Recursive Methods and Problem Solving

Chris Kiekintveld

CS 2401 (Fall 2010)

Elementary Data Structures and Algorithms

Review: Calling Methods

```
int x(int n) {
      int m = 0;
      n = n + m + 1;
      return n;
}

int y(int n) {
      int m = 1;
      n = x(n);
      return m + n;
}
```

What does y(3) return?

Calling Methods

- Methods can call other methods
- Can a method call itself?
- Yes! This is called a recursive method (function)
- “A method within a method”


LEONARDO DICAPRIO
YOUR MIND
IS THE SCENE
OF THE CRIME
INCEPTION
FROM THE DIRECTOR OF THE DARK KNIGHT
EXPERIENCE IT SUMMER 2010 IN IMAX
InceptionMovie.com

Example

```
void test(int n) {
  if (n > 0) {
    System.out.println(n);
    test(n-1);
    System.out.println(n);
  }
}
```

Trace the execution of test(4)

Example (pt 2)

```
void test(int n) {
  if (n > 0) {
    System.out.println(n);
    test(n-1);
    System.out.println(n);
  }
}
```

Trace the execution of test(-4)

Terminology

- A recursive method is any method that calls itself
- Base case
  - VERY IMPORTANT
  - Stops the recursion (prevents infinite loops)
  - Solved directly to return a value ***without*** calling the same method again

Recursive Definitions

- Directly recursive: method that calls itself
- Indirectly recursive: method that calls another method and eventually results in the original method call
- Tail recursive method: recursive method in which the last statement executed is the recursive call
- Infinite recursion: case where every recursive call results in another recursive call

Tracing a Recursive Method

- As always, go line by line
- Recursive methods may have ***many copies***
- Every method call creates a new copy and transfers flow of control to the new copy
- Each copy has its own:
  - code
  - parameters
  - local variables

Tracing a Recursive Method

- After completing a recursive call:
  - Control goes back to the calling environment
  - Recursive call must execute completely before control goes back to previous call
  - Execution in previous call begins from point immediately following recursive call

Factorial Numbers

- Factorial numbers (i.e., n!) defined recursively:
  - factorial(0) = 1
  - factorial(n+1) = factorial(n) * n+1
- Examples
  - 0! = 1
  - 1! = 1 * 1 = 1
  - 2! = 2 * 1 * 1 = 2
  - 3! = 3 * 2 * 1 * 1 = 6
  - 4! = 4 * 3 * 2 * 1 * 1 = 24

Iterative Factorial Method

```
public static int fact(int num) {
   int tmp = 1;
   for (int i = 1; i <= num; i++) {
      tmp *= i;
   }
   return tmp;
}
```

Trace fact(5)

Recursive Factorial Method

```
public static int fact(int num)
{
    if (num == 0)
        return 1;
    else
        return num * fact(num - 1);
}
```

Trace fact(5)

