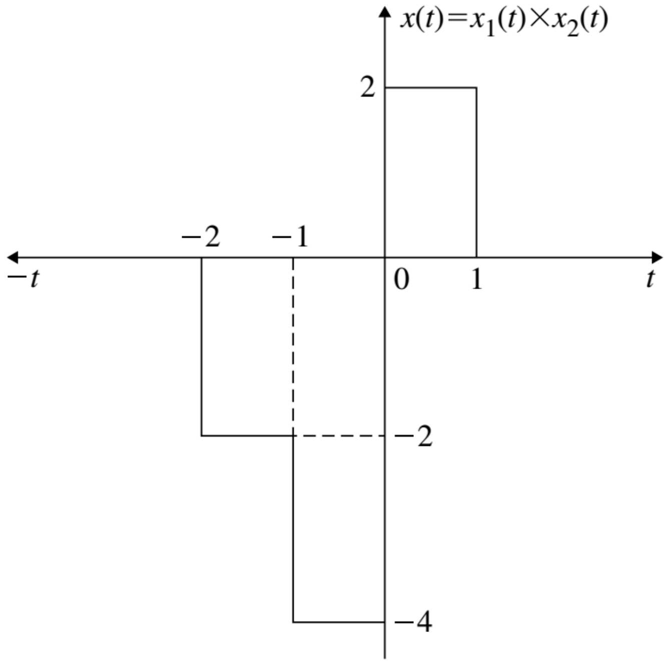


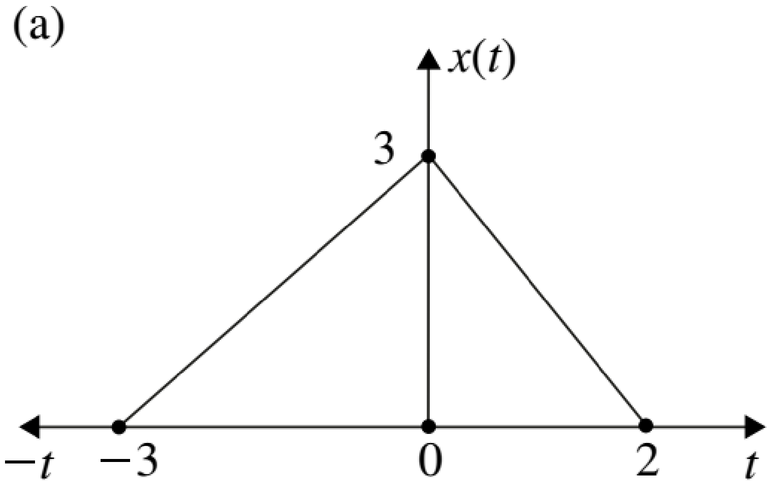
Table 1.2 Multiplication of two CT signals

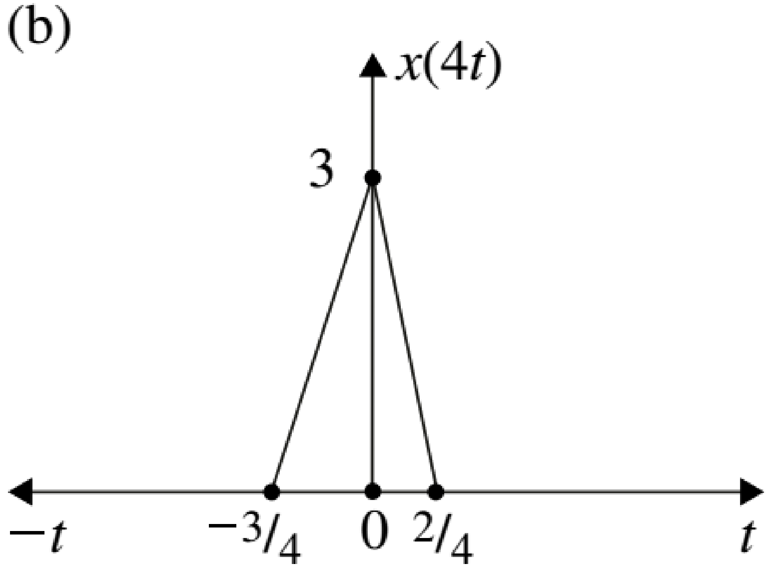
t	-3	-2	-1	0	1	2
$x_1(t)$	0	1	2	2	0	0
$x_2(t)$	1	-2	-2	1	3	0
$x(t) = x_1(t) \times x_2(t)$	0	-2	-4	2	0	0



