Book "1" Linear Algebra

chapter "1" Matrices and Systems of equations

1.1 Systems of linear equations

linear : خطي → $x^1, y^1, \ldots$ (variables)

$x^2 = 4$, X not linear

System of linear equations

unknowns : $x_1, x_2, x_3, \ldots x_n$
(variables)

coefficient : $a_{11}, a_{12}, a_{24}, \ldots$
ثوابت

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1$$

(constant)

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \ldots + a_{mn}x_n = b_m$$

m : equation
n : unknowns (variables) } → m x n

note

If $b_1, b_2, b_3, \dots b_m = 0 \Rightarrow$

homogenous system.

example:

$x_1 + 2x_2 - x_3 = 2$

$4x_1 - 2x_2 - 7x_3 = 0$

sol: it is a linear system.

2 equations

3 unknowns $\Rightarrow$ 2X3 system

non-homgenous. system (sys.)

$x_1 + 2x_2 - x_3 - 2 = 0 \Rightarrow x_1 + 2x_2 - x_3 = 2$

ex

$x + y = 0$

$-x - 3y = 0$ $\Rightarrow$ homgenous system

$x + 2y = 0$ 3X2 sys.

2x2 System

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

ex 1 $x_1 + x_2 = 2$

$x_1 - x_2 = 2$, solve the system

sol:

$$x_1 + x_2 = 2$$
$$x_1 - x_2 = 2 \quad / \text{add}$$

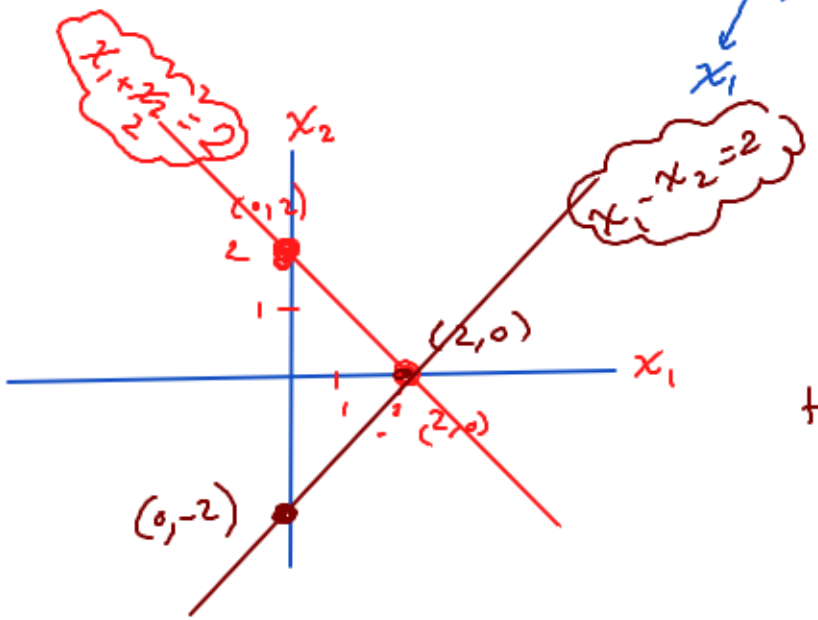
$$2x_1 = 4 \Rightarrow \boxed{x_1 = 2}$$

$x_1 + x_2 = 2$

$2 + x_2 = 2$

$\boxed{x_2 = 0}$

the solution (sol.) is : $(2, 0)$ (x_1, x_2)



from the graph the solution is (2,0)

$x_1 + x_2 = 2 \longrightarrow x_1 = 0 \rightarrow x_2 = 2 \rightarrow (0,2)$

$x_2 = 0 \rightarrow x_1 = 2 \rightarrow (2,0)$

$x_1 - x_2 = 2 \longrightarrow x_1 = 0 \rightarrow x_2 = -2 \rightarrow (0,-2)$

$x_2 = 0 \rightarrow x_1 = 2 \rightarrow (2,0)$

the two lines intersect at a point.

→ this system has a unique solution

(consistent) متسق

ex2 solve the system

$x_1 + x_2 = 2$

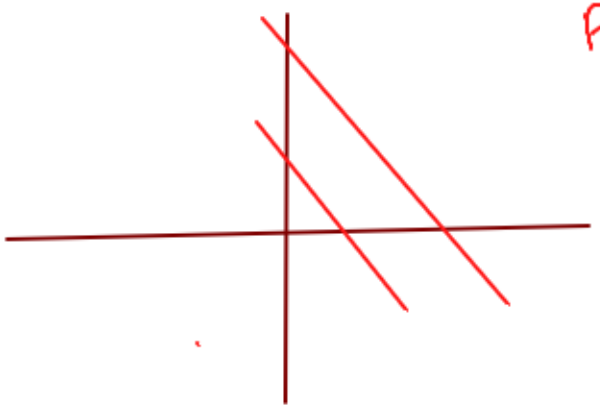
$x_1 + x_2 = 1$ → ضرب -1

Sol:

$$\begin{array}{r} x_1 + x_2 = 2 \\ -x_1 + x_2 = -1 \quad / \text{add} \\ \hline 0 = 1 \end{array} \rightarrow \times$$

this system has no solution.

(inconsistent) غير متسق



Parallel lines

ex 3

solve the system

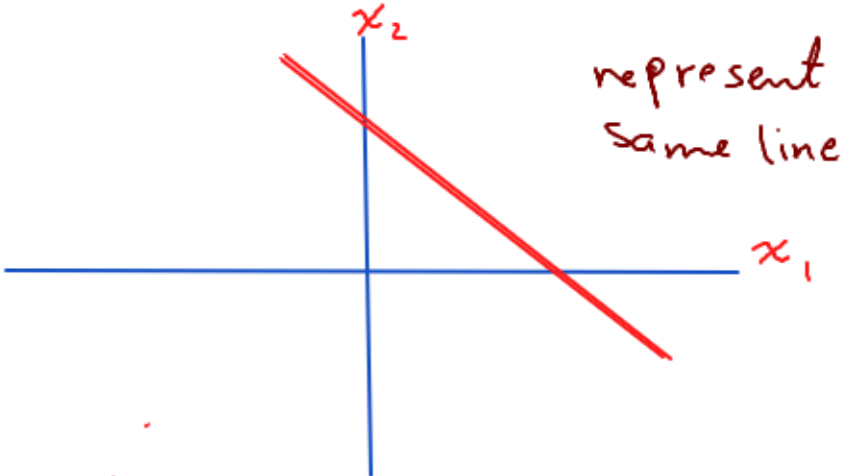
$$x_1 + x_2 = 2$$
$$-x_1 - x_2 = -2$$

sol:

$$\begin{array}{r} x_1 + x_2 = 2 \\ -x_1 - x_2 = -2 \\ \hline 0 = 0 \end{array} \quad / \text{add} \Rightarrow$$

this system has infinitly many solutions. عدد لانهائي من الحلول

(consistent)



represent the same line.

the solutions :-

(0, 2)
(2, 0)
(1, 1)
(5, −3)
(−3, 5)
(10, −8)
(−8, 10)
⋮
⋮

consistent system : If the system has at least one solution

inconsistent system : If the system dosent have a solution