Recursive Factorial Trace

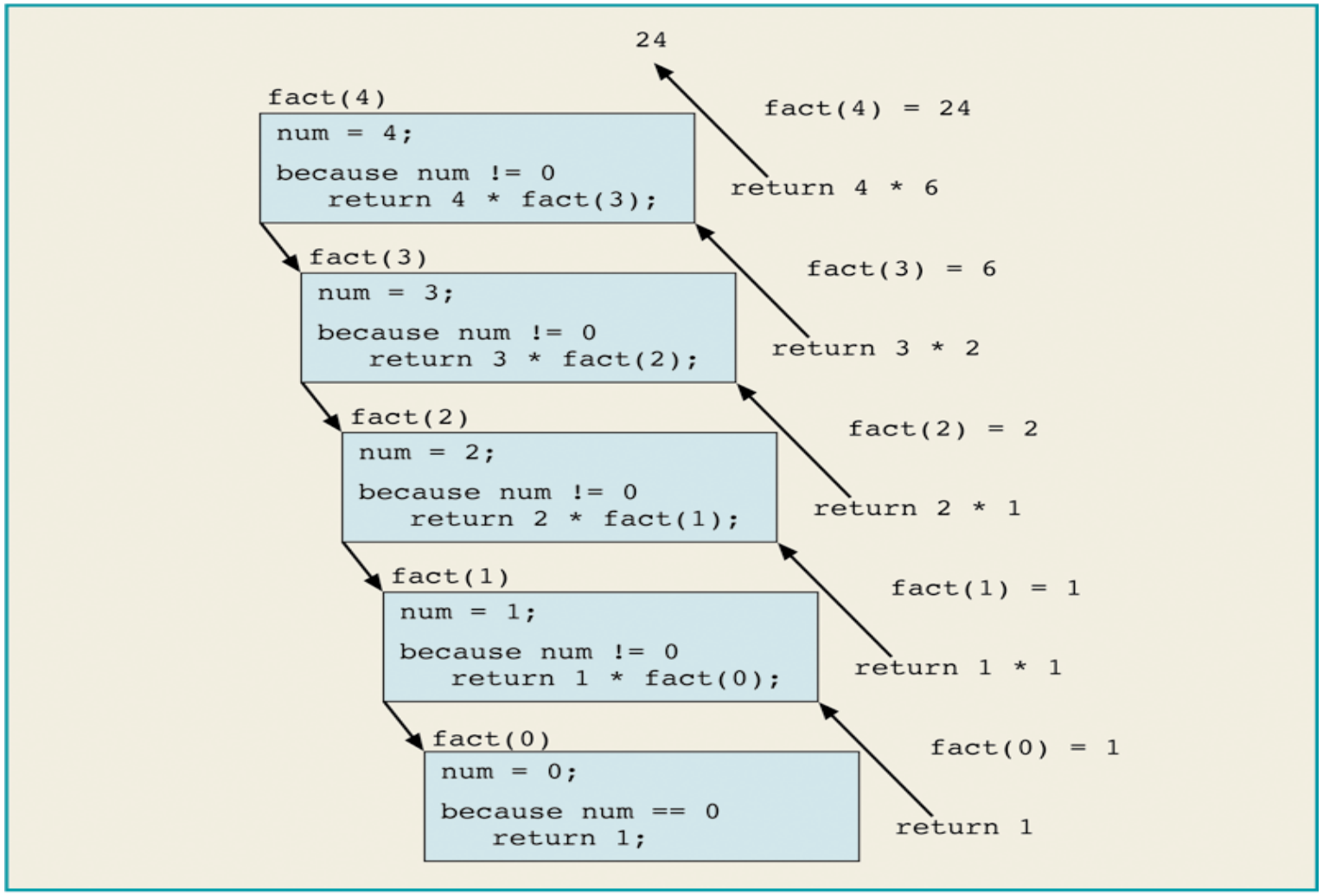


Figure 14-1 Execution of the expression `fact(4)`

Recursion or Iteration?

- **Moral:** There is usually more than one way to solve a problem!
  - Iteration (loops to repeat code)
  - Recursion (nested function calls to repeat code)
- Tradeoffs between two options:
  - Sometimes recursive solution is easier
  - Recursive solution is often slower

Today's Topic: Recursion (Continued)

Facts about Recursion

- Recursive methods call themselves
- Each call solves an identical problem
  - The code is the same!
  - Successive calls solve smaller/simpler instances
- Every recursive algorithm has at least one base case
  - A known/easy to solve case
  - Often, when we reach 1 or 0

Designing Recursive Algorithms

- General strategy: “Divide and Conquer”
- Questions to ask yourself
  - How can we reduce the problem to smaller version of the same problem?
  - How does each call make the problem smaller?
  - What is the base case?
  - Will you ***always*** reach the base case?

Similarities

- Closely related to recursive definitions in math
- Also closely related to proof by induction
  - Inductive proofs work in “reverse”
  - Start by proving a base case
  - Then show that if it is true for case n, it must also be true for case n+1

Exercise

Write a recursive function that prints the numbers 1…n in descending order:

```
public void descending(int n) {



}
```

Exercise

Write a recursive function that prints the numbers 1…n in descending order:

```
public void descending(int n) {
  if (n <= 0) return;
  System.out.println(n);
  descending(n-1);
}
```

Exercise

Write a recursive function to perform exponentiation return x^m, assuming m >= 0

```
public int exp(int x, int m) {



}
```

Exercise

Write a recursive function to perform exponentiation return x^m, assuming m >= 0

```
public int exp(int x, int m) {
  if (m == 0) { return 1; }
  if (m == 1) { return x; }
  return x * exp(x, m-1);
}
```

```
public static boolean  p(string s, int i, int f)
  if (i < f) {
    if (s[i] == s[f]) {
      return p(s, i+1, f-1);
    } else {
      return false;
    }
  } else {
    return true;
  }
}
```