Time Scaling of CT Signals

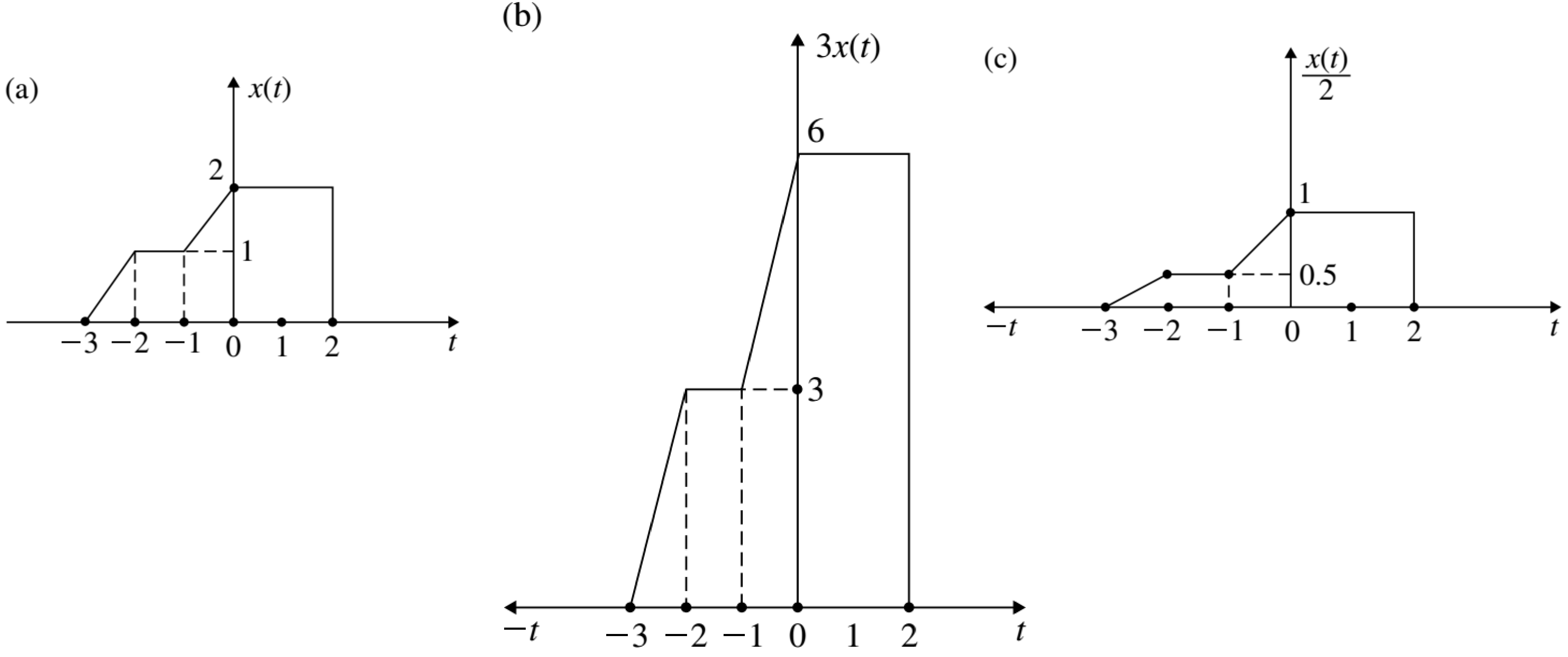
- The compression or expansion of a signal in time is known as time scaling





$x(at)$ is time compressed by a factor a and $x(\frac{t}{a})$ is time expanded by a factor a.

Amplitude Scaling of CT Signals



(c)

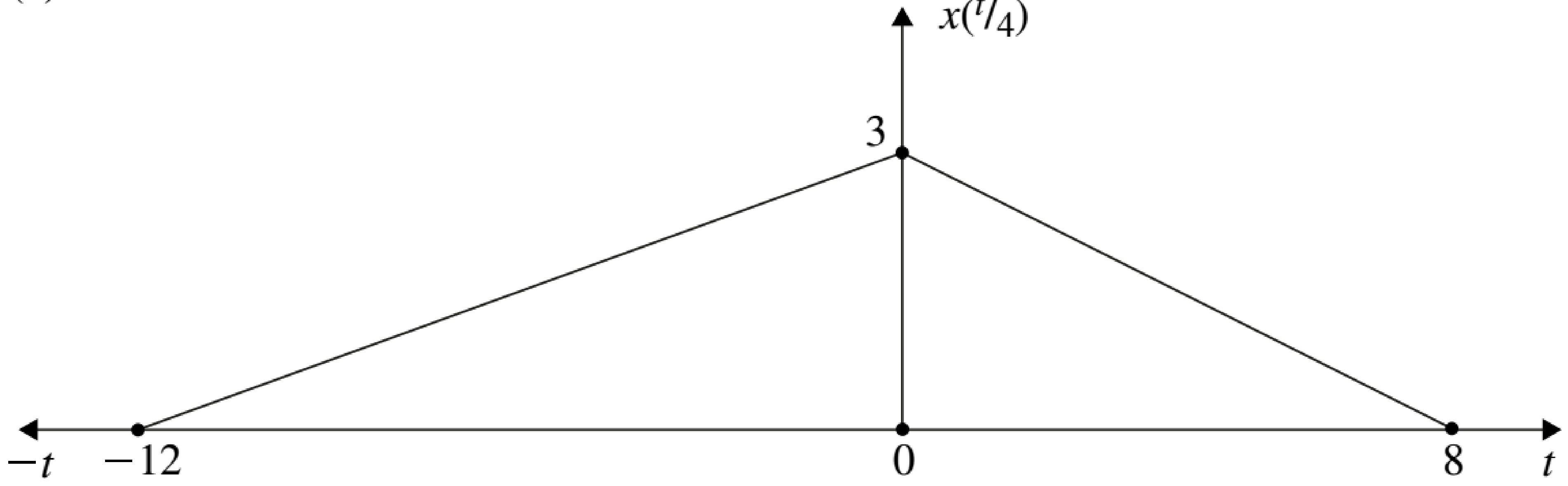
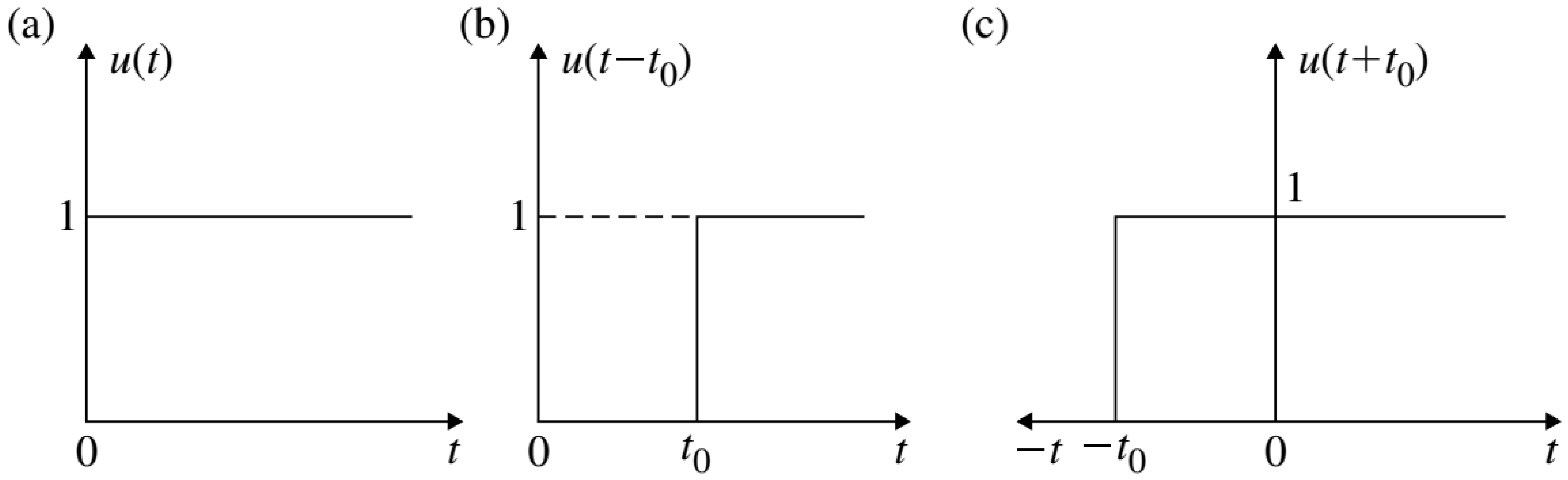