What does p(s,0,s.length-1) return

a) if s ="UTEP'

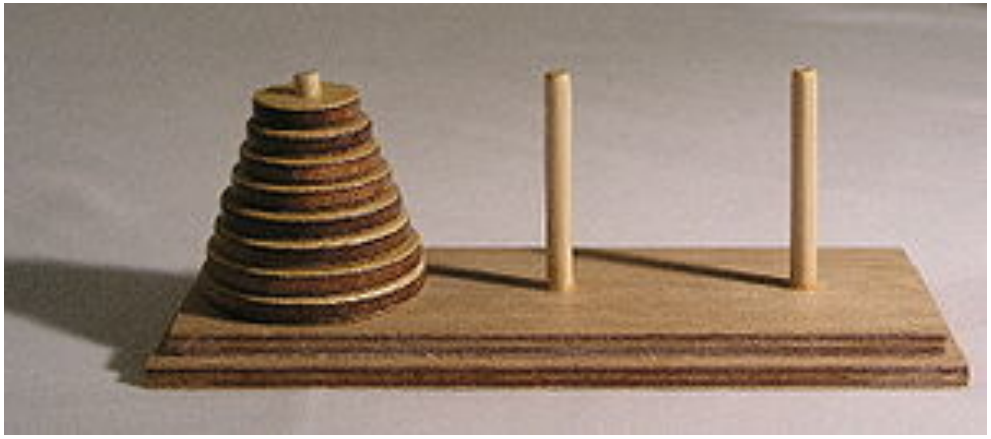
b) if s ="SAMS'

c) if s ="kayak"

d) if s= "ABBA"

Towers of Hanoi

- The legend of the temple of Brahma
  - 64 golden disks on 3 pegs
  - The universe will end when the priest move all disks from the first peg to the last



The Rules

- Only move one disk at a time
- A move is taking one disk from a peg and putting it on another peg (on top of any other disks)
- Cannot put a larger disk on top of a smaller disk
- With 64 disks, at 1 second per disk, this would take roughly 585 billion years