Fig. 1.26 Time scaling of CT signals

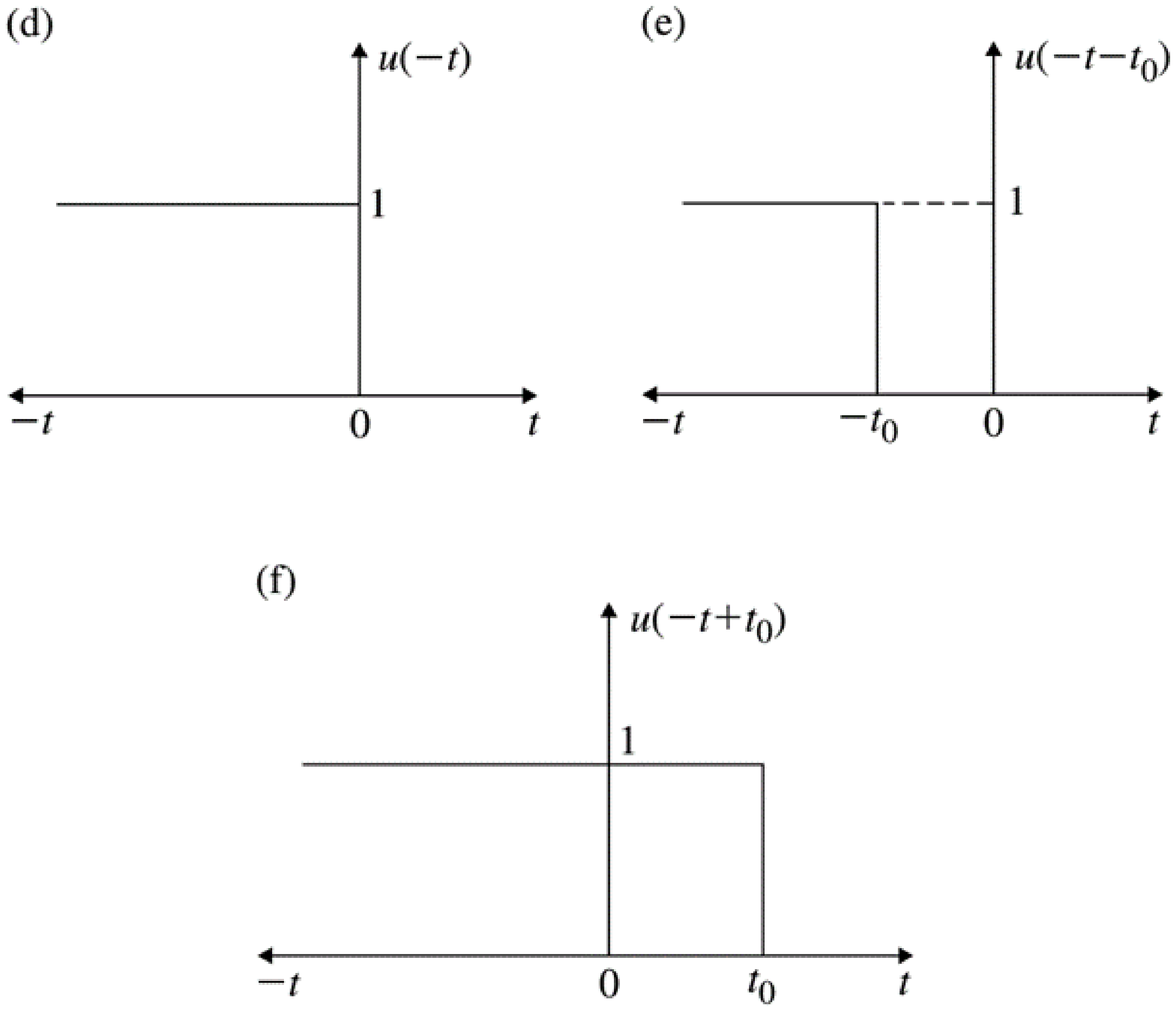
Time Shifting of CT Signals

Summary of Shifting of CT signal

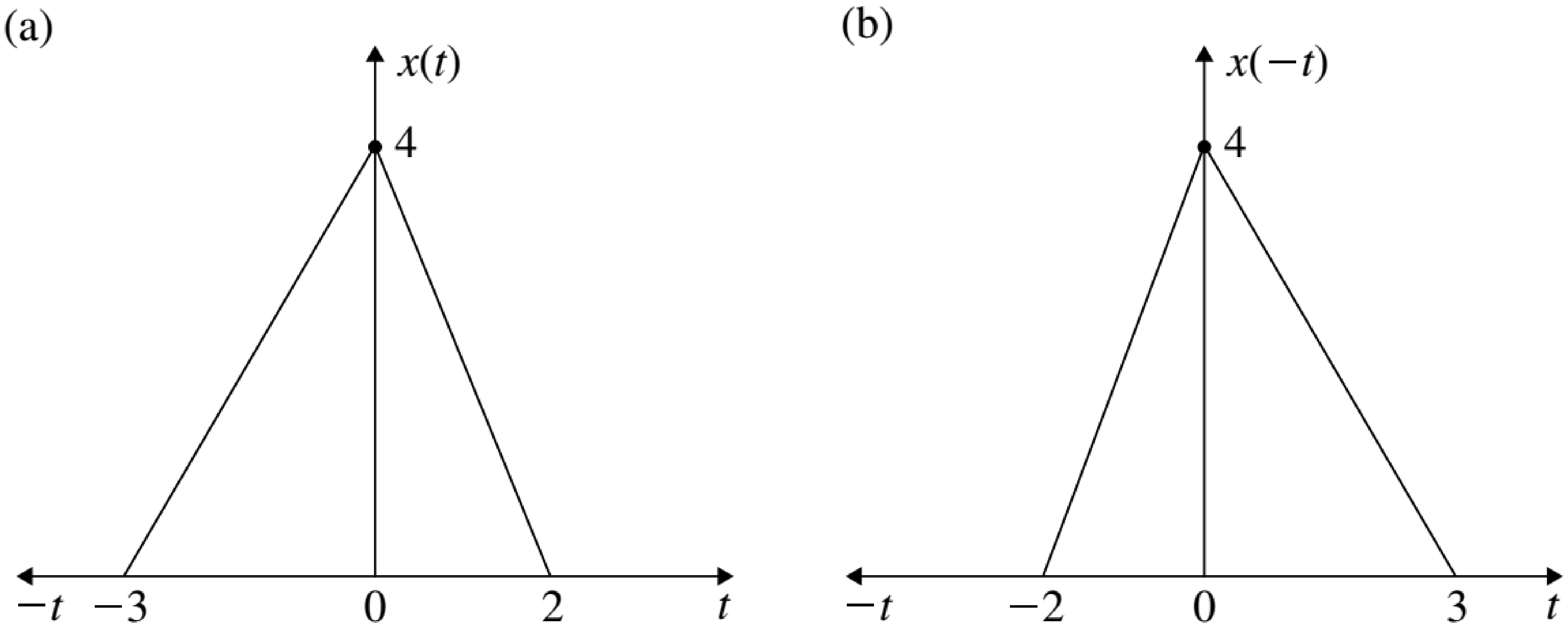
1. **It $x(t)$ is given, then $x(t+t_0)$ is plotted by shifting $x(t)$ to the left by t_0.**
2. **It $x(t)$ is given, then $x(t-t_0)$ is plotted by shifting $x(t)$ to the right by t_0.**
3. **It $x(-t)$ is given, then $x(-t-t_0)$ is plotted by shifting $x(-t)$ to the left by t_0.**
4. **It $x(-t)$ is given, then $x(-t+t_0)$ is plotted by shifting $x(-t)$ to the right by t_0.**
5. **In general for $x(t+t_0)$ and $x(-t-t_0)$ the time shift is made to the left of $x(t)$ and $x(-t)$, respectively, by t_0. For $x(t-t_0)$ and $x(-t+t_0)$ the time shift is made to the right of $x(t)$ and $x(-t)$, respectively, by t_0.**



(a)
u(t)
1
0
t
(b)
u(t−t0)
1
0
t0
t
(c)
u(t+t0)
1
−t
−t0
0
t

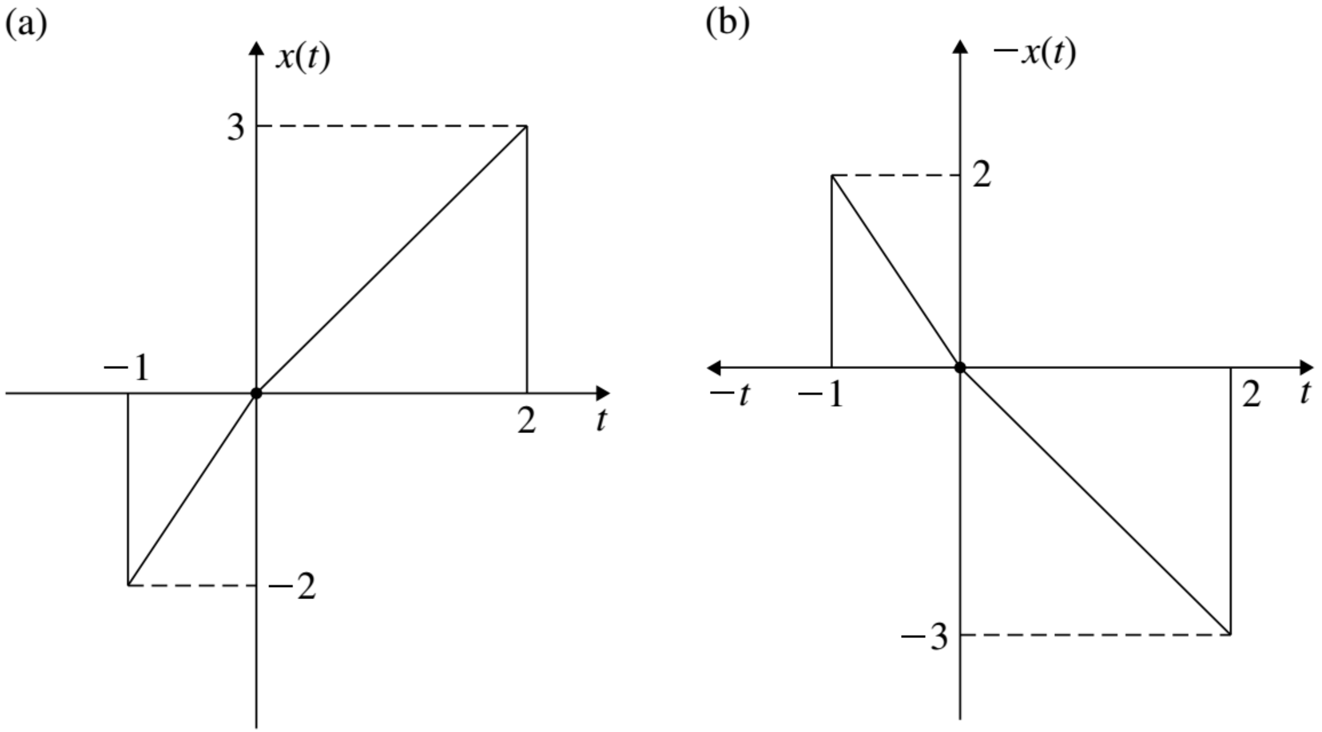

(d)
$u(-t)$
1
$-t$
0
t
(e)
$u(-t-t_0)$
1
$-t$
$-t_0$
0
t
(f)
$u(-t+t_0)$
1
$-t$
0
t_0
t

Signal Reflection or Folding



Inverted CT Signal

- The inverted signal −x(t) is obtained by inverting its amplitude. By this the signal above the horizontal axis (time axis) comes below the axis and vice versa.



Multiple Transformation

Consider the following signal:

$$y(t) = Ax\left(\frac{-t - t_0}{a}\right)$$

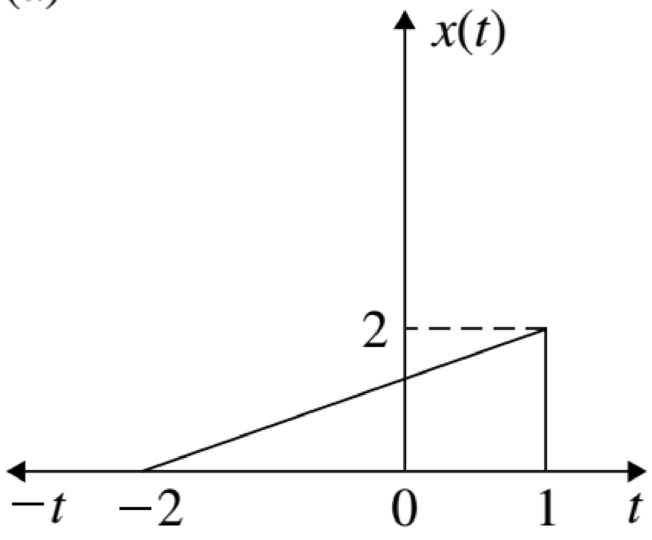
The following sequence of transformation is followed:

1. $y(t)$ is written in the following form:

$$y(t) = Ax\left(-\frac{t}{a} - \frac{t_0}{a}\right)$$

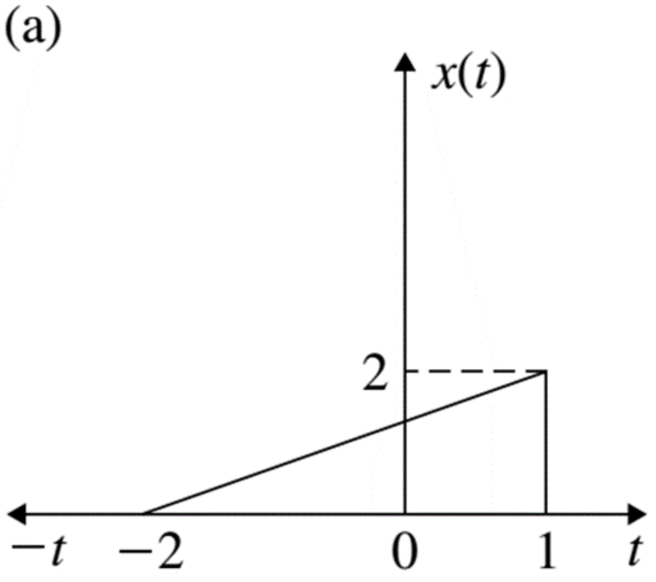
2. Plot $x(t)$.
3. Plot $Ax(t)$ using amplitude scaling.
4. Plot $Ax(-t)$ using time reversal.
5. Plot $Ax(-t - \frac{t_0}{a})$ by shifting $Ax(-t)$ to the left by $\frac{t_0}{a}$ (time shifting).
6. Plot $Ax(-\frac{t}{a} - \frac{t_0}{a})$ by time expansion.

(a)



■ Example 1.3

Consider the signal $y(t) = 5x(-3t + 1)$ where $x(t)$ is shown in Fig. 1.30a. Plot $y(t)$ and $-y(t)$.