Towers of Hanoi: Three Disk Problem

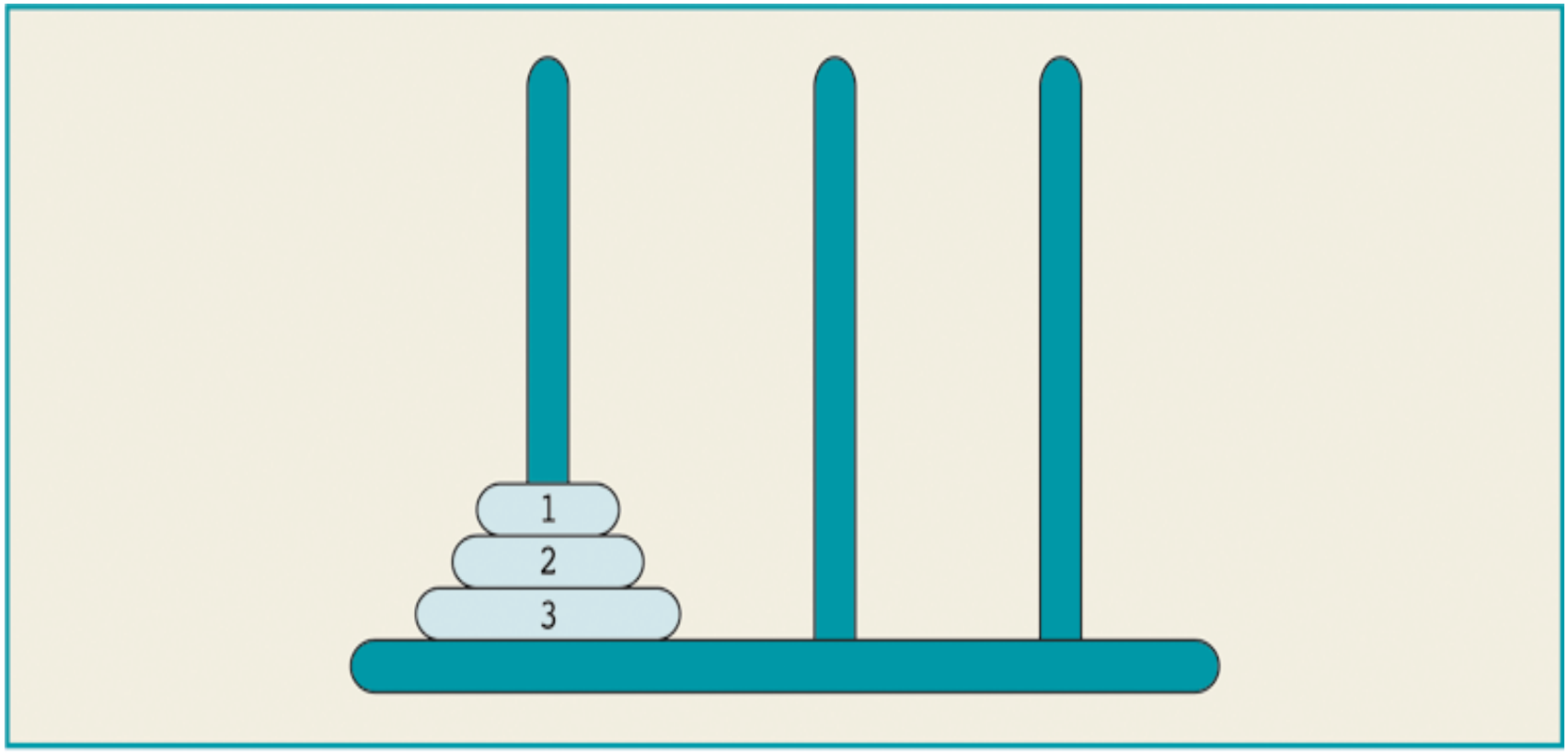