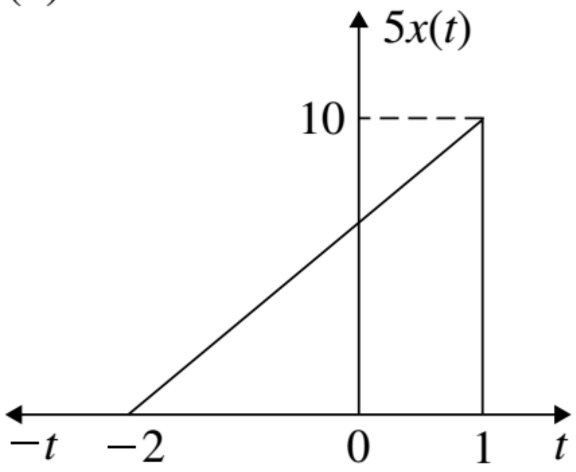


Solution:

$$y(t) = 5x(-3t + 1)$$

1. The given signal $x(t)$ is represented in Fig. 1.30a.
2. The signal $x(t)$ is amplitude scaled and plotted in Fig. 1.30b.

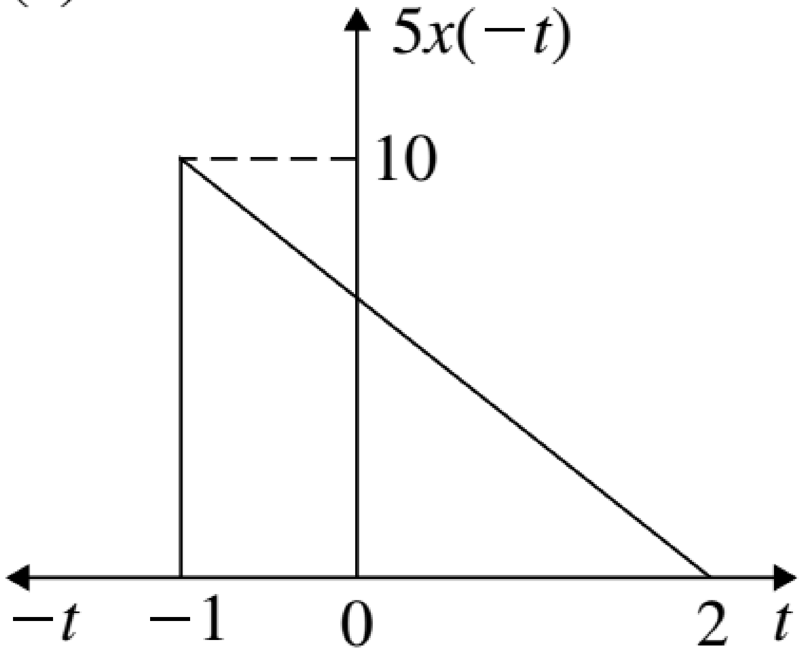
(b)



Amplitude scaling

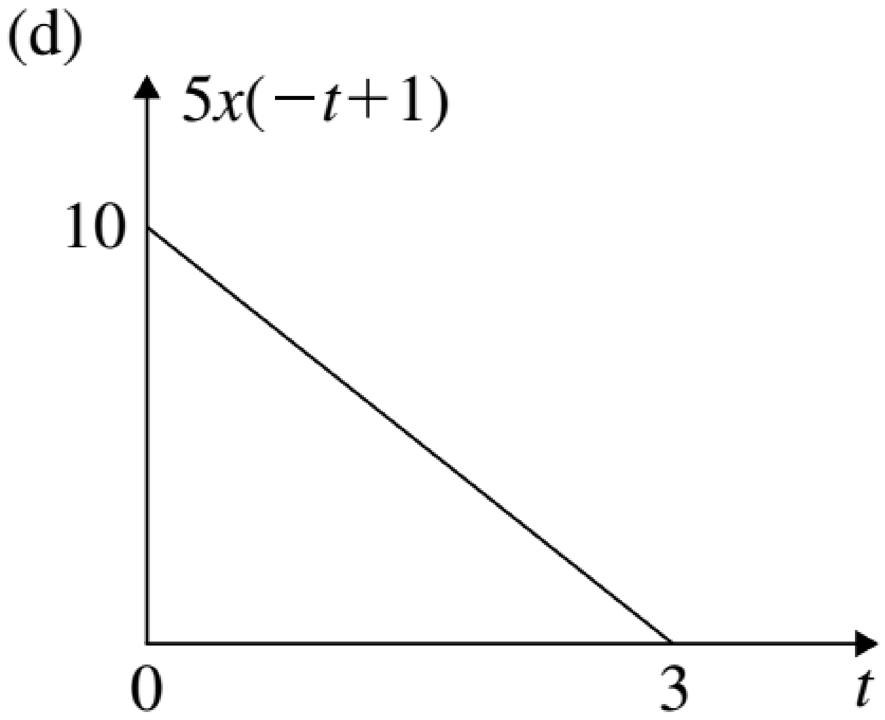
3. $5x(-t)$ is obtained by folding $5x(t)$ in Fig. 1.30b and is plotted in Fig. 1.30c.

(c)



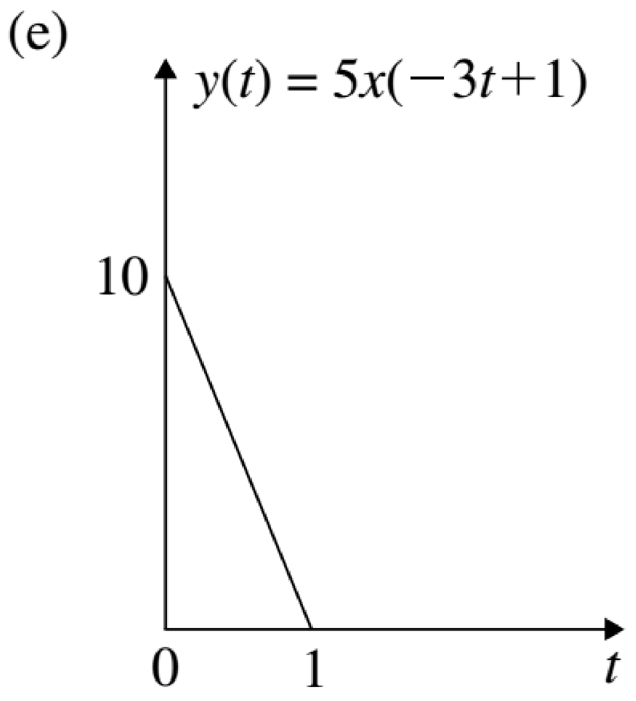
Time reversal

4. $5x(-t)$ is time shifted by one unit to the right and $5x(-t+1)$ is obtained and shown in Fig. 1.30d.



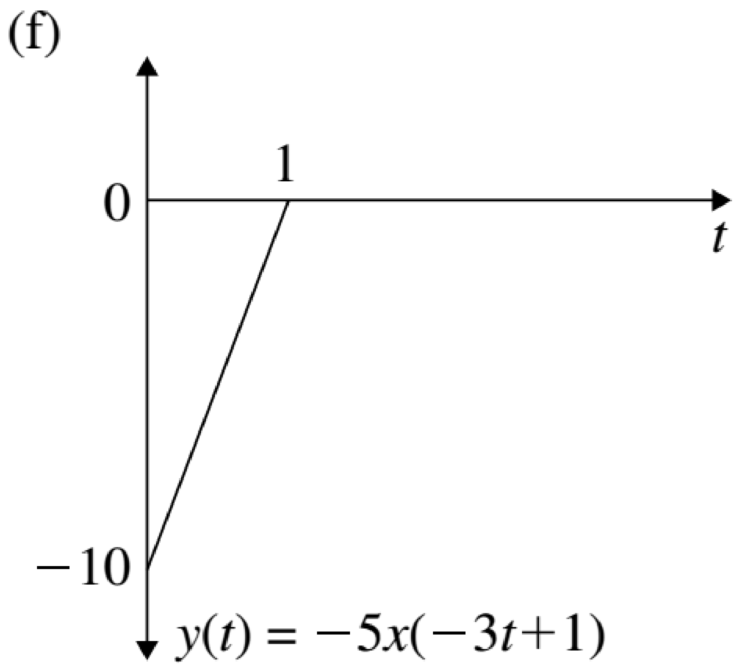
Time shifting

$5x(-t+1)$ is time compressed by a factor 3 and $5x(-3t+1)$ is obtained. This is shown in Fig. 1.30e.



Time scaling
(compression)

6. $5x(-3t+1)$ amplitude inverted to get $-5x(-3t+1)$. This is shown in Fig. 1.30f.