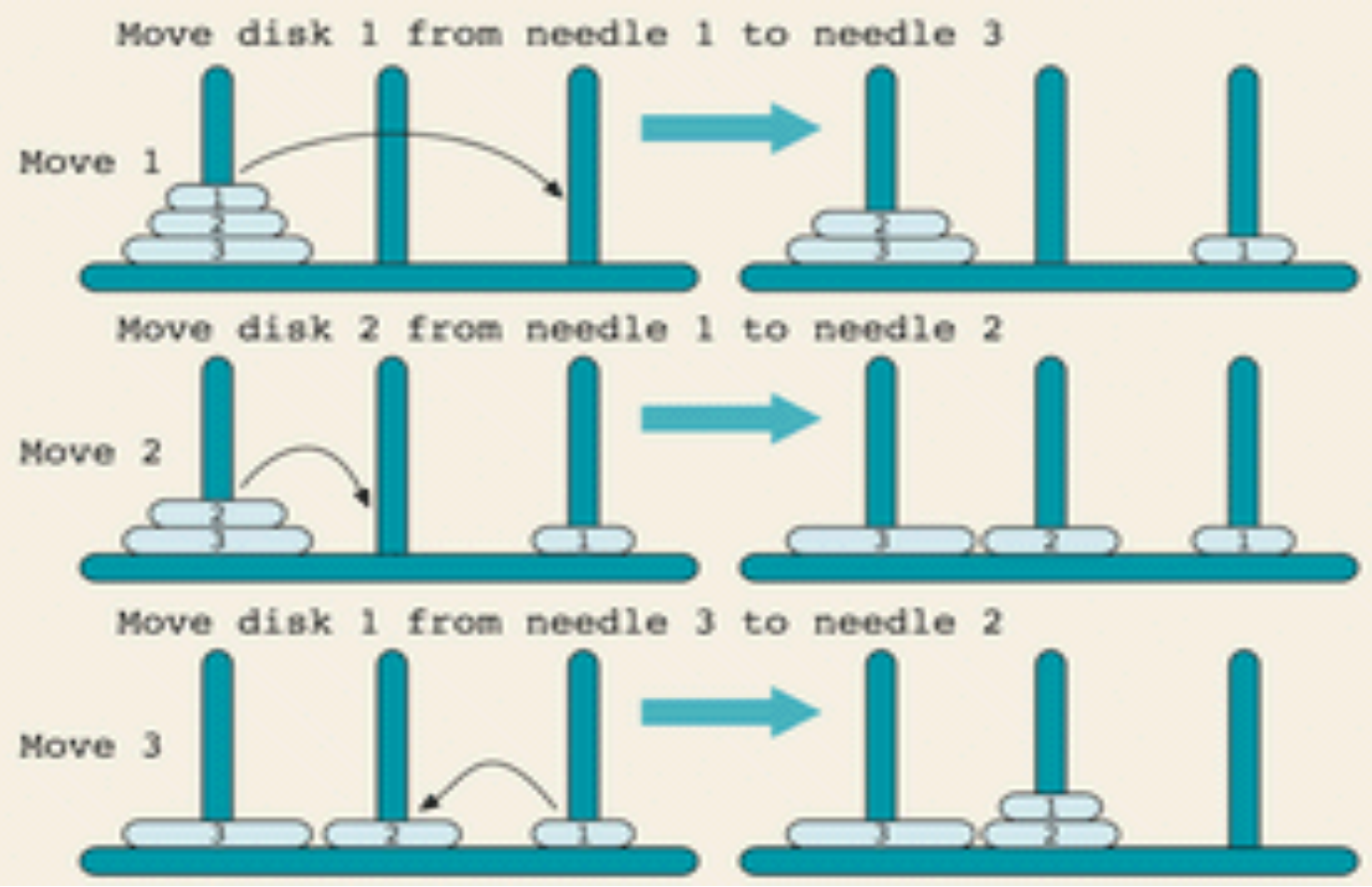
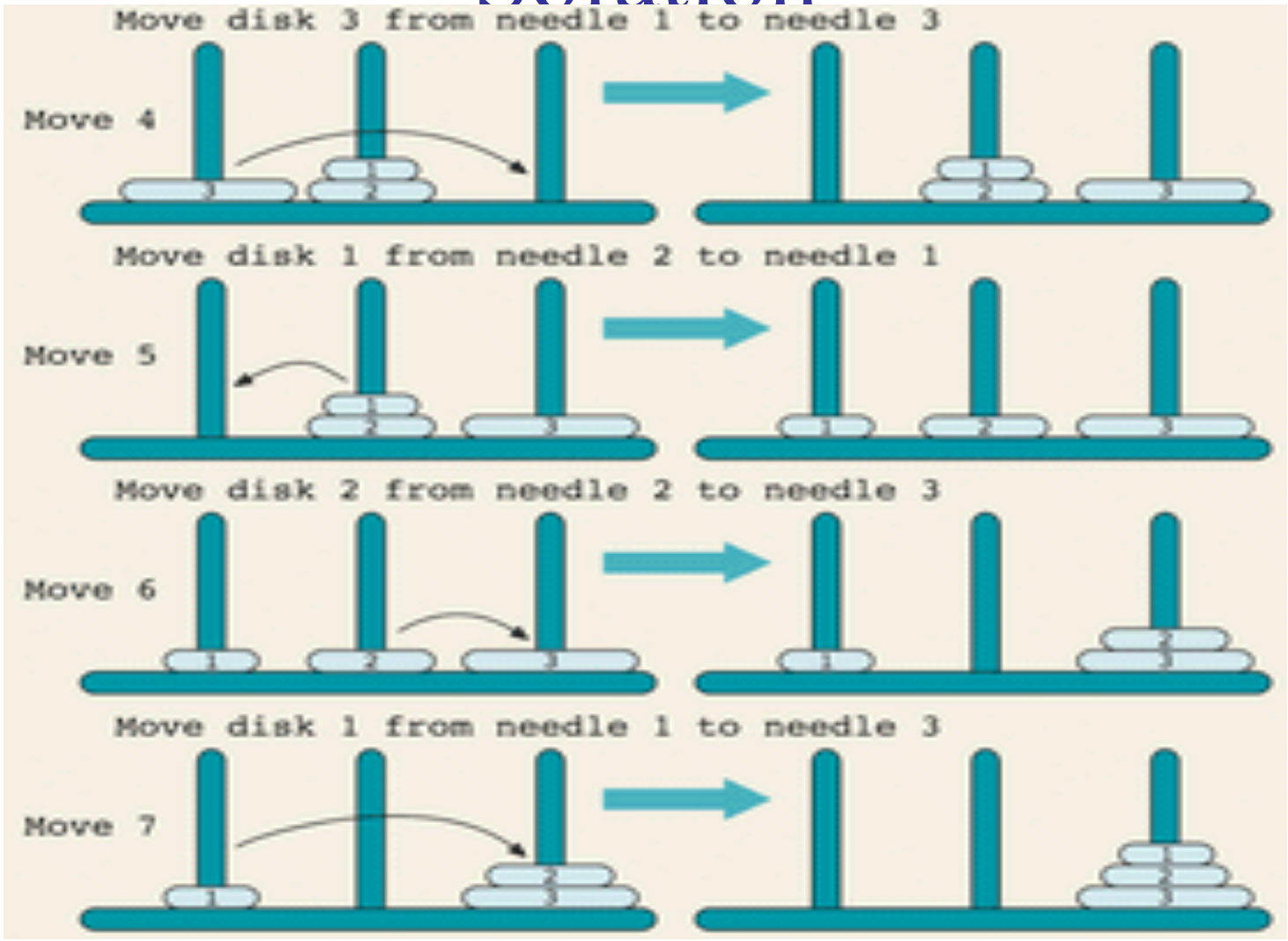