Amplitude inverted

■ Example 1.4

Consider the signal

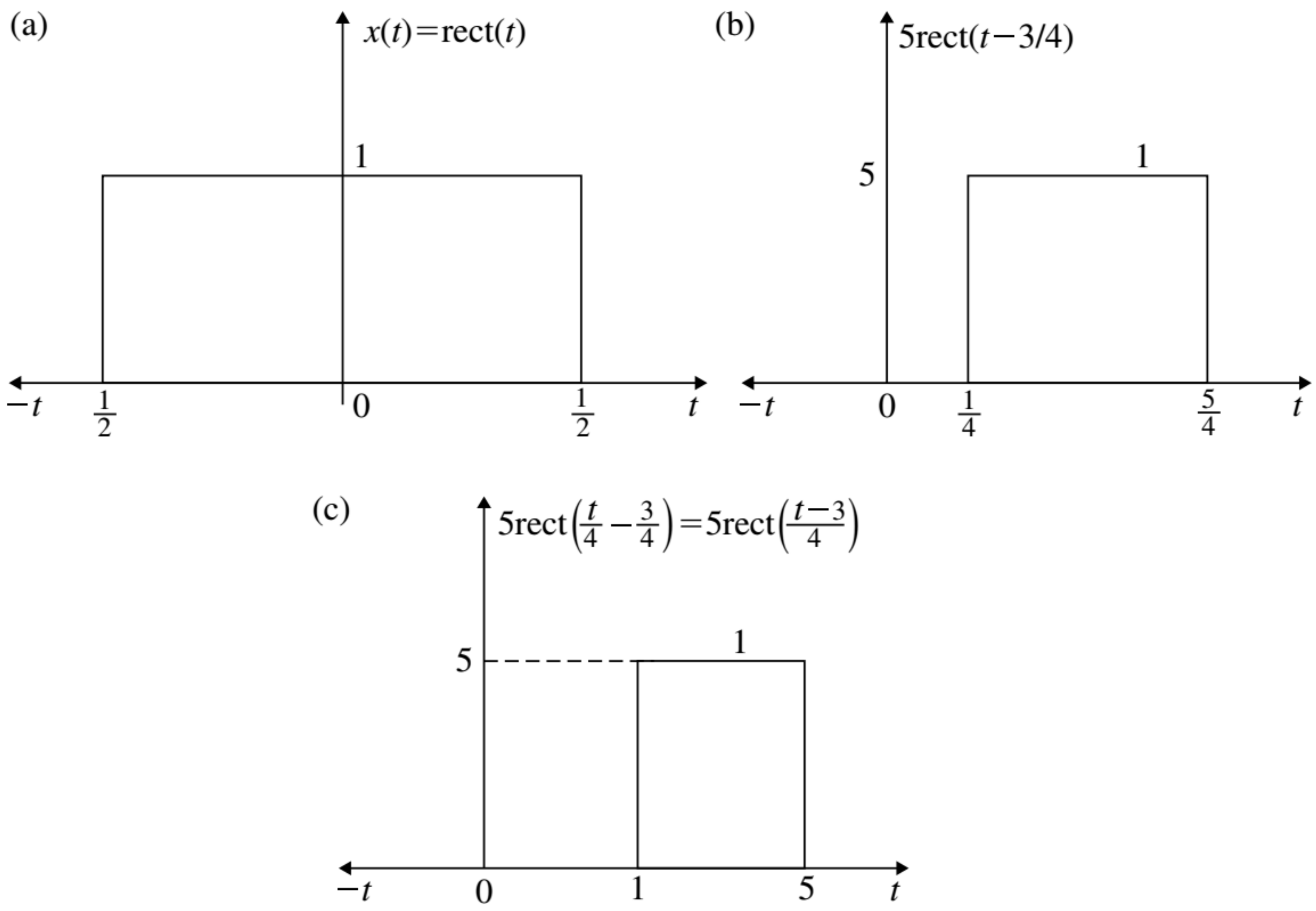
$$x(t) = \text{rect}(t)$$

Plot $y(t) = 5\text{rect}(\frac{t-3}{4})$.

Solution:

$$x(t) = 5\text{rect}\frac{(t-3)}{4}$$

1. $x(t)$ can be written as $x(t) = 5\text{rect}\left(\frac{t}{3} - \frac{3}{4}\right)$. The plot of $\text{rect}(t)$ is shown in Fig. 1.31a.
2. The time delayed ($t_0 = 3/4$) signal is right shifted by 3/4 and with its amplitude multiplied by 5 is shown in Fig. 1.31b.
3. The time shifted signal represented in step 2 is to be time expanded by a factor
4. This is shown in Fig. 1.31c as $y(t) = 5\text{rect}\frac{(t-3)}{4}$.


(a)
x(t)=rect(t)
1
−t
1/2
0
1/2
t
(b)
5rect(t−3/4)
5
1
−t
0
1/4
5/4
t
(c)
5rect(t/4 − 3/4)=5rect((t−3)/4)
5
1
−t
0
1
5
t

■ Example 1.5

For the signal shown in Fig. 1.32a, sketch

$$y(t) = -3x\left(-\frac{2}{3}t + 1.5\right)$$

Solution:

1. $x(t)$ is sketched as shown in Fig. 1.32a.
2. By time reversal $x(-t)$ is obtained and sketched as shown in Fig. 1.32b.
3. By amplitude scaling and inversion $-3x(t)$ is obtained and is shown in Fig. 1.32c.
4. The signal obtained in step 3 is right shifted by $t = 1.5$ and $-3x(-t + 1.5)$ is shown in Fig. 1.32e.
5. By time scaling expanded by 3/2, we get $-3x(-(2/3)t + 1.5)$ which is shown in Fig. 1.32f.

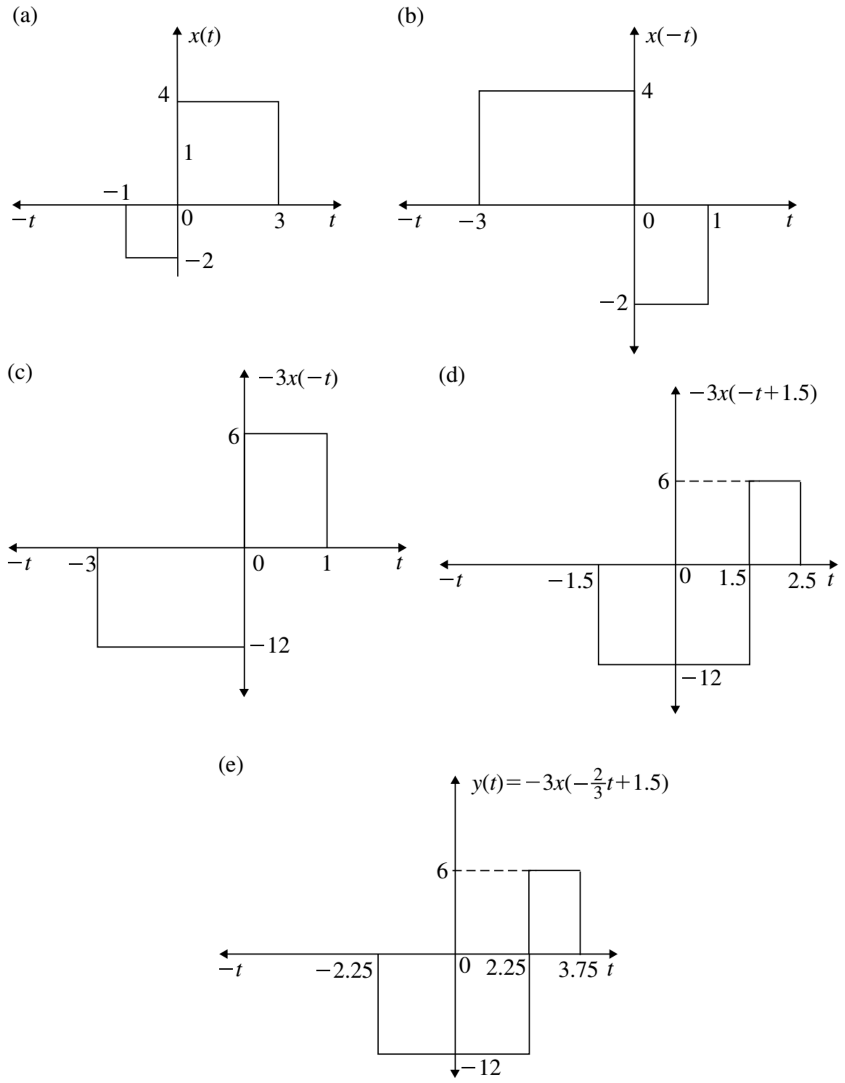


Fig. 1.32 Sketch of $y(t) = -3x\left(-\frac{2}{3}t + 1.5\right)$