Figure 14-6 Tower of Hanoi problem with three disks

Towers of Hanoi: Three Disk Solution



Towers of Hanoi: Three Disk Solution



Four disk solution (courtesy wikipedia)

Recursive algorithm idea

- Final step is to move the bottom disk from peg 1 to peg 3
- To do this, the other n-1 disks must be on peg 2
- So, we need an way to move n-1 disks from peg 1 to peg 2
- Base case: moving the smallest disk is easy (you can always move it to any peg in one step)

Pseudocode

```
solveTowers (count, source, destination, spare) {
  if (count == 1) {
    move directly
  }
  else {
    solveTowers(count-1, source, spare, destination)
    solveTowers(1, source, destination, spare)
    solveTowers(count-1, spare, destination, source)
  }
}
```

Today's Topic: Recursion (Continued)

Recursive Fibonacci

$$\text{rFibNum}(a,b,n) = \begin{cases} a & \text{if } n = 1 \\ b & \text{if } n = 2 \\ \text{rFibNum}(a,b,n-1) + \text{rFibNum}(a,b,n-2) & \text{if } n > 2. \end{cases}$$

Recursive Fibonacci (continued)

```
public static int fib(int n) {
    if (n <= 2) {
        return 1;
    }
    return fib(n - 1) + fib(n - 2)
}
```

Recursive Fibonacci (continued)

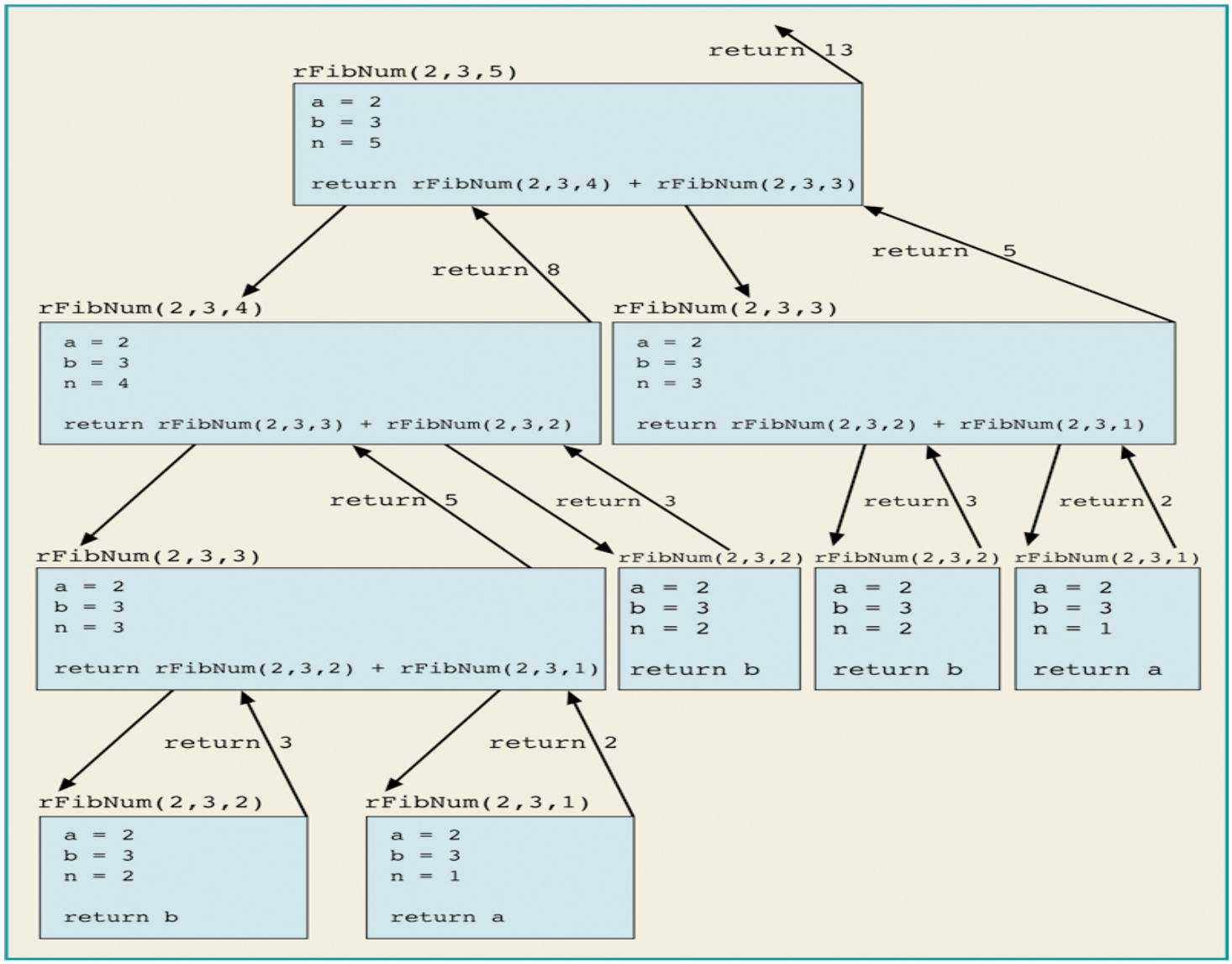


Figure 14-5 Execution of the expression `rFibNum(2, 3, 5)`

Iterative Solution

- Recursive solution repeats many computations, so it is very inefficient. An iterative approach:

```
public static int fib(int n) {
    if (n <= 2) return 1;
    int f1 = f2 = 1;
    for (int i = 3; i <= n; i++) {
        int tmp = f1 + f2;
        f1 = f2;
        f2 = tmp;
    }
    return f2;
}
```

Improved Recursion

```
int[] cache = new int[n];
public static int fib(int n) {

    if (cache[n] <= 0) {
        cache[n] = fib(n-2) + fib(n-1);
    }
    return cache[n];
}
```

Bugs?

Improved Recursion (debugged)

```
int[] cache = new int[n+1];
public static int fib(int n) {
    if (n <= 2) return 1;
    if (cache[n] <= 0) {
        cache[n] = fib(n-2) + fib(n-1);
    }
    return cache[n];
}
```

Search

- Design a method that returns true if element n is a member of array x[] and false if not
- Iterative approach:

```
public boolean search(int[] x, int n) {
    for(int i = 0; i < x.length, i++) {
        if (x[i] == n]) return true;
    }
    return false;
}
```

Recursive Search

- A recursive variant:

```
boolean search(int[] x, int size, int n) {
    if (size > 0) {
        if (x[size-1] == n) {
            return true;
        } else {
            return search(x, size-1, n);
        }
    }
    return false;
}
```

Faster Search

- The problem: these methods are slow
- Recall the phone book example
- “Linear search” – need to look at every element
- “Binary search” is much faster on sorted data

Binary Search Pseudocode

```
search(phonebook, name) {
   if only one page, scan for the name
   else
      open to the middle
      determine if name is before or after this page
      if before
         search (first half of phonebook, name)
      else
         search (second half of phonebook, name)
```

```
boolean binarySearch(int[] x, int start, int end, int n) {
   if (end < start) return false;
   int mid = (start+end) / 2;
   if (x[mid] == n) {
       return true;
    } else {
       if (x[mid] < n) {
          return search(x, mid+1, end, n);
       } else {
          return search(x, start, mid-1, n);
       }
    }
}
```

Programming Example

```
int test(String s, int last) {
  if (last < 0) {
    return 0;
  }
  if (s.charAt(last) == "0") {
    return 2 * test(s, last-1);
  }
  return 1 + 2 * test(s, last-1);
}
```

Trace test("01101", 4)

What does method test do?

Exercise

Write a recursive function convert a decimal number into a binary number, printing the binary number

```
public static void decToBin(int num){



}
```

Exercise
Decimal to Binary

```
public static void decToBin(int num)
{
     if (num > 0)
     {
          decToBin(num / 2);
          System.out.print(num % 2);
     }
}
```